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Behaviour of Rotating Field in Slip-Ring Induction Motors with Chopper Control.

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BEHAVIOUR OF ROTATING FIELD IN SLIP-RING INDUCTION MOTORS WITH CHOPPER CONTROL

BY

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ABSTRACT:

Stator currents of slip-ring induction motor having chopper control in the rotor circuit, are no more sinusoidal. The analysis of the air-gap resultant field, due to such currents, results in a different expression for the established rotating field. The derived expression is examined in order to study the rotating field behaviour under chopper control. In addition to some of the well known behaviours, a new one had been distinguished and proved by constructing the geometrical pattern of the air-gap resultant field. An effective winding factor is introduced to give the effect of nonsinusoidal stator currents on the magnitude of the resultant m.m.f. harmonics.

0. Nomenclature:

'a' := reference phase;

F_a := momentary maximum m.m.f. per phase, sinusoidal current;

F_m := absolute maximum m.m.f. per phase, sinusoidal current;

$F_R(x)$:= resultant m.m.f. of the three phases, sinusoidal currents;

F_{Rn} := peak of the n th harmonic of the resultant field;

f := supply frequency;

f_n := frequency of the n th harmonic of the resultant m.m.f. ;

i_a := instantaneous stator current per phase;

I_a := stator current per phase without ...per, rms sinusoidal value;

...mum of rectangular stator current per phase;

... th harmonic winding factor;

... th harmonic effective winding factor ...hopper effect;

... order;

...ous speed corresponding to ...equency;

..., is a lecturer at the Faculty of ... University, El-Mansoura, Egypt

T_{ph} := number of turns per phase;
$2p$:= number of poles;
α := phase spread;
ω := angular speed = $2\pi f$ electrical radians/sec. ;
F'_a , F'_m , $F'_R(x)$ and F'_{Rn} have the same definitions as above but under chopper effect.

1. Introduction:

It has recently shown [1] that a reliable speed control can be attained by chopper action in the rotor circuit of a 3-phase, slip-ring induction motor. Usually the 3-phase rectifier which is connected to the slip rings, supplies the rotor current in d.c. form to the chopper circuit. This circuit consists mainly of an electronic switch connected across a control resistance . The chopper action results in a contactless adjustement of the control resistance and in turn the motor speed can be controlled.

Neglecting the magnetising current and leakage reactances, and assuming an ideal rectification the alternating stator current per phase follows the rectifier action in the corresponding phase. Figure 1 shows the instantaneous variation of the stator currents regardless of the number of pulses of the rectifier. It is seen that the phase current is a square wave and appears only through the angular interval $2\pi/3$ of the sinusoidal half-wave duration. For any instant, it is noticed that only two phases complete the current circulation while the third one is shut off according to the rectifier action.

Accordingly, the nature of the phase m.m.f. and the resultant m.m.f. due to the three phases may be affected. The following study aims to clear up this effect.

2. Theoretical Analysis:

In a three phase slip-ring induction motor under symmetrical conditions the m.m.f. established by a full-pitch distributed phase-winding is trapezoidal. Due to the sinusoidal stator current, this m.m.f. has a gradually pulsating nature and can be represented by its Fourier Series as:

$$F_a(x) = (4/\pi) F_a \sum_{n=1}^{n=\infty} (1/n) K_{wn} \sin(nx) \quad . . (1)$$

where

$$K_{wn} = \sin(n\alpha/2) / (n\alpha/2) \quad . . . . (2)$$

and

$$F_a = \pm (i_a T_{ph} / 2p) \quad \text{AT/pole}$$

Equation (1) gives the spatial distribution of the phase m.m.f. at a fixed point x in the air-gap according to the instantaneous value of the corresponding current which is;

$$i_a(t) = \sqrt{2}\ I_a \sin(\omega t) \quad . \quad . . . . (3)$$

Consequently, the resultant m.m.f. in the stator space due to the three phase m.m.fs. can be found to have the form

$$F_R(x) = \frac{3}{2} \cdot \frac{4}{\pi} F_m \left[\frac{K_{w1}}{1} \sin(x - \omega t) + \frac{K_{w5}}{5} \sin(5x + \omega t) + \frac{K_{w7}}{7} \sin(7x - \omega t) + \frac{K_{w11}}{11} \sin(11x + \omega t) + \ldots \right] \quad . . . . (4)$$

where

$$F_m = \pm (\sqrt{2}\ I_a T_{ph} / 2p) \qquad \text{AT/pole/phase} \quad (5)$$

In this case the maximum fundamental of this rotating field is 3/2 of the maximum fundamental m.m.f. per phase.

In case of slip-ring induction motors with rotor chopper the stator current per phase is not sinusoidal. It has a rectangular form for a period of $2\pi/3$ and becomes zero for a period $\pi/3$ before it reverses its direction. Accordingly, the phase m.m.f. loses its gradually pulsating nature to pulsate in a step-wise manner following, thereby, the phase current variation. This current can be represented as:

$$i'_a(t) = \sum_{n=1}^{n=\infty} K_n I \sin(n\omega t) \quad . . . . (6)$$

where

$$K_n = (4 / n\pi) \cos(n\pi/6) \quad . . . . (7)$$

This factor K_n relates the n th current harmonic peak to the maximum of the rectangular current. Accordingly, the momentary maximum m.m.f. per phase is

$$F'_a = i'_a(t)\ T_{ph} / 2p = F'_m \sum_{n=1}^{n=\infty} K_n \sin(n\omega t) \quad . . (8)$$

where

$$F'_m = \pm (I\ T_{ph} / 2p) \qquad \text{AT/pole/phase} \ . . . . . (9)$$

Therefore, the spatial distribution of phase m.m.f. under chopper action; for phase 'a'

$$F'_a(x) = \frac{4}{\pi} F'_m \sum_{n=1}^{n=\infty} [K_n \sin(n\omega t)][(1/n) K_{wn} \sin(nx)]$$

and for the other two phases

$$F'_b(x) = \frac{4}{\pi} F'_m \sum_{n=1}^{n=\infty} [K_n \sin n(\omega t - 120)][(1/n) K_{wn} \sin n(x - 120)]$$

$$F'_c(x) = \frac{4}{\pi} F'_m \sum_{n=1}^{n=\infty} [K_n \sin n(\omega t + 120)][(1/n) K_{wn} \sin n(x + 120)]$$

The resultant m.m.f. at a given point x in the air-gap will be

$$F'_R(x) = F'_a(x) + F'_b(x) + F'_c(x)$$

in which the n th harmonic term is

$$F'_{Rn}(x) = \frac{4}{n\pi} F'_m K_n K_{wn} \cdot \frac{1}{2} \Big[3 \cos n(x - \omega t) - (\cos n(x + \omega t) + \cos n(x + \omega t - 120) + \cos n(x + \omega t + 120)) \Big]$$

In this relation the negative term is equal to zero, consequently

$$F'_R(x) = \sum_{n=1}^{n=\infty} \frac{3}{2} \cdot \frac{4}{n\pi} K_n K_{wn} F'_m \cos n(x - \omega t) \quad . . . . (10)$$

This equation gives the spatial distribution of the resultant air-gap m.m.f. in an induction motor having chopper control in the rotor circuit. The fundamental peak is shown to be

$$F'_{R1} = \frac{3}{2} \cdot \frac{4}{\pi} \cdot K_1 K_{w1} F'_m \quad . . . . . . (11)$$

The factor $K_1 = (4/\pi)\cos(\pi/6)$, relates the maximum fundamental current to the maximum rectangular current per phase.

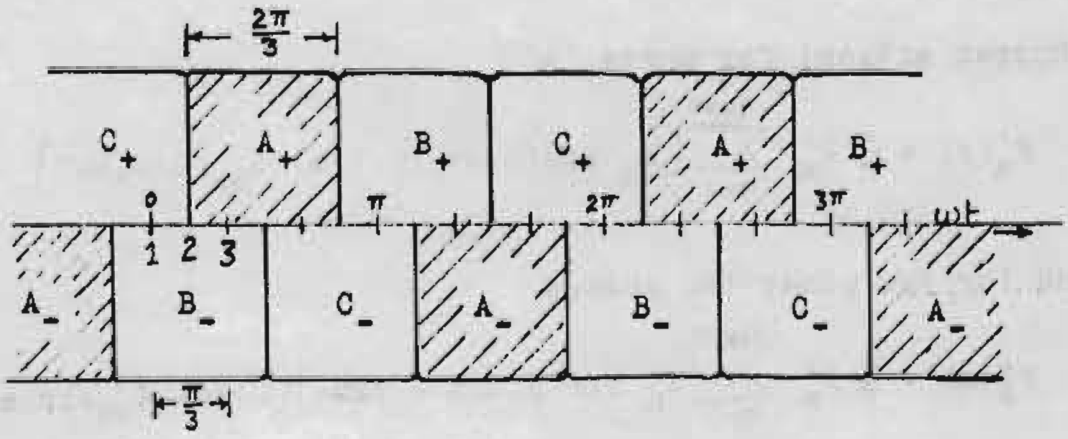
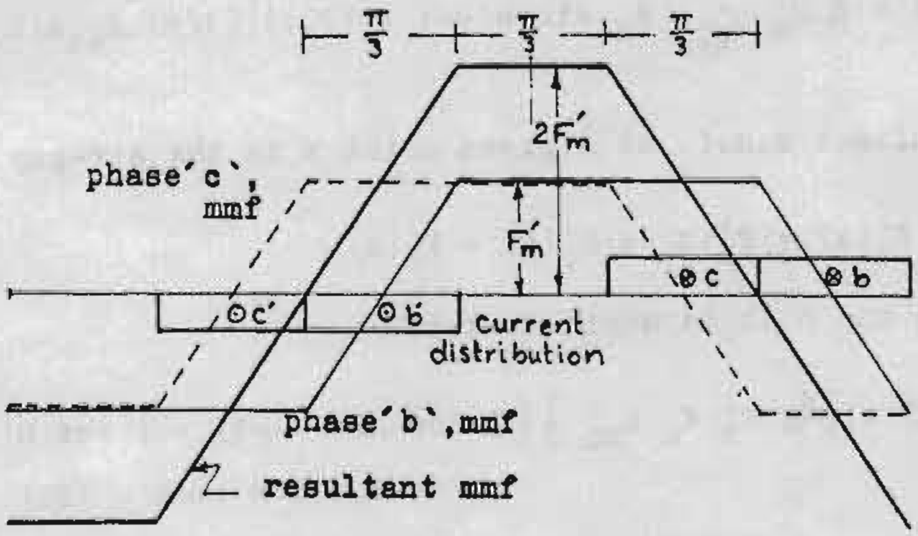
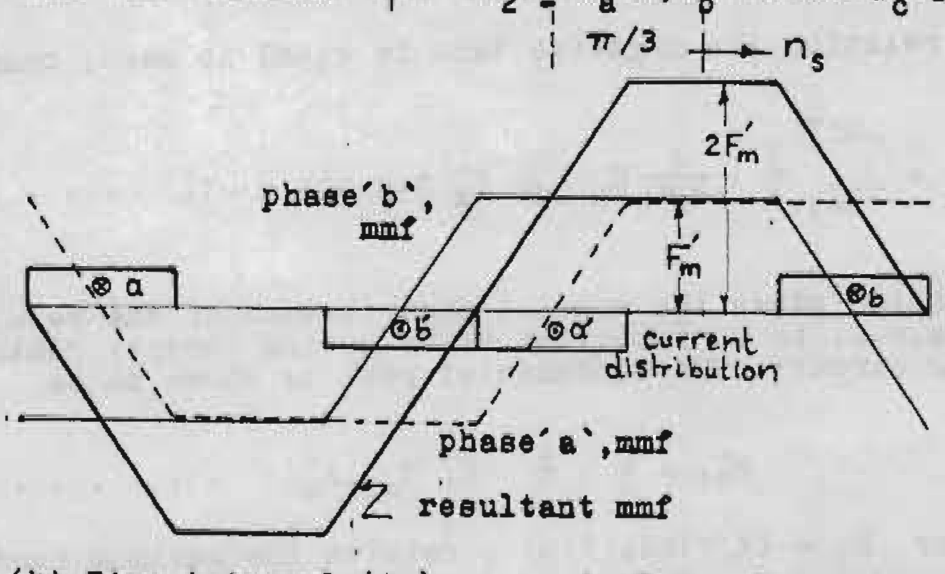


Fig. 1 : Stator currents.



(a) Time intrval t_1 - $(t_2)_-$: i_a=0, i_b=-I and i_c=I



(b) Time interval $(t_2)_+$ - t_3 : i_a=I, i_b=-I and i_c=0

Fig. 2 : Geometrical pattern of the resultant field.

Thus, the maximum fundamental m.m.f. per phase is

$$F'_{a1} = (4/\pi) K_1 F'_m \quad \ldots\ldots\ldots (12)$$

and the maximum fundamental of the resultant field is 3/2 of F'_{a1} . The effect of a distributed winding on the produced m.m.f. is still maintained through K_{w1} .

3. Resultant Field Geometrical Pattern:

The geometrical pattern of the resultant rotating field, given by eq.(10), can be obtained for a given moment with the help of the individual m.m.f. distribution of the three phases according to the instantaneous currents,Fig.1 . The resultant waveforms at two different instants are given in Fig.2 . The phase winding is assumed to be 2-pole full pitch winding with 60° phase spread.

To follow the resultant field rotation the time interval within the time instants t_1 and $(t_2)_-$ may be considered. During this interval the instantaneous currents ($i_c = I$, $i_b = -I$, while $i_a = 0$) remain constant. The current sheets and the corresponding phase m.m.f. distributions will also remain fixed in position which result in a standing resultant m.m.f. wave during this interval. The resultant m.m.f. is trapezoidal in form and has a maximum value of $2F'_m$. While the m.m.f. distribution of phase 'c' lags that of phase 'b' by a phase spread α , the resultant m.m.f. distribution has its position mid way between them.

At the instant $(t_2)_+$, i_c is replaced by i_a at the same magnitude and direction to have the current sheet of phase 'a' instead of that of phase 'c'. It follows that the m.m.f. distribution of phase 'a' will lead that of phase 'b' by the phase spread α. Accordingly, the resultant field is obliged to travel the same angular distance α in a step-wise manner to have a position mid way between the available phase m.m.fs.

This new position corresponds to the time t_3 and will be held fixed until the following negative current replacement is attained. Thus, the successive positive and negative current replacements will cause the resultant m.m.f. to advance each time an angular distance equal to a phase spread α .

Accordingly, the geometrical pattern of the resultant field, Fig.2 , will rotate an angular distance of 2π through one cycle of the current. It means that the resultant field will rotate by the synchronous speed around the air-gap but in a step-wise manner.

4. Behaviours of Resultant Field under Chopper Control:

The resultant field of a slip-ring induction motor with rotor chopper control is characterised by eq.(10). An examination of the physical interpretation of this equation shows :

(a) The well known relation between the n th harmonic peak of the resultant field and the corresponding harmonic peak of the phase m.m.f. is still valid, namely

$$F'_{Rn} = (3/2)(4/n\pi)K_n \; F'_m \; K_{wn} \quad \ldots\ldots (13)$$

It is obvious that the n th harmonic peak of the resultant field is 3/2 of the corresponding harmonic peak of the phase m.m.f. and remains constant during rotation.

(b) The term cos $n(x - \omega t)$ in eq.(10) indicates that the n th harmonic is a travelling wave having at any angular position x a magnitude which varies sinusoidally at a frequency $f_n = nf$. The effect of this variation over the whole stator periphery is to cause the corresponding resultant m.m.f. harmonic to travel at the synchronous speed. This means that all harmonics will rotate with the same speed in the same direction. This new character offers a great advantage to the motor performance; the torques produced by the individual harmonics are all in the same direction. Negative torques will not exsist.

(c) The factor K_n gives the n th harmonic peak of the stator current in terms of the maximum value of the rectangular current, eq.(7) . If a slip-ring induction motor under and without chopper control has the same maximum stator current, i.e. $I = \sqrt{2}\, I_a$, it is seen from Table (1) that K_1 forces the fundamental peak of current , or m.m.f. , in the first case by 10 % due to chopper.

(d) An effective winding factor K'_{wn} may be introduced by combining K_n to the winding factor K_{wn} to give

$$K'_{wn} = K_n \cdot K_{wn} = \frac{4}{n\pi} \cdot \frac{\sin(n\alpha)}{(n\alpha)} \quad , \ldots . (14)$$

Accordingly, the resultant field expression becomes:

$$F'_R(x) = \sum_{n=1}^{n=\infty} \frac{3}{2} \cdot \frac{4}{n\pi} K'_{wn} \; F'_m \cos n(x - \;\; t)$$

$$\ldots\ldots (15)$$

Table (1)

Harmonic order, n	K_n	K_{wn}	K'_{wn}
1	1.103	0.955	1.053
3	0.000	0.637	0.000
5	-0.221	0.191	-0.042
7	-0.158	-0.136	0.021
9	0.000	-0.212	0.000
11	0.100	-0.087	-0.009
13	0.085	0.073	0.006
15	0.000	0.127	0.000

It can be proved that K'_{wn} is the harmonic peak ratio of the rotating field waveform which maintains its trapeziodal shape during rotation. Accordingly, it relates the n th harmonic peak of the resultant m.m.f. distribution to the actual maximum. The effective winding factor gives, in addition to the effect of a distributed phase winding on the resultant m.m.f., the effect of nonsinusoidal stator currents under chopper control.

The existance of a given harmonic in the resultant m.m.f. is expected if the value of K'_{wn} differs from zero. It can be seen from table (1) that all triple harmonics do not exist; a result which is also well known about normal rotating field established by sinusoidal currents. On the other hand all higher harmonics are reduced by this factor to about 8.5% - 22 % of their normal values.

5. Conclusion:

The analysis of the resultant field in the air-gap of a slip-ring induction motor under chopper control results in a different expression of the established rotating field. The examination of this expression shows that some of the well known behaviours of the rotating field due to sinusoidal stator currents still valid.

The derived expression reveals that all harmonics rotate in the same direction with the same synchronous speed. Consequently, the spatial distribution of the resultant m.m.f. maintains its shape and absolute maximum during its rotation at the synchronous speed. Due to this character negative torques are not expected in such slip-ring induction motor.

The graphically obtained geometrical pattern of the rotating field shows that its trapezoidal waveform or maximum value does not change. It is found that it travels around the air-gap in a step-wise manner with the synchronous speed.

The derived expression introduces an effective winding factor , K'_{wn} , which includes the effect of the nonsinusoidal stator currents. This factor is essentialy the harmonic peak ratio of the resultant m.m.f. waveform. Numerical values of K'_{wn} result in a remarkable increase in the magnitude of the fundamental m.m.f. and in a great reduction in the magnitudes of the corresponding higher harmonics. The n th harmonic in the resultant field exists only if the value of this factor differs from zero .

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