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Wave Propagation on High Voltage Power Lines.

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WAVE PROPAGATION ON HIGH VOLTAGE POWER LINES

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ABSTRACT:

Travelling waves in power systems is of importance in solving problems relating to power-line carrier communication, protection of very long lines, energisation of unloaded lines and calculation of recovery voltages on circuit breakers under fault conditions. This paper presents comprehensive analyses for wave propagation on single-phase and on three-phase overhead power transmission lines. Using modal quantities, the wave equation are solved. Natural modes are illustrated to clarify the propagation of carrier signals on single circuit lines as well as on double circuit lines. Different transmission line configurations are considered. The effect of power line transposition on carrier modes is also included.

1.   INTRODUCTION:

Electrical energy is trnsmitted by O.H.T.L. from one point to another in the form of electromagnetic wave propagation[1,2]. Any transmission line can be thought of as a waveguide where the field is external to the guiding metal, i.e. the conductors. In the case of energy propagation, this field corresponds to the induction field. The amount of the power transmitted and the particular mode of propagation depends on these fields[3]. For this, the effects of the electric and magnetic fields are considered here in terms of the primary line parameters[4], e.g. inductance, resistance, capacitance and conductance per unit length of the line (R,L,C and G). But if the conductors of the T.L. are employed as conventional power line carrier[5,6,7], the fields surrounding these conductors can be divided into two components, namely; the induction field and the radiation field[3].

The most convenient method presently used for solving propagation performance and the problem of power-line carrier (PLC) on T.L. is based on the concept of natural modes[8,9,10]. Thus, by understanding the propagation modes it is possible to select frequencies and coupling phases for PLC channels on long transmission lines[6,8,11].

2. WAVE PROPAGATION ON TRANSMISSION LINES:

2.1 Propagation on single-phase two-wire lines:

Considering an infinitely small length dx of a two-wire line, the voltage and current at the input and output of the section are as shown in Fig.1. Suppose that R,L,C and G represent the primary parameters per unit length, thus

$$V-(V+dV) = R \, dx \, I + j\omega L \, dx \, I$$

or

$$-\frac{dV}{dx} = (R + j\omega L) \, I$$

and

$$I-(I+dI) = G \, dx \, V + j\omega C \, dx \, V$$

or

$$-\frac{dI}{dx} = (G + j\omega C) \, V$$

......(1)

By differentiating these equations with respect to x, then

$$\frac{d^2 V}{dx^2} = (R + j\omega L)(G + j\omega C) \, V = \gamma_e^2 \, V$$

and

$$\frac{d^2 I}{dx^2} = (G + j\omega C)(R + j\omega L) \, I = \gamma_i^2 \, I$$

......(2)

where ;

γ is known as the propagation constant;

e and i subscripts denote voltage and current respectively.

Equations (2) have solutions of the form

$$V = A \exp(-\gamma_e x) + B \exp(\gamma_e x)$$

and

$$I = C \exp(-\gamma_i x) + D \exp(\gamma_i x)$$

......(3)

where A,B,C and D are constants whose values depend on the conditions existing at the ends of the line. Thus, if x = 0 then

$$V = V_s = A + B$$

where V_s is the sending end voltage.

Assuming an infinitely long line, then

$$V = 0 \text{ as } x \longrightarrow \infty \quad \therefore \quad B \longrightarrow 0 \quad \text{and} \quad V_s = A$$

Therefore, for an infinitely long line, the voltage and current at any point are

$$V = V_s \exp(-\gamma_e x)$$

and

$$I = C \exp(-\gamma_i x)$$

Considering this case, i.e. infinite line length, then

$$- \frac{dV}{dx} = (R + j\omega L) I = \gamma_e V_s \exp(-\gamma_e x)$$

$$\therefore \qquad I = \gamma_e V_s \exp(-\gamma_e x)/(R + j\omega L)$$

$$= V_s \exp(-\gamma_e x)/(\sqrt{\frac{R + j\omega L}{G + j\omega C}}) \qquad \text{(from eqn.2)}$$

$$= V_s \exp(-\gamma_e x)/ Z_c$$

$$= I_s \exp(-\gamma_e x) \qquad \qquad \qquad \dots\dots(4)$$

where:
$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \text{ is known as the characteristic impedance of the line;}$$

$C = I_s = V_s/Z_c$; and $\gamma_i = \gamma_e$.

For loss-free lines or with high frequency on transmission line, we have $\omega L \gg R$ and $\omega C \gg G$. Therefore;

$$Z_c = \sqrt{\frac{L}{C}} \left[1 - j(\frac{R}{2\omega L} - \frac{G}{2\omega C})\right] \approx \sqrt{\frac{L}{C}} \qquad \dots\dots(5)$$

Since we are dealing, at the moment, with a single-phase (infinitely long) line, the propagation constants γ_e and γ_i are the same, γ , say. Thus, from equation (2) the general expression for the propagation constant   becomes

$$\gamma = \frac{R}{2} \sqrt{\frac{C}{L}} + j\omega \sqrt{LC} = \alpha + j\beta \qquad \dots\dots(6)$$

where:
α is the attenuation constant $= \frac{1}{2}R\sqrt{\frac{C}{L}} = \frac{1}{2}\frac{R}{Z_c}$ Np/unit length

β is the phase-shift constant $= \omega\sqrt{LC}$ rad/unit length

γ , α and β , together with the characteristic impedance Z_c, are known as the secondary coefficients of the line. Thus, if the values of the primary coefficients are known at a particular frequency, the secondary coefficients for that frequency can be calculated.

2.2 Wave propagation on multi-conductor T.L.:

The differential equations describing electromagnetic wave propagation on a multi-conductor transmission system are[4]:

$$\left.\begin{array}{l} - \frac{\delta [v]}{\delta x} = [L] \frac{\delta [i]}{\delta t} + [R][i] \\[2mm] - \frac{\delta [i]}{\delta x} = [C] \frac{\delta [v]}{\delta t} + [G][v] \end{array}\right] \qquad \dots\dots(7)$$

where x is the distance along the line, measured from the sending end; [i] and [v] are column matrices of line currents and voltages; and [L] , [R] , [C] and [G] are square matrices.

The Fourier transform converts the partial differential equations (7) into ordinary differential equations, which are functions of frequency. Then, equations (7) becomes:

$$-\frac{d[V]}{dx} = [R(\omega)][I] + j\omega[L(\omega)][I] = [Z(\omega)][I]$$

and

$$-\frac{d[I]}{dx} = [G(\omega)][V] + j\omega[C(\omega)][V] = [Y(\omega)][V]$$

$$\left.\right\}\ \ldots\ldots(8)$$

where [V] and [I] are the Fourier transform of [v] and [i] respectively. The second derivative w.r.t. x, of [V] and [I] gives,

$$\frac{d^2[V]}{dx^2} = [Z(\omega)][Y(\omega)][V]$$

and

$$\frac{d^2[I]}{dx^2} = [Y(\omega)][Z(\omega)][I]$$

$$\left.\right\}\ \ldots\ldots(9)$$

For an infinitely long line, equations (9) are solved in the form

$$[V] = [A]\ \exp(-[\gamma_e]\ x)$$

and

$$[I] = [B]\ \exp(-[\gamma_i]\ x)$$

$$\left.\right\}\ \ldots\ldots(10)$$

where $[\gamma_e]$ and $[\gamma_i]$ are square matrices and [A] and [B] are column vectors of constants to be determined by the boundary conditions.

Expansion of $\exp(-[\gamma_e]\ x)$ using Cayley-Hamilton theorem and then differentiating the resulting expansion, the following result is obtained.

$$\frac{d\ \exp(-[\gamma_e]\ x)}{dx} = -[\gamma_e]\ \exp(-[\gamma_e]\ x)$$

Similarly

$$\frac{d\ \exp(-[\gamma_i]\ x)}{dx} = -[\gamma_i]\ \exp(-[\gamma_i]\ x)$$

Differentiating eqn.(10) twice, we get;

$$\frac{d^2[V]}{dx^2} = [\gamma_e]^2\ [V]$$

and

$$\frac{d^2[I]}{dx^2} = [\gamma_i]^2\ [I]$$

$$\left.\right\}\ \ldots\ldots(11)$$

Substituting from eqn.(11) into eqn.(9) produces

$$[\gamma_e]^2 = [Z(\omega)] \, [Y(\omega)]$$

$$[\gamma_i]^2 = [Y(\omega)] \, [Z(\omega)]$$

$$\ldots\ldots(12)$$

Therefore, current and voltage propagation constants $[\gamma_e]$ and $[\gamma_i]$, are defined in terms of line parameters. Also, the first derivative of $[I]$ in eqn.(10) gives,

$$\frac{d[I]}{dx} = -[\gamma_i] \, \exp(-[\gamma_i]x) \, [B] = -[\gamma_i] \, [I]$$

Equating this equation to the second equation of (8) gives;

$$-[\gamma_i] \, [I] = [Y(\omega)] \, [V]$$

or,

$$[V] = [Y(\omega)]^{-1} \, [\gamma_i] \, [I]$$

Since at any point along the line the voltage and current must be related by the characteristic impedance of the line, then

$$[V] = [Z_c] \, [I]$$

giving

$$[Z_c] = [Y(\omega)]^{-1} [\gamma_i]$$

$$= [Y(\omega)]^{-1} \left\{ [Y(\omega)] \, [Z(\omega)] \right\}^{\frac{1}{2}}$$

$$\ldots\ldots(13)$$

At the sending end of the infinitely-long line, $[V_s] = [Z_c] \, [I_s]$ and $[A] = [V_s]$ and $[B] = [I_s]$

$\therefore$

$$[A] = [Z_c] \, [B] \quad \text{or} \quad [B] = [Z_c]^{-1} \, [A]$$

Therefore, the voltage and current at any point along the infinitely long line may be expressed in terms of the characteristic impedance and their values at the sending end. Thus

$$[V] = [V_s] \, \exp(-[\gamma_e]x)$$

$$[I] = [Z_c]^{-1} \, [V_s] \, \exp(-[\gamma_i]x) = [I_s] \, \exp(-[\gamma_i]x)$$

$$\ldots\ldots(14)$$

Equations (14) are the solution of equations (9).

As shown before, the difference in the propagation constants for current and voltage is determined by the order of multiplication of Z and Y matrecis in equation (12).

If the line is considered as transposed 3-phase line, the self impedance and admittance values are the same for each phase. The mutuals between the phases will also be the same for each phase. This means that both impedance

and admittance of the line are symmetric. Hence

$$[Z(\omega)] = [Z(\omega)]_t \quad \text{and} \quad [Y(\omega)] = [Y(\omega)]_t$$

and

$$[Z(\omega)] \cdot [Y(\omega)] = [Z(\omega)]_t \cdot [Y(\omega)]_t$$

$$= \left\{ [Y(\omega)] \cdot [Z(\omega)] \right\}_t$$

$$\therefore \quad [\gamma_e] = [\gamma_i]_t \qquad \qquad \dots\dots(14a)$$

and, owing to symmetry, $[\gamma_e] = [\gamma_i] = [\gamma]$

In this case(transposed 3-phase line), the propagation constant matrices for voltage and current are equal and equation (14) becomes

$$\left.\begin{array}{l} [V] = [V_s] \, \exp(-[\gamma]x) \\[2mm] [I] = [Z_c]^{-1} \cdot [V_s]\exp(-[\gamma]x) = [I_s]\exp(-[\gamma]x) \end{array}\right\} \quad\dots\dots(15)$$

2.3 Solution of wave equation using modal quantities:

With mutual coupling between the phases of the transmission line, the modal analysis is used to study the propagation problem

Expansion of "$\exp(-[\gamma_e]x)$" to a matrix exponent in numerical terms can be affected in different ways. Sylvester's Formula is straightforward and produces interesting physical results. For the case where $[\gamma_e]$ is a 3 by 3 matrix, the results are

$$\exp(-[\gamma_e]x) = \frac{\exp(-\gamma_1 x)}{(\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)} \cdot \left\{ [\gamma_e] - \gamma_2 \, [U] \right\}\left\{ [\gamma_e] - \gamma_3 \, [U] \right\} +$$

$$\frac{\exp(-\gamma_2 x)}{(\gamma_2 - \gamma_1)(\gamma_2 - \gamma_3)} \cdot \left\{ [\gamma_e] - \gamma_1 \, [U] \right\}\left\{ [\gamma_e] - \gamma_3 \, [U] \right\} +$$

$$\frac{\exp(-\gamma_3 x)}{(\gamma_3 - \gamma_1)(\gamma_3 - \gamma_2)} \cdot \left\{ [\gamma_e] - \gamma_1 \, [U] \right\}\left\{ [\gamma_e] - \gamma_2 \, [U] \right\}\dots(16)$$

In equation (16), $[U]$ is a 3x3 unit matrix, γ_1, γ_2 and γ_3 are the eigenvalues of the $[\gamma_e]$ matrix, which are the values that satisfy the characteristic eqn. derived from the voltage propagation constant matrix. Therefore, the analysis of propagation on overhead transmission lines, conducted completely in phase quantities, requires an eigenvalue analysis when actually producing general numerical results.

If eqn. (16) is substituted in eqn.(14) the result will still appear rather formidable. If $[e_1]$ is an eigenvector of the $[\gamma_e]$ matrix, the following equation is valid;

$$[\gamma_e][e_1] = \gamma_1 \cdot [e_1] \qquad\qquad \dots\dots(17)$$

where γ_1 is the eigenvalue associated with the eigenvector $[e_1]$.

By assuming that $[V_s]$ in eqn.(14) to be equal to $[e_1]$, for a specific example, it is evident that each term in eqn.(16) will be multiplied by $[e_1]$. For example, equation (18) is a portion of the required operations; using (17);

$$\{[\gamma_e] - \gamma_3 [U]\} [e_1] = [\gamma_e] [e_1] - \gamma_3 [e_1]$$

$$= \gamma_1 [e_1] - \gamma_3 [e_1]$$

$$= (\gamma_1 - \gamma_3) [e_1] \qquad \ldots\ldots(18)$$

The term $(\gamma_1 - \gamma_3)$ is just a number, not a matrix. It may, therefore, be used in the equations without regard to the order of multiplication as is required in matrix operations. Repeated use of eqn.(16) produces

$$\exp(-[\gamma_e]x) [e_1] = \exp(- \gamma_1 x) [e_1] \qquad \ldots\ldots(19)$$

From the above example, the simplicity of eqn.(16) is apparent. If a voltage proportional to the eigenvector of the $[\gamma_e]$ matrix is applied to the system, this voltage will propagate with only attenuation and phase shift as described by eqn.(19). The propagation constant for this signal will equal the eigenvalue of $[\gamma_e]$ associated with the eigenvector $[e_1]$ described in(17).

In an even more general manner than previously illustrated, any voltage applied to the system can be broken into the sum of three components proportional to the eigenvectors of $[\gamma_e]$ as

$$[V_s] = [e_1] + [e_2] + [e_3] \qquad \ldots\ldots(20)$$

As in the previous example, if this voltage is applied to eqn.(14), using eqn.(16) and (19), the result is

$$[V] = [e_1] \exp(-[\gamma_e]x) + [e_2] \exp(-[\gamma_e]x) + [e_3] \exp(-[\gamma_e]x)$$

$$= [e_1] \exp(-\gamma_e x) + [e_2] \exp(-\gamma_e x) + [e_3] \exp(-\gamma_e x)$$

$$\ldots\ldots(21)$$

This shows that, any arbitrary signal applied to the system (T.L.) will propagate as three distict natural signals or in three natural modes of propagation, and the three signals will be proportional to the eigenvectors of the propagation constant matrices.

3. PROPAGATION OF CARRIER SIGNAL ON P.T.L.:

3.1 Propagation of carrier modes:

The most convenient method presently known for solving propagation problems on P.T.L. with high frequency signal, is based on the concept of natural modes described above. The natural mode analysis allows one to determine the factors which govern the ratio of power mode to the total input power of the carrier signal, and to estimate the propagation performance of PLC signal on P.T.L.

The idea of modes of propagation of PLC has physical significance. For example, if a given carrier-frequency power was fed from a transmitter into a single-conductor line, it could be observed that this power rapidly decreases with distance, following the exponential law (3). At a distance of about 10 miles approximately 1% of the input power could be extracted from the line[9]. In this case there exists one mode of propagation, called the "ground mode", which is characterized by a high attenuation. For the same input power fed into only one conductor of a 2-phase line, the presence of the second deenergised conductor would extend the range of the 1% signal from 10 miles to about 50 miles. For this system (2-conductor line), the ground mode is attenuated in the first few miles and a second mode between the two conductors carries the signal at larger distances. But if the same input power fed into only one phase conductor of a three-phase horizontal line, the 1% signal can be obtained from any of the phase conductors at a distance of about 250 miles. At this distance, nearly equal signals appear on the two outer phases and on the center phase (the return path), the signal is somewhat higher. This is called "line-to-line mode", which is characterized by the lowest attenuation, and plays a very important role in long PLC systems.

3.2 Power line carrier modes on horizontal 3-phase lines:

In the case of mode analysis, a system (T.L.) is specified in terms of phase self- and mutual-impedances. Thus, the mode analysis is completely general in that the R,L,C, and G matrices include all system parameters.

Now, a numerical example is presented for calculating high-frequency natural-mode impedances and current or voltage components, and, also, the ratio of the total mode power to the input power.

For a lossless 3-phase horizontal line, with no transposition, the eqns. are:

$$Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 = V_1$$
$$Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3 = V_2 \qquad \ldots\ldots(22)$$
$$Z_{31}I_1 + Z_{32}I_2 + Z_{33}I_3 = V_3$$

Equations (22) can be written in the form

$$A\,I_1 + B\,I_2 + C\,I_3 = V_1$$
$$B\,I_1 + D\,I_2 + B\,I_3 = V_2$$
$$C\,I_1 + B\,I_2 + A\,I_3 = V_3$$

Or, in matrix form;

$$\begin{bmatrix} A & B & C \\ B & D & B \\ C & B & A \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad \dots\dots(23)$$

where A and D are the self surge-impedances, and B and C are the mutual surge-impedance between the phase conductors, determined by the line geometry [9,12].

For the determination of natural mode components, consider a very long single-conductor line, with relatively low losses, connected to a high frequency generator impressing a voltage and current $V_{(s)}$ and $I_{(s)}$ respectively, as shown in Fig.2. Thus, at the sending end

$$V_{(s)} = Z_{(s)} \, I_{(s)}$$

But, the voltage and current decrease exponentially with distance x from the generator, thus

$$V_{(x)} = V_{(s)} \, \exp(-\alpha x) \quad \text{and} \quad I_{(x)} = I_{(s)} \, \exp(-\alpha x)$$

Therefore, $\quad Z_c = \dfrac{V_{(x)}}{I_{(x)}} \;$ = constant for a simple, reflection-free line.

This is feasible for three sets of phase voltages and currents of a long reflection-free line, for which

$$V^n_{k(x)} = V^n_{k(s)} \, \exp(-\alpha^n x)$$

and $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \dots\dots(24)$

$$I^n_{k(x)} = I^n_{k(s)} \, \exp(-\alpha^n x)$$

Then, $\qquad \dfrac{V^n_{k(x)}}{I^n_{k(x)}} = Z^n_c \qquad \text{and} \qquad V^n_{k(x)} = Z^n_c \, I^n_{k(x)}$

where k = 1, 2, 3 designates the phase number and, n = 1, 2, 3 designates the particular set corresponding to natural mode of propagation. These modes are characterized by quite different values of attenuation coefficient α^n, and different propagation velocities. At a large distance from the generator, more than 50 miles, the current and voltage of one mode would be much larger than those of the other two (see section 3.1). The mode which permits a long distance PLC is called "mode 1".

The phase current and voltage of each mode n are related by expression (24) and, at the same time, must satisfy the line equation (23). Thus;

$$
\begin{bmatrix} A & B & C \\ B & D & B \\ C & B & A \end{bmatrix} \begin{bmatrix} I_1^n \\ I_2^n \\ I_3^n \end{bmatrix} = Z_c^n \begin{bmatrix} I_1^n \\ I_2^n \\ I_3^n \end{bmatrix} = \begin{bmatrix} V_1^n \\ V_2^n \\ V_3^n \end{bmatrix} \qquad \ldots\ldots(25)
$$

For horizontal 3-phase line, the line equation reduces to[9]:

$$
(C - A + Z_c^n)\left[2B^2 - (D - Z_c^n)(C + A - Z_c^n)\right] = 0
$$

Then, if the values Z_c^n satisfying the latter line equation as eigenvalues of the square matrix, these values are the natural mode characteristic impedance.[9] Hence;

Mode 1 $Z_c^1 = \dfrac{A + C + D}{2} - \dfrac{\sqrt{E^2}}{2}$

Mode 2 $Z_c^2 = A - C$ $\ldots\ldots(26)$

Mode 3 $Z_c^3 = \dfrac{A + C + D}{2} + \dfrac{\sqrt{E^2}}{2}$

where $E^2 = A^2 + 8B^2 + C^2 + D^2 + 2(AC - AD - CD)$.

For each value of Z_c^n, equations (26), the ratios of mode n currents and therefore voltages of various phase conductors are

$$
p = \frac{I_2^1}{I_1^1} = \frac{I_2^1}{I_3^1} = \frac{V_2^1}{V_1^1} = \frac{V_2^1}{V_3^1} = \frac{Z_c^1 - A - C}{B}
$$

$$
\frac{I_3^2}{I_1^1} = \frac{V_3^2}{V_1^1} = -1
$$

$$
\frac{I_2^2}{I_1^2} = \frac{V_2^2}{V_1^2} = 0 \qquad\qquad\qquad \ldots\ldots(27)
$$

$$
q = \frac{I_2^3}{I_1^3} = \frac{I_2^3}{I_3^3} = \frac{V_2^3}{V_1^3} = \frac{V_2^3}{V_3^3} = \frac{Z_c^3 - A - C}{B}
$$

where p and q are the center-phase natural-mode coefficients.

In any particular problem, the values of mode phase current or voltage can be calculated from the actual values. Thus, if the phase currents are known, then from equation (27);

$$I_1^1 + I_1^2 + I_1^3 = I_1$$

$$I_2^1 + I_2^2 + I_2^3 = pI_1^1 + 0 + qI_1^3 = I_2$$

$$I_3^1 + I_3^2 + I_3^3 = I_1^1 - I_1^2 + I_1^3 = I_3$$

In matrix form;

$$
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}
=
\begin{bmatrix} 1 & 1 & 1 \\ p & 0 & q \\ 1 & -1 & 1 \end{bmatrix}
\cdot
\begin{bmatrix} I_1^1 \\ I_1^2 \\ I_1^3 \end{bmatrix}
\qquad \ldots\ldots(28)
$$

By replacing the I's in equation (28) by V's, the relations are established between the phase and the mode voltages. The columns of the coefficient matrix in (28) are called "eigenvectors" of the system. Thus, for each mode impedance Z_c^n, equations (26), (27) and (28) determine the mode phase current and voltages. Thus, for phase 1,

$$V_1 = V_1^1 + V_1^2 + V_1^3 \quad \text{mode voltage}$$

and $\ldots\ldots(29)$

$$I_1 = I_1^1 + I_1^2 + I_1^3 \quad \text{mode current}$$

So, the results of mode analysis can be written as given by Table 1.

Table 1 : Results of modal analysis of a 3-phase T.L.

Phase	Mode 1 I_1^1 or V_1^1	Mode 2 I_1^2 or V_1^2	Mode 3 I_1^3 or V_1^3
1	1	1	1
2	p	0	q
3	1	-1	1
Impedance	Z_c^1	Z_c^2	Z_c^3

The power delivered to the line is $P_{in} = I_1V_1 + I_2V_2 + I_3V_3$.

It is useful to calculate the mode coupling efficiencies, which mean the ratio of the total mode power P^n to the input power. Hence;

$$\text{Mode } n \text{ power} = P^n = P_1^n + P_2^n + P_3^n$$

$$= I_1^n V_1^n + I_2^n V_2^n + I_3^n V_3^n$$

∴. The mode n coupling efficiency $\eta^n = \dfrac{P^n}{P_{in}}$

For a long PLC link, the best coupling arrangement would be that which produces maximum mode 1 power with minimum mode attenuation $\exp(-\alpha^n x)$.

3.3   Carrier modes propagation on a double-circuit line :

As before, there are three modes of propagation which applies to single-circuit horixontal line. Mode 1 out going on the center conductor, returning on both outer conductors, is characterized by the lowest attenuation. Mode 2 outgoing on one outer conductor, returning on the other outer conductor,gives medium attenuation. Mode 3, the ground mode, outgoing on all other conductors, returning through the ground gives very high attenuation.

Each mode has its own velocity of propagation. Mode 1 travels close to the speed of light, mode 2 is about 1.5 % slower and, mode 3 is somewhat slower again.

For Double-circuit lines, there exist 6 modes. Mode 1 is the ground mode. The other five modes may be discussed as follows.

For a given mode, the parameters define the voltage on each conductor compared to that on the lower left conductor (called conductor 1) as shown in Fig.3. In the case of mode 2, for a particular 230-kV, double-circuit line, if there is 1 volt on conductor 1, there is 1.35 volts on conductor 2, and 1 volt, 180° out of phase, on conductor 4, -0.77 volt on conductor 6, etc, as shown in table 2.

Table 2: Measured modal parameters of a 230-kV, double-circuit line

Conductor number	Mode number					
	1	2	3	4	5	6
1	not measured	1.00	1.00	1.00	1.00	1.00
2		1.35	-0.25	0.00	-2.49	-1.39
3		0.77	-1.31	-1.32	1.38	1.06
4		-1.00	1.00	1.00	1.00	-1.00
5		-1.35	-0.25	0.00	-2.49	1.39
6		-0.77	-1.31	1.32	1.38	-1.06

The relative mode velocities with respect to the velocity of mode 6 is found by

$$v^n = \frac{2\,\ell\,f}{2\,\ell\,f + c} \qquad\qquad \dots\dots(30)$$

where   n = 2, 3, 4, 5;

v^n = velocity of mode n compared to velocity of mode 6 (assumed to be equal to c); and ℓ is the line length in miles; f is the frequency of inter-ference between modes in Hz; and c is the velocity of light in miles per sec. Therefore, the velocity of mode 2 = 99.5% of that of mode 6, velocity of mode 3 = 99.6% of that of mode 6 and velocities of modes 4 and 5 are approximately equal to that of mode 6.

For measured and computed attenuation on a 120-mile line (not transposed) with line-to-line intercircuit coupling and line-to-ground coupling[8], it would appear that the use of top-to-top (3 to 6), or bottom-to-bottom (1 to 4) coup-ling, Fig.3, for the intercircuit line-to-line case would be optimum, and that there is no obvious choice for line-to-ground coupling.

3.4 Propagation on ground-return carrier circuit:

The symmetrical-component treatment of ground-return carrier transmission on 3-phase lines, are given by the analysis of voltage into positive, negative and zero-sequence components, and using corresponding propagation constant and characteristic impedance with each component. Therefore, if a carrier voltage E_s is applied between phase 1 and ground, Fig.4, the voltage to ground on phases 1, 2 and 3 at a distance x from the sending point are:

$$\left.\begin{aligned}
E_1 &= \frac{E_s}{1 + 2Z'}\exp(-\gamma_0 x) + \frac{2Z'}{1 + 2Z'}\,E_s\,\exp(-\gamma x) \\[2mm]
E_2 &= \frac{E_s}{1 + 2Z'}\exp(-\gamma_0 x) + \frac{Z'}{1 + 2Z'}\,E_s\,\exp(-\gamma x) \\[2mm]
E_3 &= E_2
\end{aligned}\right\} \quad\dots\dots(31)$$

and the corresponding currents are:

$$\left.\begin{aligned}
I_1 &= \frac{1}{Z_0}\cdot\frac{E_s}{1 + 2Z'}\exp(-\gamma_0 x) + \frac{2Z'}{Z(1+2Z')}\,E_s\,\exp(-\gamma x) \\[2mm]
I_2 &= \frac{1}{Z_0}\cdot\frac{E_s}{1 + 2Z'}\exp(-\gamma_0 x) - \frac{Z'}{Z(1+2Z')}\,E_s\,\exp(-\gamma x) \\[2mm]
I_3 &= I_2
\end{aligned}\right\} \quad\dots\dots(32)$$

where $Z' = \dfrac{Z}{Z_0} + \dfrac{Z_0 + Z_2}{Z + Z_2}$ as given by Reference 13;

γ_0 = zero-sequence propagation constant;

Z_0 = zero-sequence characteristic impedance (of all 3 phases in parallel with ground return);

γ = positive or negative-sequence propagation constant($=\sqrt{ZY}$);

Z = positive or negative sequence characteristic impedance;and
Z_2= load impedance to neutral at coupling point.

The first term in equations (31) and (32) is a zero-sequence term. The attenuation of the zero-sequence terms is high on lines without ground wires, because of the high resistivity of the ground return path. These terms become negligible on long lines in comparison with the positive and negative sequence terms at a certain distance from coupling point, and propagation takes place between the coupled phase and the other two. Therefore, the attenuation per unit of distance is greater on short line-to-ground channels than on long line-to-ground ones.

3.5 Effect of P.T.L. transposition on carrier modes:

Since the number of transposed, high voltage lines are now far less than used to be in the past, the attenuation and impedance of the carrier circuit may be strongly dependent on which phase conductor is selected for coupling. Thus, the carrier signals are resolved into several modes with different propagation constant and modal parameters. These parameters are sufficient for a full understanding of carrier propagation on a long untransposed line.

On multi-transposed lines, for example, the three phases may be considered identical. Therefore, only two methods of coupling are used; line-to-line and line-to-ground. This assumption of symmetry is not true when the number of transpositions is drastically reduced. For a 230-km, 500-kV line, which has a transposition 100 km from the transmitter end, with a mode 1 signal applied, the result[11] are shown by the curves of Fig.5. Thus, the two center phase curves show that, the mode 1 component is attenuated 6 dB more than for the untransposed line. The other three quarters of the incident mode 1 power is converted to mode 2 at the transposition, and decrease or decrease the outer phase signals at the receiving terminal. If the single transposition is located near the middle of a long line, mode 3 vanishes. As to mode 2, the simplest is to consider that it may be in or out of phase with mode 1.

At the point of transposition, the sum of the incident and reflected phase quanitities must be equal to the refracted (transmitted) quantities. Thus, with the aid of Fig.6, we have for the currents

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} + \begin{bmatrix} I'_1 \\ I'_2 \\ I'_3 \end{bmatrix} = \begin{bmatrix} I''_3 \\ I''_1 \\ I''_2 \end{bmatrix}$$

or,
$$\begin{bmatrix} I_k \end{bmatrix} + \begin{bmatrix} I'_k \end{bmatrix} = \begin{bmatrix} I''_{tk} \end{bmatrix} \qquad \ldots\ldots(33)$$

and, similarly, for the voltages

$$\begin{bmatrix} V_k \end{bmatrix} + \begin{bmatrix} V'_k \end{bmatrix} = \begin{bmatrix} V''_{tk} \end{bmatrix} \qquad \ldots\ldots(34)$$

where k = 1, 2, 3; designates the phase sequence to the left of the trans-
position and the subscript t indicates that the phase sequence is transposed.
Equations (33) and (34) directly calculate the reflected quantities in
terms of the incident values. This reflects less than 1% of the incident
power. Therefore, the transposition induces a reduction of the received sig-
nal by about 6 dB[10,11]. This occurs because a transposition acts as a mode
converter and, in this case, the preponderantly mode 1 incident power is
converted into 23.6% for mode 1 and 74.5% for mode 2 of the reflected power.
Before reaching the receiving end, the signal of mode 2 is much lower than
the signal of mode 1 and this causes the apparent loss.

4.   CONCLUSIONS :

A comprehensive analysis for the solution to wave propagation on trans-
mission lines is introduced. Single and multi-phase lines, with single- and
double-circuit structures, are considered. In all but the single-phase case,
matrix algebra is employed, i.e. natural modes. The paper investigates both
power and carrier wave propagation problems, and gives attention to the
effect of line transposition when studying the propagation of carrier signals.

All solutions given, however, are for the steady state case; only one
frequency is regarded throughout the solutions. This is so done because there
is no general solution to the wave equation for arbitrary waveshapes, even
for the simplest case.

The main conclusion which can be drawn out of this investigation is that,
although some simplification may be possible for particular cases, the
generalized solutions require digital computer facilities for evaluation.

Regarding the effects on the propagation of carrier wave signals of
line transposition, this is shown to depend on the number of transposition
points along the whole length of the line. The attenuation and impedance
of the carrier circuit may also be dependant on which phase conductor is
selected for coupling.

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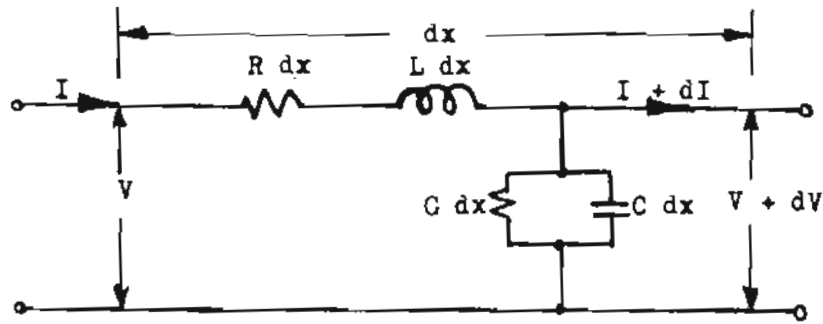
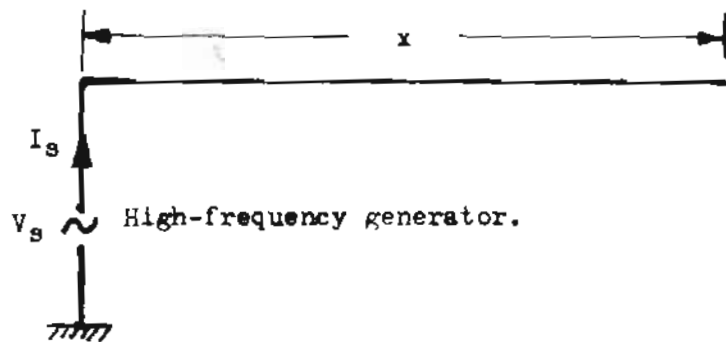Fig. 1. Infinitely small length of T.L. parameters.

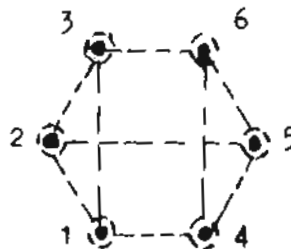Fig. 2. Single conductor of a T.L. connected to high-frequency generator.
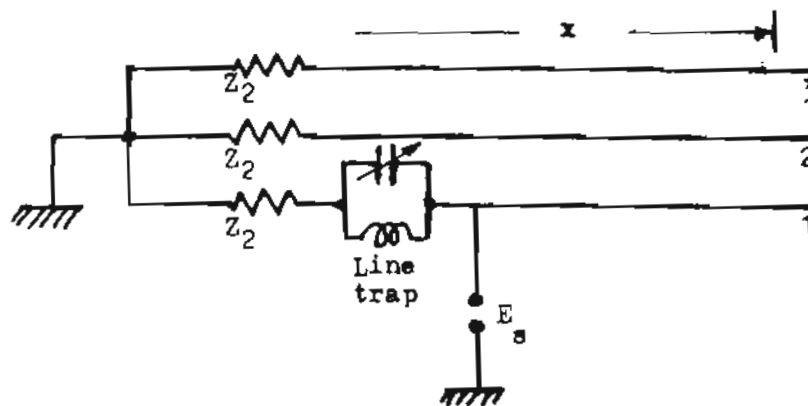

Fig. 3. Double-circuit line configuration.

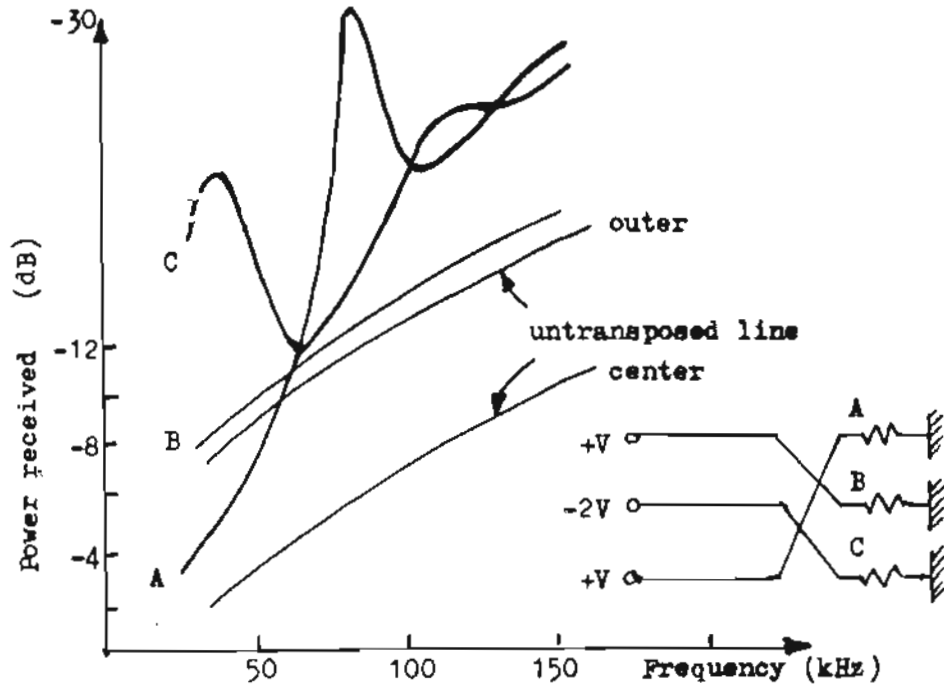Fig. 4. Configuration assumed in discussion of propagation on ground return circuits.
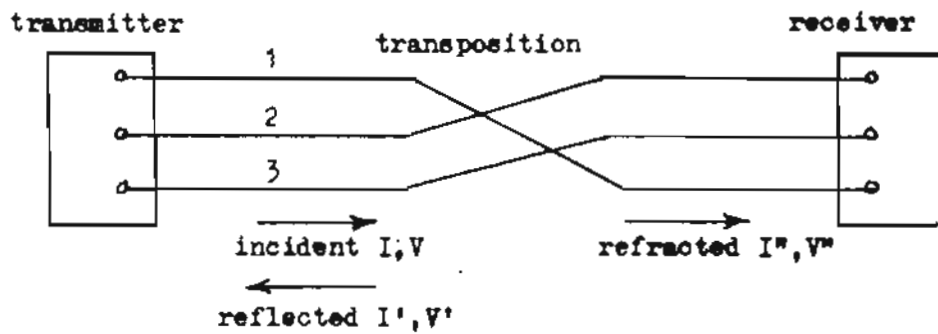
Fig. 5. Effect of transposition on mode 1 signal.

Fig. 6. P L C link with one transposition.