

12-6-2021

Building Techniques of Multi-Subconductor Transmission Lines.

Reda El-Dewieny

Assistant Professor., Electrical Systems and Machines Department., Faculty of Engineering., El-Mansoura University., Mansoura., Egypt.

Follow this and additional works at: <https://mej.researchcommons.org/home>

Recommended Citation

El-Dewieny, Reda (2021) "Building Techniques of Multi-Subconductor Transmission Lines.," *Mansoura Engineering Journal*: Vol. 4 : Iss. 2 , Article 3.

Available at: <https://doi.org/10.21608/bfemu.2021.187347>

This Original Study is brought to you for free and open access by Mansoura Engineering Journal. It has been accepted for inclusion in Mansoura Engineering Journal by an authorized editor of Mansoura Engineering Journal. For more information, please contact mej@mans.edu.eg.

BUNDLING TECHNIQUES OF MULTI-SUBCONDUCTOR
TRANSMISSION LINES

BY

Reda M. K. El-Dewieny, B.Sc., Ph.D.

ABSTRACT:

A great number of transmission lines in which each phase consists of multi-subconductors are now present in many practical systems. This paper introduces three different bundling techniques which can be used effectively to obtain the parameters of such lines in terms of phase quantities.

Examples are given to illustrate the application of these techniques. A method for eliminating the earth wire(s) while preserving its effect, based on the well known matrix reduction, is also introduced. A general comparison between the bundling methods is given and it is shown that, one of these methods require more computation time than the others though it may be easier in programming.

1. INTRODUCTION:

The recent advent of high-speed digital computers, with large core storage, has made it possible to solve by direct numerical methods^{1,2} large varieties of physical problems. The application of digital computers to such problems requires the mathematical formulation of the problem to be accomplished in a logical and organized manner.

Of these problems are the 'bundling' techniques which are performed for polyphase transmission lines in which each phase (and may be the earth wire) consists of multi-conductors, each known as a subconductor. This type of line is now present in many practical systems and it is necessary, for many purposes, to be able to calculate the parameters^{3,4,5} of such a bundle and, consequently, the series impedance (Z) and shunt admittance (Y) matrices of the line.

It is the aim of this paper to describe some methods, which are suitable for digital computers, for bundling the subconductors of multiconductor lines. Two of these methods, namely those based on linear transformation and diakoptics, use matrix techniques but require first the computation of the series and shunt parameters (self and mutual) for each single conductor in the configuration. The third method, which uses the concept of the geometric-mean distance⁶, enables us however, to calculate the bundle parameters

The author is a lecturer at the Department of Electrical Systems and Machines Engineering, Faculty of Engineering, University of El-Mansoura, El-Mansoura, Egypt.

directly from the line geometry.

Having bundled the subconductors of each phase into one equivalent conductor, the order of the Z and Y matrices now equals to three times as many rows and columns as there are power circuits in the line, plus one for the earth wire(s). Then, on the assumption that, throughout its length, the earth wire remains at zero potential, its presence can be eliminated from the matrices by the normal method of matrix elimination, thus reducing the order of each matrix to that of the number of power phases (6 for a double-circuit line).

Throughout this paper, only the impedance matrix Z is considered. However, the methods apply equally well for the shunt admittance matrix Y. Examples are included for illustrative purposes.

Before proceeding with the bundling techniques, however, the calculation of the series impedance, for each single conductor in the configuration, is briefly discussed.

2. CALCULATION OF LINE SERIES IMPEDANCE:

The elements of the series-impedance matrix Z, for each single conductor in the line, consist of the self and mutual impedances between conductors. Following the method described in Ref.6, the mutual term between the mth and pth conductors at points M and P of Fig.1 is given by

$$Z_{mp} = \frac{\alpha^2}{2\pi} \ln\left(\frac{d'_{mp}}{d_{mp}}\right) + \frac{\alpha^2}{\pi} \int_0^{\infty} \frac{\mu \cos[\gamma(x_m - x_p)] \exp[-\gamma(y_m + y_p)]}{\mu\gamma + (\gamma^2 + \beta^2)^{\frac{1}{2}}} d\gamma \quad \dots(1)$$

The self term consists of two parts; an internal impedance calculated by Galloway et al⁴, and an external impedance derived from eqn.(1) with p = m as

$$Z_{mm} = \frac{\alpha^2}{2\pi} \ln\left(\frac{d'_{mm}}{d_{mm}}\right) + \frac{\alpha^2}{\pi} \int_0^{\infty} \frac{\mu \exp(-2\gamma y_m)}{\mu\gamma + (\gamma^2 + \beta^2)^{\frac{1}{2}}} d\gamma \quad \dots(2)$$

where d_{mm} = the radius of the mth conductor ;

$\alpha^2 = j\omega\mu_0$ (ω = angular frequency, $\mu_0 = 4\pi \times 10^{-7}$); and

$\beta^2 = j\omega\mu\mu_0\sigma$ (μ = relative permeability of conductor material, σ = earth's conductivity).

In the equations above, the integral terms are seen to be functions of the earth's conductivity. So, if the earth is perfectly conducting, i.e. $\sigma = \infty$, the integrals in both equations disappear, leaving only the logarithmic terms.

If, however, the earth has finite conductivity, i.e. $\sigma \neq \infty$, the integral terms are involved. Since the exponential terms in both integrals decay rapidly (y_m and y_p are the heights of conductors above earth), they

ensure that the main contribution to the integrals is from a small interval in the neighbourhood of $\gamma = 0$, say $0 \leq \gamma \leq \eta$. (Values for η in the range 0.1 to 0.3 would be adequate⁶.) With this fact taken into consideration, the simplicity of the equations is thus retained. (See Section 6.)

The following section describes the sequence in which the conductors in a multi-subconductor transmission line are to be numbered. This sequence of conductor numbering is particularly useful when the bundling by linear transformation (Section 4) is adopted.

3. NUMBERING OF CONDUCTORS IN A MULTI-SUBCONDUCTOR LINE:

For the mathematical formulation to be accomplished in an organized manner, the conductors should be numbered in the manner shown in Fig.2. This shows two examples. Firstly, a double-circuit line with duplex conductors on one circuit and single conductors on the other with two earth wires, Fig.2(a). Secondly, a single-circuit line with quad-conductors and one earth wire, Fig.2(b). The conductors must be numbered in the following way:

Firstly, the power conductors are numbered in the order "a" phase, "b" phase, "c" phase, for the first power circuit, followed by a similar arrangement for the second circuit, conductors 1 - 6 in Fig.2(a). It should be noted that, even though some power circuits may contain bundled conductors, only one conductor of each phase of each circuit is numbered at this point. It is immaterial which particular subconductor in a phase bundle is treated in this manner.

Subconductors to be bundled are numbered next, conductors 7 - 9 in Fig.2(a), and conductors 4 - 12 in Fig.2(b). The earth wires are numbered last, e.g. conductors 10 and 11 in Fig.2(a) and conductor 13 in Fig.2(b).

Now, if the series and mutual parameter terms for each single conductor in the line are already available and assembled in a matrix, the order of elements within the matrix must be as to correspond to the above conductor numbering system. If however they are to be calculated, it is necessary to do so from the start.

4. BUNDLING METHOD USING LINEAR TRANSFORMATION:

Let C be the total number of conductors forming the transmission line. Hence the elementary series-impedance matrix Z_{CC} giving the self and mutual terms for each single conductor, will contain as many rows and columns as there are conductors in the line. Assuming i_C to be the current through each conductor, the steady-state equations, i_C in matrix form, are

$$E_C = Z_{CC} i_C \quad \dots\dots(3)$$

where

- E_C = a column vector representing the voltage drops along the conductors;
- Z_{CC} = a square matrix; and i_C = a column vector.

Since the voltage drop across each bundle is the same as for each subconductor in the same bundle, we can construct a voltage column vector which contains as many elements as there are bundles in the line. Denoting this by E_B , it is possible to construct a connection matrix, D_{CB} , such that

$$E_C = D_{CB} E_B \quad \dots(4)$$

D_{CB} will be referred to as the 'voltage connection matrix'. It consists of as many rows as there are conductors in the line and as many columns as there are bundles. The elements of D_{CB} will be either 1 or 0, and with the system of numbering described above, it will consist (for symmetric lines) of unit matrices, a zero row vector and zero column vector(s).

Similarly, if the terms giving the total currents in the various bundles are assembled and represented by a column vector I_B , then we can write

$$I_B = F_{BC} I_C \quad \dots(5)$$

F_{BC} is the current connection matrix and, with the system of numbering adopted in Section 3, it will also consist, for symmetric lines, of unit matrices, zero row vector(s) and a zero column vector. In fact, it can be shown that

$$F_{BC} = D_{BC}^t \quad \dots(6)$$

where t indicates transposition. Equations (5) and (6) give

$$I_B = D_{BC}^t I_C \quad \dots(7)$$

From equations (3) and (7),

$$I_B = D_{BC}^t Z_{CC}^{-1} E_C \quad \dots(8)$$

Substituting for E_C from eqn.(4) into eqn.(8), we obtain

$$I_B = D_{BC}^t Z_{CC}^{-1} D_{CB} E_B \quad \dots(9)$$

Since I_B and E_B are bundle quantities, the matrix resulting from the multiplication $D_{BC}^t Z_{CC}^{-1} D_{CB}$ is the inverse of the line series-impedance matrix of the power phases and the earth wire. Therefore, we obtain

$$E_B = Z_{BB} I_B \quad \dots(10)$$

where

$$Z_{BB}^{-1} = D_{BC}^t Z_{CC}^{-1} D_{CB}$$

Z_{BB} now represents the series impedance matrix for the bundled system.

Having bundled the subconductors of each phase into one equivalent conductor, the order of Z_{BB} now equals to the number of phases plus one for the earth wire(s). Then, on the assumption that, throughout its length, the earth wire remains at zero potential, its presence can be eliminated from the matrix by the well known method of matrix elimination (see the Appendix), thus reducing the order of Z_{BB} to that of the number of phases.

Example 1:

Consider the line configuration of Fig.3. The conductors are numbered in the way described in Section 3. Since there are 13 conductors in the configuration, the order of Z_{CC} is (13x13), E_C is (13x1) and i_C is (13x1), where, from eqn.(3):

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \cdot \\ \cdot \\ \cdot \\ E_{13} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & Z_{1(13)} \\ Z_{21} & Z_{22} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & Z_{2(13)} \\ Z_{31} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{13(1)} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & Z_{13(13)} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \cdot \\ \cdot \\ \cdot \\ i_{13} \end{bmatrix}$$

The dots here represent elements. With reference to Fig.3, eqn.(4) gives

$$E_B = \begin{bmatrix} E_a \\ E_b \\ E_c \\ E_f \\ E_g \\ E_h \\ E_e \end{bmatrix} \quad \text{and} \quad D_{CB} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where E_e = voltage of the earth wire. Note the construction of matrix D_{CB} .

For phase a, the total current is

$$I_a = i_1 + i_7$$

Similarly, for the other phases

$$I_b = i_2 + i_8$$

$$I_c = i_3 + i_9$$

⋮

$$I_h = i_6 + i_{12}$$

and for the earth wire

$$I_e = i_{13}$$

Equation (5) yields

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_f \\ I_g \\ I_h \\ I_e \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ \vdots \\ \vdots \\ i_{13} \end{bmatrix}$$

which can also be obtained by applying equation (7).

On inverting Z_{CC} and applying eqn.(10), and then inverting the resulting matrix, the matrix Z_{BB} of the bundled system is obtained. The order of Z_{BB} is now (7x7); the elements of the first six rows and columns are those corresponding to the six phases of the bundled line and the remaining row and column correspond to the earth bundle (or wire). It remains now to eliminate these to obtain the final series impedance matrix of the line.

5. BUNDLING TECHNIQUE USING THE METHODS OF DIAKOPTICS:

In this method, bundling is performed in steps and not in one step, as in the previous method. In each step, two subconductors are taken and confined into a single equivalent conductor using the methods of diakoptics^{2,7}.

Having computed the conductor (or elementary) series-impedance matrix, the two conductors to be bundled are in effect short circuited by a zero impedance. This may be expressed in mathematical terms by establishing a new row and a new column which are added to the matrix. These new row and column are then eliminated from the matrix in the manner described in the Appendix for the earth wire. This results in a matrix which contains a redundant row and column which are discarded.

The process just described forms one step. This process is repeated, for other subconductors, until bundling is completed. The matrix thus obtained contains three times as many rows and columns as there are power circuits in the line, plus one row and column for the earth bundle. The final matrix for the bundled system is obtained when these are eliminated.

The elements of the new row and column, which are established at every bundling step, are obtained from the elements of the rows and columns corresponding to the conductors to be bundled. Each off diagonal element of this new row is the numerical difference between the elements of the rows corresponding to the conductors to be bundled. The new diagonal element is the difference between the elements of the new row, corresponding to these two conductors to be bundled. Since the original matrix is symmetric, the new row and column are also symmetric. The method is more clarified by the following example.

Example 2:

Consider the elementary series-impedance matrix

$$Z_{CC} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \end{matrix} \quad \dots(11)$$

where Z_{mp} and Z_{mm} represent mutual and self impedances, respectively, of the individual conductors. It is now required to combine conductors 1 and 3 which are assumed to be bundled. The matrix is symmetrical.

Forming the new row and column as described above, and adding them to the matrix (11), we obtain

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{11} - Z_{13} \\ Z_{21} & Z_{22} & Z_{23} & Z_{21} - Z_{23} \\ Z_{31} & Z_{32} & Z_{33} & Z_{31} - Z_{33} \\ Z_{11} - Z_{31} & Z_{12} - Z_{32} & Z_{13} - Z_{33} & Z_{11} + Z_{33} - 2Z_{13} \end{bmatrix} \end{matrix}$$

Eliminating the new row and column and noting that the third row and third column are those to be discarded, we obtain

$$Z_{BB} = \begin{bmatrix} Z_{11} - \frac{(Z_{11} - Z_{13})^2}{Z_{11} + Z_{33} - 2Z_{13}} & Z_{12} - \frac{(Z_{11} - Z_{13})(Z_{12} - Z_{13})}{Z_{11} + Z_{33} - 2Z_{13}} \\ Z_{21} - \frac{(Z_{21} - Z_{23})(Z_{11} - Z_{31})}{Z_{11} + Z_{33} - 2Z_{13}} & Z_{22} - \frac{(Z_{21} - Z_{23})^2}{Z_{11} + Z_{33} - 2Z_{13}} \end{bmatrix}$$

In the two methods of bundling discussed up to now, it is obvious that same results are obtained if the process of eliminating the earth wire(s) proceeds that of bundling the phase subconductors.

6. BUNDLING USING THE CONCEPT OF GEOMETRIC-MEAN DISTANCE:

Using this concept, the bundling is effected directly while the self and mutual terms of the line parameters are being calculated from the line geometry. Referring to Fig.4, and assuming equal currents in the subconductors and an infinitely conducting earth, the effective mutual impedance between n subconductors and any other remote conductor p , is from eqn.(1)

$$\frac{\alpha^2}{2\pi} \sum_{i=1}^n \frac{1}{n} \ln \left(\frac{d'_{ip}}{d_{ip}} \right) = \frac{\alpha^2}{2\pi} \ln \left[\frac{\prod_{i=1}^n (d'_{ip})}{\prod_{i=1}^n (d_{ip})} \right]^{1/n}$$

i.e. the impedance is expressed in terms of the geometric-mean distances

$$\left[\prod_{i=1}^n (d'_{ip}) \right]^{1/n} \quad \text{and} \quad \left[\prod_{i=1}^n (d_{ip}) \right]^{1/n}, \quad \text{as is the usual practice.}$$

With finite earth conductivity, however, and taking into consideration the assumption that $0 \leq \delta \leq \eta$, mentioned in Section 2, the integral term reduces to

$$\frac{\alpha^2}{2\pi} \int_0^{\eta} \left[\frac{\mu \cos(\delta \bar{x}) \exp(-\delta y_p)}{\mu \delta + (\delta^2 + \beta^2)^{\frac{1}{2}}} \frac{1}{n} \sum_{i=1}^n \exp(-\delta y_i) \right] d\delta$$

where \bar{x} is the average horizontal distance between the n subconductors and the conductor p . Taking y to be the arithmetic mean = the geometric mean vertical distance of the subconductors within one phase and earth, for

$1 \leq i \leq n$, the above integral becomes

$$\frac{\alpha^2}{\pi} \int_0^{\eta} \frac{\mu \cos(\delta \bar{x}) \exp[-\delta(y+y_p)]}{\mu \delta + (\delta^2 + \beta^2)^{\frac{1}{2}}} d\delta$$

For the effective self-impedance of a phase having n subconductors, the external part is, from eqn.(2) and Fig.4,

$$\frac{\alpha^2}{2\pi} \ln \left[\frac{\prod_{i=1}^n (d'_{ii})}{\prod_{i=1}^n (d_{ii})} \right]^{1/n} + \frac{\alpha^2}{\pi} \int_0^{\eta} \frac{\mu \exp(-2\delta y)}{\mu \delta + (\delta^2 + \beta^2)^{\frac{1}{2}}} d\delta$$

As already pointed out, it can be seen that the simplicity of eqns. (1) and (2) is retained when using the concept of the geometric-mean distance.

Again, it now remains to eliminate the presence of the earth wire(s) from the impedance matrix in the manner described in the Appendix.

7. COMPARISON:

Assuming same equations (e.g. (1)&(2) together with Ref.4) are used to compute the conductor self and mutual parameters, the results obtained using the geometric-mean distance would be comparable to those obtained by the matrix-bundling techniques of Sections 4 and 5 of this paper. This is obvious since the accuracy of the results will depend almost entirely, on the equations³⁻⁶ used to determine the conductor self and mutual parameters.

While the bundling methods using matrix techniques require the pre-determination of the various conductor parameters, that employing the concept of geometric-mean distance enables us to obtain the phase quantities directly from the line geometry.

Regarding the computation time required to complete the bundling of multi-subconductor transmission lines, the method based on linear transformation would take longer time than that based on diakoptics, since the former includes matrix inversions. That based on direct evaluation of the geometric-mean distance appears to reduce the overall time required to obtain the parameters in terms of phase quantities. However, it is obvious that programming is relatively easier for the first method, that based on linear transformation, than for the other methods.

8. CONCLUSIONS:

Modeling multi-subconductor transmission lines for the purpose of phase analysis of power systems is presented. Three methods of bundling subconductors are given.

The first is based on linear transformations of the elementary conductor parameter matrices. It requires at least two matrix inversion operations. The order of the matrices to be inverted depends on the number of conductors and on the number of power phases being involved.

The second method is based on the methods of diakoptics and has the feature of bundling only two conductors at a time. This is computationally more desirable if it is skilfully programmed.

The third bundling technique, that based on the concept of geometric mean distance, does not require the predetermination of the elementary conductor-parameter matrices. It seems that this method saves more time. However, more detailed conclusions will be reported.

9. REFERENCES:

1. G.W. Stagg and A.H. El-Abiad, "Computer methods in power system analysis", Book, McGraw-Hill Company, 1968.
2. A. Brameller, M.N. John and M.R. Scott, "Practical diakoptics for electrical engineers", Book, Chapman & Hall, 1969.
3. R.H. Galloway, W.B. Shorrocks and L.M. Wedepohl, "Calculation of electrical parameters for short and long polyphase transmission lines", Proc. IEE, Vol.111, No.12, Dec. 1964.

4. N. Mullineux and J.R. Reed, "Calculation of electrical parameters for short and long polyphase transmission lines", Proc. IEE, Vol. 112, No. 4, 1965.
5. Khuong D. Tran and Jean Robert, "New study of parameters of EHV multi-conductor transmission lines with earth return", Trans. IEEE, PAS, May 1971, pp 452-458.
6. M.J. Battison, S.J. Day, N. Mullineux, K. Parton and J. Reed, "Some effects of the frequency dependance of transmission-line parameters", Proc. IEE, Vol. 116, No. 7, July 1969.
7. J.P. Bickford, "Investigation into the use of resistors for the reduction of switching overvoltages with particular reference to line energisation", Ph.D. Thesis C.N.A.A., U.K., 1968.

10. APPENDIX:

Having obtained the impedance matrix Z_{BP} for the bundled system, the steady-state equation of the line can be partitioned so that the current and voltage variables of power bundles are separated from those of the earth bundle (or wire). Denoting the former by the subscript p and the latter by the subscript e, we obtain, from eqn.(10)

$$\begin{bmatrix} E_p \\ \text{---} \\ E_e \end{bmatrix} = \begin{bmatrix} Z_{pp} & | & Z_{pe} \\ \text{---} & | & \text{---} \\ Z_{ep} & | & Z_{ee} \end{bmatrix} \begin{bmatrix} I_p \\ \text{---} \\ I_e \end{bmatrix}$$

where Z_{pp} and Z_{ee} are square matrices;
 Z_{pe} is a column vector; and
 Z_{ep} is a row vector.

Since the potential of the earth wire(s) is assumed to be zero throughout its length, then

$$E_e = 0 = Z_{ep} I_p + Z_{ee} I_e$$

from which

$$I_e = -Z_{ee}^{-1} Z_{ep} I_p$$

so that

$$\begin{aligned} E_p &= Z_{pp} I_p + Z_{pe} I_e \\ &= (Z_{pp} - Z_{pe} Z_{ee}^{-1} Z_{ep}) I_p \\ &= Z'_{pp} I_p \end{aligned}$$

where Z'_{pp} is the reduced impedance matrix of the line and contains three times as many rows and columns as there are power circuits in the line.

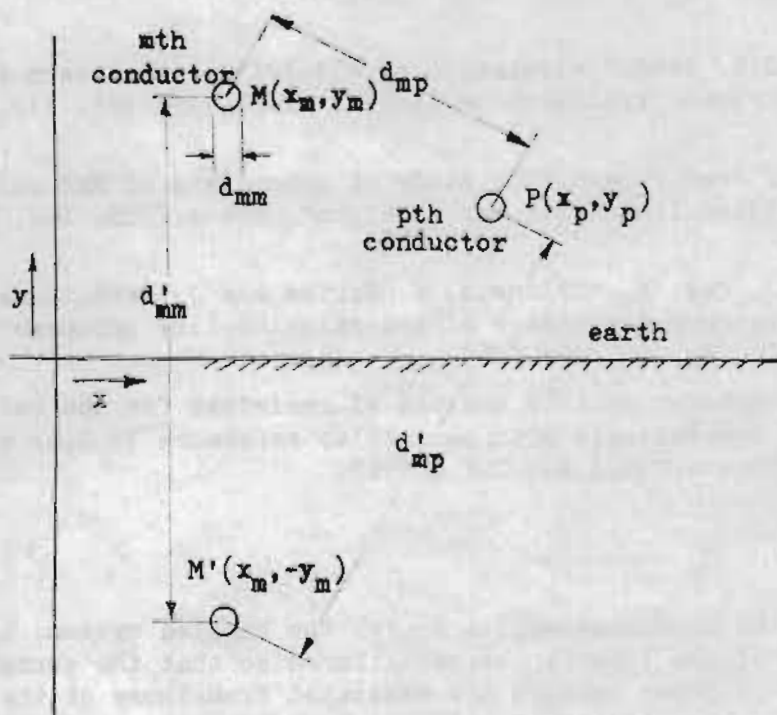


Fig. 1. Schematic of conductor co-ordinates

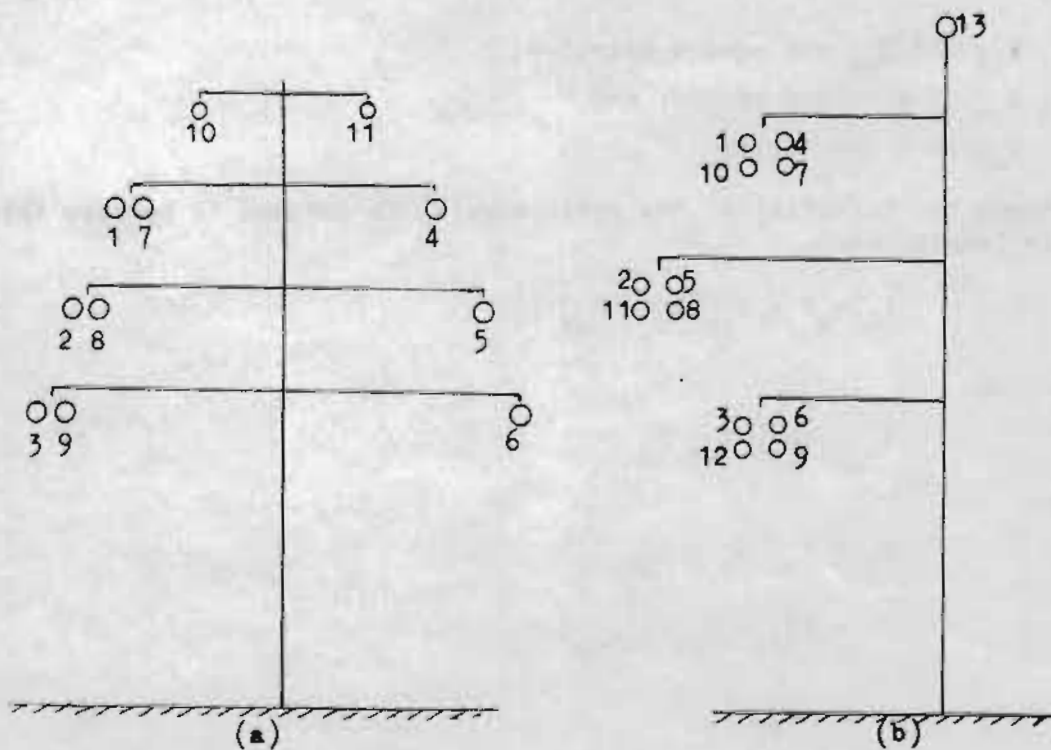


Fig. 2. Examples for illustrating the numbering of conductors

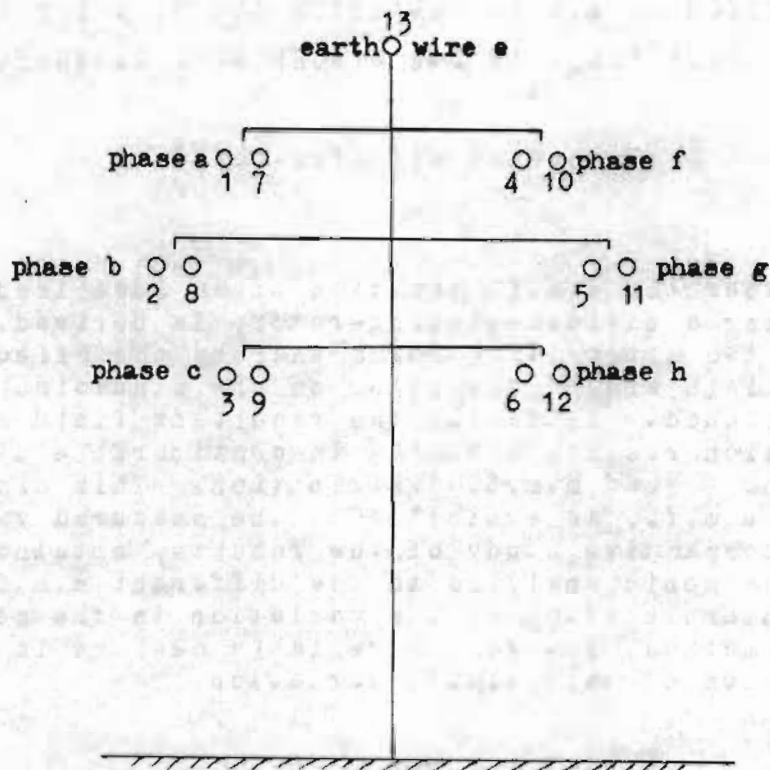


Fig. 3. Conductor numbering of double-circuit line with duplex conductors for each phase and one earth wire (Example 1).

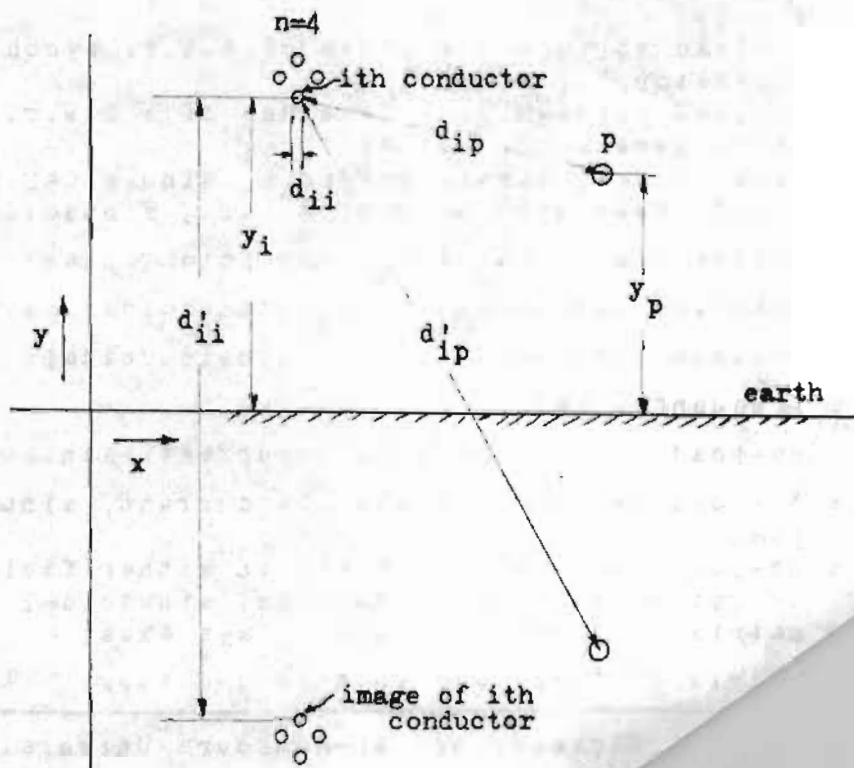


Fig. 4. Illustrating the concept of