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DERIVATION OF ECONOMICAL LOAD DISPATCHING BY ASSIGNING THE INCREMENTAL TRANSMISSION LOSSES GRAPHICALLY

BY

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SYNOPSIS:

The purpose of this article is to suggest a new graphical verification of the incremental transmission losses by means of which curves relating the power output for the various generating units and the incremental cost of received power are constructed and plotted point by point. Thus a curve for the total power output against the incremental cost of received power can be plotted.

This proposed graphical technique is applied for power systems with different systems of reasonable number of buses and generating thermal units.

1) INTRODUCTION:

The problem of economical load sharing calculations for interconnected power systems has been one of gradual evaluations. Early investigations neglect generally the transmission losses in the power network, and it is shown that if each load increment was assigned to the unit that could produce it at the lowest cost then the lowest overall cost of generation would be realized.

As systems increased in size and complexity and as hydro generation moved farther and farther from the load centres, transmission losses become an important factor and can not be neglected. The system is then operating at minimum cost when the incremental cost of delivered energy is the same for all the plants in the system.

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There are several approaches to the economical dispatching calculations that take into consideration transmission losses. The loss formula expression method (B - constants method) was the first widely used approach. By this method, several assumptions are made to obtain the B-constants equation. These assumptions permit the calculation of the B-constants based on a base load flow calculation, to be valid for other system loading conditions.

As digital computers and network analysers became in use, economical load sharing calculations are available using the non-linear programming methods. The results obtained are more accurate because most of the assumptions made for loss formula coefficients determination not existing using these methods.

The data required for economical load sharing calculations is summarized as follows:

) Electrical system data

1- Impedance diagram for the transmission and subtransmission networks.

2- Daily load cycle for the typical ordinary operation.

3- Load duration curve for period of operation considered.

) Plant Data

1- Thermal characteristics of units and in partioular the incremental fuel rate data of all units.

2- Cost of fuel at various plants in pounds per 10^6 BTU.

3- Determination of an expression for incremental production cost and power for the various plants.

2) COMPUTATION OF THE LOSS FORMULA BY THE USE OF DIGITAL COMPUTER ACCORDING TO THE LEAST SQUARES METHOD.

The general loss formula is in the form:(1)

$$P_L = P_m B_{mn} P_n + B_{no} P_n + B_{oo}$$

The calculation of B_{no} and B_{oo} by the equations

$$B_{no} = 2\, P^{o}_{LK} \quad (R_{Gn-LK} - i'_j\, R_{Lj-Lk})\, K_{nk} + (i''_j \quad R_{Lj-Lk})H_{nk}$$

where

P^{o}_{Lk} = real power value of load k when $i_L = o$

$$K_{nk} = \frac{1}{V_n\, V_{Lk}} \left((1+s_n\, s_{Lk}) \cos \Theta_{n-Lk} + (s_n - s_{Lk}) \sin \Theta_{n-Lk}\right)$$

$$H_{nk} = \frac{1}{V_n\, V_{Lk}} \left((1+s_n\, s_{Lk}) \sin \Theta_{n-Lk} + (s_{Lk} - s_n) \cos \Theta_{n-Lk}\right)$$

Also

$$B_{oo} = \text{Real } (i^{o}_{j} \quad Z_{Lj-Lk} i_k^{\ o})$$

would be quite lengthy, since these equations require the determination of the self and mutual impedances between loads. In this item, we describe a method of calculating the B_{no} and B_{oo} terms which does not require the determination of the self and mutual impedances between loads. The method discussed involves the use of circuit theory together with a least-squares solution for w', B_{no}, and B_{oo}.

The philosophy behind this method involves the concept of using w', B_{no}, and B_{oo} to fit the formula to observed transmission loss data.

To understand the method,[(1)] we will first briefly review the idea of multiple correlation with a least-squares criterion. Consider the following function for which y is defined as

$$y = a_1\, x_1 + a_2\, x_2 \tag{1}$$

It is desired to determine a_1 and a_2 when given observed values of y for various values of x_1 and x_2 such that the summation of the squares of the residuals is a minimum. A residual is determined as the deviation of the computed value from the observed value. Thus, for case 1 the residual v is given by:

$$v_1 = a_1 x_1^{(1)} + a_2 x_2^{(1)} - y_1 \qquad (2)$$

where

y_1 = observed value for case 1

$x_1^{(1)}$ = value of x_1 for case 1

$x_2^{(1)}$ = value of x_2 for case 1

Assuming that we have four cases, we have

$$v_2 = a_1 x_1^{(2)} + a_2 x_2^{(2)} - y_2 \qquad (3)$$

$$v_3 = a_1 x_1^{(3)} + a_2 x_2^{(3)} - y_3 \qquad (4)$$

$$v_4 = a_1 x_1^{(4)} + a_2 x_2^{(4)} - y_4 \qquad (5)$$

The method of least squares results in a solution for the desired coefficients such that the sum of the squares of the residuals is a minimum. Thus, it is desired that:

$$S = \text{sum of squares of residuals}$$

$$= v_1^2 + v_2^2 + v_3^2 + v_4^2 = \text{minimum} \qquad (6)$$

A necessary condition that S = minimum is that

$$\frac{\partial S}{\partial a_1} = 0 \qquad (7)$$

$$\frac{\partial S}{\partial a_2} = 0 \qquad (8)$$

From equation (6) we have:

$$\frac{\partial S}{\partial a_1} = 2\frac{\partial v_1}{\partial a_1} v_1 + 2\frac{\partial v_2}{\partial a_1} v_2 + 2\frac{\partial v_3}{\partial a_1} v_3 + 2\frac{\partial v_4}{\partial a_1} v_4 \qquad (9)$$

$$\frac{\partial S}{\partial a_2} = 2\frac{\partial v_1}{\partial a_2} v_1 + 2\frac{\partial v_2}{\partial a_2} v_2 + 2\frac{\partial v_3}{\partial a_2} v_3 + 2\frac{\partial v_4}{\partial a_2} v_4 \qquad (10)$$

From equations (2) → (5) we note

$$\frac{\partial v_1}{\partial a_1} = x_1^{(1)} \quad (11)$$

$$\frac{\partial v_2}{\partial a_1} = x_1^{(2)} \quad (12)$$

$$\frac{\partial v_3}{\partial a_1} = x_1^{(3)} \quad (13)$$

$$\frac{\partial v_4}{\partial a_1} = x_1^{(4)} \quad (14)$$

Substituting equations (11) to (14) into equation (9), we obtain

$$\frac{\partial S}{\partial a_1} = 2\, x_1^{(1)}\, v_1 + 2\, x_1^{(2)}\, v_2 + 2\, x_1^{(3)}\, v_3 + 2\, x_1^{(4)}\, v_4 = 0 \quad (15)$$

Similarly,

$$\frac{\partial S}{\partial a_2} = 2\, x_2^{(1)}\, v_1 + 2\, x_2^{(2)}\, v_2 + 2\, x_2^{(3)}\, v_3 + 2\, x_2^{(4)}\, v_4 = 0 \quad (16)$$

Substituting the definitions for v_1, v_2, v_3, and v_4 into equation (15), we obtain

$$\frac{\partial S}{\partial a_1} = 2\, x_1^{(1)}(a_1 x_1^{(1)} + a_2 x_2^{(1)} - y_1) + 2x_1^{(2)}(a_1 x_1^{(2)} + a_2 x_2^{(2)} - y_2) + 2x_1^{(3)}(a_1 x_1^{(3)} + a_2 x_2^{(3)} - y_3) + 2x_1^{(4)}(a_1 x_1^{(4)} + a_2 x_2^{(4)} - y_4) \quad (17)$$

Grouping the coefficients of a_1 and a_2 and dividing by (2), equation (17) becomes

$$a_1(x_1^{(1)}\, x_1^{(1)} + x_1^{(2)}\, x_1^{(2)} + x_1^{(3)}\, x_1^{(3)} + x_1^{(4)}\, x_1^{(4)}) + a_2(x_1^{(1)}\, x_2^{(1)} + x_1^{(2)}\, x_2^{(2)} + x_1^{(3)}\, x_2^{(3)} + x_1^{(4)}\, x_2^{(4)}) = x_1^{(1)}\, y_1 + x_1^{(2)}\, y_2 + x_1^{(3)}\, y_3 + x_1^{(4)}\, y_4 \quad (18)$$

Similarly, equation (10) becomes

$$a_1(x_2^{(1)}\, x_1^{(1)} + x_2^{(2)}\, x_1^{(2)} + x_2^{(3)}\, x_1^{(3)} + x_2^{(4)}\, x_1^{(4)}) +$$

$$a_2(x_2^{(1)}\, x_2^{(1)} + x_2^{(2)}\, x_2^{(2)} + x_2^{(3)}\, x_2^{(3)} + x_2^{(4)}\, x_2^{(4)})$$

$$= x_2^{(1)}\, y_1 + x_2^{(2)}\, y_2 + x_2^{(3)}\, y_3 + x_2^{(4)}\, y_4 \qquad (19)$$

The values of a_1 and a_2 obtained from the solution of the normalized simultaneous equations (18) and (19) satisfy the necessary condition that the summation of squares of the residuals is a minimum.

For the general cases, define y as:

$$y = \sum_{j=1}^{r} a_j\, x_j \qquad (20)$$

for which there are r coefficients to be determined. Designate by n the number of observations. The summation of the squares of the residuals is given by:

$$S = \sum_{i=1}^{n} v_i^2 \qquad (21)$$

where $$v_i = \sum_{j=1}^{r} a_j\, x_j^{(i)} - y_i \qquad (22)$$

The necessary condition that S is a minimum is given by:

$$\frac{\partial S}{\partial a_k} = 0 = 2 \sum_{i=1}^{n} \frac{v_i}{a_k}\, v_i \qquad (23)$$

Where $k = 1, 2, \ldots\ldots r$

From equation (22), we have

$$\frac{\partial v_i}{\partial a_k} = x_k^{(i)} \qquad (24)$$

Substituting equation (24) and (22) into equation (23),

$$\frac{\partial S}{\partial a_k} = \sum_{i=1}^{n} \left(\sum_{j=1}^{r} a_j x_j^{(i)} - y_i\right) x_k^{(i)} = 0 \qquad (25)$$

Collecting coefficients of a_j, we obtain the following normalized simultaneous equationsto solve for the r unknown coefficients:

$$\sum_{j=1}^{r} \left(\sum_{i=1}^{n} x_j^{(i)} x_k^{(i)}\right) a_j = \sum_{i=1}^{n} y_i x_k^{(i)} \qquad (26)$$

Having reviewed the method of least squares, let us consider the application of this method to the determination of the loss-formula coefficients.

But:

$$P_L = P_m B_m P_n + B_{no} P_n + B_{oo}$$

$$= P_m A_{mn} P_n - P_m H_{mn} (f_m - f_n) \rho_n$$

$$+ P_m K_{mn} P_n w' + B_{no} P_n + B_{oo} \qquad (27)$$

The quantities A_{mn} and $H_{mn} (f_m - f_n)$ may be easily calculated as indicated by the following equation:

$$\text{Real } \frac{a_m + b_m}{2} = d_m = R_{Lk - Gm} l_k'$$

$$f_m = R_{Gm - Lk} l_k''$$

$$f_n = R_{Gn - Lk} l_k''$$

$$K_{mn} = \frac{1}{V_m V_n}((1 + s_m s_n) \cos \theta_{mn} + (s_m - s_n) \sin \theta_{mn})$$

$$H_{mn} = \frac{1}{V_m V_n}((1 + s_m s_n) \sin \theta_{mn} + (s_n - s_m) \cos \theta_{mn})$$

$$A_{mn} = K_{mn} (R_{Gm} - Gn - d_n - d_m)$$

Knowing P_L from load flow data, we then write the driving function as

$$P_L - P_m A_{mn} P_n + P_n H_{mn} (f_m - f_n) P_n =$$

$$w' (P_m K_{mn} P_n) + B_{no} (P_n) + B_{oo} (1) \qquad (28)$$

From equation (28), it is desired to obtain w', B_{no}, and B_{oo} by the method of least squares. Since it is necessary to determine (n + 2) unknowns, it is suggested that 2(n + 2) cases be considered in order to avoid distorting the loss formula to fit a small number of cases exactly. In comparing equation (28) to equation (20) let

$$y = P_L - P_m A_{mn} P_n + P_m H_{mn} (f_m - f_n) P_n \qquad (29)$$

$$a_j = w', B_{no}, B_{oo} \qquad (30)$$

$$x_j = P_m K_{mn} P_n, P_n, 1 \qquad (31)$$

The solution of equation (28), according to the least-squares method, may be efficiently accomplished by the use of an automatic digital computer. In order to caleulate a loss formula by these methods, the computer flow design is shown in the figure. Steps 1, 2, and 7 have been changed. In steps 3 and 4 the values of s_m, θ_m, and V_m correspond to the average of the on-peak and off- peak data.

Data

R_{Lj-Gn}, R_{Gm-Gn}, i'_{Lj}, i''_{Lj}, V_n, θ_n

$$d_n = \left(\frac{\text{On peak } i'_{Lj} - \text{off peak } i'_{Lj}}{\text{on peak } i_L - \text{off peak } i_L}\right) R_{Lj-Gn}$$

$$f_n = \left(\frac{\text{on peak } i''_{Lj} - \text{off peak } i''_{Lj}}{\text{on peak } i_L - \text{off peak } i_L}\right) R_{Lj-Gn}$$

$$K_{mn} = \frac{1}{V_m V_n}((1+s_m s_n) \cos \theta_{mn} + (s_m - s_n) \sin \theta_{mn}$$

$$H_{mn} = \frac{1}{V_m V_n}((1+s_m s_n) \sin \theta_{mn} + (s_n - s_m) \cos \theta_{mn}$$

$$A_{mn} = (R_{Gm-Gn} - d_m - d_n) K_{mn}$$

$$H_{mn}(f_m - f_n)$$

By multiple correlation through least squares criterion obtain w', B_{no}, and B_{oo} from solution of $w'(P_{mn} K_{mn} P_n) +$

$$B_{no}(P_n) + B_{oo}(1) = P_L - P_m A_{mn} P_n - P_m H_{mn}(f_m - f_n) P_n$$

$$B_{mn} = A_{mn} + w' K_{mn} - H_{mn}(f_m - f_n)$$

Computer flow diagram for general loss formula calculation.

3) GRAPHICAL EVALUATION OF INCREMENTAL TRANSMISSION LOSSES.

In this case, the graphical solution of the coordination equations used for simple power systems with reasonable number of buses and generating thermal units; can take place to give for the different generating units curves relate the power output against the incremental cost of received power.

These curves are to be obtained and plotted point by point. This a curve for the total power output and the incremental cost of received power can be obtained. The method of having curves for the individual units is as follows:

Knowing ($\frac{dF}{dP}$ - P) curve for the generating units as shown in Fig. 1, and having the value of $\frac{\partial P_L}{\partial P}$

Where $\frac{dF}{dP}$ is the incremental fuel rate in BTU/MWh

P is the output power in MW.

P_L is the transmission losses.

Then, at a certain value of power P_1 if:

$oa = P_1$ in power units.

$$ab = 1 - \frac{\partial P_L}{\partial P_1} \qquad (32)$$

$$ac = \left(\frac{dF}{dP}\right)_{P_1}$$

and $eb = 1$

Therefore, referring to Fig. 1, $\triangle\triangle$ abe & acd are similar

$$\frac{ab}{ac} = \frac{be}{cd}$$

$$cd = \frac{(be)(ac)}{ab} = \frac{\left(\frac{dF}{dP}\right)_{P_1}}{(1 - \partial P_1/\partial P_1)} \qquad (33)$$

$$= \lambda$$

λ = Incremental production cost of received power.

So, the horizontal distance (cd) in $\frac{dF}{dP}$ units will represent λ.

Repeating the same process for different values of power and plotting the results obtained we can have the power output against the incremental cost of received power.

The incremental transmission losses can be determined graphically for simple systems as given below:

3.1 Two bus bar system

For the two bus bar system shown in Fig. 2, the incremental transmission losses can be expressed either as a function of the B-constants or as a function of voltages and phase angles. The expressions for each case are:

(i) $\frac{\partial P_L}{\partial P_1} = 2\ B_{11}\ P_1$ losses are function of B constants.

$$\& \frac{\partial P_L}{\partial P_2} = 2\ B_{22}\ P_2 \qquad (34)$$

(ii) $\frac{\partial P_L}{\partial P_1} = \frac{2 \tan \theta_{12}}{K + \tan \theta_{12}}$ losses are function of voltages and phase angles.

$$\& \frac{\partial P_L}{\partial P_2} = \frac{-2 \tan \theta_{12}}{K - \tan \theta_{12}} \qquad (35)$$

where $\theta_{12} = \theta_1 - \theta_2$

K = constant = ratio between reactance and resistance of the transmission line.

(a) First case:

For graphical determination of incremental transmission losses as it is expressed by equation 3, consider Fig. 3.

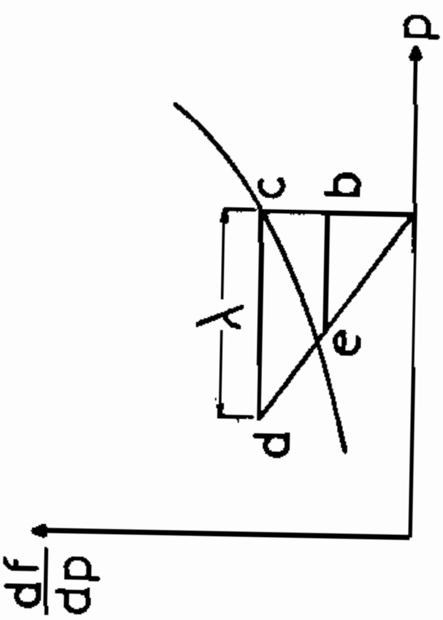

FIG.(1)

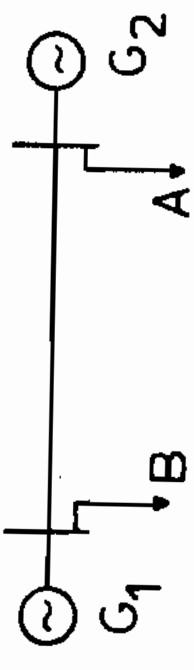

FIG.(2)

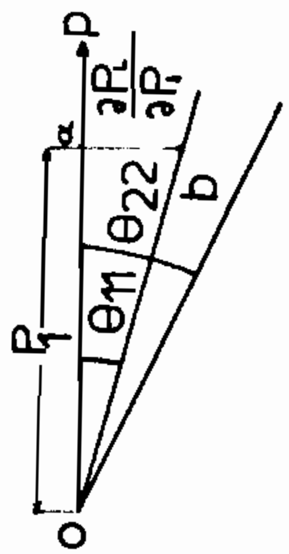

FIG.(3)

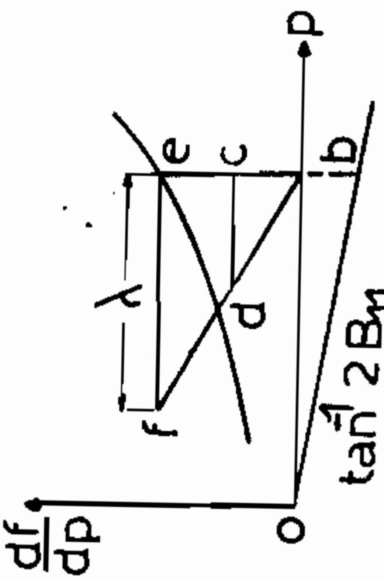

FIG.(4)

Let $\theta_{11} = \tan^{-1} 2 B_{11}$

$\theta_{22} = \tan^{-1} 2 B_{22}$

Then, at a certain value of power P_1

$$ab = P_1 \tan \theta_{11} = 2 B_{11} P_1 = \frac{\partial P_L}{\partial P_1}$$

The complete process for economical load sharing calculations in this case will be as shown in Fig. 4.

$$ac = 1 - \frac{\partial P_L}{\partial P_1}$$

$$ef = \lambda = \frac{dF / dP}{(1 - \partial P_L / \partial P_1)}$$

(b) Second case:

If the losses are computed as a function of voltages and phase angles then,

$$\frac{\partial P_L}{\partial P_1} = \frac{2 \tan \theta_{12}}{K + \tan \theta_{12}}$$

For graphical determination of $\frac{\partial P_L}{\partial P_1}$ in this case consider Fig. 5:

$oa = 1, \; bc = K, \quad cd = 2$

$ac = \tan \theta_{12} + K$

From the similarity of $\triangle\triangle$ abc & acd we have:

$$be = \frac{ab \cdot cd}{ac} = \frac{2 \tan \theta_{12}}{K + \tan \theta_{12}}$$

So, be will represent $\partial P_L / \partial P_1$.

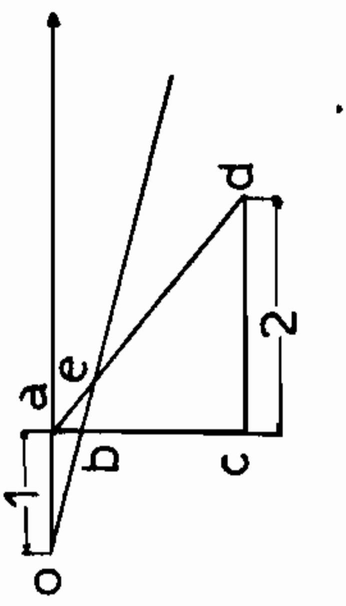

FIG.(5)

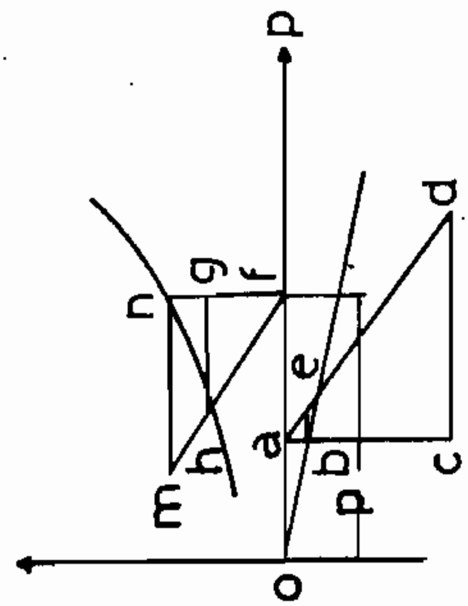

FIG.(6)

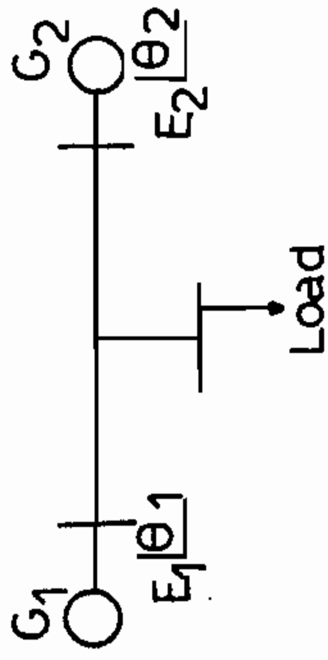

FIG.(7)

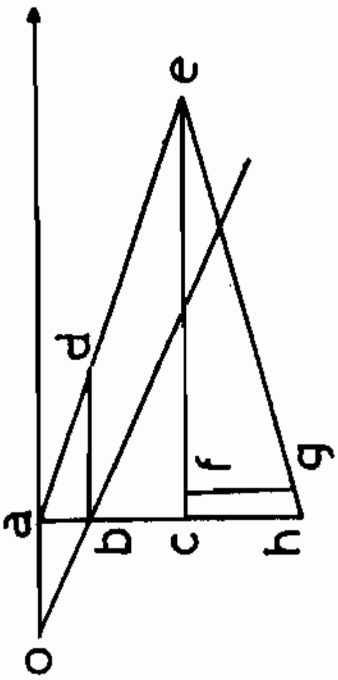

FIG.(8)

The complete process for incremental production cost of received power-output determination, this case is shown in Fig. 6.

mn will represent λ corresponding to P_1 So, the curve of $\lambda \to P_1$ can be plotted, in a similar way $\lambda \to P_2$ is to be determined. The sum of the 2 curves gives $\lambda \to$ total power and by these curves, we can obtain P_t as a function of P_1 and P_2.

3.2 Three bus bar system

For the 3-bus bar system shown in Fig. 7, the expressions for incremental transmission losses are:

(i) As a function of B-constants

$$\frac{\partial P_L}{\partial P_1} = 2B_{11} P_1 + 2B_{12} P_2$$

$$\& \frac{\partial P_L}{\partial P_2} = 2B_{22} P_2 + 2B_{12} P_1 \tag{36}$$

(ii) As a function of voltages and phase angles:

$$\frac{\partial P_L}{\partial P_1} = \frac{4K \tan \frac{\theta_{12}}{2}}{(K + \tan \frac{\theta_{12}}{2})2} \tag{37}$$

where K = constant

$= X_{12} / R_{12}$

Equation (4) can be determined graphically as follows: Consider Fig. 8 then,

$oa = 1, \quad ab = \tan \frac{\theta_{12}}{2}$

$bc = K \quad ac = K + \tan \frac{\theta_{12}}{2}$

and $bd = ac$ & $\perp^r$ to ac

$\triangle\triangle$ abd and ace are similar. Then,

$$ce = \frac{bd \cdot ac}{ab}$$

$$= \frac{(K + \tan\frac{\theta_{12}}{2})^2}{\tan\frac{\theta_{12}}{2}}$$

If ef = 4 K, ch = 1

From the similarity of $\triangle\triangle$ efg and ech we have

$$fg = \frac{ch \cdot ef}{} = \frac{1(4K)}{(K+\tan\frac{\theta_{12}}{2})^2} \tan\frac{\theta_{12}}{2}$$

So, fg represents $\frac{\partial P_L}{\partial P_1}$

The same procedure as in case of 2 bus bar system is to be adopted for the determination of incremental cost of received power against output power curve.

In a similar way $\lambda - P_2$ is determined. Then, $P_t \rightarrow P_1$ and P_2 can be obtained.

- CONCLUSIONS:

A rapid, logic and a new graphical derivation of the coordination equations applicable for power systems with certain reasonable number of buses and generating thermal units, is suggested to take place to yield for any number of generating units curves relating the power output against the incremental cost of received power. Thus, simply, acurve for the total power output and the incremental cost of received power can be deduced.

The incremental transmission losses which depends on the incremental production cost of received power can be assigned graphically for two cases:

Firstly for two bus bar system as revealed by figures 3,4 and 5 if the incremental transmission losses can be expressed either as a function of the B-constants or as a function of voltages and phase angles. (As expressed mathematically by equations 3 and 4).

Secondly for three busbar system:

This case is depicted in figures 7 and 8 and the incremental transmission losses are expressed mathematically by equations (5) and (6) as a function of the B-constants and voltages and phase angles.

Thus, the incremental cost of received power against output power characteristic is derived.

In a similar way for the other units this relation is plotted.

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