## Mansoura Engineering Journal

Volume 2 | Issue 2 Article 4

12-1-1977

# Analysis of Electric Power Systems with Untransposed Transmission Lines.

#### Mohamed Mohamed Tantawy

Assistant Professor., Electrical Power & Machines Department., Faculty of Engineering., El-Mansoura University., Mansoura., Egypt., mtantawi@mans.edu.eg

#### S. Farrag

Demonstrator., Electrical Power & Machines Department., Faculty of Engineering., El-Mnofia University., Mnofia., Egypt.

Follow this and additional works at: https://mej.researchcommons.org/home

#### **Recommended Citation**

Mohamed Tantawy, Mohamed and Farrag, S. (1977) "Analysis of Electric Power Systems with Untransposed Transmission Lines.," *Mansoura Engineering Journal*: Vol. 2: Iss. 2, Article 4. Available at: https://doi.org/10.21608/bfemu.2021.185487

This Original Study is brought to you for free and open access by Mansoura Engineering Journal. It has been accepted for inclusion in Mansoura Engineering Journal by an authorized editor of Mansoura Engineering Journal. For more information, please contact mej@mans.edu.eg.

## ANALYSIS OF ELECTRIC POWER SYSTEMS WITH UNTRANSPOSED TRANSMISSION LINES

BY

## M. Tantawy & S. Farrag

## ABSTRACT:

Untransposition in O.H.T.L. creates the unequiality of T.L. phase impedances. Several problems are arised due to the out -of- balance of the phase impedances.

The main of which is that it affects on the system performance. For the seek of system analysis with untransposed T.L., the modeling of system element in matrix form is given and the modeling of the power system as a whole is also cleared.

The methods for the reduction of the out-of-balance are discussed and the flow-chart for the problem analysis is given.

It could be noticed that transposition is the best method for the out-of-balance reduction, but it reduces the reliability of the system.

#### 1- INTRODUCTION:

Unsymmetrical lines are in common use because of the more convenient mounting on towers or for the purpose of keeping the average hight above ground of the conductors as low as practicable in order to minimize hazards due to lightning. This creates the problem that the transmission line impedances become so out-of-balance that affect on the generator

<sup>\*</sup> Lecturer, Faculity of Engineering, El-Mansoura Univ.

max Demonestrator, Faculty of Engineering, El-Mnofia Univ.

ratings, unbalanced voltages at the receiving end, increased losses within the transmission network, and may cause false tripping in a line circuit breakers.

Although transposition is the most convenient method for compensating the out-of-balance as well as to minimize the transmission losses, however, it is not desirable in extra and ultra-voltages because of: reduction of reliability, added cost, and complexity in the design of the transposing towers.

## SYMBOLS USED:

Z<sub>ik</sub> = series impedance element.

Z's shunt capacitive element (impedace form.)

 $y_{ik}$  : series admittance element.

 $y_{ik}$  = shunt capacitive admittance element.

Y = self element - nodal admittance.

Z<sub>11</sub> = driving point and transfer impedance element.

V, = branch voltage drop

.; = branch current

' = node voltage

I = node current.

= permittivity

f = generator internal voltage.

' = earth resistivity

= angular frequency, rad./sec.

f = frequency, cycles/seconds.

h<sub>i</sub> = height of conductor i in feet.

dik = distance in feet between conductors i and k.

Dik = distanca in feet between conductor i and the image
 of conductor k.

(M.R., = geometric mean radius of conductor i or bundle in feet.

rad, = radius of ith conductor = dik for i = k

N = number of circuits.

n = number of conductors.

#### POWER SYSTEM MODELING:

Generally, any power system consists mainly of alternators, transmission line, power transformers, and loads.

The method of representation (for the seek of power system computations) for each of them will be illustrated.

## 3.1. Transmission Line Matrices: -

The characteristics of a multi-conductor T.L. can be defined by its series impedance matrix (Z) per unit length, and its shunt admittance matrix (y) per unit length.

The matrix equations for an "n" conductor line are: 3.1.1. Series Matrix:-

The voltage current relation in matrix form for the series impedance of T.L. is:

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{1n} \\ z_{21} & z_{22} & z_{nn} \\ \vdots & \vdots & \vdots \\ z_{n1} & z_{n2} & z_{nn} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix}$$

or [v] = (Z).(i) .....3.

- Where, (V) is the series voltage drop along the line
  - (i) is the line current.
  - (Z) T.L. impedance including the earth effect and equals:

= (r) + (
$$\Delta$$
r) + ) { ( $x_g$ ) + ( $x_s$ ) + ( $\Delta$ x)} ...3.3

- (r) = diagonal matrx of conductor resistance.
- (xg) = diagonal matrix of conductor internal reactance.
- $(x_r)$  = square reactance matrix due to line geometry.
- (Ar), (Ax) = square matrices calculated from Carson's earth correction formula.

 $(x_g)$ ,  $(x_g)$  are combined in the one computation and equation  $(x_g)$  is reduced to,

$$Z_{ii} = r_{ii} + \triangle r_{ii} + J (x_{ii} + \triangle x_{ii}) \qquad \dots \qquad 3.4$$

$$Z_{ik} = \triangle r_{ik} + J (x_{ik} + \triangle x_{ik}) \qquad \dots \qquad 3.5$$

If the impedances are calculated in ohms/mile then:-

r, = resistance of conductor at system frequency(ohms/mile)

$$m_{0.5} = 0.7411 \ 10^{-3} \ \text{W. } \log_{10} \frac{1}{\text{GMR}_{1}}$$
 ..... 3.6

$$c_{i,k} = 0.7411 \quad 10^{-3} \quad \text{W. } \log_{10} \frac{1}{d_{i,k}} \qquad \dots 3.7$$

$$= 0.2528 \quad 10^{-3} \quad \text{W} + 2.599 \quad 10^{-7} \quad \text{W} \quad \text{h}_{1} \quad \sqrt{f/P} \quad \dots \quad 3.8$$

$$= 0.7411 \ 10^{-3} \ \text{W.} \ \log_{10} 2162 \ \sqrt{l/f} + 2.599 \ 10^{-7} \ \text{W.h.} \sqrt{f/g}$$

$$0.2528 \ 10^{-3} \cdot \text{W} + \left[ (-2.599 \ 10^{-7} \cdot \frac{D_{ik}}{2} \cos \theta_{ik}) \ \sqrt{f/P} \ \right] \text{W}$$

$$e_{ik} = 0.7411 \ 10^{-3}.W. log_{10} \ 2162 \sqrt{P/f} + \left[ (2.599 \ 10^{-7}. \frac{D_{1k}}{2} cos \right]$$

The terms Ar, Ax are corrective values derived from corrective formulae for the self and mutual impedances with north return.

Mg. 1 shows the arrangement of T.L. conductors.

## .....2. Shunt Matrix:-

The voltage - current relation for the shunt components of the T.L. in matrix form

$$\begin{bmatrix} v_{1} \\ v_{2} \\ \vdots \\ v_{n} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1n} \\ z_{21} & z_{22} & \dots & z_{2n} \\ \vdots & \vdots & \vdots \\ z_{n1} & z_{n2} & \dots & z_{nn} \end{bmatrix} \begin{bmatrix} i_{1} \\ i_{2} \\ \vdots \\ i_{n} \end{bmatrix} \dots 3.12$$

 $\begin{bmatrix} \mathbf{i} \end{bmatrix} = \begin{bmatrix} \mathbf{y} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{v}} \end{bmatrix}$ 

where,

[v] is the line voltage.

[i] is the shunt capacitive current.

$$[\vec{y}] = [\vec{z}]^{-1}$$

and is defined by the following equation: -

$$[\vec{y}] = [0] + 32\pi \omega \in [A]^{-1}$$
 ...... 3.14

or in impedance form

$$\begin{bmatrix} \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \end{bmatrix} - \begin{bmatrix} \frac{1}{2\pi\omega} \in \mathbf{A} \end{bmatrix}$$
 ...... 3.15

A is a geometric matrix defined by,

$$A_{1k} = \log_{e} \frac{D_{1k}}{d_{1k}} \qquad 3.16$$

$$\therefore z_{1k} = 0 - \int (\frac{25.76}{w} \log_{10} \frac{D_{1k}}{d_{1k}}) \qquad 3.17$$

$$z_{1i} = 0 - \int (\frac{25.76}{w} \log_{10} \frac{D_{1i}}{rad_{1}}) \qquad 3.18$$

## .1.3. Effect of earth wires on T.L. matrices:-

Generally, the [2] and [y]matrics for a T.L. be of the rder of (3N + G), (where N is the number of T.L. parallel ircuits and G is the number of earthwires.)

In a transmission line with grounded earthwires, he values of the elements of the impedance matrix "7" can be addited to include the effect of earth wires as will be leared.

The series matrix with one grounded earthwire is:-

By the use of matrix Algebra, the elements of Z matrix in eqn. 3.19 are modified as follows:-

$$Z_{ij}$$
 (new) =  $Z_{ij}$  -  $Z_{ik}$  .  $Z_{kk}^{-1}$  .  $Z_{kj}$  ...... 3.20

For T.L. with multi earthwires, the eqn. (3.20) can be simply repeated to have finally a 2-matrix of order 3x3

## 3.1.4. Conductor Bundling:-

For transmission line with bundle conductors, the phase impedance and admittance matrices may be initially calculated and then bundling can be taken into consideration as: if conductors i, k are bundled - (i.e. energised by the same voltage). then the series matrix is,

Subistracting tow i from row k in (3.21).

$$\begin{bmatrix} \mathbf{v_i} \\ \mathbf{z_{ii}} \\ \mathbf{z_{ki}} - \mathbf{z_{ii}} \\ \mathbf{z_{kk}} - \mathbf{z_{ik}} \end{bmatrix} \begin{bmatrix} \mathbf{i_i} \\ \mathbf{i_k} \\ \mathbf{i_k} \end{bmatrix} \dots 3.22$$

Applying equation (3.20) on equation (3.22), row and colum representing conductor k can now be eleminated.

Similarly, for [y] matrix, the bundling of conductors can be considered as:-

N.B.: the general rule may be described as:-

Column k is added to column i, then discard column k. row k is added to row i.

The total current in the bundle is  $(i_1 + i_k)$ , row k and  $V_k$  can now be discarded.

The same techniques may be applied to the "capacitive" shunt admittance matrix.

Simillarly, for the case of T.L. with parallel circuits can be considered to have an equivalent single circuit line.

## 3.1.5. Application of Symmetrical Components on T.L. Matrices

To determine the unbalanced currents in the terminal equipment, the system can be analysed to its symmetrical component impedances.

For T.L. the phase impedance matrix [2] must be transfered to phase impedance sequence matrix [2] by carrying out a spinor trans formation as:-

$$\begin{bmatrix} z_{8} \end{bmatrix} = \frac{1}{3} \quad \begin{bmatrix} c \end{bmatrix} \begin{bmatrix} z \end{bmatrix} \begin{bmatrix} c_{1} \end{bmatrix} \qquad 3.23$$
where:
$$\begin{bmatrix} c \end{bmatrix} \text{ is the spinor transform } = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^{2} \\ 1 & a^{2} & a \end{bmatrix}$$

$$\begin{bmatrix} c_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}, a = -\frac{1}{2} + j \frac{\sqrt{3}}{2}$$

[2] is the phase matrix with earthwire eleminated.

Similarly,

$$[y_s] = \frac{1}{3} [c][y][c_1] \qquad \dots \qquad 3.24$$

& For a multi-circuit line of order 3 N

$$\begin{bmatrix} z_{e \ 11} & z_{s \ 1n} \\ \vdots & \vdots & \vdots \\ z_{s \ n1} & z_{s \ nn} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} c & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & c \end{bmatrix} \begin{bmatrix} z_{11} & \cdots & z_{1n} \\ \vdots & \vdots & \vdots \\ z_{n1} & z_{nn} \end{bmatrix} \begin{bmatrix} c_{1} & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & c_{1} \end{bmatrix}$$

## 3.1.6. T.L. Modelling: -

The transmission line can be represented by the nominal equivalent circuit:-

Shown in Fig. (3)

the loop equations for the T circuit shown are:-

Substituting from (3.32), (3.26), (3.29) in (3.30), (3.31)

$$[I_1] = [y] [V_1] + [y/2] [V_1] + [-y] [V_2]$$

$$[I_2] = [-y] [V_1] + [y] [V_2] + [y/2] [V_2]$$

in matrix form: -

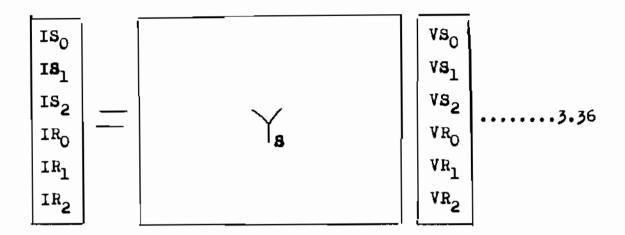
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y + \frac{y}{2} & -y \\ -y & y + \frac{y}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \dots \dots 3.33$$

For a three conductor line the matrix will be of order 6 x 6 as:-

. . . . . . . . . 3 . 34

IS and IR are sending and receiving end currents respectively. VS and VR are sending and receiving end voltages respectively.

Applying spinor transformation (equation 3.24) on equation (3.34) to get the admittance sequence matrix for the T.L. to be:-

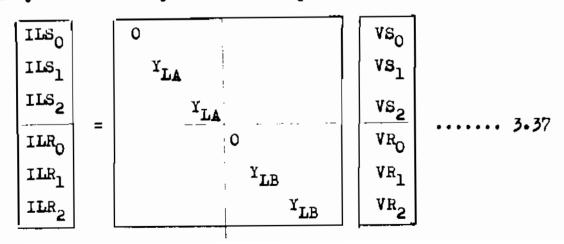


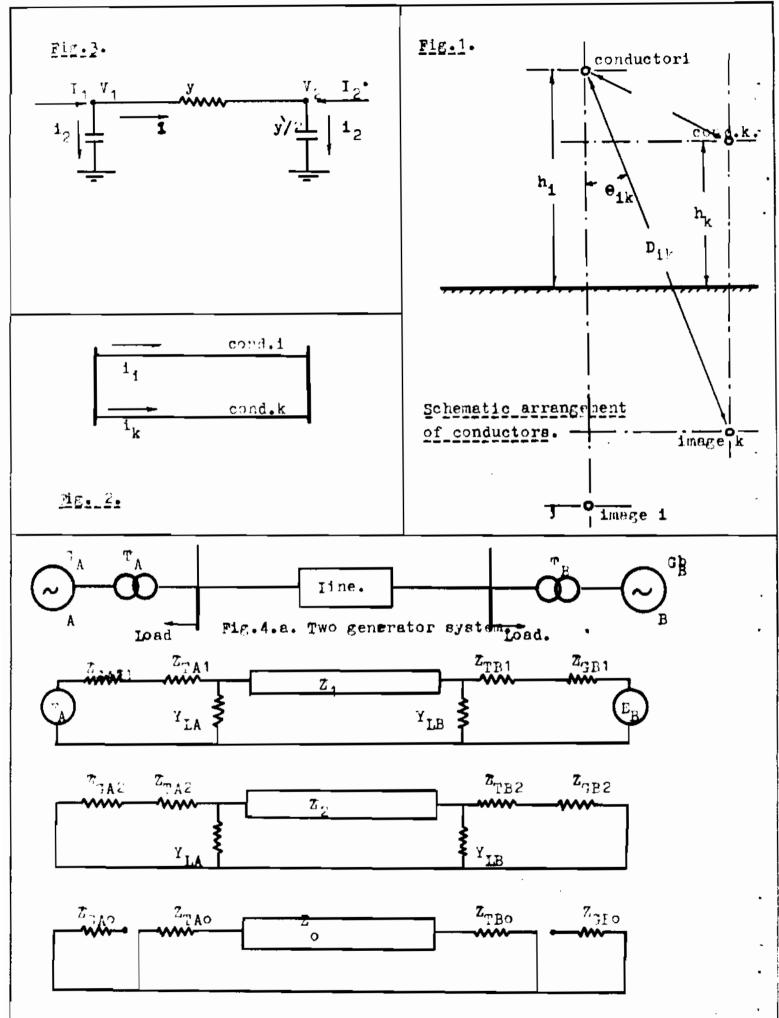
For the particular system shown in Fig. (4.a). Fig. (4.b) gives the equivalent sequence impedance diagrams, and the following assumptions are valid:-

- i) The impedance of zero-sequence path to the loads can be paralleled with the zero-sequence impedance of the transformers TA and TB.
- ii) The load positive and negative sequence admittances are assumed to be equal and include impedances of transformers, and other equipments between the line terminals and the loads.
- iii) The transformer positive and negative sequence impedances are equal.
  - iv) The generator positive sequence impedances are represented by the synchronous impedances.

## 3.2. Addition of Terminal Loads:-

The characteristic equation for the load in matrix form and by the use of symmetrical components is:-

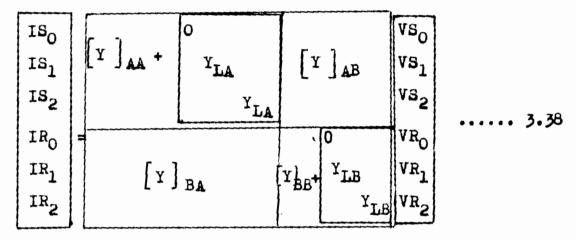




Pir. 1.1 .: Symmetrical component circuits.

where the load sequence currents are ILS and ILR at the sending and receiving ends of the line respectively.

Equations (3.35), (3.36) can be added directly to have,



The node currents IS and IR include the load currents.

The line voltages VS and VR are the sequence voltages.

Equation (3.38) can be inverted to be:-

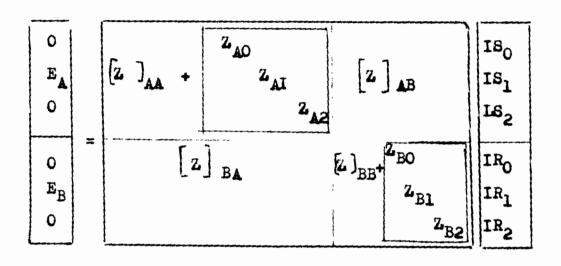
## 3.3. Addition of Transformer and Generator Impedances:-

The generator and transformer series voltage drop may be represented as follows.

At sending end (A):-

$$\begin{bmatrix} O & - & VS_O \\ E_A & - & VS_1 \\ O & - & VS_2 \end{bmatrix} = \begin{bmatrix} Z_{TAO} \\ Z_{TA1} + Z_{GA1} \\ Z_{TA2} + Z_{TA2} \end{bmatrix} \begin{bmatrix} IS_O \\ IS_1 \\ IS_2 \end{bmatrix} \cdots 3.40$$

So, the generator and transformer impedance matrices are added to equation (3.39) resulting in:-

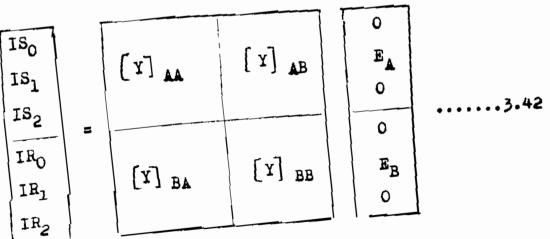


......3.41

where:

$$Z_{AO} = Z_{TAO}$$
 $Z_{BO} = Z_{TBO}$ 
 $Z_{A1} = Z_{TA1} + Z_{GA1}$ 
 $Z_{A2} = Z_{TA2} + Z_{GA2}$ 
 $Z_{B1} = Z_{TB1} + Z_{GB1}$ 
 $Z_{B2} = Z_{TB2} + Z_{GB2}$ 

equation (3.41) can be inverted to be:-



## 4. TERMINAL EQUIPMENT UNBALANCE FACTORS:

#### 4.1. Difinitions:-

For power system with unbalanced untransposed transmission line, we can define the following unbalance factors:-

## At T.L. sending-end.:-

- a) Negative-sequence unbalance factor for generator current (MS2), which is the ratio between negative-sequence current (IS<sub>2</sub>) and the positive-sequence current (IS<sub>1</sub>).
- b) Zero-sequence unbalance factor for transformer current ( $MS_0$ ), which is the ratio between zero-sequence current ( $IS_0$ ) and the positive-sequence current ( $IS_1$ ).
- c) The terminal negative-sequence unbalance voltage of the line (NS<sub>2</sub>), which is the ratio between negative-sequence voltage and the positive-sequence voltage.
- d) The terminal zero-sequence unbalance voltage of the line  $(NS_0)$ , which is the ratio between zero-sequence voltage and the positive-sequence voltage.

Similarly, at T.L. receiving-end, we have:-

- e) Negative-sequence unbalance factor for generator current (MR2).
- f) Zero-sequence unbalance factor for transformer current(MR)
- g) The terminal negative-sequence unbalance voltage of the line  $(NR_2)$ .
- h) The terminal zero-sequence unbalance voltage of the line  $(NR_{\Omega})$

## 4.2. Methods of reduction of system unbalance: -

The unbalance caused by untransposed transmission line is affected by the following different methods:-

I. Making complete or partition transposition for T.L.

- II. Locating series capacitor banks at different distances along the T.L.
- III. Insertion of unequal series impedances into the line.
  - IV. Adjusting transformers tapping.
  - V. Increasing the impedance of the terminating apparatus.
  - VI. Insulating earthwires.

## 4.3. Calculation of Unbalance Factors:-

The solution of equation (3.42) for different values of internal voltages and power angles gives for each value the different unbalance factors:-

#### 4.3.1. The Unbalance Currents:-

## 4.3.2. The Unbalance Voltages:-

the terminal sequence voltages are:-

and

therefore, the terminal sequence unbalance voltage factors are:

$$NS_{2} = VS_{2}/VS_{1}$$

$$NS_{0} = VS_{0}/VS_{1}$$

$$NR_{2} = VR_{2}/VR_{1}$$

$$NR_{0} = VR_{0}/VR_{1}$$

## 5 . Transmission Line Losses: -

The transmission line losses can be computed by the aid of transmission line currents and voltages at both ends of the line as follows:-

the components of current at sending-end are:-

& at receiving - end:-

So, the power at sending-end is

$$P_s + jQ_s = (V_{s0} \cdot I_{0S} + V_{S1} \cdot I_{1S} + V_{S2} \cdot I_{2S}) \cdot \dots 5.3$$
  
and the power at the receiving—end is:-

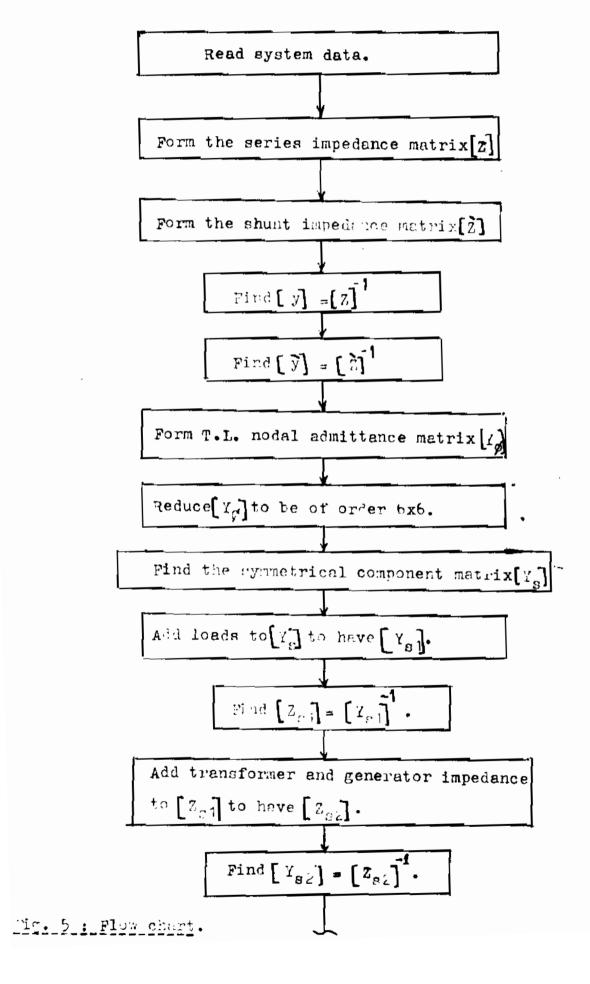
$$P_{R} + J Q_{R} = (V_{RO} \cdot I_{OR} + V_{R1} \cdot I_{1R} + V_{R2} \cdot I_{2R}) ... 5.4$$

therefore, the transmission line losses is the difference between the sending and receiving end powers.

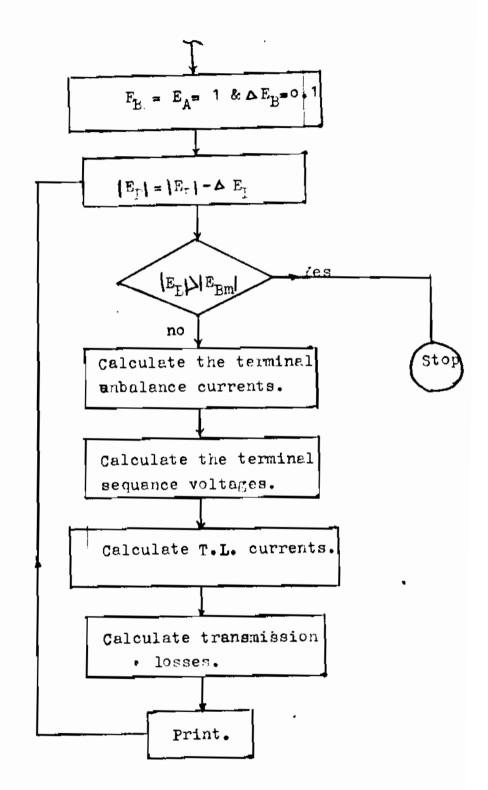
Losses = 
$$[(P_S + \int_{S}^{Q}) - (P_R + \int_{S}^{Q})] \qquad \dots \qquad 5.5$$

#### 6. FLOW-CHART:

Figure 5 shows a complete flow chart for the analysis (in matrix form) of the untransposed T.L. problem in electric power network, also the calculation of the corresponding transmission line losses.



Cont.



#### 7-CONCLUSIONS:

This paper provides a complete analysis for the representation of the electric power systems in matrix forom which is suitable for the analysis of the untransposed T.L. in problems in electric power system. The modeling given is also required for load flow studies, stability analysis, etc.

The methods adopted for out of balance reduction are illustrated it is seen that by these methods have the disadvantage of increasing the transmission line losses. So, it is prefered to make transpostions an T.L. to decrease the out of balance and the decrease also the T.L. losses, but it has a main dis-advantage of reducing the system reliability.

## 8 -REFERENCES:

- 1. BUTTERWORTH, S. "Electrical characteristics of over-head lines" (ERA 1954).
- 2. CLARKE, E. "Circuit Analysis of A.C. Power systems" Vol. 1 (Book).
- 3. CARSON, I.R. "Wave propagation in overhead wires with ground return" Bell syst. Tech. Journal 1926.
- 4. SUBBA Rac. V.S. "Balancing bundle-conductor transmission Line constants without transpositions" Proc. IEE Vol. 112. No. 5, May 1965, pp 931-940.
- 5. HESSE, M.H. "Electromagnetic and electrostatic transmission line parameters by digital computers" IEEE trans. Vol. 81, June 1963, pp. 282-91.
- 6. HESSE, M.H. "Circulating currents in paralleled untransposed multi-circuit lines I-Numerical Evaluations" IEEE. PAS May 1965, pp. 801-11

- 33. Mansoura Bulletin December 1977.
- 7. HOLLEY, H., COLFMAN, D., SHIPLEY, R.B. "Untransposed E.H.V. line computations" IEEE trans. PAS. Vol. 28. March 1964, pp. 291-296.
- 8. SHIPLEY, R.B., COLEMAN, D., HOLLEY, H. "Power circuit Representation for digital studies" IEEE trans. PAS April 1964, pp. 376-380.
- 9. COLEMAN, D., WATTS, F., SHIPLEY, R. "Digital calculation of overhead Transmission-Line. Constants."

  Trans. IEEE. Feb. 59, pp. 1266-1268.
- 10. GROSS, E., HESSE, M.H. "Electromagnetic Unbalance of Untransposed Transmission Lines" IEEE Trans., PAS, Vol. 72, December 1953, pp. 1323-36.