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E. El-Konyaly
Electrical Power & Machine Department, Faculty of Engineering, Mansoura University, Mansoura, Egypt.

A. El-Desouky
Electrical Power & Machine Department, Faculty of Engineering, Mansoura University, Mansoura, Egypt.

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OPTIMAL STABILIZATION OF LINEARIZED SYSTEMS VIA RECEDING HORIZON TECHNIQUE

by

E. El-Konyaly and A.I. El-Desouky
FACULTY OF ENGINEERING
ELMANSOURA UNIVERSITY

ABSTRACT:

The paper investigates the role of Horizon length 'N' in tuning the dynamical behaviour of discrete time systems. The paper introduces ,as well, a new method for solving the receding horizon matrix differential equation, which aviods solution of linear matrix equation usually encountered. An efficient fast and rigid algorithm is developed which is well suited for on-line control using micro-processors.

INTRODUCTION:

Many recent puplications have considered application of modern control theory in the continuous or discrete data forms to improve and/or optimise the performance of dynamical systems. However, improved solution methods are usually needed for on-line control using micro-processors.

One of the modern design techniques that has found practical application is the infinite time regulator problem[1]. The technique assumes that the resultant closed-loop system is stable and possesses certain desirable damping characteristics so that the practical performance of the system will be satisfactory. This technique can be interpreted as the minimization of the cost functional

$$J_1 = \lim_{N\to\infty} \sum_{i=0}^{N} x'(i)\, \underline{Q}\, x(i) + u'(i)\, \underline{R}\, u(i) \tag{1}$$

subject to the system equation

$$x(i+1) = \underline{A}\, x(i) + \underline{B}\, u(i) \tag{2}$$

where $x \in R^n$, $u \in R^m$, $A \in R^{n\times n}$, and $B \in R^{n\times m}$ are system matrices, $'$ implies transpose, $Q \in R^{n\times n}$ and $R \in R^{m\times m}$ are positive semi difinite and positive difinite symmetric matrices, respectively, N is the number of samples. Complete controllability is essential for the resultant feedback signals to be effective. The optimal control law is given by

$$u(i) = -R^{-1} B' (A')^{-1} [P(i) - Q] \tag{3}$$

where,

$$P(i) = Q + A' P(i+1) [I + B' R^{-1} P(i+1)]^{-1} A \tag{4}$$

with $P(N) = 0$ (5)

This recursive relation renders constant feedback gains as $N \to \infty$ (2,3).

Following the above procedure two difficulties arise:

(1) the matrix Q which is a function of system matrices must be computed. Different techniques are available for the calculation of the matrix Q (4,5), however, the methodology itself is time consuming which might pose some limitation for on-line purpose (6).

(2) The solution of equation (4) is another time consuming process.

Recent development have shown that a finite time interval (in contrust with the infinite time interval) can be used to calculate stable feedback gains . This strong finding together with the receding horizon notion (7), present a powerful method for calculating stabilising feedback gains.

The paper adapts the receding horizon notion to devise a new technique for optimal stabilisation of discrete data systems. The asymptotic behaviour of the closed-loop system under study is examined using this suggested algorithm.

DEVELOPMENT OF THE SOLUTION METHOD:

Given the discrete linear system described by equation(2). It is desired to find the control sequence

$$u(0), u(1), \ldots \ldots \ldots u(N-1) \quad (6)$$

that minimizes the modified cost functional

$$J_2 = \sum_{i=0}^{N-1} u'(i)\, R\, u(i) \quad (7)$$

subject to system's equation (2) and the boundary constraint

$$x(N) = 0 \quad (8)$$

where $R \in R^{m \times m}$ positive definite matrix to be specified and N is the horizon length. The control law for this case can be easily found to be

$$u(i) = -R^{-1} B' M^{-1}(N)\, A\, x(i), \qquad N \geq n+1 \quad (9)$$

where M(N) can be obtained from[7];

$$\left.\begin{aligned} M(i+1) &= A^{-1} M(i) (A')^{-1} + B R^{-1} B' \\ M(0) &= 0 \end{aligned}\right\} \quad (10)$$

Equation (10) can be solved by successive substitution so that

$$M(N) = \sum_{i=0}^{N-1} A^{-i} B R^{-1} B' (A')^{-i} \quad (11)$$

Now, the problem is reduced to the selection of particular value for N such that a desired eigen-values pattern and consequently a specific response for the closed-loop system is obtained.

In the conventional regulator problem, the matrix Q enables the designer to locate the closed-loop poles in the prescribed postion ,he desires,inside the unit circle. In receding horizon method the matrix Q is absent. However,the matrix $M^{-1}(N)$ will affect directly the feedback law of equation (9). Cosequently, the horizon time N is directly affecting the solution.

Kleinman[3] and Kown[7] have shown that the number of samples N should be $N \in [n+1, \infty]$,where n is the dimension of the system. In classical design procedure the choice of N=n+1 results in a very costly gains (as these gains are large). Conversely as N grows the convergence properties of the algorithm are impaired. A proper upper limit for N is one of the objective points in this study.

ALGORITHM:

Given the continuous time system

$$\dot{x}_1 = A_1 x_1 + B u_1 \tag{12}$$

which is to be controlled using a micro-processor as shown in figure (1) . If a sampling period is chosen properly then

$$x_1((i+1).\Delta) = e^{A_1\Delta} x_1(i\Delta) + \int_{i\Delta}^{i\Delta+\Delta} e^{A_1(i\Delta+\Delta-\tau)} B_1 u_1(i\Delta)d\tau$$

$$= e^{A_1\Delta} x_1(i\Delta) + \int_0^{\Delta} e^{A_1\delta} d\delta\, B_1 u_1(i\Delta) d\delta \tag{13}$$

where the variable of integration is replaced by

$$\delta = i\Delta + \Delta - \tau$$

The equivalent discrete system for the system described by equation (13) is given by equation (2). The sampling period must satisfy the inequality

$$\Delta \leqslant 1/2 \left| Re\ (\lambda_{max}) \right|$$

for good reproduction of signals[1].

where λ_{max} is the maximum eigen-value of the system matrix A_1.

The solution algorithm can be summerized as follows.

Step 1: Set $R = I$, $\Delta = 1/2 \left| Re(\lambda_{max}) \right|$, h (step size)$= 0.1 \left| Re(\lambda_{max}) \right|$, $N = n+1$

Step 2: Compute the matrices A and B as follows

$$A = S^j \quad , \quad B = \left(\sum_{i=0}^{j} S^j \right) B$$

where

$$S = \left(I - \frac{h}{2} A + \frac{h^2}{12} A^2 \right)^{-1} \left(I + \frac{h}{2} A + \frac{h^2}{12} A^2 \right)$$

and

$$j = \Delta / h$$

(consult Ref. (8) for further details)

Step 3: Compute

$$M(N) = \sum_{i=0}^{N-1} A^{-i} B R^{-1} B' (A')^{-i}$$

$$K = - R^{-1} B' M^{-1}(N) A$$

(consult flow chart Figure(2))

Step 4: Set $N = N+1$ and repeat the process until $\|K\|_2 = \left(\sum_{ij} K_{ij}^2 \right)^{1/2}$ reaches a constant value.

NUMERICAL EXAMPLE:

Data for a continuous time system which represents a linearized power system model, is given as;

$$A_1 = \begin{bmatrix} 0. & 1. & 0. & 0. \\ -22.5 & 0. & -47.4 & 0. \\ -0.086 & 0. & -.195 & .129 \\ 85.7 & 0. & -822. & -20. \end{bmatrix} , \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1000. \end{bmatrix}$$

The eigen-values of the matrix A_1 are

$+0.25 \pm j4.96$

$-10.4 \pm j3.31$

The sampling period is given by

$$=1/2|Re\,\lambda_{max}| \qquad =0.048 \text{ sec.}$$

The equivalent discrete system is calculated and is found to be.

$$A = \begin{bmatrix} .998 & -.009 & -.002 & 0 \\ .216 & .998 & .454 & 0 \\ .001 & 0 & .996 & -.001 \\ -.9 & .004 & 8.687 & 1.2 \end{bmatrix} , \qquad B = \begin{bmatrix} 0 \\ 0.001 \\ -.006 \\ 10.677 \end{bmatrix}$$

The open-loop poles are

$1.04 \pm j0.04$

0.99 and 1.12 indicates initially unstable system.

The state feedback gains are

$$K = [\,0.3 \quad 0.06 \quad 0.5 \quad -0.04\,]' \qquad \text{at } N = 22n$$

The closed-loop poles are found to be

$0.94 \pm j\,0.04$

0.89

0.97

Observe that only the unstable poles are shifted.

The eigen-value loci as well as the varation of $\|K\|$ as N changes are shown in figure(3) for $N \in [n+1\ ,\ 25n]$,where n is the order of the matrix A.

DISCUSSION AND OBSERVATION:

(1) The control energy $\|K\|$ is monotonically decreasing function of the horizon length N that is

i- As $N \to 0$ $\|K\| \to \infty$

ii-As $N \to \infty$ $\|K\| \to$ constant

(2) For small values of N , the closed-loop poles are forced deeply towards the origin of the unit circle with large

imaginary parts indicating fast response but the margin of the transient behaviour is slightly improved.

(3) As N increases ,the margin of the transient response is improved (see figure (3)).

(4) As N increased the closed-loop poles move towards the circumference of the unit circle with reduction in their imaginary parts.For N as large as 25n ,the gain K approaches its constant value indicating that no more eigen-values movements can be achieved and the process terminated. It is interesting to note that the stable open-loop poles hold their original locations while the unstable poles assume new location inside the stable region of the Z-plane.

CONCLUSION:

The receding horizen technique is applied successfully to the design of discrete data systems. Light is focused on the relation between control energy variation and eigen-values distribution and the horizen length since, the state weighting matrix Q is absent. It is concluded that the horizen length N can be used, to play the role of Q ,to tune the response of the closed-loop system while offering a dramatic computational advantages over the conventional regulator problem. The technique is suitable for online applications.

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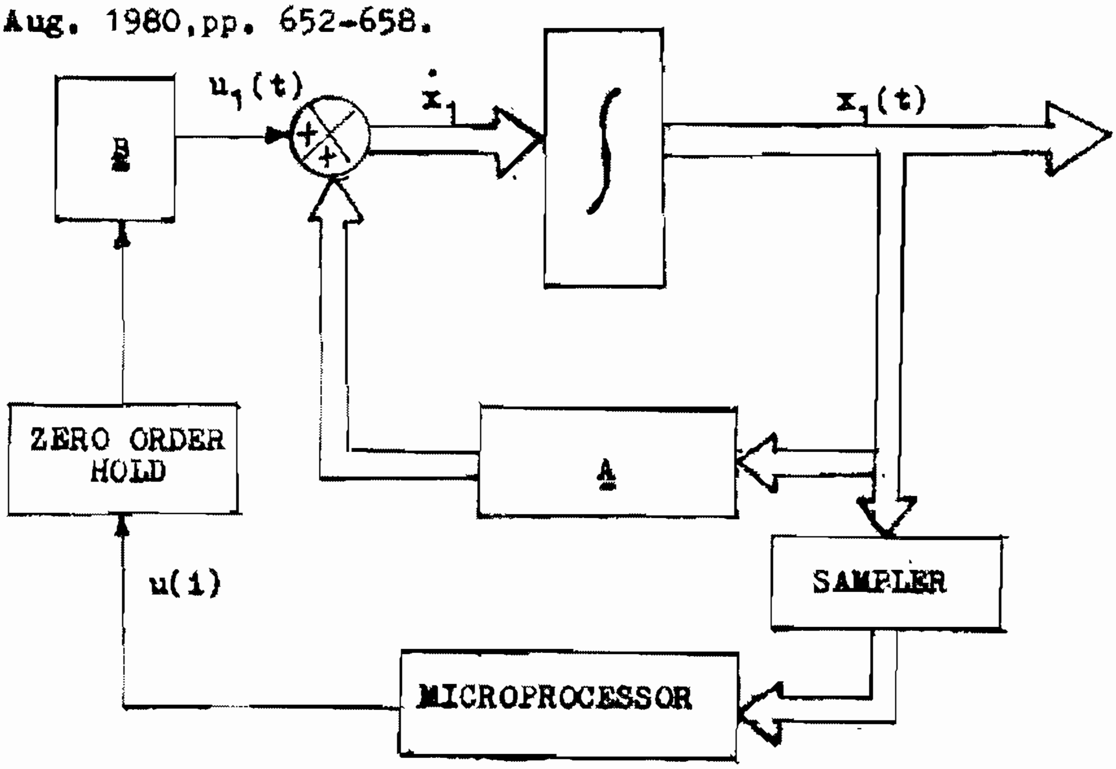

FIGURE (1) SAMPLED DATA CONTROL SYSTEM

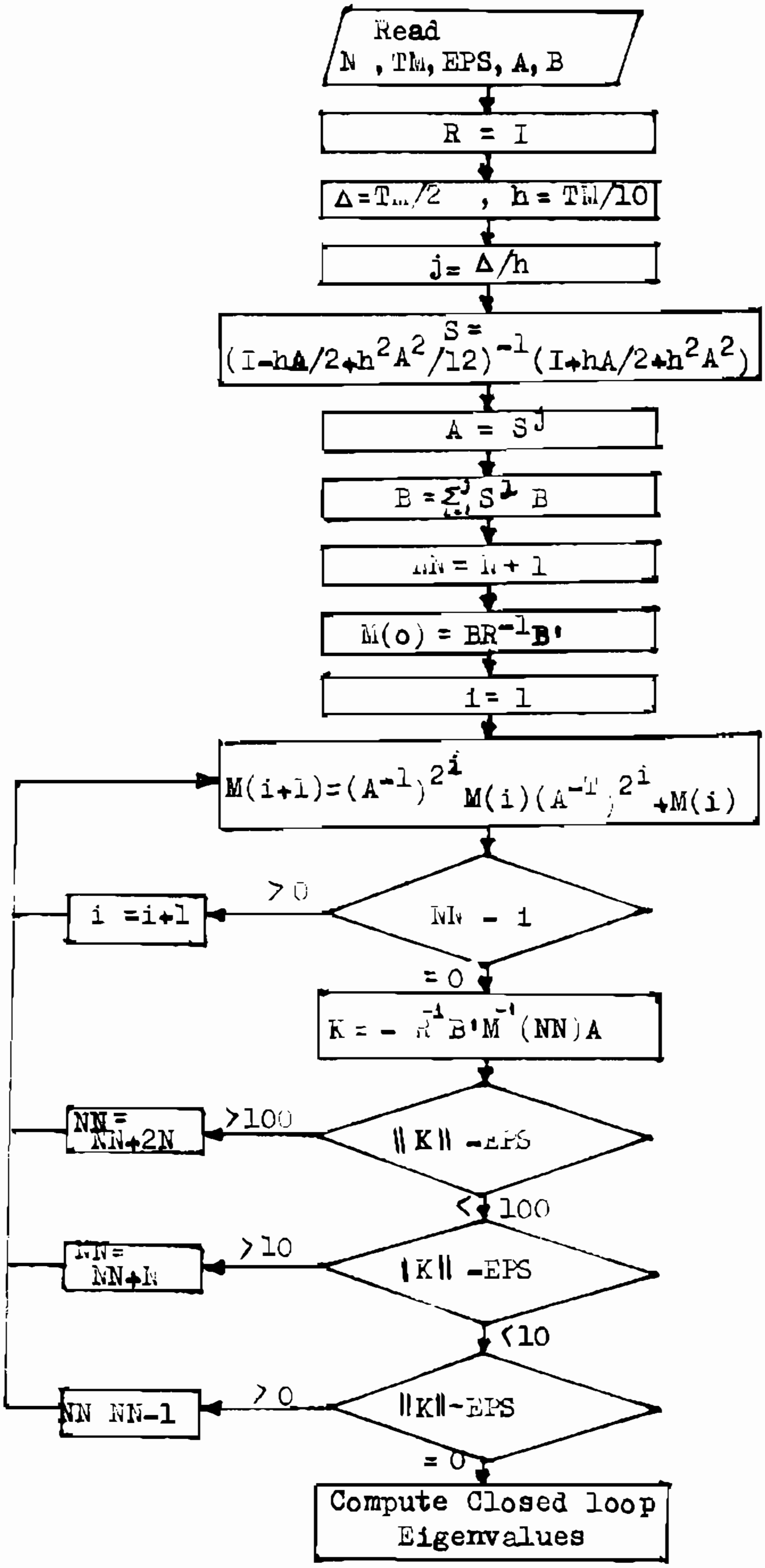

Fig.(2) Design of The Receding Horizon Controller

For Discrete Time Systems

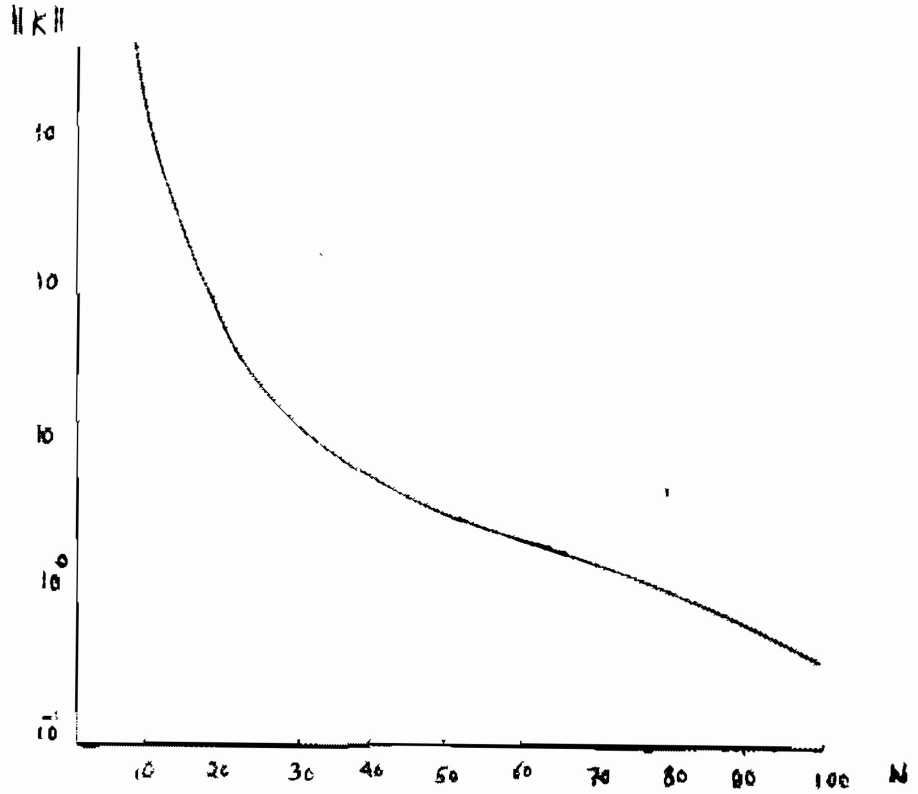

Fig.(3) a- Control Energy Variations

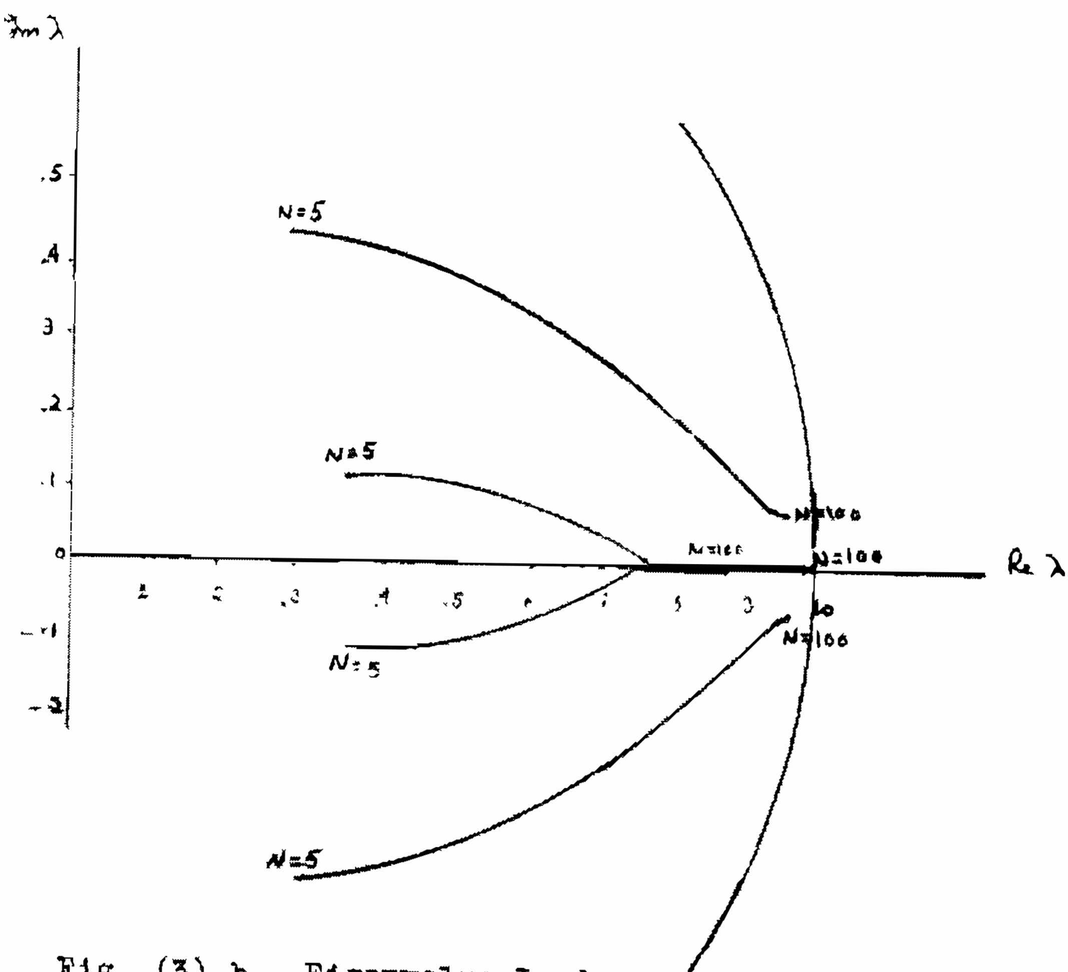

Fig. (3) b- Eigenvalue Loci