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Partially-Full Pipe Crossings.

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PARTIALLY-FULL PIPE CROSSINGS

BY

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ABSTRACT

The design of partially-full pipe crossing includes mainly the determination of the pipe diameter and the corresponding upstream water depth.

One of the two equations required for the design purposes was derived from momentum relationships. The second equation was developed from data analysis of conducted laboratory experiments.

The suggested two equations can be easily applied for design purposes and also for estimating the discharge under free and submerged flow conditions.

INTRODUCTION

Pipe crossings are relatively economical, easily built and are commonly used in irrigation and drainage projects, where it is required to transport water from one location to another, traversing various topographic features along the way.

In conveyance structures such as road crossings, inverted syphons, aqueducts, canal outlets, drain inlets, etc., pipes may or may not be subjected to internal hydrostatic pressure. Partially-full pipes are required mainly in road crossings inorder to allow for the passage of weeds, trash or ice.

The hydraulic design of a pipe crossing flowing full consists of selecting a pipe diameter that will result in a maximum velocity allowable for the local existing transitions. The maximum upstream invert elevation of the pipe is then determined by subtracting the pipe diameter and all the existing head losses from the upstream normal water surface elevation in the canal. Such a design has been studied satisfactorily, well known in practice and may be found in many referenceal.

Existing methods for the hydraulic calculations of partially-full pipe crossings are relatively complicated. The characteristics of the flow are controlled by many variables, including the inlet geometry, slope, size, roughness, approach and tail water conditions, etc. For an adequate determination of such a flow, Chow[4] recommended laboratory or field investigations.

A theoretical study developed from momentum relationships is presented herein to evaluate the discharge in a partially-full, horizontal and straight pipe crossing. Based on the

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results of conducted experiments, another equation required for design purposes is suggested to be used in combination with the discharge equation. The suggested simple equations gave good agreement with the measured parameters and may be applied for both design and flow measuring purposes under free and submerged flow conditions.

MOMENTUM THEORY

A theoretical flow discharge equation will be developed herein for a horizontal straight pipe as shown in Fig. 1. For steady flow state, the momentum equation can be written between sections 1 and 2 in the direction of flow as follows:

$$F - F_f = \frac{\gamma Q}{g} (\beta_2 V_2 - \beta_1 V_1) \qquad ......(1)$$

in which F = the resultant force of the pressure distribution between the two flow cross sections, F_f = the friction or drag force acting on the surface of the control volume, Q = the water discharge, γ = specific weight of the water, g = the acceleration of gravity, β_1 and β_2 = momentum coefficients for the two flow sections, V_1 and V_2 = the average velocity of flow at sections 1 and 2 respectively.
When uniform velocity distribution is assumed and the friction force is neglected, equation (1) may be written as follows:

$$F = \frac{\gamma Q}{g} (V_2 - V_1) \qquad ......(2)$$

By assuming hydrostatic pressure distribution,

$$F = \gamma a_1 z_1 - \gamma a_2 z_2 - R$$

$$= \gamma a_1 z_1 - \gamma a_2 z_2 - \gamma (a_1 z_1 - az)$$

$$F = \gamma az - a_2 z_2 \qquad ......(3)$$

in which a = the area of flow in the pipe section corresponding to a water level equals to the upstream water level, i.e. = the area of flow in the pipe to a depth = y, a_2 = the area of flow at section 2, z and z_2 = the depth of centroid below water surface for a and a_2 respectively, and

a_1 = the area of flow at section 1.

Taking into consideration that:

$$V_1 = \frac{Q}{a_1}; \qquad V_2 = \frac{Q}{a_2}$$

The momentum equation may be written as follows:

$$az - a_2 z_2 = \frac{Q^2}{g} (\frac{1}{a_2} - \frac{1}{a_1}) \qquad ......(4)$$

Solving equation (4) for the discharge,

$$Q = \sqrt{\frac{a_1 a_2 \, (az - a_2 z_2) g}{(a_1 - a_2)}} \qquad \ldots\ldots(5)$$

When the downstream water surface does not influence the upstream water surface, i.e., when the hydraulic control is at the upstream end of the pipe, the contracted water depth at section 2 is considered critical ($\simeq 0.67\ y$).
When the downstream water surface is raised to a stage that causes some increase in the contracted depth and consequently in the upstream water surface, the right hand side of equation 5 should be corrected by a variable coefficient, α , depending on the degree of submergence. For a given discharge, $\alpha = 1$ when the downstream water surface is equal to, or lower than the water surface at the contracted critical section.

Thus, equation 5 may be written as follows:

a) when free flow exists, i.e., $y_2 = y_c$, the critical depth,

$$Q = \sqrt{\frac{a_1 . a_c \ (a.z - a_c z_c) . g}{(a_1 - a_c)}} \qquad \ldots\ldots(6)$$

in which a_c = the area of flow for the critical depth, z_c = the depth of centroid below water surface for a_c.

b) for submerged flow conditions, i.e., $y_2 > y_c$

$$Q = \alpha \sqrt{\frac{a_1 a_2 \ . \ (az - a_2 z_2) . g}{(a_1 - a_2)}} \qquad \ldots\ldots(7)$$

For the general case, considering that

$$\frac{a}{a_2} = K, \quad \frac{a_1}{a_2} = K_1, \quad \frac{z}{z_2} = \psi \qquad \ldots\ldots(8)$$

equations 5 and 8 yield to

$$Q = \alpha . m \, a_2 \sqrt{2g \, z_2} \qquad \ldots\ldots(9)$$

in which m = a coefficient of discharge and is calculated with the assumption that $y_2 = 0.67\ y$, from

$$m = \sqrt{\frac{K_1 \ (K \psi - 1)}{2 \ (K_1 - 1)}} \qquad \ldots\ldots(10)$$

For the case of free flow, equation 9 yields

$$Q = m\ a_c \sqrt{2g.Z_c} \qquad ......(11)$$

The results of experimental investigations, conducted within the limits L/D = 8-20 showed that the coefficient, m, may be assumed = 0.96 as a mean value for initial calculations. In the case of a partly full crossing, value of the ratio y/D is usually assumed within the range 0.75 - 0.85 and for that ratio, the value of m is closely equal to 0.96. For a depth of water, y_2, flowing through a pipe of diameter = D, values of a_2 and z_2 may be easily calculated by using the following table:

y_2/D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
ϕ_a	0.041	0.112	0.198	0.293	0.393	0.498	0.587	0.674	0.745	0.785
ϕ_z	0.045	0.080	0.120	0.165	0.210	0.260	0.310	0.365	0.425	0.500

where, $a_2 = \phi_a \cdot D^2$

$z_2 = \phi_z \cdot D$

From the above table, the calculated values of y/D are plotted against $a\sqrt{z}/D^{2.5}$ on a logarithmic scale, as shown in Fig. 2. It is apparent that the plot may be approximated by two straight lines, intersecting nearly, at a point, where y/D = 0.67.

The case discussed herein belongs to the straight continuous line, shown in Fig. 2, where the ratio of y_2/D does not exceed 0.67. For that limited range of y_2/D values, the functional relationship defined by equation 12 may be used.

$$\frac{a_2\sqrt{z_2}}{D^{2.5}} = 0.65\left(\frac{y_2}{D}\right)^{1.85} \qquad ......(12)$$

or

$$a_2\sqrt{z_2} = 0.65 \cdot y_2^{1.85} \cdot D^{0.65} \qquad ......(13)$$

Combining equations 9 and 13, an equation for submerged flow discharge may be written as follows,

$$Q(m^3/sec) = 2.77\ \alpha\ (y_2)^{1.85} \cdot D^{0.65} \qquad ......(14)$$

where m is taken as 0.96.

Equation 14 may be rewritten in terms of y instead of y_2 as follows,

$$Q(m^3/sec) = 1.32\ \alpha\ y^{1.85} \cdot D^{0.65} \qquad ......(15)$$

in which y = the upstream water depth, measured from the invert pipe level, $y = 1.5\ y_2$

The coefficient of submergence, α , may be calculated from $\alpha = (\frac{y_c}{y_2})^{1.85}$

Equation 14 may also be written in terms of y_c as follows,

$$Q(m^3/sec) = 2.77\ (y_c)^{1.85}\ .\ D^{0.65} \qquad ......(16)$$

Equation 16 may be used for calculating the critical water depth, y_c, corresponding to a discharge Q flowing through a partially-full pipe of diameter D. For the case, where y_c and D are known, the discharge may be calculated from equation 16 for free flow and submerged flow conditions.

EXPERIMENTAL VERIFICATION

Experimental investigations were conducted in the Hydraulics Laboratory of the University of Baghdad, IRAQ. The writer performed a set of experiments on pipes laid inside a horizontal floor flume with a rectangular cross section 30 cm. wide. The arrangement of performance of tests is shown in Fig. 3 and Fig. 4. A series of tests were conducted on a single pipe of diameter 10 cm and for two different pipe materials: aspectos and polyethelene. Another series of tests were conducted on two polyethelene pipes of diameter 7.5 cm. each.

The mentioned series of tests were conducted on two different pipe lengths, L = 80 and 150 cm and thus the ratio of (L/D) values ranged within 8-20.

Water was supplied by a recirculating constant head system and the discharge, Q, was measured by a calibrated 90° sharp edged triangular weir. For every type of pipes, the experiments were repeated for three different discharges ranging between 3500 and 1200 cu. cm. per sec. In each experiment of constant discharge, the water level downstream, the pipe was kept at about (0.80 to 0.85) D and then was lowered gradually by means of an inclined weir of variable crest, located at the flume end.

The depth of water upstream and downstream of the pipe were measured by point gages, reading to the nearest 0.1 mm. Values of the discharge coefficient, m, were calculated for the data resulting from the experiments by applying equation 10. It seemed clearly that the values of discharge coefficient, m, are affected mainly by the ratio y/D. The plot of m against y/D given in Fig. 5, shows that the scattering of m values is nearly limited within the range 0.92 to 0.99. The plot shown in Fig. will yield a straight line relationship within the applied range of y/D = 0.9 - 0.6. The resulting empirical equation is

$$m = 1.135 - 0.22 \frac{y}{D} \qquad ......(17)$$

For the case of partially-full pipe crossings, the design ratio of y/D is usually taken within 0.75 to 0.85. For such a case and for other initial hydraulic calculations, a mean value of m = 0.96 will produce satisfactory results for practical use.

When the downstream water level influnces the water depth, y, upstream of the pipe, another equation is required for determining the coefficient of submergence, α , included in the discharge equation. As indicated by Skogerboe[3], the ratio of any flow depth measured upstream from the contracted depth, y_2, can be used as a submergence ratio. Developing a submerged flow discharge equation in flow-measuring flumes, Skogerboe developed a dimensionless parameter by trial and error and used it as a pi-term in the dimensional analysis. The developed pi-term, $(y_1-y_3)/y_2$ was correlated by Skogerboe with another pi-term expressed by the submergence, y_3/y_1. The writer made an analogous study on the generated experimental data in order to find such a correlation for the case discussed herein. As a final result, a logarithmic plot of $\frac{2(y - y_3)}{y_c}$ against $\frac{y_c}{y_3}$ was found to yield a straight line relationship.

For submerged flow conditions, the empirical equation resulting from Fig. 6 is

$$y = y_3 + \frac{y_c}{2} \left(\frac{y_c}{y_3}\right)^{2.33} \qquad ......(18)$$

in which y, y_c and y_3 are measured from the invert pipe level.

In case, when $y_3 = y_c$, the above formula yield to

$$y = 1.5\, y_c \qquad ......(19)$$

Equation 18 is suggested to be used in combination with the discharge equation 16 for the hydraulic calculations of partially-full pipe crossings. The presented equations may be simply applied for both free-flow and submerged conditions.

To make a comparison between the measured parameters, y and Q, and that calculated from equations 15, 16 and 18, a sample of the generated data with such a comparison is given in Table 1.

NUMERICAL EXAMPLES

The design of a partially-full pipe crossing mainly includes the determination of the pipe diameter, D, and the corresponding upstream water depth, y. The given prameters usually include the water discharge, Q, and the downstream water depth, y_3, and the ratio y/D.

a) For free-flow condition, i.e., when the downstream water depth, y_3 is less than the critical, y_c, equations 16 and 19 are used, as illustrated in the following example.

Example (1):

The following data are given:
the design discharge, $Q = 2.4\ m^3/sec$.
the downstream water depth, $y_3 = 0.75$ m.
the ratio of y/D value is about 0.80
The values of y and D may be determined as follows:
By trial and error, assume D = 1.5 m.
From equation 16, $Q = 2.77\ y_c^{1.85}\ D^{0.65}$, we obtain:

$y_c = 0.80$ m.

$y = 1.5\ y_c = 1.20$ m (for free flow condition)

y/D= 0.80

(i.e. the assumption of the D value meets the design requirements).

b) For the submerged condition, i.e., when the downstream water depth, y_3, exceeds the critical depth, y_c, equations 16 and 18 are used, as illustrated by the following example.

Example (2):

The following data are given:
the design discharge $Q = 2.4\ m^3/sec$.
the downstream water depth, $y_3 = 1.20$ m.
the ratio of y/D value is about 0.80.

The values of y and D may be determined as follows:
By trial and error, assume D = 1.70 m.
From equation 16, $y_c = 0.768$ m., i.e. submerged flow condition.

From equation 18, $y = y_3 + \frac{y_c}{2}\left(\frac{y_c}{y_3}\right)^{2.33}$, one obtains

$$y = 1.34 \text{ m.}$$

$$\frac{y}{D} = \frac{1.34}{1.70} = 0.79$$

(within the range of required ratio).

To use the partially-full pipe crossing as a measuring device for the flowing discharge, it is necessary to measure only the upstream flow depth, when free flow exists, whereas the two depths upstream and downstream the pipe must be measured under submerged-flow conditions.

The discharge may be calculated from formula 16,

$$Q = 2.77\ y_c^{1.85}\ D^{0.65},$$

and y_c may be calculated from formula 18,

$$y = y_3 + \frac{y_c}{2}\ (\frac{y_c}{y_3})^{2.33}$$

Example (3):

The following data are given for submerged flow conditions. Upstream and downstream water depths are 1.35 and 1.10 m respectively.
Diameter of pipe crossing = 1.70 m.

The discharge may be calculated as follows.

From the equation $y = y_3 + \frac{y_c}{2}\ (\frac{y_c}{y_3})^{2.33}$

$$y_c = 0.868 \text{ m}$$

$$Q = 3 \text{ m}^3/\text{sec}.$$

The discharge, Q, may also be calculated from equation 15,

$$Q = 1.32\,\alpha\ y^{1.85}\ D^{0.65} = 3 \text{ m}^3/\text{sec}$$

in which $\alpha = (\frac{y_c}{0.67y})^{1.85} = 0.9266.$

CONCLUSION:

Partially-full pipes may be required mainly in road crossings for the passage of floating materials such as weeds, trash or ice. Two fundamental equations have been suggested for the hydraulic calculations of such crossings under both free flow and submerged flow conditions.

The first equation was based upon the momentum relationships and is written as

$$Q(\text{m}^3/\text{sec}) = 2.77\ (y_c)^{1.85}\ D^{0.65} \qquad \text{......(16)}$$

The second empirical equation was developed from the analysis of expirimental data and is written as

$$y = y_3 + \frac{y_c}{2}\ (\frac{y_c}{y_3})^{2.33} \qquad \text{.......(18)}$$

The above two equations may be easily used in combination for the design purposes of such crossings and also for estimating the flowing discharge. It seems clearly from the given numerical solved examples that the required parameters can be found by simple calculations.

Experimental results, obtained for L/D ratio within the range 8 to 20, indicate that the observed data are closely equal to the theoretical values computed by the above two suggested equations.

APPENDIX I- REFERENCES

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3. Skogerboe, Gaylord V., and Hyatt, M. Lyon, "Analysis of Submergence in Flow Measuring Flumes", Journal of the Hydraulics Division, ASCE, Vol. 93, No. HY4, Proc. Paper 5348, July, 1967, pp. 183 - 200.

4. Ven Te Chow, "Open Channel Hydraulics", Chapter 17, McGRAW-HILL Book Company, New York, 1959.

APPENDIX II- NOTATION

The following symbols are used in this paper:

a = area of cross section;
D = diameter of pipe;
F = hydrostatic force;
F_f = frictional or drag force;
g = gravitational acceleration;
L = length of pipe;
m = coefficient of discharge related to free flow;
Q = flow rate or discharge;
v = mean water velocity;
y = water depth;
z = depth of centroid below water surface;
α = coefficient of discharge for submergence;
β = momentum correction coefficient;
γ = specific weight of water;
ϕ_a, ϕ_z = coefficients;

subscript 1, 2 and 3 - value at section 1, 2 and 3 respectively; subscript c - value at critical section.

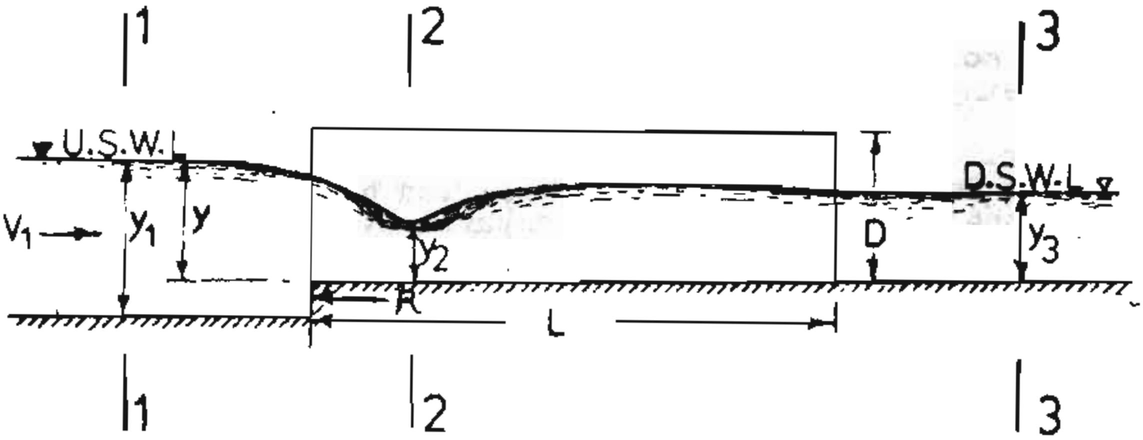

FIG. 1 _ DEFINITION SKETCH FOR PARTIALLY-FULL PIPE CROSSING.

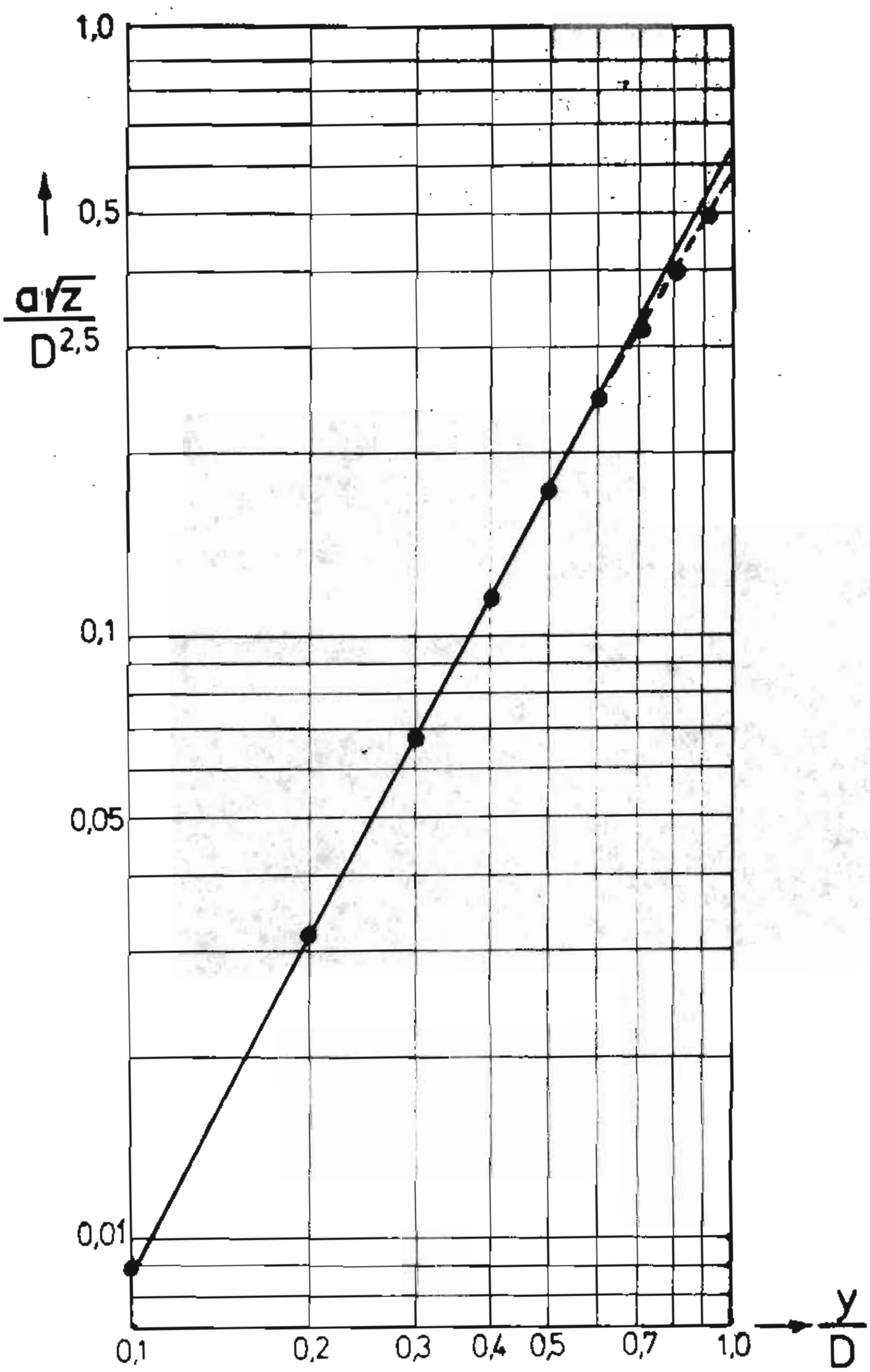

FIG. 2 _ RELATION BETWEEN $\frac{y}{D}$ and $\frac{a\sqrt{z}}{D^{2,5}}$

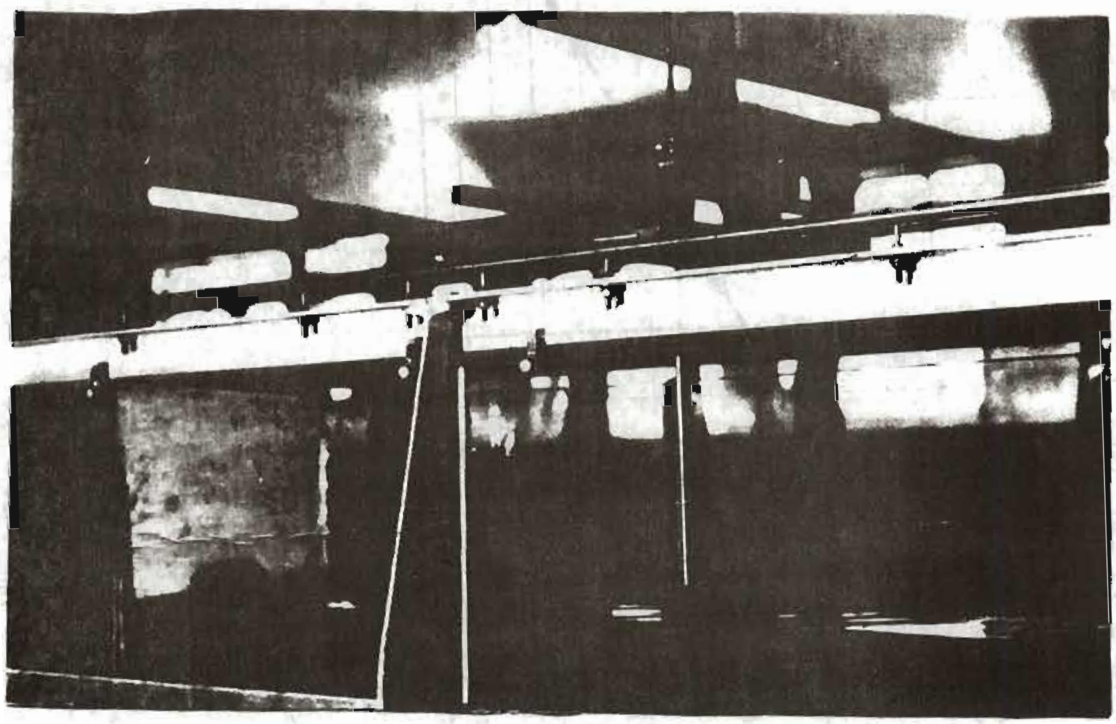

FIG. 3 - UPSTREAM END OF THE PIPE

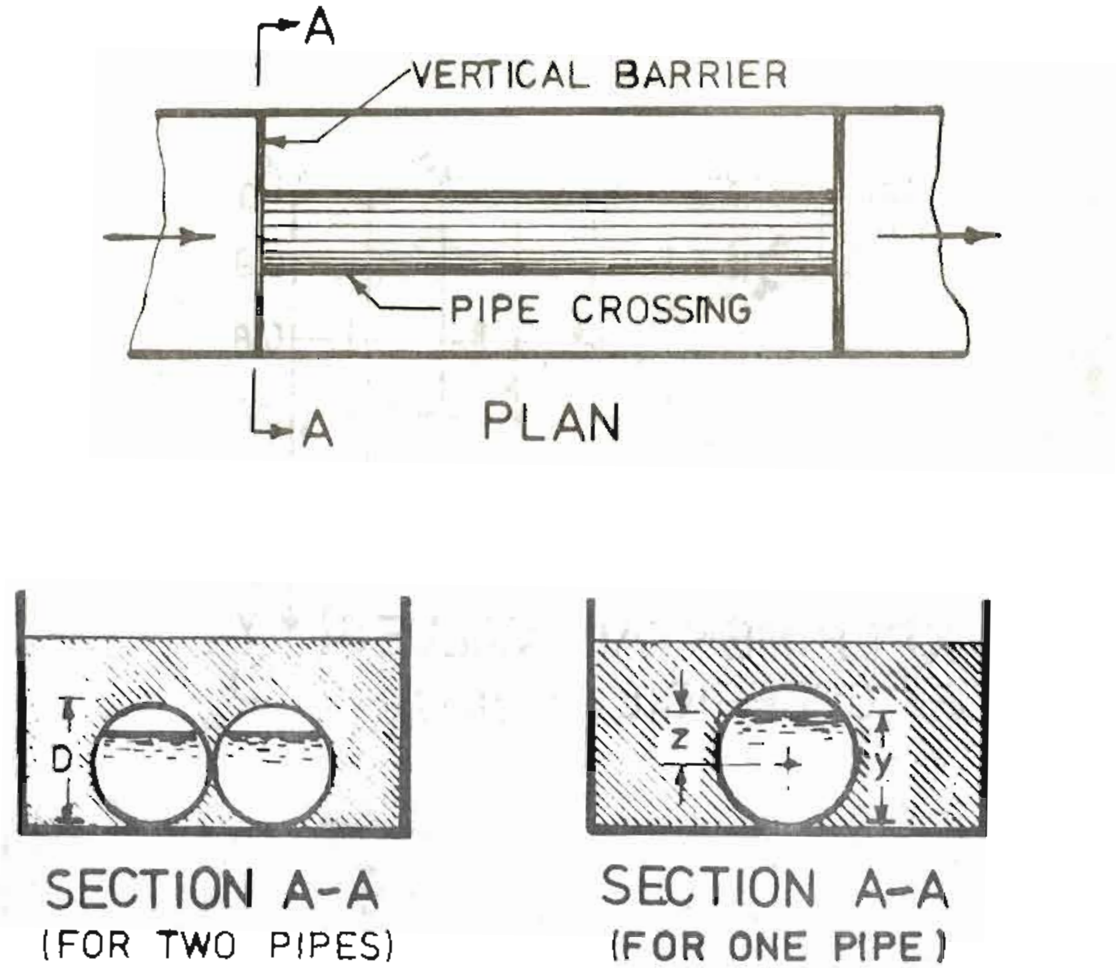

FIG. 4 – EXPERIMENTAL PIPE CROSSING

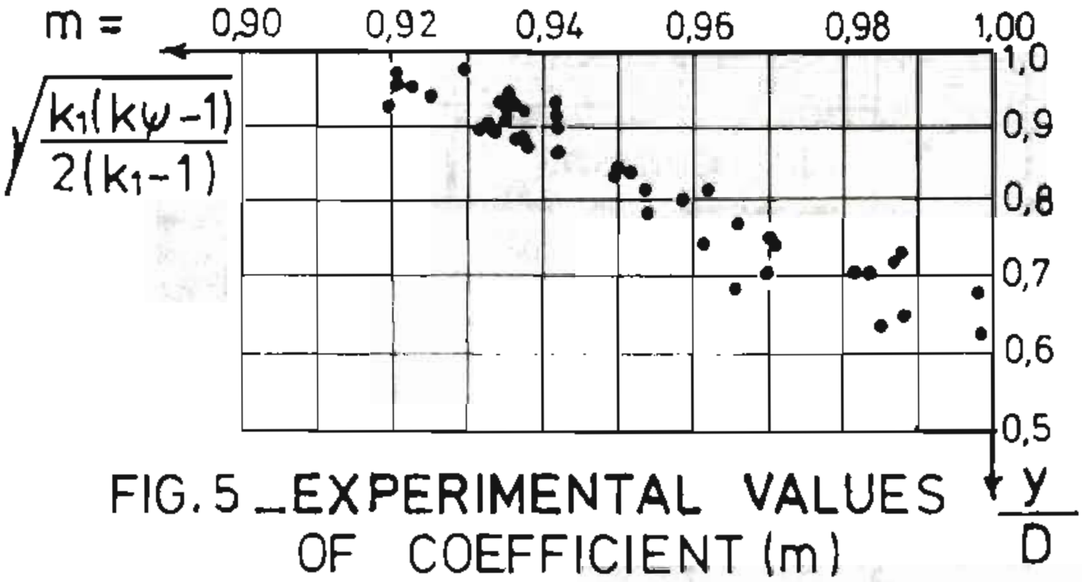

FIG. 5 _ EXPERIMENTAL VALUES OF COEFFICIENT (m)

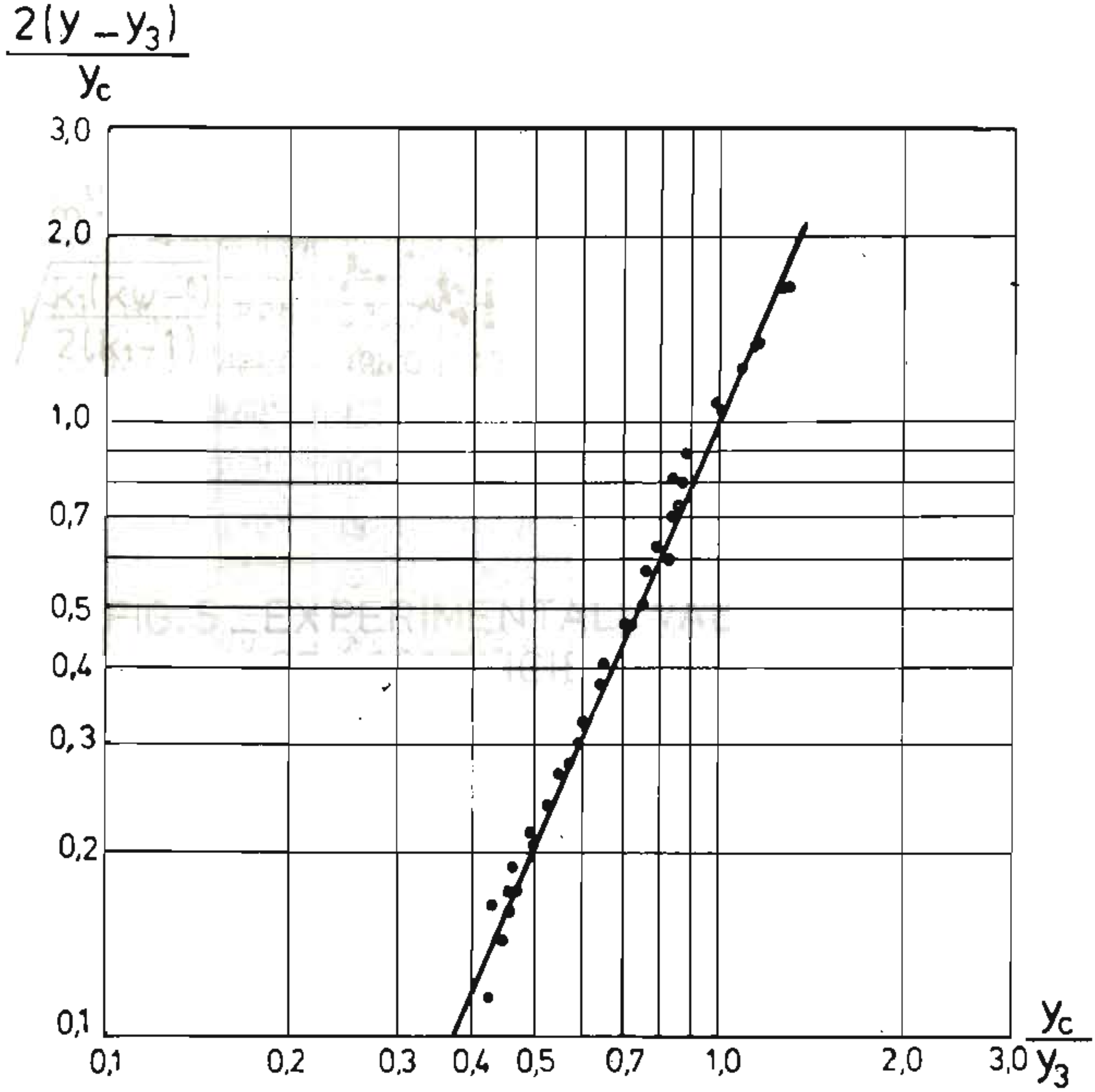

FIG. 6 _ RELATION BETWEEN $\frac{y_c}{y_3}$ and $\frac{2(y - y_3)}{y_c}$

TABLE 1 – SAMPLE TABULATION FOR THE COMPARISON BETWEEN MEASURED AND CALCULATED PARAMETERS

NO.	MEASURED DATA			CALCULATED DATA				REMARKS
	Q, cm^3/sec	y, cm.	y_3, cm.	y_c, cm (eq. 16)	y, cm (eq. 18)	α	Q, (eq. 15)	
1	1970	6,83	5,37	4,468	6,82	0,955	1963	ONE PIPE (ASPECTOS)
2		7,09	5,80		7,01	0,891	1968	
3		7,51	6,55		7,46	0,801	1968	L = 80 cm.
4		7,90	7,15		7,89	0,730	1970	D = 10 cm.
5		8,20	7,55		8,20	0,681	1969	
6		8,45	7,85		8,45	0,644	1968	
7		8,81	8,28		8,81	0,596	1968	
8		9,12	8,67		9,14	0,560	1971	
9		9,45	9,05		9,48	0,524	1970	
1	1212	4,20	3,60	2,61	4,22	0,870	1210	2 PIPES (POLYETHELENE)
2		4,46	4,05		4,52	0,779	1211	
3		4,87	4,50		4,87	0,662	1211	
4		5,19	4,95		5,24	0,588	1210	L = 150 cm.
5		5,60	5,38		5,62	0,511	1210	D = 7,5 cm.
6		5,99	5,75		5,96	0,451	1210	
7		6,38	6,18		6,36	0,401	1209	
8		6,66	6,47		6,63	0,371	1211	