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YIELD LINE VERSUS FINITE ELEMENT ANALYSES OF BEAMLESS SLABS

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Introduction :

In recent years beamless slabs, i.e. flat plates, have become widely used and increasingly economical as the cost of field labour has increased relative to materials. With the abscence of beams, this system utilizes the simplest possible formwork which can be constructed in minimum time with minimum field labour. It has the advantage of giving the highest freedom in the layout of columns, partitions and small openings while providing the minimum storey height for a certain clear headroom.

Several methods for the design of beamless slabs are available. Some of these methods are based on the elastic theory such as :

i) Flat slab method(1)***,

ii) The finite difference method(2),

iii) The finite element method(3),

While other methods are based on the ultimate theory such as :

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*** Numbers between brackets indicate reference numbers in the list of references.

IV) Hillurburg strip method[4],
V) kemp's method[5],
VI) The yield line theory[6],

For irregular column arrangement, which is the case in many residential buildings, the most practical methods in day design are the yield line theory and the finite element method- if a finite element program is available.

In this paper the design moments obtained using these two practical methods are compared against each other for the typical floor in an actual building.

Yield line analysis of slabs

As mentioned before, the yield line analysis is an ultimate load technique. In this method one should first predict the crack (yield line) pattern of the slab at collapse. These cracks are assumed straight lines which devide the slab into non yielding segments. Each segment rotates about an axis of rotation which is a line of zero deflection. Positive cracks start from the pole (the point of maximum deflection), and extend towards the points of intersection of axes of rotation. Negative created around supports between adjacent slabs.

In order to calculate the moments the following three methods may be used :

i) The work method,
ii) The equilibrium method,
iii) A combination between work and equilibrium methods.

In the work method the external work done, which is essentially the external load multiplied by the deflection of its c.g., is made equal to the internal work. which equals to the projection of the developed moment multiplied by the rotations that take place i.e.

$$Ld \times o = M \times L \times \theta$$

Where :

Ld : is the total applted external load.
o : is the deflection under the c.g. of the load.
M : is the value of the developed moment per meter.
L : is the projection length of the yield line on the axis of rotation.
θ : is the angle of rotation of the considered segment that takes place.

From this equation the value of the moments can be calculated. For more details of the yield line theory see reference (6).

Finite Element Analysis of Slabs

The finite element method is considered the most suitable numerical procedure of analysis for slabs. Variations in thickness, irregular shapes, openings and curved boundaries present no special difficulties in the solution. Also variations in conditions at the boundaries are permitted.

In this method, the slab is subdivided into an adequate number of segments(elements) which may vary in shape and size. The degree of discretization is increased where sharp variations in moments are expected. A relatively coarse mesh may be used in the regular zones. The subdivision process is essentially an engineering judgment on how accurate the simple displacement model, assumed within the element, approximate the true solution.

The finite element used in this paper is a quadrilateral plate bending element which has been developed by Fellipa eta/.[7] The element is subdivided internally by the program into four triangles as shown in figure(1). Within each subdivion the displacement function is assumed cubic. This allows for a linear variation of moments inside the triangles. The stiffness matrix is formed for each triangle and assembled to form the stiffness of the equadrilateral element. After condensation of the degrees of thefreedom of the interior node 0 , the element stiffness matrix becomes 12 x 12 .
The accuracy of this element has been proven high due to the compatability of slopes along the boundaries.

Example considered

A 33 storey office building is to be constructed in Agouza zone of Giza province. The building is ( ) x ( ) with the longer side facing the Nile River. After a comparative study of possible structural systems, a decision has been made to use flat plate construction for the typical office floor. Light drywall partitions and plastic flooring are used. The live load is taken 300 kg/m . The design dead loads and live load add up to 1 ton/m .

The typical floor is analysed using the plate bending element and the yield line theory. The finite element idealization is shown in figure (2).
The assumed crack pattern for the floor is shown in figure (3).

Comparison of results

The slab moments in the transversal and longitudinal directions obtained using the finite element method are plotted in figures (4) and(5) respectively for the largest panel S1 . The analysis indicates that the maximum negative bending moment is 4.3 m.t/m and the maximum positive moment is 3.1 m.t/m.

In the yield line analysis an appropriate ratio between the negative and the positive moments in the panel should be assumed. This ratio is commonly taken between 1.0 and 1.5. Application of the work method to the crack pattern shown in figure (3) for three ratios (R) between M(-ve)/(+ve) gives the following results:

(R)	M(+ve) m.t/m	M(-ve) m.t/m
1.0	3.5	3.5
1.2	3.3	4.0
1.5	3.0	4.47

If the first ratio is adopted a violation of the finite element requirements occurs in the negative moment zone. However, with a redistribution of about 18% between negative and positive moments, the yield line results fulfill the requirements of the adjusted finite element solution. On the other hand, if the design is made according to the second or the third ratios, the steel reinforcements satisfy the requirements of the finite element solution.

Conclusion

The finite element method and the yield line theory have been applied to analyse a typical floor in an actual building. The two methods gave close results. The differences can be taken care of by adequate allowance for redistribution of bending moments. A comparison between the two methods is held :

a) The finite element analysis results in moments at various locations in the slab. The yield line theory gives max. values only of positive and negative moments for an assumed ratio between them .
b) The finite element solution includes the deflections under service loads while the yield line theory does not offer any values .
c) The finite element method requires the availability of a computer program and the yield line analysis required hand calculations only .

Experience is mandatory for the application of both methods, in the mesh shape for a finite element analysis and in assuming the critical crack pattern for the yield line analysis.

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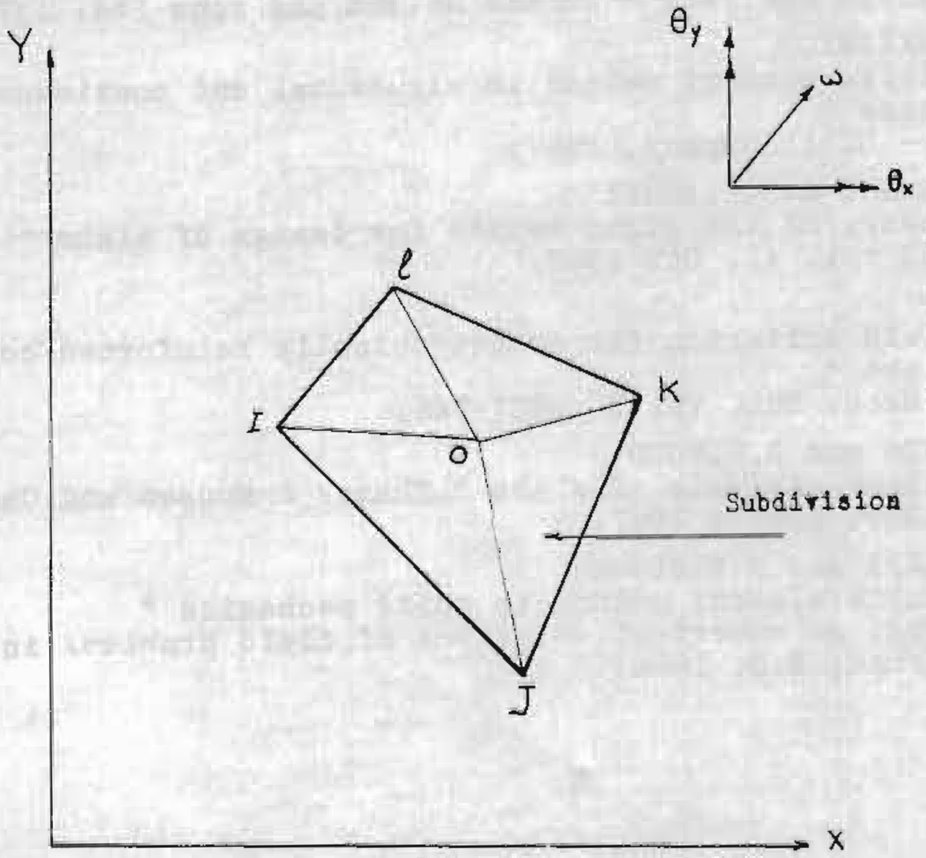

Figure (1) Quadrilateral plate bending element used in analysis

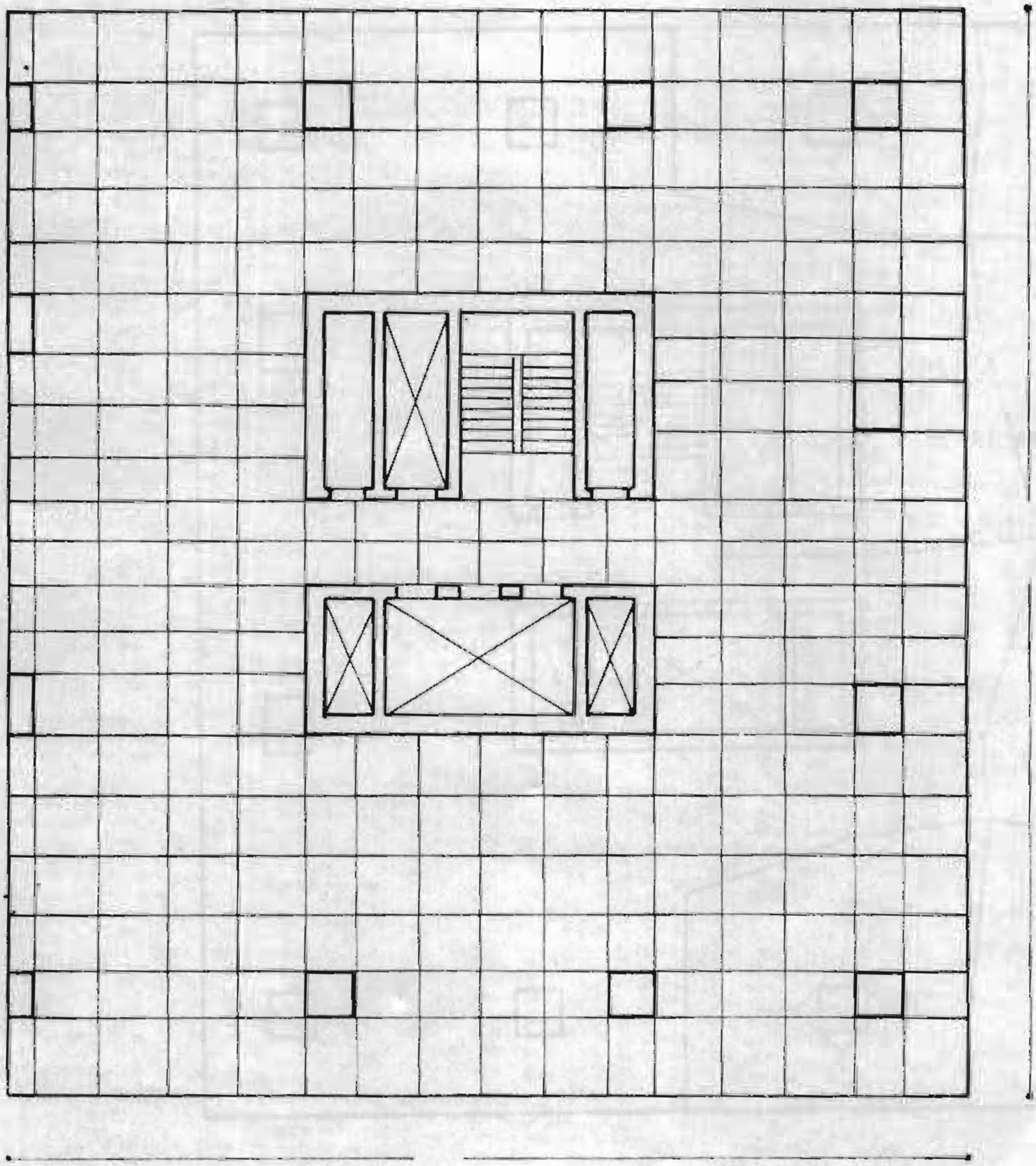

Figure (2) Plan Of The Typical Floor Showing The Finite Element Mesh.

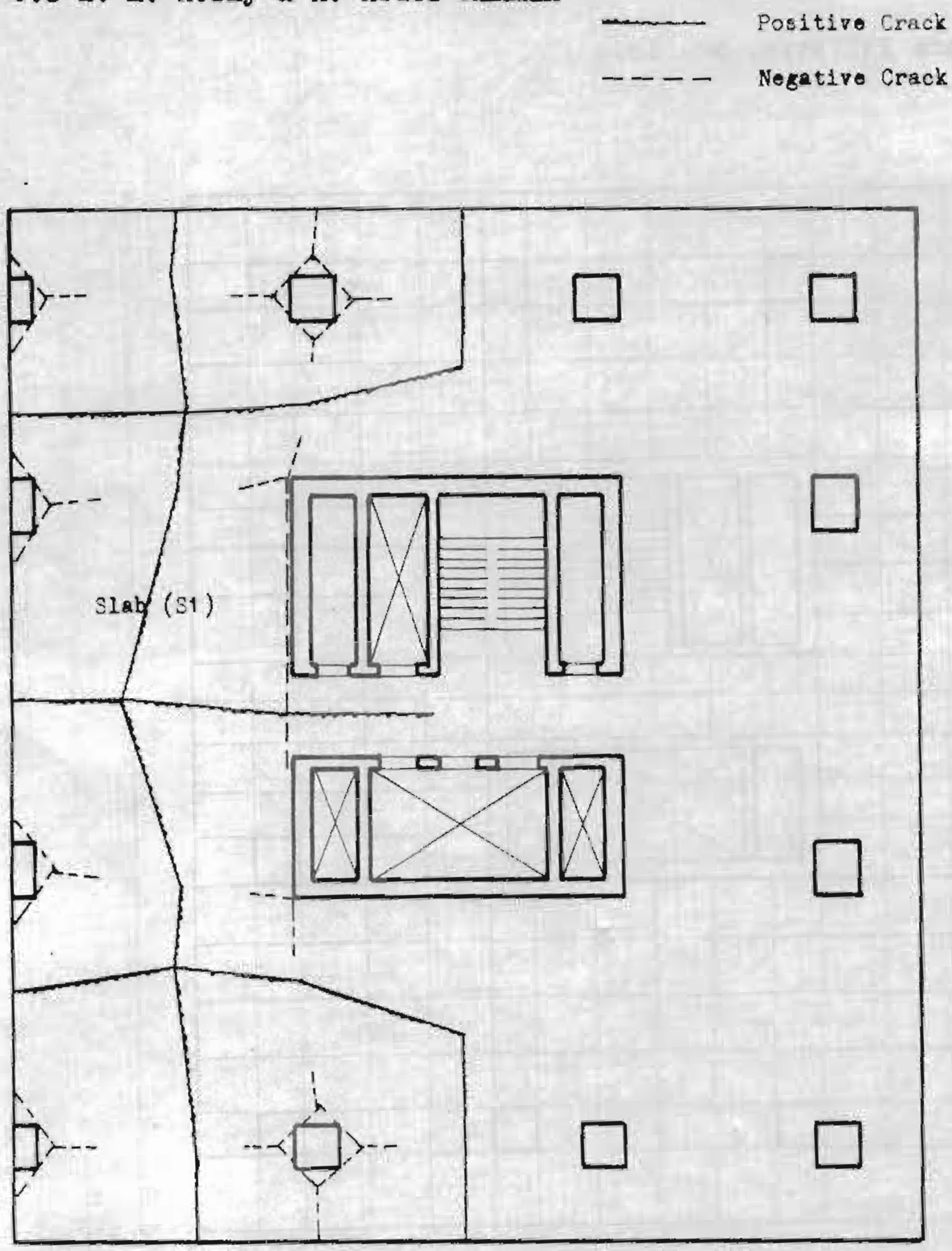

Figure (3) Plan Of The Typical Floor Showing The Assumed Crack Pattern

Fig.(4) F.E. bending moments in the transversal direction

for slab S1

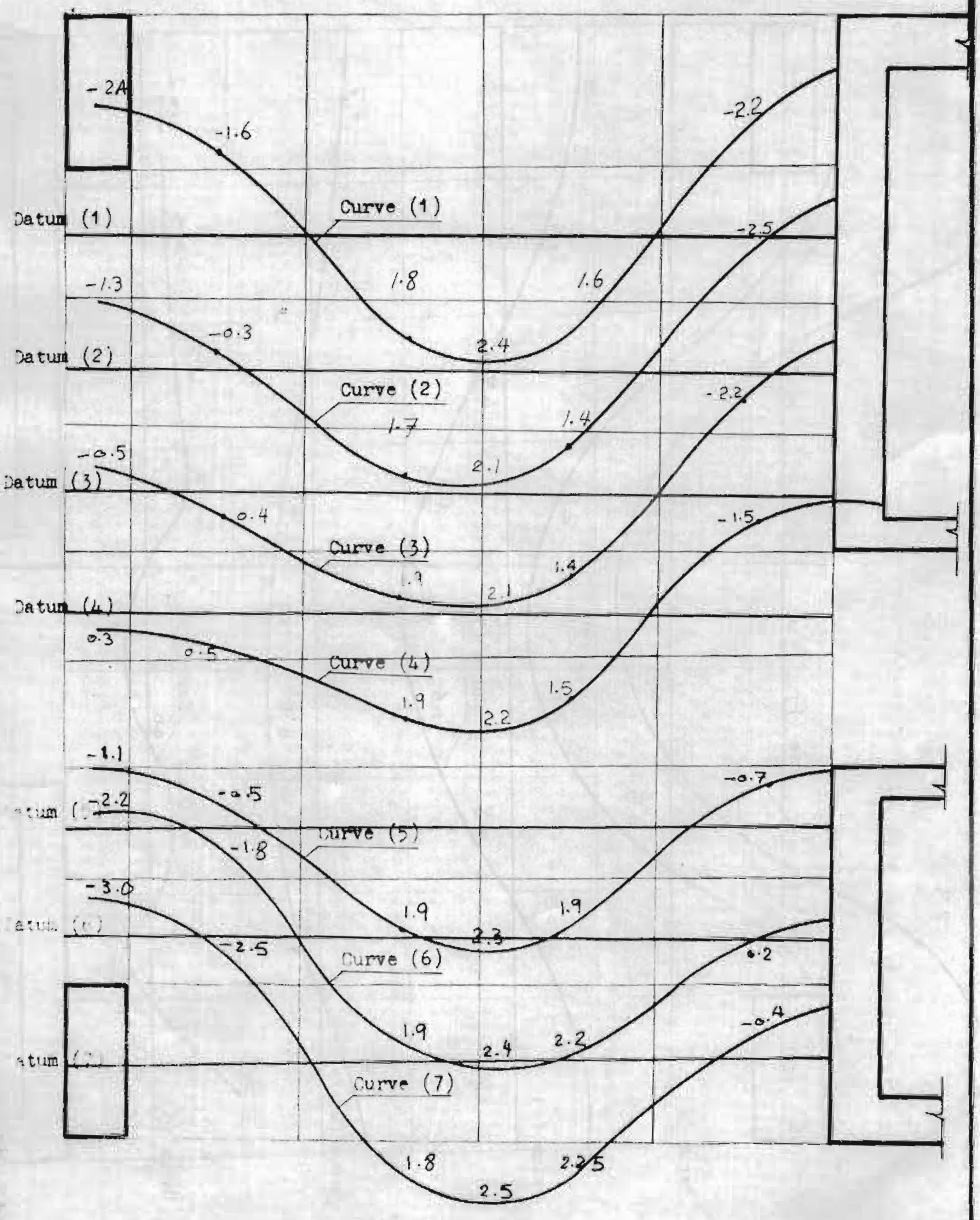

Fig.(5) F.E. moments in the longitudinal direction for slab S1.