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"AN INVESTIGATION OF SOME TURBULENT BOUNDARY LAYER PARAMETERS"

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A B S T R A C T

In turbulent boundary layer problems, the velocity defect law suggested by Coles consists of two-universal functions; the law of the wall and the law of the wake. This defect law agrees very well with experiments for turbulent boundary layers, only at momentum Reynolds number $Re_{\delta^{**}} > 6000$.

In the present work the wake function is modified to satisfy in such away simulatanously the normalizing conditions stated by Coles and the variation in the empirical constant $\varkappa$ with $Re_{\delta^{**}}$ (without any restrictions) and the velocity profile parameter P.

The new modification in $\varkappa$ and wake functions provides the possibility of studying the velocity profile and the different parameters controlling the behaviour of the turbulent boundary layer efficiently.

NOMENCLATURE

c_∞	free stream velocity	m/sec
$\bar{c}$	velocity at the outer edge of the boundary layer	m/sec
c^*	friction velocity, $\sqrt{\tau_w/\rho}$	m/sec
c_f	local skin friction coefficient, $\tau_w/\rho\frac{\bar{c}^2}{2}$	
c_x	velocity of the fluid inside the boundary layer in x-direction	m/sec
c_y	velocity component in y-direction	m/sec

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H_{12}	boundary layer form parameter, δ^{*}/δ^{**}	
I	boundary layer shape parameter, $\int_0^\infty \left(\frac{\bar{c} - c_x}{c_\tau}\right) . d\left(\frac{y c_\tau}{\delta^{*} \bar{c}}\right)$	
P	the velocity profile parameter	
p	free stream static pressure bar	
$Re_{\delta^{**}}$	Reynolds number based on the momentum thickness $\frac{\bar{c}.\delta^{**}}{\nu}$	
x, y	coordinates	
$w(\frac{y}{\delta})$	wake function	
δ	boundary layer thickness, m	
δ^{*}	displacement thickness of the boundary layer m $\int_0^\infty (1 - \frac{c_x}{\bar{c}}).dy$	
δ^{**}	momentum thickness of the boundary layer, m $\int_0^\infty \frac{c_x}{\bar{c}} (1 - \frac{c_x}{\bar{c}}).dy$	
τ	shear stress in boundary layer N/m^2	
τ_w	wall shear stress	
μ	dynamic viscosity poise	
ν	kinematic viscosity m^2/sec	
Λ	Euler number, $= -\frac{1}{\bar{c}} \frac{d\bar{c}}{dx} \delta^{**}$	
Π	pressure gradient parameter	
$\varkappa$	emperical constant	

1- INTRODUCTION:

ATTEMPTS have been made by many workers in the past 40 years to predict turbulent boundary layer growth and separation in two-dimensional flow. No complete theoretical solution has yet been valid due to the difficulties in obtaining a clear picture of the mechanism of turbulent motion. Additional empirical formula, based on experimental results, were usually introduced to the mathematical basis for boundary layer investigation, based on momentum or energy equations. Accordingly the calculations of turbulent boundary layer are semi-emperical in nature.

A principle assumption for calculating turbulent boundary layer is that, the velocity profiles can be described by a single-parameter. This assumption greatly simplifies the study of the turbulent boundary layer.

Many authors devised two-parameter methods, among them Rotta[1], Gruschwitz[2] and Peter[3], in order to give a better description of the velocity profile than can be offered by uniparameter method.

Buri[4] introduced a different shape factor to describe the velocity profile, and different assumption for the wall shearing stress. A.E. Von Doenhoff and Titervin[5] proposed another approximate method for calculating the important values that control the behaviour of turbulent boundary layer.

Coles (1956)[6] suggested that the turbulent boundary layer with adverse pressure gradient can be described with a profile consisting of the law of the wall and the law of the wake. The skin friction coefficient (through C_τ) and the velocity profile parameter P were employed as parameters. This velocity law of Coles is taken therefore as a basis to this study, with the introduction of some new developments in order to suit the variation in the empirical constant $\varkappa$ with $Re_{\delta^{**}} \gtrless 6000$.

2- THE LAW OF THE WAKE AND THE LAW OF THE WALL IN TURBULENT B.L.

After an extensive survey of mean velocity profile in various two-dimensional incompressible turbulent boundary layer flows, it is proposed to represent the profile by a linear combination of two universal functions. One is the well-known law of the wall, the other the law of the wake which is characterized by the profile at a point of separation or reattachment. These functions are considered to be established empirically, by a study of the mean-velocity profile, without reference to any hypothetical mechanism of turbulence. The development of a turbulent boundary layer is ultimately interpreted in terms of an equivalent wake profile, which supposedly represents the large eddy structure and is consequence of the constraint provided by the inertia. This equivalent wake profile is modified by the presence of a wall, at which a further constraint is provided by viscosity. The wall constraint, although it penetrates the entire boundary layer, is manifested chiefly in the sublayer flow and in the logarithmic profile near the wall.

The historical development of the law of the wall, shows that, in the hands of Prandtl, Von KÂRMÂN[7] and others, included a simple dimensional argument which has not lost its usefulness.

Let C_x (x,y) and C_y(x,y) be the mean velocities in a considerable turbulent shear flow which is steady and two-dimensional on the average. The flow exerts a shearing stress $\tau_w(x)$ on a smooth impereable wall at rest, at Y = 0. For a fluid of constant density, a friction velocity C_τ (x) is defined by:

$$\rho \cdot C^2 = \tau_w \qquad \text{........(2.1)}$$

Suppose that the mean-velocity profile of that flow is found to be adequately represented by a relationship $\phi(C_x, y, \delta, \tau_w, \mu, \rho) = 0$, in an obvious notation, and that this relationship is found in some region, near the surface, to be independant of the characteristic length δ. It follows from the principles of dimensional analysis, without any explicit assumptions about the nature of the turbulence, that in this region the equation

$$\frac{C_x}{C_\tau} = f\left(\frac{y \cdot C_\tau}{\nu}\right) \qquad \text{........(2.2)}$$

must be satifeid

As several writers in the field have pointed out, equation (2.2) is an implicit equation for C_{τ} (hence for τ_w) where ρ, μ, and $C_x(y)$ are given.

Before the development of the mixing analogy, the function in Eq. (2.2) was sometimes taken as a power law, for the lack of a better representation. The sublayer, that is, the region where viscous stress is predominate, was treated separately by means of the plausible assumption of a linear velocity profile very near the wall. In this approximation

$$\frac{\partial C_x}{\partial y} = \frac{C_x}{y} = \frac{\tau_w}{\mu} = \frac{C_{\tau}^2}{\nu}$$

and therefore

$$\frac{C_x}{C_{\tau}} = \frac{y \cdot C_{\tau}}{\nu}.$$

The mixing analogy of Prandtl (1926) and the similarity hypothesis of Von Kàrmàn (1932) [7] had provided an equation $\frac{\partial C_x(x,y)}{\partial y} = \frac{C_{\tau}(x)}{\varkappa \cdot y}$ for the mean velocity in the fully turbulent region, with the integral $\frac{C_x}{C_{\tau}} = \frac{1}{\varkappa} \ln(\frac{y}{y_o(x)}) + C$.

The unspecified characteristic length $y_o(x)$ can be chosen equal to $\frac{\nu}{C_{\tau}}$ as a part of the dimensional argument already mentioned. Therefore the above equation takes the form:

$$\frac{C_x}{C_{\tau}} = \frac{1}{\varkappa} \ln(\frac{y \cdot C_{\tau}}{\nu}) + C \qquad \text{........(2.3)}$$

in which $\varkappa$ and C are two empirical constants to be determined experimentally - the numerical values given to these constants are: $0.39 < \varkappa < 0.41$ and $C = 5.1$. Equations(2.3) represents the universal law of the wall specially for values of $\frac{y.C_{\tau}}{\nu} > 50$. On the other hand, the predominance of laminar shear near the wall requires $\frac{C_x}{C_{\tau}}$ to approcach $\frac{y.C_{\tau}}{\nu}$ as y - approches zero.

3- Velocity Defect Law:

The description just given of mean-velocity profile in a turbulent shear flow may be summarized in the formula:

$$\frac{C_x}{C} = f\left(\frac{y.C_{\tau}}{\nu}\right) + h(x,y) \qquad ....:(3.1)$$

where the function h is arbitrary except that it is negligibly small in some finite region rear the wall - say for $(y/\delta) <$ 0.1, where δ is the shear flow thickness.

For certain special cases frequently encountered (e.g. uniform pipe and channel flow and the b.L. on a flat plate in a uniform flow) equation (3.1) is found experimentally to have the special form:

$$\frac{C_x}{C_{\tau}} = f\left(\frac{y.\ C_{\tau}}{\nu}\right) + g(P,\ y/\delta) \qquad ....(3.2)$$

where P is a parameter which is independent of x and y. Profile similarity in terms of the argument (y/δ) is usually expressed by a relationship known as the velocity defect law, or more properly the moment defect law. Outside the sublayer, it is an immediate consequence of the logarithmic variation of "f" in equation (3.2) that:

$$\frac{\bar{C} - C_{\tau}}{C_{\tau}} = F(P,\ y/\delta) \qquad ....(3.3)$$

with $C_x = \bar{C}$ at $y = \delta$.

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According to experimental evidence from many sources, the defect function $F(P, y/\delta)$ in a given flow is insensitive to roughness at the wall. Despite that there is a small dependence of the defect law on the turbulence level in the external stream.

4- The Wake Function:

The essential element is not to study the defect function "F" in Eq. (3.3), but to study the origional function $g(P, y/\delta)$ in Eq. (3.2), which gives the logarithmic law of the wall. An extensive survey of experimental data at large Reynolds numbers leads to the cruical conclusion that this function can be reduced directly to a second universal similarity law. Therefore, Eq. (3.2) may be written in the form:

$$\frac{C_x}{C_\tau} = f\left(\frac{y.C_\tau}{\nu}\right) + \frac{P(x)}{\varkappa}\, w(y/\delta) \qquad ....(4.1)$$

where P is a profile parameter and the function $w(y/\delta)$ will referred to as the "law of the wake". If P does not depend on x, then both $g(p, y/\delta)$ in Eq. (2.5) are function of y/δ only. This is the property assigned to "equilibrium flows" by F. Clauser [8] (flows with a defect law, that is, a flow for which the parameter P is constant).

In order to test the hypothesis of the universal wake function in Eq. (4.1), it is necessary first to define the thickness δ and to specify some normalizing factor for W. The maximum value of w will occur very rearly at $y/\delta = 1$, minimum value is at $y/\delta = 0$ and the area under the curve is equal to unity. Therefore, it has been subjected to the following normalizing conditions:

$$W(0) = 0 \ , \quad W(1) = 2$$

$$\int_0^2 (y/\delta)\, dW = 1 \qquad \text{.......(4.2)}$$

A diagramtic representation of Eq. (4.2) which states the normalizing conditions is shown in Fig. (1).

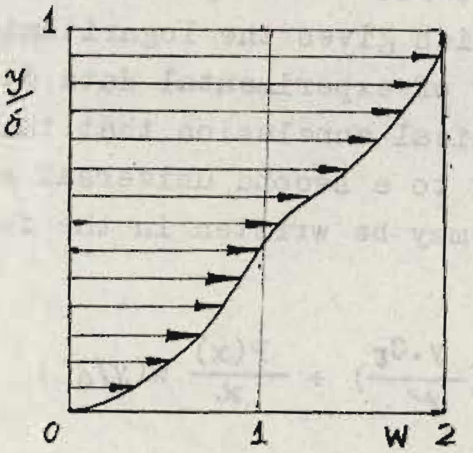

Fig. (1): Representation of the normalizing condition

5- The Velocity Profile For Fully Turbulent Boundary Layers

From the previous deriviations, the mean-velocity profile can be represented well by a linear combination of the two universal functions, the law of the wake Eq. (2.3), and the law of the wall Eq. (2.5). Neglecting the departure of the flow in the sublayer from the logarithmic wall law, then

$$\frac{C_x}{C_\tau} = \frac{1}{\varkappa} \ln \left(\frac{y.C_\tau}{\nu}\right) + C + \frac{P(x)}{\varkappa} . W(y/\delta) \quad \text{...(5.1)}$$

At the outer edge of the boundary layer, at $y = \delta$, the velocity component $C_x = \bar{C}$. Substituting with these values at the outer of the b.L., and applying the normalizing condition, $W(1) = 2.$ in Eq. (5.1), therefore

$$\frac{\bar{C}}{C_{\tau}} = \frac{1}{\varkappa} \cdot \ln\left(\frac{\delta . C_{\tau}}{\nu}\right) + C + \frac{2\,P(x)}{\varkappa} \qquad \ldots\ldots\ldots(5.2)$$

Equation (5.2) represents the relation between the profile parameter P and the local skin friction coefficient ($C_f = \frac{2C_{\tau}^2}{\bar{C}^2}$).

Substracting Eq. (5.1) from Eq. (5.2), there results is

$$\frac{C - C_x}{C_{\tau}} = -\frac{1}{\varkappa} \ln(y/\delta) + \frac{P}{\varkappa} \quad 2 - W(y/\delta) \ \ldots\ldots(5.3)$$

The velocity profile in that form describs a defect law with a defect function, $F(P, y/\delta)$, equals to the right hand side of Eq. (5.3) which depends, at x = const., on a single parameter.

From Eq. (5.3), the velcoity distribution in the boundary layer is given by

$$\frac{C_x}{\bar{C}} = \underbrace{1 + \sqrt{\frac{\tau_w}{\rho . \bar{C}^2}} \left[\frac{1}{\varkappa} . \ln\left(\frac{y}{\delta}\right) - \frac{2P}{\varkappa} \right]}_{I} + \underbrace{\sqrt{\frac{\tau_w}{\rho . \bar{C}^2}} . \frac{P}{\varkappa} W(y/\delta)}_{II}$$

$$\ldots\ldots\ldots(5.4)$$

The first term "I" (Eq. 5.4) represents the law of the wall and the second "II" indicats the law of the wake from which Cole's velocity profile consists. Figure (2) stats Coles velocity profile, the dashed line represents the law of the wall, Eq;(2.3). The dash-point line denotes the wake-like structure represented in Eq. (4.1). The associated velocity defect ($C_\infty - \bar{C}$) is given by $C_\tau . P/\varkappa \left[2- W(y/\delta) \right]$, and the intercept at y = 0 of the equivalent wake profile therefore differs from the velocity in the external stream by an amount $2 . C_\tau . P/\varkappa$. Since the turbulent motion in the outer part of a boundary layer is effectively unrestricted and the process of intrainment of non-turbulent fluid takes place by processes very similar to those observed in wakes and jets, the boundary layer may be viewed as wake flow, into which a solid thin plate is placed at the central plane, the velocity defect of the wake being $\left[C_\infty - \bar{C} = 2. C_\tau \cdot P/\varkappa \right]$ at the center. At the surface of the plate the boundary conditions of vanishing velocity and molecular friction are to be satisfied. These conditions impose an additional constraint on the flow, whose effect is to modify the mean velocity profile as shown by the solid line in Fig. (2). Near the plate, where the mean wake velcoity is nearly constant, the constraint provided by viscosity produces a flow pattern as described by the similarity law of wall flow.

6- Numerical Solution of the Velocity Profile:

6.1- Mathematical Correlation on the Emperical Wake Formula

In order to solve Eq. (5.3) numericaly, the profile parametes P, the empirical const. $\varkappa$, the dimensionless wall shear stress $\tau_w/\rho.\bar{C}^2$, and the wake function $w(y/\delta)$ must be known.

For the profile parameter P values are given between zero (for very strong accelerated flow) and infinity (separation), all inbetween values can be described.

The empirical constant $\varkappa$, which regarded as = 0.4, was found by Thimpson [9] to be a function of $R_{e_{\delta^{**}}}$ only for a turbulent boundary layer over a flat plate with zero incident. Following are values given by him:

$$\left.\begin{aligned} \varkappa &= 0.4 \left(\frac{6000}{Re_{\delta^{**}}}\right)^{1/8} && \text{for } Re_{\delta^{**}} < 6000 \\ \varkappa &= 0.4 && \text{for } Re_{\delta^{**}} \geq 6000 \end{aligned}\right\} \quad \ldots\ldots.(6.1)$$

while for boundary layers with definite pressure gradient $\left(\frac{dp_\infty}{dx} \neq 0\right)$, Thimpson found that this relation is a function of $Re_{\delta^{**}}$ and P in the form

$$\begin{aligned} \varkappa &= 0.4 \left(\frac{6000}{Re_{\delta^{**}}}\right)^{1/8 \exp(0.55^2 - P^2)} && \text{for } Re_{\delta^{**}} < 6000 \\ \varkappa &= 0.4 && \text{for } Re_{\delta^{**}} \geq 6000 \end{aligned} \qquad (6.2)$$

Substituting with P = 0.55 in Eq. (6.2), gives the case of flat plate boundary layers.

To calculate the dimensionless wall shear stress Eq. (4.2) is to be solved for the friction velocity C_τ and profile parameter P.

The wake function $w(y/\delta)$ is given by Coles in a table form and later by E.Strehle [10] in empirical formula which satisfy the first normalizing condition w(0) = 0 but not the second condition and gives for w(1) = 1.9. This result has no agreements with Coles's assumption, therefore a mathematical correlation was doen on Strehl's formula and leads to:

$$w(y/\delta) = 2.124(y/\delta)^2 + 14.344(y/\delta)^3 - 30.027(y/\delta)^4 + 20.527(y/\delta)^5 - 4.968(y/\delta)^6 \quad \text{........(6.3)}$$

and satisfies both values for normalizing conditions.

6.2- The Sublayer Region

The sublayer region where viscous stress is predominant and the flow departs from the logarithmic wall law and consequantly from Colse"s velocity profile, for this region the dimensionless group $y.C_\tau/\nu < 50$. This region is defined with the following formula

$$\frac{y.C_\tau}{\nu} = \frac{Re_{\delta^{**}}\sqrt{\tau_w/\rho\bar{c}^2}}{\delta^{**}/\delta} \cdot (y/\delta)$$

from which

$$(y/\delta)_{sublayer} = 50\left[\frac{\delta^{**}/\delta}{Re_{\delta^{**}}\sqrt{\tau_w/\rho.\bar{c}^2}}\right] \quad \text{........(6.4)}$$

Equation (6.4) limits the value of the sublayer region which is equal = 0.1 (y/δ).

6.3- Boundary Layer Parameters

The mean-velocity profile in turbulent boundary layers defined by Eq.(5.4) is the convenient form to evaluate the different turbulent boundary layer parameters. These parameters are such that the dimensionless displacament thickness δ^*/δ, the dimensionless momentum thickness δ^{**}/δ, the form parameter $H_{12} = \delta^*/\delta^{**}$ and the shape parameter I.

6.3.1- Displacement thickness:

Generally, the dimensionless displacement thickness is defined by equation

$$\delta^*/\delta = \int_0^1 (1 - \frac{C_x}{\bar{C}}) / \, d(\frac{y}{\delta}) \quad \ldots\ldots\ldots(6.5)$$

Substituting for $C_x/\bar{C}$ in Eq. (6.5) its value given by Coles Eq. (5.4) and noting that $1/\varkappa\sqrt{\tau_w/\rho\,\bar{C}^2}$ = const, and integrating by parts, gives

$$\delta^*/\delta = \frac{1}{\varkappa}\sqrt{\frac{\tau_w}{\rho\,\bar{C}^2}}\cdot(1 + P) \quad \ldots\ldots\ldots(6.6)$$

which is the dimensionless displacement thickness as a function of the velocity profile parameter P.

6.3.2- Momentum Thickness

The dimensionless momentum thickness is defined by a similar equation

$$\delta^{**}/\delta = \int_0^1 \frac{C_x}{\bar{C}}(1 - \frac{C_x}{\bar{C}}).\, d(y/\delta) \quad \ldots\ldots\ldots(6.7)$$

Equation (6.7) may be performed to have the following form

$$\frac{\delta^{**}}{\delta} = \int_0^1 (1 - \frac{C_x}{\bar{C}}).d(y/\delta) - \int_0^1 (1 - \frac{C_x}{\bar{C}})^2.\, d(y/\delta) \quad (6.8)$$

Substituting for the velocity distribution in the boundary layer $C_x/\bar{C}$ Colse's value, Eq. (5.4) and noting the definition of the displacement thickness and integrating by parts, follows

$$\frac{\delta^{**}}{\delta} = \frac{\delta^*}{\delta} - \frac{2}{\varkappa^2}\cdot\frac{\tau_w}{\rho\cdot\bar{C}^2}\Big\{1 - P\int_0^1 \big[2 - w(y/\delta)\big]\ln(y/\delta).$$

$$d(y/\delta) + \frac{P^2}{2}\int_0^1\big[2 - w(y/\delta)\big]^2.\, d(y/\delta)\Big\} \ldots\ldots(6.9a)$$

thus

$$\frac{\delta^{**}}{\delta} = \frac{\delta^{*}}{\delta} - \frac{2}{\varkappa^2}\frac{\tau_w}{\varrho \bar{c}^2}\left(1 + K_1 P + K_2 P^2\right) \quad \text{.... (6.9b)}$$

The numerical values of K_1 and K_2 are obtained through analyzation of equation (6.9a), their magnitude are $K_1 = 1.6$ and $K_2 = 0.761$, so that

$$\frac{\delta^{**}}{\delta} = \frac{1}{\varkappa}\sqrt{\frac{\tau_w}{\varrho \bar{c}^2}}\,(1 + P) - \frac{2}{\varkappa^2}\frac{\tau_w}{\varrho \bar{c}^2}\left(1 + 1.6P + 0.761P^2\right) \quad \text{..... (6.10)}$$

Thus Eq.(6.10) gives the momentum thickness as a function of the velocity profile parameter P.

The convential form parameter $H_{12} = \frac{\delta^{*}}{\delta^{**}}$ is defined also as a function of the profile parameter P by dividing Eq.(6.6) by (6.10), that yields

$$H_{12} = \frac{1}{1 - \frac{2}{\varkappa}\sqrt{\frac{\tau_w}{\varrho \bar{c}^2}}\left(\frac{1 + 1.6P + 0.761P^2}{1 + P}\right)} \quad \text{..(6.11)}$$

This relation given by Eq.(6.11) is useful to predict another convenient parameter, which is known as shape parameter "I", described by

$$I = \frac{2}{\varkappa}\,\frac{(1 + 1.6P + 0.761P^2)}{(1 + P)} \quad \text{....... (6.12)}$$

which represents explicit relation between the shape parameter "I" and the profile parameter "P".

6.3.3 THE PRESSURE GRADIENT PARAMETER Π, AND EULER NUMBER Λ

The pressure gradient parameter Π represents the ratio between the pressure forces and the viscous forces at the wall, also known as Hagen number. It is given in the emoirical formula:

$$\Pi = (P - 0.55).(1.60456 + 0.420645P) \quad ....(6.13)$$

This parameter is also depends on the profile parameter P.

The preceeding parameters lead to the definition of Euler Number Λ. This number is the ratio between the local pressure forces and the inertia force. For turbulent boundary layer it is defined as follows:

$$\Lambda = -\frac{1}{\bar{c}}\frac{d\bar{c}}{dx}\delta^{**},$$

and is given in relation with the pressure gradient parameter II and the dimensionless wall shear stress in the following form

$$\Lambda = \Pi \frac{\left[\frac{\tau_w}{\rho \bar{c}^2}\right]}{H_{12}} \quad .... (6.14)$$

Now the form of Eq.(6.14) is adequate for numerical computation through computer program.

6.3.4 THE SLOPE OF THE MOMENTUM THICKNESS $d\delta^{**}/dx$

Performing VON-Karman's momentum equation give an explicit form for the slope of the momentum thickness. Moreover, the Euler number is introduced in the momentum equation to include the effect of the pressure gradient on the slope. This form is:

$$\frac{d\delta^{**}}{dx} = (2 + H_{12})\Lambda + \frac{\tau_w}{\rho \bar{c}^2} \quad .... (6.15)$$

The foregoing form is valid for both laminar and turbulent boundary layers. A computer program was made to give the numerical solution of the above equations. These were translated into a FØRTRAN-4 language.

7. REPRESENTAION AND DISCUSSION OF RESULTS

Results obtained from the computer program are classified and represented in chart form. Some of the results obtained will be reported and discussed as follows:

7.1 VARIATION OF THE FORM PARAMETER WITH EULER NUMBER Λ

Figure(3)represents the variation in the form parameter $H_{12} = \delta^{*}/\delta^{**}$, with Euler number, Λ, and $Re_{\delta^{**}}$ as a parameter. The figure consists of five curves for boundary layer at five values of the momentum thickness Reynolds numbers. The dash point line represents boundary layers at velocity profile parameter P = 0.0 or boundary layers at very accelerated flow with negative pressure gradient ($dp/dx < 0$).

7.2 THE LOCAL SKIN FRICTION WITH EULER NUMBER

Fig.4 illustrate the relation between the local skin friction coefficient $c_f/2 = \tau_w / \rho \bar{c}^2$ and Euler number with the $Re_{\delta^{**}}$ as parameter. The figure contains curves for turbulent boundary layers at twelve values from $Re_{\delta^{**}}$ as represented in Fig.4.

For boundary layers at the same $Re_{\delta^{**}}$ the skin friction coefficient decreses for Euler number increase because of the increase of the form parameter with Euler number increase. For boundary layer at the same Euler number, the local skin friction coefficient decreases with $Re_{\delta^{**}}$ increase, For example, in flat plate boundary layers, where $\Lambda = 0.0$, the local skin friction coefficient decreases from $c_f/2 = 0.0055$, to $c_f/2 = 0.0007$, for $Re_{\delta^{**}} = 10^6$. The dash-point line represents boundary layers at velocity profile parameter P = 0 or very accelerated boundary layer.

7.3 THE PRESSURE GRADIENT PARAMETER AND THE SHAPE PARAMETER FOR EQUILIBRIUM BOUNDARY LAYERS

The pressure gradient parameter II could be driven exact from the energy equation. The parameter "I" was found to be a suitable shape parameter for turbulent boundary layers velocity profiles, so that exists a relation $I = I(\Pi)$. Another dependance exists between the pressure gradient parameter Π and the velocity profile parameter P. Figure 5 presents this relationship in which Π increses with the increase of P. The curves intersects the vertical axis at P = 0.55 and $\Pi = 0$. This point represents the case of turbulent boundary layers on flat plate with zero pressure gradient. For $P < 0.55$, which presents boundary layers with negative pressure gradient, Π has negative values as shown in Fig.5.

Figure 6 gives the relation between Π and I for boundary layers at $Re_{\delta^{**}} > 6000$. It was found that the pressure gradient parameter II lies between ($-0.5 < II < 250$) for equilibrium boundary layers.

7.4 PREDICTION OF THE MOMENTUM THICKNESS

Figure 7 represents the relation between Euler number and the slope of the momentum thickness $d\delta^{**}/dx$ with Re_{δ} as a parameter. The chart contains five curves for boundary layers ar $Re_{\delta^{**}} = 3.\ 10^2,\ 10^3,\ 10^4,\ 10^5$ and 10^6.

For the same values of Λ = constant, the values of $d\delta^{**}/dx$ increase with the decrease of $Re_{\delta^{**}}$. Each curve can be devided into two devisions. First devision, the slope of the curve is positive and approximately constant, this means the increse of the slope $d\delta^{**}/dx$ with the increase of Λ. Second devision, the slope of the curve becomes sharp, that means large increase in the slope of $d\delta^{**}/dx$ for small increase in .

This designing charts are of practical use in the prediction of the momentum thickness, by applying the isocline method. This estimation method gives an indication about the skin friction coefficient over smooth surface.

CONCLUSIONS

The present work deals with the theoretical study of the velocity profile and other parameters which affect the behaviour of the turbulent boundary layer. This was achieved using the modified Coles velocity law. As a result of the present investigation the following conclusions are obtained:

1- The modified Cole's velocity law in its new formula becomes valid for turbulent boundary layers at $Re_\delta > 6000$.

2- The wall velocity component is very large near the wall and decreases with the increase of y/δ. Also it decreases with the increase of the velocity profile parameter P.

3- The wake velocity component is very small near the wall and increases with the increase of y/δ. Also its value increases with the increase of P until the velocity profile becomes a pure wake velocity profile near the separation.

4- For boundary layers at the same $Re_{\delta^{**}}$, the velocity decreases with the increase of the velocity profile parameter P.

5- For boundary layers at the same velocity profile parameter, the velocity profile increase with the increase of $Re_{\delta^{**}}$.

6- The velocity profile parameter increases with the increase of Euler number Λ and $Re_{\delta^{**}}$.

7- The form parameter H_{12} increases with the increase of Euler number Λ but it decreases with the increase of $Re_{\delta^{**}}$ and it takes value $H_{12} \approx 4$ near separation.

8- The local skin friction coefficient $c_f/2$ decreases with the increase of both Euler number Λ and $Re_{\delta^{**}}$.

9- The velocity profile parameter and shape parameter I are increasing with the increase of the pressure gradient parameter Π.

10- The slope of the momentum thickness $d\delta^{**}/dx$ increases with the increase of Euler number Λ and decrease with the increase of $Re_{\delta^{**}}$.

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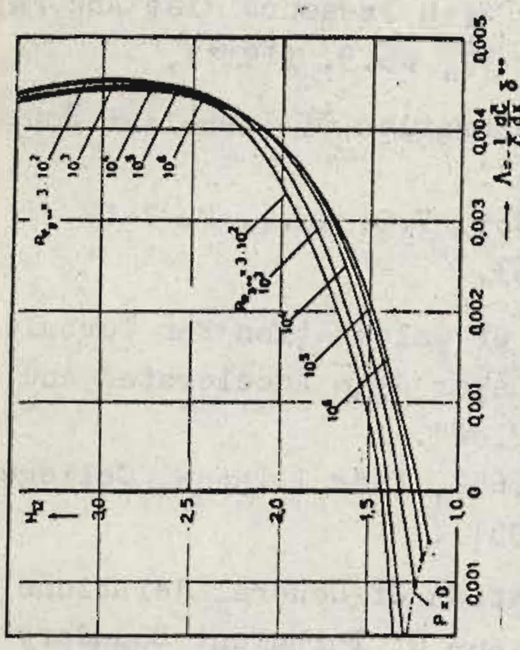
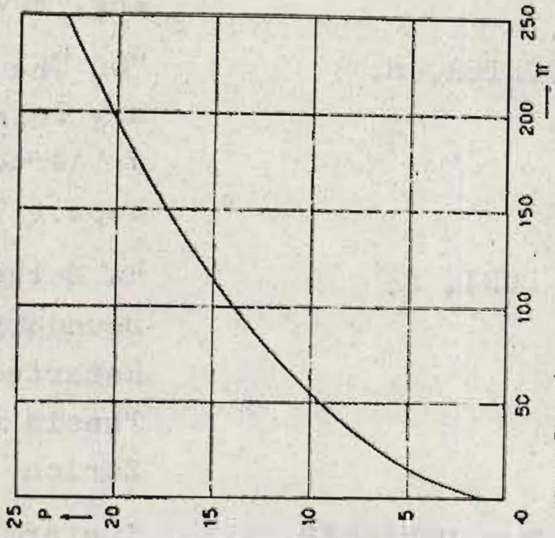
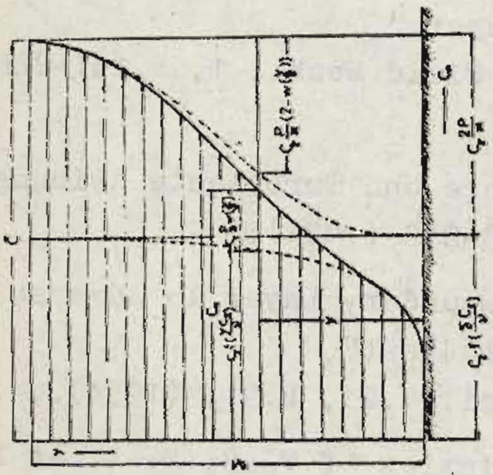


Figure(3) Form parameter versus Euler number with momentum thickness Reynolds number as parameter

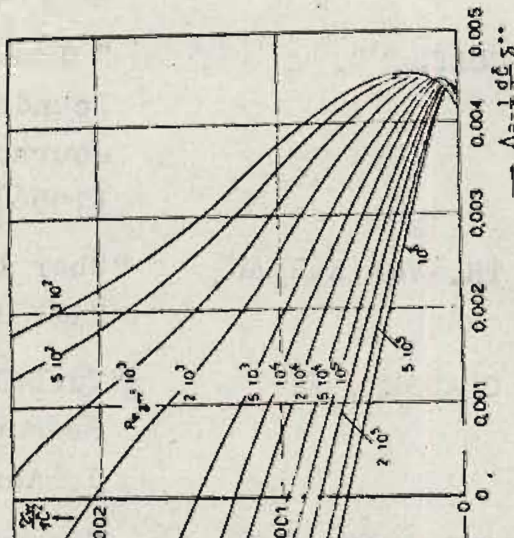


Figure(5) Velocity profile parameter versus pressure gradient parameter

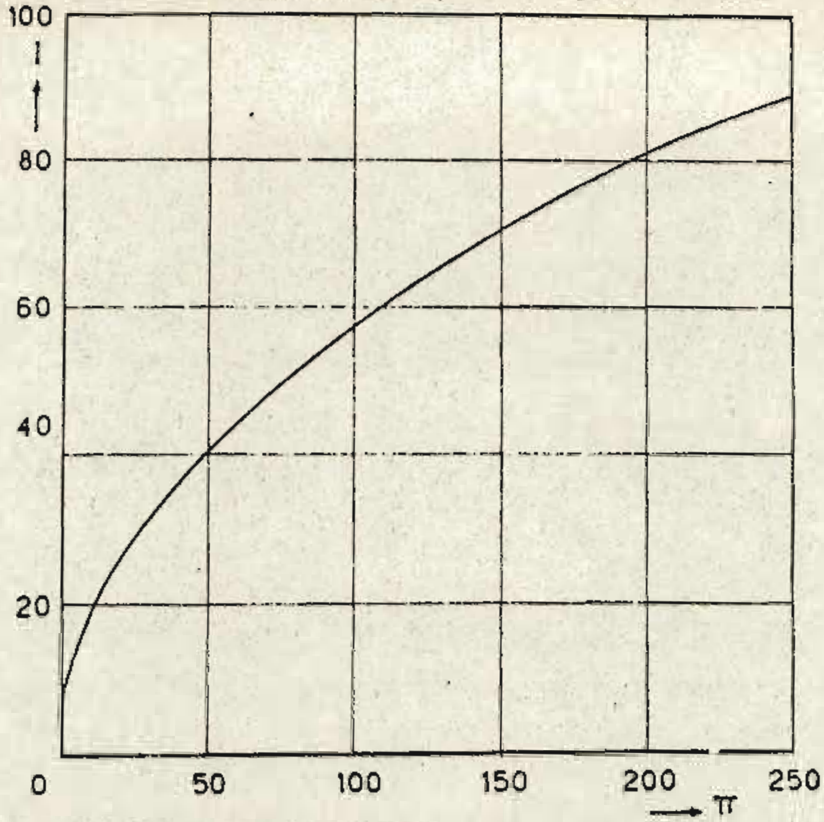


.... Velocity distribution of wall law
– – – Velocity distribution of wake law
—— Total mean velocity profile

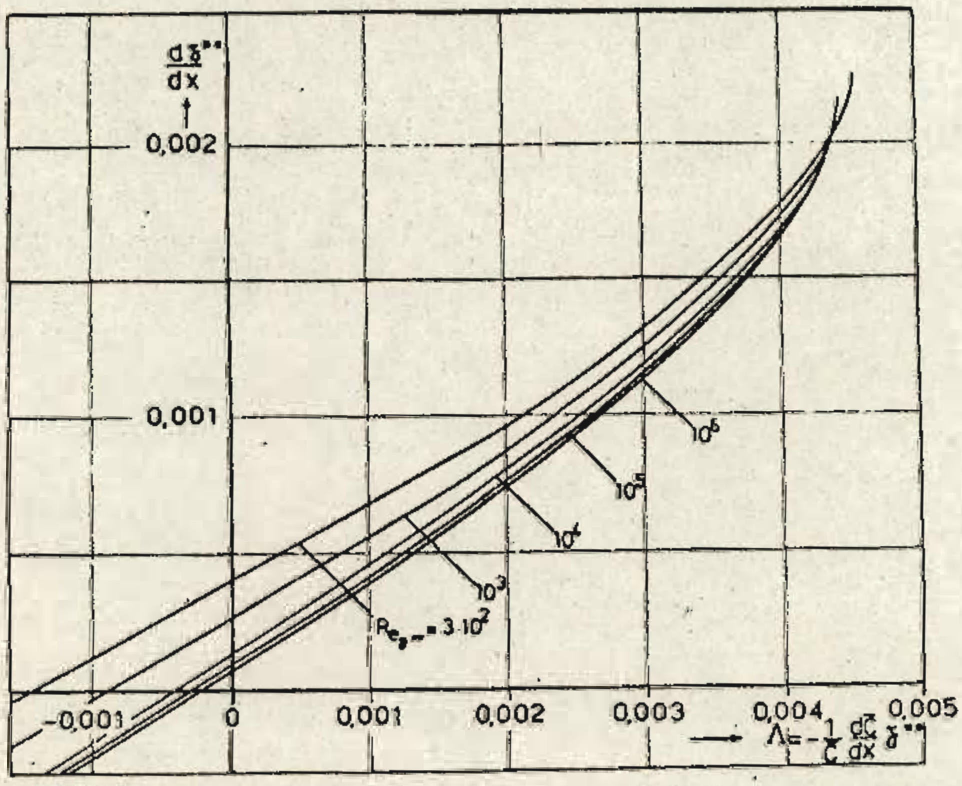
Figure(2) Hypothetical velocity profile of turbulent boundary layer



Figure(4) Local skin friction coefficient versus Euler number with momentum thickness Reynolds number is parameter



Figure(6) Shape parameter versus pressure gradient parameter for turbulent boundary layers at $R_{e_{\delta^{**}}} \geq 6000$



Figure(7) Slope of the momentum thickness versus Euler number with momentum thickness Reynolds number as parameter