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INITIAL TORSIONAL STIFFNESS OF REINFORCED CONCRETE L-BEAMS UNDER COMBINED LOADS

By

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1. INTRODUCTION:

Extensive research has been conducted in the past three decates to investigate the behaviour of concrete beams under torsion. Torsion in concrete structures rarely occurs without other actions. Usually flexure, shear, and axial forces are also present. Agreat many of the more recent studies have attempted to establish the laws of interactions that may exist between torsion and other structural actions.

Because of the large number of parameters involved, some effort is still required to assess reliably all aspects of this complex behaviour. The available test data of reinforced concrete beams, subjected to combined bending, shear and axial forces, have dealt mainly with the strength in combined loading.

The aspect of torsional stiffness in combined loading has been dealt in few papers. Chinenkov (1) discussed qualitatively the torsional stiffness in combined bending and torsion.

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Pandit and Warwaruk (2) disussed the effect of flexure on the initial torsional stiffness of reinforced concrete beams of rectangular cross-section. Sharma and Pandit (3) investigated the torsional stiffness of concrete beams tested under axial compression, biaxial bending couples and torsion.

This paper describes the tests carried out on 18 plain and reinforced concrete beams under bending, shear and torsion. The measured initial torsional stiffness has been compared with an emperical expression for the initial torsional stiffness of reinforced concrete beams subjected to combined bending, shear and torsion.

2- TORSIONAL STIFFNESS

The torsional stiffness of a beam is defined as the twisting moment required to produce a unit angle of twist per unit length.

$$S = T/\theta \quad (1)$$

For a homogeneous material

$$S = K\,G \quad (2)$$

The torsional constant K for elastic material is given by

$$K = \frac{2}{G\theta} \iint \phi_e \, dx.dy \quad (3)$$

and for plastic material , it can be given by

$$K = \frac{2}{G.\theta} \iint \phi_p \, dx.dy \quad (4)$$

where ϕ_e and ϕ_p are elastic and plastic stress functions respectively.

The initial torsional stiffness referes to the initial slope of the torque-twist curve. It is given by the equation.

$$S_i = \lim_{T \to 0} \left[\frac{dT}{d\theta} \right] \qquad (5)$$

3. TEST PROGRAMME

Tests on eighteen beams were carried out in various combinations of torsion, bending and shear. The test specimens included plain and reinforced concrete beams. All beams had an L cross-section. Table (1) shows the scope of the tests:

TABLE 1 Testing programme

Serial No.	Load Combination at mid-span	Croup A plain	Group B reinforced	Group C reinforced	Total No. of specimens
1	Pure torsion	6	-	-	6
2	Torsion,bending and shear	-	6	-	6
3	Torsion and bending	-	-	6	6
Total		6	6	6	18

The details of reinforcement and dimensions of the beams are given in table (2). All the beams had an overall length of 1.2 m. The concrete in all the beams had a proportion of 1 : 1.5 : 3 with a water-cement ratio of 0.45. The test specimens and control cubes were compacted on vibrating tables and cured under 90 percent humidity.

4. TEST SET-UP AND TESTING PROCEDURE:

All specimens were tested on a M.A.N. 150 m.Kg torsion testing machine. The test set up is shown in fig. (1). The twisting moment was applied by the machine while the bending loads were applied by means of a hydraulic jack. A load cell was inserted between the hydraulic Jack and the point of application of the load in order to measure its value. For specimens of group (C), where two concentrated loads were applied,a spreader beam was used.

Two especial fittings were manufactured from steel plastes and angles which were fixed to the machine head in order to connect the beam ends to the machine. The machine can apply only pure torsion, in order to apply the vertical loads as well, the end fittings were resting on roller bearings which were generously greased to minimize friction. The left head of the machine is fixed while the right end rotates to give uniform twisting moment on the beam. Each specimen in group (B and C) was tested to failure by applying the loads in a series of increments. In each increment the vertical load was first increased and then the twisting moment. For each increment readings for vertical deflection and rotation were taken. The beams of group A , plain concrete beams, were tested under pure torsion only.

5. TEST RESULTS

The main test results are given in Table 3, 4 and shown in

Fig. 4 - 6 . All these curves are characterised by an initial straight part upto cracking, followed by a curved portion in the post cracking stage (for reinforced beams), where the decreasing slope shows loss of stiffness.

The presented torque-twist curves indicate that the initial torsional stiffness of reinforced concrete beams is reduced due to the presence of flexural moment and shear. The initial torsional stiffness of beams in group B tested under the combined action of bending, shear and torsion is about 56 % of that for pure torsion. While the initial torsional stiffness of the beams in group C tested under flexure and torsion is about 52 % of that for pure torsion.

The torsional stiffness in combined loading may be expressed by the following equation

$$S_i = \frac{S_{io}}{1 + \frac{M}{M_u} - \frac{V}{V_u}} \qquad (6)$$

Hsu, T.T.C. (4) showed that a good estimate of the initial torsional stiffness of a reinfroced beam in pure torsion can be obtained for a corresponding plain concrete beam.

The values of S_i computed from equation 6 and those determined from test results are shown in table 4. The average value of the ratio $\frac{S_i \text{ test}}{S_i \text{ computed}}$ is 1.027 and the standard deviation is 0.075. There is a reasonable agreement between computed and test values justifying the validity of equation 6.

6. CONCLUSIONS.

The effect of flexural moment and shear is to reduce the initial stiffness as shown by equation 6. The torsional stiffness up to cracking torque for the combined loading can be computed from equation 6, as the torque-twist curves are approximately linear upto cracking torque.

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APPENDIX I. - NOTATION.

A_s = area of tensile steel reinforcement;

A_w = area of one leg of stirrup;

A'_s = area of compression steel reinforcement;

B = width of flange;

b = width of web;

d = depth to centroid of tensile steel;

f_c = cube strength of concrete in compression;

f_{sy} = yield stress of tensile steel;

f_{wy} = yield stress of stirrup steel;

G = modulus of rigidity of concrete

K = torsion constant;

M = applied bending moment;

M_u = assumed ultimate flexural moment in pure bending $= 0.9\,d\,A_s f_{sy}$;

V = applied flexural shear

V_u = ultimate shear capacity of beam = 11 f'_c

ρ_s = tensile steel ratio $= A_s/bd$;

ρ_w = stirrup steel ratio $= 0.99\,A_w/bd\,(x_1/S)$;

x_1 = smaller centre to centre dimension of a closed rectangular stirrup;

s = spacing of stirrups;

S = torsional stiffness

S_{io} = initial torsional stiffness of the beam subjected to pure torsion

S_i = initial torsional stiffness of a reinforced concrete beam subjected to combined bending, shear and torsion.

T = applied torques;

T_{cr} = cracking torque of concrete beams;

T_u = ultimate torque of concrete beams;

θ = angle of twist per unit length;

θ_{cr} = angle of twist immediately before cracking;

θ_u = angle of twist at failures;

t = overall depth of beam;

t_s = thickness of flange.

TABLE 2 : Details of specimens and reinforcing steel

Beam	Dimensions					Stirrups				Longitudinal bars			
	b cm	t cm	B cm	t_s cm	$\frac{t}{b}$	Ø mm	S cm	ρ_w %	f_y kg/cm^2	A_s'	A_s	ρ_s %	fy kg/cm^2
A_1						-	-	-	-	-	-	-	-
B_1, C_1	7.5	7.5	12	1.5	1.0	4	5	0.28	2700	2Ø6	2Ø6	1.26	2700
A_2						-	-	-	-	-	-	-	-
B_2,C_2	7.5	10.5	13.8	2.1	1.4	6	5	0.185	2700	2Ø6	3Ø6	1.26	2700
A_3						-	-	-	-	-	-	-	-
B_3,C_3	7.5	13.5	15.6	2.7	1.8	4	5	0.14	2700	2Ø6	2Ø6 + 1Ø8	1.27	2700
A_4						-	-	-	-	-	-	-	-
B_4,C_4	7.5	16.5	17.4	3.3	2.2	4	5	0.112	2700	2Ø6	3Ø8	1.3	2700
A_5						-	-	-	-	-	-	-	-
B_5,C_5	7.5	19.5	19.2	3.9	2.6	4	5	0.095	2700	2Ø6	2Ø8 + 1Ø10	1.35	2700
A_6						-	-	-	-	-	-	-	-
B_6,C_6	7.5	22.5	21.0	4.5	3.0	4	5	0.081	2700	2Ø6	4Ø8	1.3	2700

TABLE 3 : Main test results

Beam	f_c Kg/cm^2	T_{cr} Kg.m	θ_{cr} $rad/cm \times 10^{-4}$	S_i $Kg.cm^2 \times 10^6$	T_u Kg. m	Mode of failure
A_1	236.9	17.5	6.67	2.63	17.5	Brittle
A_2	230.0	50.0	8.55	5.85	50	"
A_3	250.85	85.0	12.06	7.05	85	"
A_4	206.54	94.0	8.29	11.33	94	"
A_5	276.0	105.0	7.68	13.67	105	"
A_6	245.4	140.0	10.7	14.0	140	"
B_1	303.65	20.0	12.72	1.59	30.5	Torsion
B_2	276.05	30.0	9.475	3.17	45	"
B_3	244.95	40.0	9.807	4.08	62	"
B_4	271.40	60.0	9.8071	6.14	90	Bending
B_5	240.20	62.0	8.17	7.59	95	"
B_6	334.35	61.0	7.460	8.18	120	"
C_1	271.40	10.0	7.404	1.35	17	"
C_2	297.85	22.0	7.314	3.0	40	"
C_3	234.60	30.0	7.4[illegible]	4.011	55	"
C_4	286.24	45.0	7.672	5.86	86	Torsion
C_5	272.20	62.0	9.142	6.78	110	"
C_6	253.46	60.0	7.812	7.68	120	"

TABLE 4 : Effect of flexure and shear on torsional stiffness.

Beam	V Kg	M Kg.m	V_u Kg	M_u Kg.m	$\frac{V}{V_u}$	$\frac{M}{M_u}$	S_{io} Kg-cm^2 x 10^6	S_i kg-cm^2 Test x 10^6	S_i kg-cm^2 Cal. x 10^6	$(\frac{S_i}{S_{io}})$ test	$\frac{S_{i\,test}}{S_{i\,cale}}$
B_1	200	100	1868	85.8	1.16	0.107	2.63	1.59	1.28	0.60	1.22
B_2	395	197.4	2802	195	1.01	0.141	5.85	3.17	3.13	0.54	1.01
B_3	700	350	3736	345	1.01	0.187	7.05	4.08	3.87	0.58	1.05
B_4	1175	587.5	4670	575	1.02	0.252	11.33	6.14	6.4	0.54	0.96
B_5	1500	750	5449	809	0.93	0.275	13.67	7.59	8.25	0.56	0.92
B_6	2000	1000	6383	1073	0.93	0.313	14.0	8.18	9.41	0.58	-0.96
C_1	-	81.25	1868	85.5	0.95	-	2.63	1.36	1.349	0.51	1.0
C_2	-	197.3	2802	195	1.01	-	5.85	3.0	2.91	0.51	1.03
C_3	-	325	3736	345	0.94	-	7.05	4.011	3.63	0.57	1.1
C_4	-	520	4670	575	0.9	-	11.33	5.86	5.96	0.52	0.98
C_5	-	812.5	5449	809	1.004	-	13.63	6.78	6.5	0.50	1.04
C_6	-	975	6383	1073	0.91	-	14.0	7.68	7.33	0.54	1.05

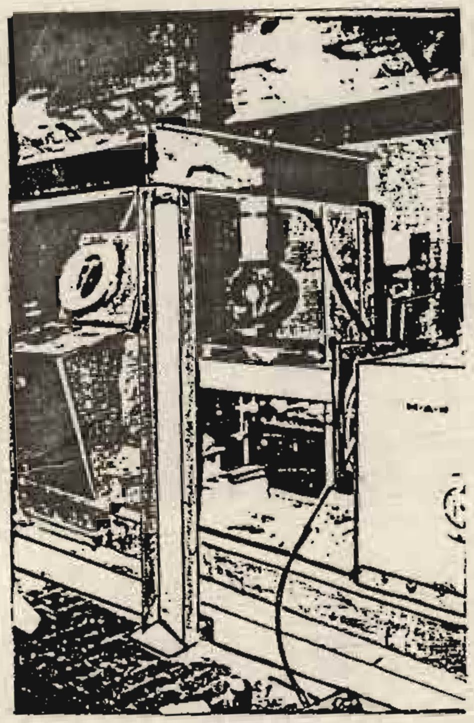

Fig. (1) General view of test set.up.

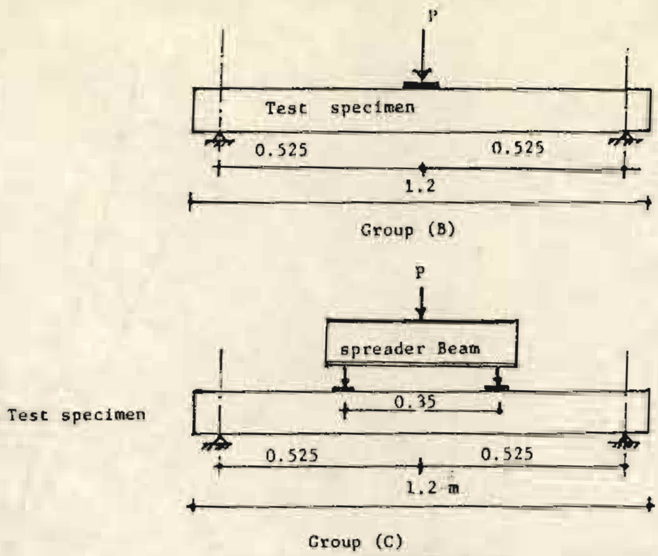

FIG. 2 Test arrangement for combined loading

$B = b_w + 3t_s$

$t_s / t = 0.2$

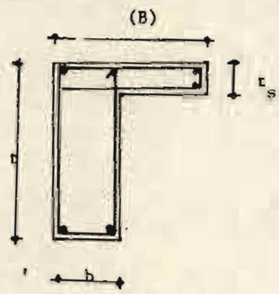

FIG. 3 Reinforcement detials for beams

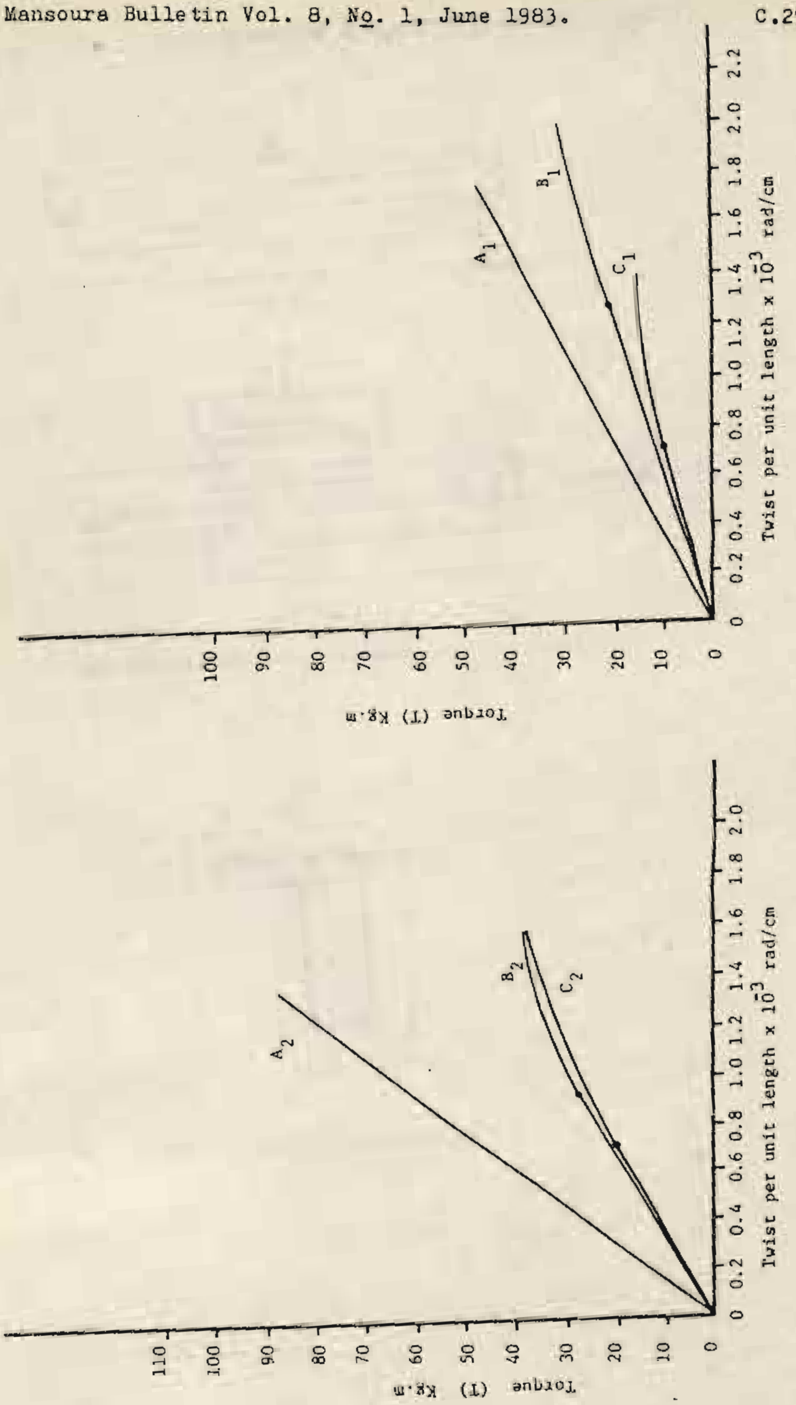

Fig. (4) Torque-Twist Curves

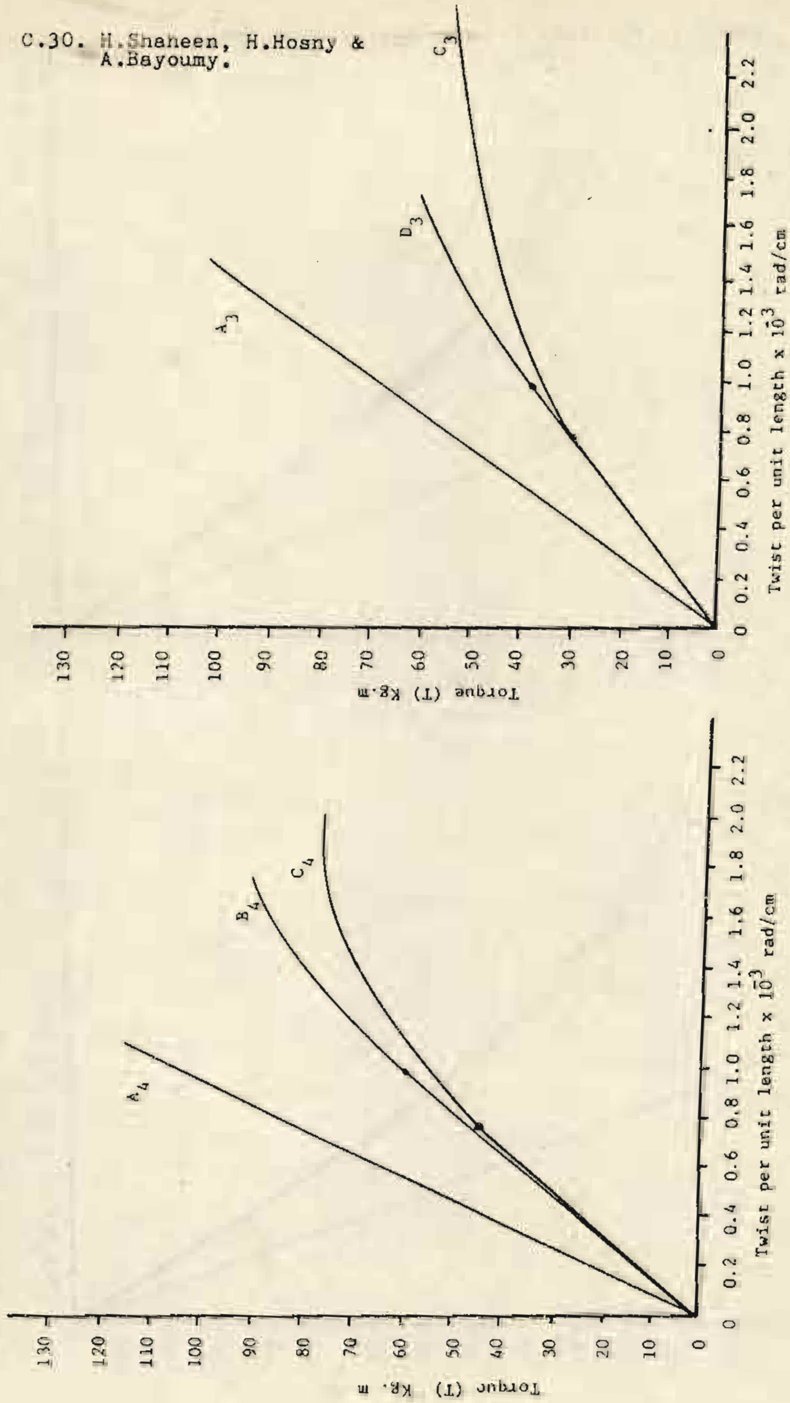

Fig. (5) Torque - Twist Curves

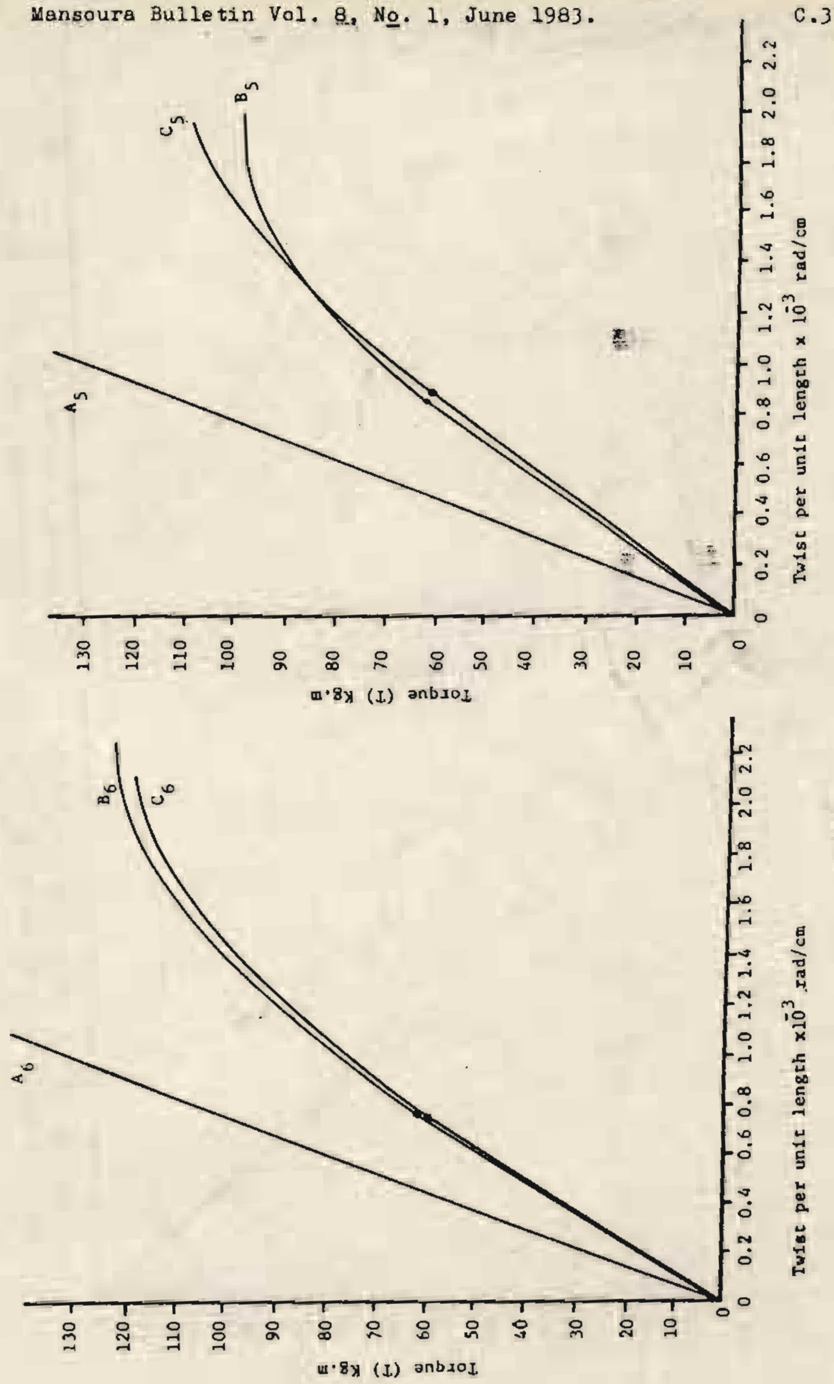

Fig. (6) Torque - Twist Curues