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THE CALCULATION OF STEP HARMONICS IN THE AIRGAP FLUX OF AN INDUCTION MACHINE

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ABSTRACT

The flux density equations for single and double slotted surface induction machines are presented. Slot harmonic components in the flux density wave-form and comparison between step harmonic for slotted and unslotted airgap surface has been driven. The pulsating torque produced by these slot harmonics and how it vary with skewing are listed. The results are compared with experemental results which proved to be in good agreement.

1- INTRODUCTION

The flux distribution in the airgap of an electrical machine is of fundamental importance in determining its efficient performance and this distribution can be represented by certain basic quantities. It is convenient to classify rotating machines into two groups those having salient poles or machines (such as turboalternator) with very large airgap and those with all the windings distributed in slots with an small airgap (such as induction machines) . In the first type of machine the effects of the slots on the two sides of the airgap are relatively unimportant , and the field may be treated in a more simple way . In the second type the airgap is small and the effect considered is very important which will be the subject of this paper .

In the past , numerous authors have derived differing expressions for the airgap flux density in the cases of single and double slotted surfaces(1,3,4,5,6,7,8,9,10) . The " Conformal Transformation" gives an accurate magnetic flux density distribution but it needs a large amount of computer time .

In this paper an approximate method is discussed to obtain the effect of slot on both sides of the airgap on the distribution of the magnetic flux. A comparison between the step harmonic components in the cases of slotted and unslotted surface is presented. The pulsating torque which produced by those slot harmonics is one of the undesirable effects discussed in this paper . The effect of skew is considered , since cogging-torque tends to be larger in the case of unskewed machines .

2- GENERAL THEORY

2-1 Airgap Permeance With Slotting

Assuming an infinite permeability of iron and radial distribution of the magnetic lines , the magnetic flux density (B) for unslotted surface and a unit potential difference is given by:

$$B = 1 / g = \text{Airgap Permeance } (P_g) \quad \ldots\ldots\ldots\ldots(1)$$

For a single slotted surface equation (1) according to Fig.(1) may be written as

$$B(\alpha) = f(\alpha) = 1 / \delta(\alpha) = P_g(\alpha) \quad \ldots\ldots\ldots\ldots(2)$$

Thus due to slotting the airgap at the point displaced by the angle α has changed by the value $\Delta(\alpha)$ which is given by

$$\Delta(\alpha) = g(\alpha) - g = 1 / f(\alpha) - g \quad \ldots\ldots\ldots\ldots(3)$$

For double slotted surface machine the airgap length (g) may written as

$$g(\alpha) = g + \Delta_1(\alpha) + \Delta_2(\alpha) = 1 / f_1(\alpha) + 1 / f_2(\alpha) - g \;.(4)$$

If the origin of the rotor slotting is displaced by the angle α_r with respect to that of the stator, eq.(4) may be expressed in the form

$$g(\alpha) = 1 / f_1(\alpha) + 1 / f_2(\alpha - \alpha_r) - g \quad \ldots\ldots\ldots\ldots(5)$$

Therefore the airgap permeance with double slotted surface and the magnetic flux density distribution is given by

$$P_g(\alpha) = \frac{f_1(\alpha)\, f_2(\alpha - \alpha_r)}{f_1(\alpha) + f_2(\alpha - \alpha_r) - g\, f_1(\alpha)\, f_2(\alpha - \alpha_r)} \quad \ldots\ldots(6)$$

The functions $f(\alpha)$ are periodic functions with the period of the angle of stator and rotor slot pitch and given as follows

$$f_1(\alpha) = a_o - \sum_{n=1}^{\infty} a_n \cos(nS_1 \alpha) \quad \ldots\ldots\ldots\ldots(7)$$

where

$$a_o = 1 / (g\, K_{g1}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

and

$$f_2(\alpha) = b_o - \sum_{n=1}^{\infty} b_n \cos(nS_2 \alpha) \quad \ldots\ldots\ldots\ldots(9)$$

where

$$b_o = 1 / (g K_{g2}) \quad ....(10)$$

K_{g1} and K_{g2} are Carter's factors for both stator and rotor respectively. The general coefficients a_n and b_n are obtained by the equations[4] .

$$a_n = (\beta / g) F_{n1} (S_1 / \lambda_1) \quad ....(11)$$

$$b_n = (\beta / g) F_{n2} (S_2 / \lambda_2) \quad ....(12)$$

where ß is a function of (S/g) obtained from the relation

$$(B_{min.} / B_{max.}) = 1 / (1 - 2\beta(S/g)) \quad ....(13)$$

and

$$F_n(s/\lambda) = (4/n\pi)\ 0.5 + \frac{(nS/\lambda)^2}{0.78 - 2(nS/\lambda)^2} \text{Sin } (1.6\ nS\pi/\lambda) \quad ....(14)$$

Substituting from equations (6)&(8) into eq.(5), the airgap permeance of double slotted surface, with good approximation, given by

$$P_g(\alpha) = 1/g \Big\{ (1 / K_{g1}K_{g2}) - (a_1/K_{g2}) \text{Cos } (S_1\alpha) - (b_1/K_{g1}) \text{Cos } (S_2(\alpha - \alpha_r)) + (a_1 b_1/2) \big[\text{Cos } (S_1\alpha + S_2(\alpha - \alpha_r)) + \text{Cos } (S_1\alpha - S_2(\alpha - \alpha_r)) \big] + ... \Big\} \quad ....(15)$$

This permeance equation for double slotted airgap, prove that the magnetic flux distribution include as basic terms harmonics corresponding to stator and rotor slots which called slot harmonic component of the airgap magnetic flux. From eq.(15) the mean airgap permeance is given by

$$P_{g_{mean}} = (1/2\pi) \int_0^{2\pi} P_g (\alpha) = 1 /(g K_{g1}K_{g2}) \quad ....(16)$$

therefore

$$B_{mean} = B_{max.} / (K_{g1} K_{g2}) \quad ....(17)$$

The resultant two sides airgap excitation factor $K_g = K_{g1} K_{g2}$ which is the same as the factor derived by Carter. Binns has obtain an more accurate factor as follows [5]

$$K_g = 0.5 (K_{g1} + K_{g2} + K_{g1} K_{g2} - 1) \quad ....(18)$$

By applying these two equations for the factor K_g on several machines with different values of (S/g) and (S/λ) of both stator and rotor it has been found that the error is nearly negligable.

2-2 The Magnetic Field With Slotted Surface

In the case of single slotted surface the magnetic field intensity (H) in the airgap at any point (α) is given by

$$H(\alpha,t) = \text{m.m.f.}(\alpha,t) \cdot P_g(\alpha) \quad \ldots\ldots\ldots\ldots(19)$$

Where m.m.f. (α,t) is the time and space behaviour of the fundamental magnetomotive force of the winding. The m.m.f. of the n^{th} order produces the flux intensity

$$H_n(\alpha,t) = \text{m.m.f.}_n \, \text{Sin}(wt \pm n\alpha) \cdot P_g(\alpha) \quad \ldots\ldots\ldots(20)$$

The airgap permeance $P_g(\alpha)$ has been expressed before by eq.(15) for double slotted surface airgap. Substituting from eq.(15) into eq.(20) the field intensity n^{th} harmonic may written in the form

$$\begin{aligned} H_n(\alpha,t) &= a_o' \, \text{m.m.f.}_n \, \text{Sin}(wt \pm n\alpha) - \\ &- (a_1' \, \text{m.m.f.}_n/2)\{\text{Sin}(wt+(S_1 \pm n)\alpha) + \text{Sin}(wt-(S_1 \pm n)\alpha)\} \\ &- (b_1' \, \text{m.m.f.}_n/2)\{\text{Sin}((w-S_2w_r)t + (S_2 \pm n)\alpha) + \\ &\quad \text{Sin}((w+S_2w_r)t - (S_2 \pm n)\alpha)\} + \ldots \quad \ldots\ldots\ldots\ldots(21) \end{aligned}$$

where

$$a_o' = 1 / g K_{g1}K_{g2} \;, \quad a_1' = a_1 / g K_{g2} \;, \quad b_1' = b_1 / g K_{g1} \quad ..(22)$$

The second term in eq. (21) is known as the first order stator slot harmonic generated by the n^{th} m.m.f. harmonic of the stator. The third term is the first order rotor slot harmonic generated by the same harmonic of the m.m.f. of the stator.

2-3 Slot Harmonic Magnetic Field

It has been found that for each different number of slots per pole there are certain harmonics which have the same winding factor as the fundamental component. such harmonics are known as slot harmonics, since their strengths are solely a function of the number of slots per pole and are in no way dependent upon the distribution. The slot harmonic order may be obtained by

$$n = (kS / p) \pm 1 \quad \text{..........................(23)}$$

where
- n the order of slot harmonics
- S the number of slots
- p the number of pole pairs
- k 1,2,3,..... or any whole number

From the infinite number of slot harmonics, only two are generally of practical importance, these two are the ones given by letting the constant k = 1 .

To study the effect of slotting on the magnetic flux distribution in the airgap of an induction machine a comparison may be made in the case of slotted and unslotted machines.

With unslotted airgap the magnetic field intensity amplitude of step harmonic of the order $n = (S/p)+1$ according to the first term of eq. (21)

$$H_{on} = m.m.f._n / g = a_o^` \, m.m.f. \, K_{g1} K_{g2} / n \quad \text{.............(24)}$$

According to the second term of eq. (21) the magnetic field intensity of n^{th} harmonic is the result of the slot harmonic due to the fundamental m.m.f. and the step harmonic of the n^{th} harmonic if m.m.f. and given by

$$H_n = m.m.f. \, a_1^` / 2 - m.m.f._n a_o^`$$

$$H_n = m.m.f. \, a_o^` \, ((a_1^`/2a_o^`) - (1/n)) \quad \text{................(25)}$$

The ratio between the resultant field harmonic of the order n in slotted machine to that in an unslotted machine is given from eq. (24) and eq. (25) by

$$m_{(S/p + 1)} = H_{on} / H_n = (\frac{a_1^`}{2a_o^`} (\frac{S}{p}+1) - 1)(1 / K_{g1} K_{g2}) \quad \text{....(26)}$$

and

$$m_{(S/p - 1)} = (\frac{a_1^`}{2a_o^`} (\frac{S}{p} - 1) - 1)(1 / K_{g1}K_{g2}) \quad \text{...........(27)}$$

In an actual machine , both stator and rotor winding carry current. Then, the slot harmonics of the first order having $(S_1/p \pm 1)$ and $(S_2/p \pm 1)$ are generated by the m.m.f. which proportional to the no-load magnetization current I_{mo} while the corresponding step field of stator and rotor m.m.f. are proportional to the stator current I_1 .

As an allestrated example , for different number of slots in a single slotted surface at no-load and full-load , the slot harmonics are calculated. The ratio between the magnetic field intensity for the case of slotted and unslotted machines are shown in Fig. (2) . It shows that the slot harmonics depend not only on the slot opening but also on the number of slots, airgap length, and slot pitch of each machine.

2-4 Pulsating Torque Calculation

Cogging torque tends to produce a pulsating torque at speed and this can get transmitted to the stator frame and produce a magnetic noise. The pulsating torque associated with cogging vary with rotor position in a periodic manner , and the periods are equal to the stator or rotor slot pitches, or submultible of these.

Cogging torque are commonly discussed in terms of permeance waves, as examplified in the well known treatise by Alger[2]. In using permeance waves, one assumes that the flux distribution in the machine airgap is simply derivable from the permeance and m.m.f. waves. The analysis doesnot account for saturation to simplify the solution.

It is possible to evaluate torque by adding the forces on individual teeth around the whole of the machine periphery, but a simpler and more revealing method is to obtain them from the energy stored in the magnetic field. The stored energy is given by

$$W_m = \frac{1}{2}\, m.m.f.\, \Phi = \frac{1}{2}\, I\, \Psi = \frac{1}{2}\Phi^2 R \quad ....(28)$$

where

Ψ is the flux linkages = $N\Phi$

Φ is the airgap flux

W_m is the stored energy

R is the reluctance of the magnetic path.

The torque due to change in the energy stored in the magnetic circuit for differential change in the rotor position $d\alpha$ is given as

$$T = \frac{dW_m}{d\alpha} = \frac{1}{2} \cdot \Phi^2 \cdot \frac{dR}{d\alpha} \Big|_{\Phi \text{ Constant}} \quad \text{...........(29)}$$

For the constant voltage system, the magnetising flux, been closely linked to the terminal voltage, can be assumed constant to a good approximation. The airgap reluctance and the magnetising current vary with α, and provide the pulsating torque. When the rotor had moved a complete cycle, the average torque and pulsating torque were evaluated by

$$T_{av.} = \frac{1}{n} \sum_{i=1}^{n} T(i) \quad \text{.......................(30)}$$

$$T_{pulsating} = T_{max.} - T_{min.} \quad \text{....................(31)}$$

where

n is the number of rotor position in a complete cycle.

$T_{max.}$ & $T_{min.}$ are the biggest and the smallest instantaneous torque in a complete rotor cycle respectively.

3- RESULTS AND DISCUSSION

The most important effect of slot harmonic flux in an induction machine is the pulsating torque. Two practical machines have been investigated by fixing search coils in the airgap on the top of stator teeth to predect the flux density and the flux pulsation due to rotor slotting. Table (1-a) shows the first and the second rotor slot harmonic components at no-load and full-load as a percentage of the fundamental for a machine 48 stator slots , 36 wound rotor slots, 4-poles , 50 C/s , and skewed one stator slot pitch. For the same machine with squirral cage rotor table (1-b) shows the harmonic components. These harmonics are smaller in the case of squirral cage rotor than the harmonics in wound rotor machines.

It has been proved that one of the reason for using skew is to reduce the slot harmonic flux and therefore the pulsating torque, magnetic noise, and viabration. The pulsating torque has been measured and calculated at standstill for the skewed test machine which proved to be in a good agreement with error less than 5 % .

A comparison between the calculated pulsating torque as a percentage of full-load torque, at starting and full-load, for skewed and unskewed machine, has been carried out. An example of this comparison is listed in table (2) .

Har. Order		No-load	Full-load
Rot.1	17	5.6	7.06
	19	6.04	6.93
Rot.2	35	1.58	1.52
	37	1.62	1.65

(a)

Har. Order		No-load	Full-load
Rot.1	28	0.5	1.5
	30	1.2	2.08
Rot.2	57	0.55	0.30
	59	0.71	0.60

(b)

Table (1) Rotor slot harmonic flux density as a percentage of the fundamental flux density

a- 36 slots wound rotor. b- 58 slots squirral-cage rotor.

Type of Rotor	Starting		Full-load	
	Skew=0	Skew=One St. slot pitch	Skew=0	Skew=One St. slot pitch
Wound	180	42	140	10.0
Squirral-cage	104	23	130	16.0

Table (2) The pulsating torque as a percentage of the full-load output torque at starting and full-load speed.

Figs. (3)&(4) show the e.m.f. waveform produced by two different search coils in the airgap of test machine for both no-load and full-load conitions. The shape of these e.m.f.s , which proportional to the flux density crossing these search coils, show clearly the effects of therotor slot harmonics. It depends on the shape of the search coil and it's position in the airgap.

4- CONCLUSION

The slot opening causes a remarkable effect on the flux density distribution in the airgap of induction machines. Those flux density harmonics due to stator and rotor slotting produce a torque ripples which calls pulsating torque. The magnitude of pulsating torque may reach, for some machines, nearly double the full-load torque. To reduce this effect it is necessary, as proved in this paper, to skew either stator or rotor. Calculating the resultant airgap field harmonics due to slotting are also necessary to determine the core losses and th magnetic noise in an induction machine.

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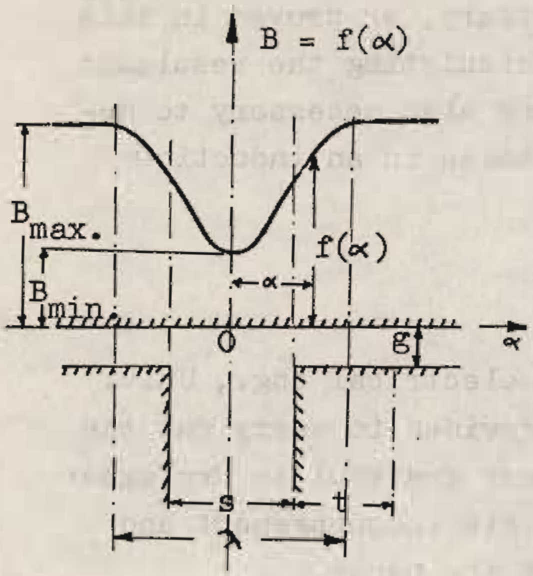

Fig. (1) Flux density variation due to slotting.

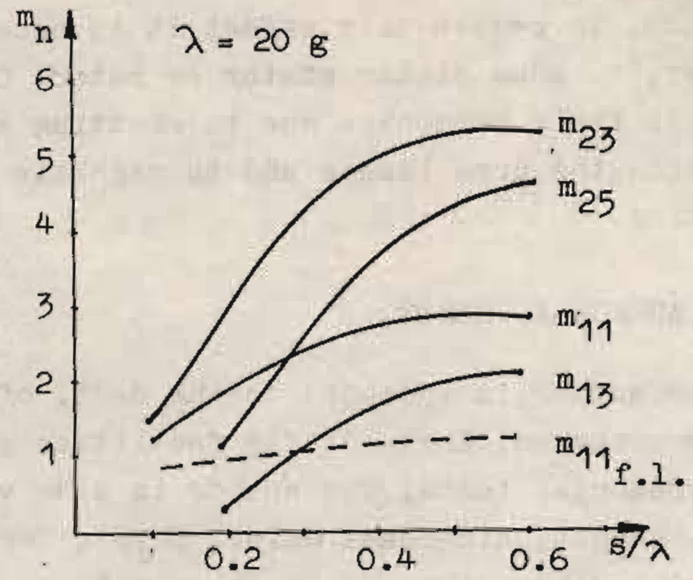

Fig. (2) The ratio between the flux density for slotted and unslotted surface induction motors.

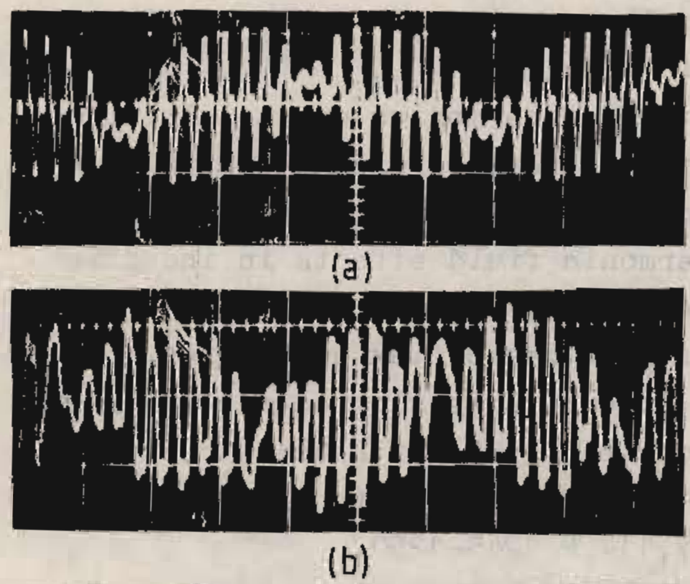

Fig. (3) E.M.F. wave - form from a small search (1/5 stator tooth top) coil.
a- at no-load.
b- at full-load.

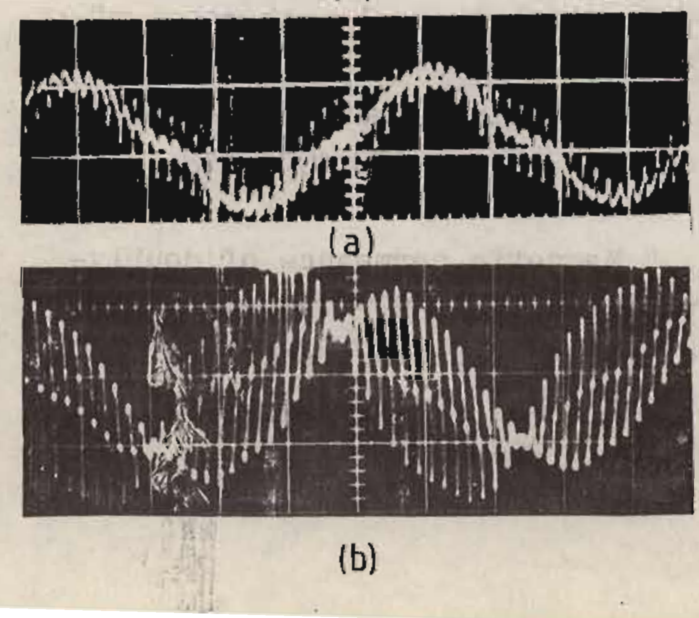

Fig. (4) E.M.F. wave- form from stator search coil (one stator tooth top)
a- at no-load.
b- at full-load.