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Computations of Water Surface Profiles in Damietta Branch.

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COMPUTATIONS OF WATER SURFACE PROFILES IN DAMIETTA BRANCH

Zidan, A.R. , Owais, T.M. , and Moharram, S.H.

ABSTRACT

The purpose of the present work is to simulate the water surface profile along Damietta branch of the river Nile and to make a comparative studies between three computational techniques used for such simulation. These methods include the standard step method, the Ezra method and the fall discharge method.

Applying the foregoing techniques and incorporating the proposed discharge of El-Salam canal both the water surface profile and water current could be predicted.

INTRODUCTION

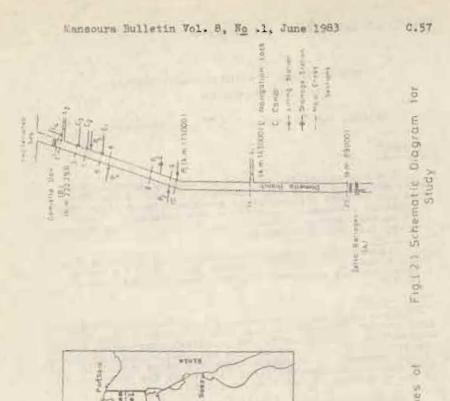
Almost all major hydraulic engineering activities in free surface flow involve the computation of gradually varied flow profile Due to its practical importence, the computation of this kind of flow has been a topic of continued interest to hydraulic engineers since Dupuit in 1848 who was perhaps the first to attempt the integration of the gradually varied flow equation.

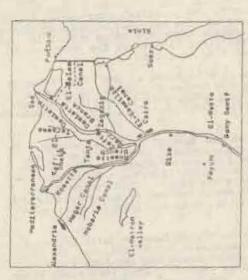
Prominent workers in this field of the computation of water surf ce profiles are Bresse (5), 1860, Bakhmetof, 1912 (1), Yon-Seggern (11) 1950, Ezra (1,5), 1954, Grimm (5), 1955, Escoffier (3), 1956, Chow (1), 1959 and Henderson (5), 1966.

At the present time the general policy of the government sims to increase the agricultural production by increasing the cultivated area which depends mainly on the river Nile Approximately 60D,000 faddans could be roclaimed in the north of Delta and Sinai. This requires 9.5 millions cubic meter of water per day from Damietta branch El-Salam canal, Fig.(1), has been proposed to carry such discharge.

The reach of Damiatta branch between Zefta barrages and Damietta dam has been chosen for this study, due to the construction of El-Salam canal and the future navigable purposes of Damietta branch. Along this reach there are many distributaries, and hydraulic structures such as pump stations and bridges Fig.(2).

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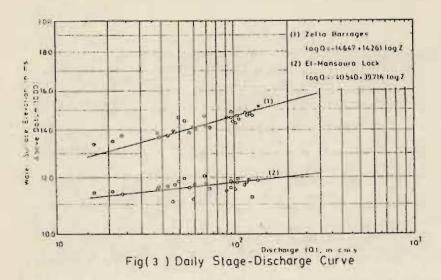
Fig(1) Map for Both Branches of the River Nits and Et-Salam Canol

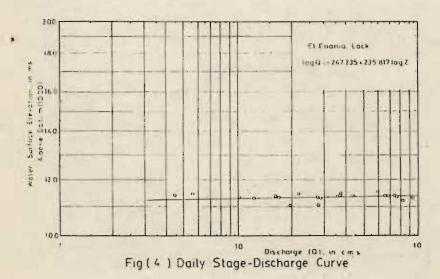
C.58. A.R. Zidan, Owsis and Mcharram Soil samples have been collected along the reach and unalysed mechanically in order to seems the particle size distribution to evaluate the Manning's roughness coefficient. Vegetations, obstructions and meandering have been considered in such evaluation (1.9). The coefficient found to be varied between 0.025 and 0.025 along the reach. Six cases have been studied: Case (1) maximum daily discharge and the corresponding nator level, with starting elevation of 1.32 m. Gase (2) average ten days discharge and the correspond-ing water level, with starting elevation of 1.16 m. Case (3) normal daily discharge and the corresponding water level, with starting elevation of 1.38 s. The starting elevations were known from the given measurements available from the Egyptian Ministry of Irrigation, they were recorded immediately upstream Demietta The next three cases are the same as those mentioned above, including the proposed discharge of El-Salam canal. From the relationship between the recorded discharges and stages as in Figs.(4,6), the new starting elevations are 1.42 s for the mexicum discharge, 1.31 s for ten days average discharge, and 1.30 s for the normal daily discharge. SECTION PROPERTIES: A survey study was prepared by Ministry of Irrigation

(7). One cross section, downstream Zetta Darrages at kilometer 89.00 was surveyed in 1980. Other cross sections were available which are; one cross section was surveyed in 1979 at kilometer 140.0, five cross sections were surveyed in 1976 along the distance between kilometer 221.0 to kilometer 222.0 at the end of the result, and one cross section surveyed in 1942 at kilometer 171.5.

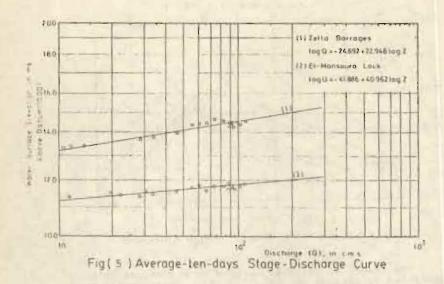
To maintain a better accuracy of calculation, addit-ional cross sections between the existing sections were calculated using linear interpolation technique. The interpolated properties of cross sections are the water alope between corresponding points on the watted parisater of every two successive major sections.

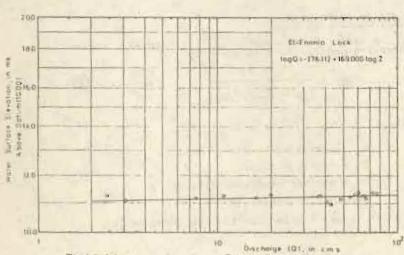
A graphical relationship between both sectional area (A) and hydraulic radius (R) against stage(Z) for all sections under investigation with and without overbank flow are given in Fig. (7).



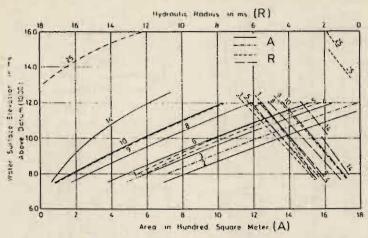


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Fig(6)Average-ten-days Stage-Discharge Curve



Fig(7) A and R vs. Water Surface Elevation

The length of reach is located by a distance between every two successive sections Fig.(2). Consequently this length could be taken as a disctance veried between 1.0 Km and 10.0 Km., twenty five sections were considered in this study.

METHOOS OF COMPUTATIONS:

The differential equation of the gradually varied flow in its general form is given by;

$$\frac{dy}{dx} = \frac{s_o - s_f}{1 + \alpha \frac{d}{dy} (v^2/2g)}$$
 (1)

in which;

So = bed slope;

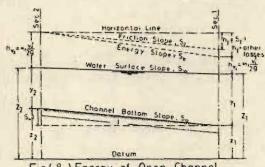
S_f = friction slope; V = average mean velocity; and

a = energy coefficient.

Oifferent forms of this equation are given in (1).

The value of energy coefficient is not equal to unity as the velocity distribution is not uniform over the channel cross section, it depends on the shape of cross section and roughness alignment of segment of cross section (1,6). For each section it may be divided of several distinct subsections of various stages of 0.5 m with constant roughness coefficient (n). For example, at Zefta $\alpha = 1.81$ for water stage of 6.0 m, 1.9 for stage 5.5 m and $\alpha = 1.53$ for stage equals 0.5 m.

The underlying theory of each method is based an Bernoullis equation, which depends on the equality of total energy head at any two sections as illustrated in Fig.(8).



Fig(8) Energy of Open-Channel

Flow $z_2 + y_2 + hv_2 = z_1 + y_1 + hv_1 + h_f + other losses (2)$ in which;

 Z_1 = stream bed elevation referred to a given detum at sec. 1;

 Z_2 = stream bed elevation reffered to a given datum at sec. 2;

y1 = depth of flow at sec. 1;

y2 = depth of flow at sec. 2;

hv1 = velocity head at sec. 1;

 hv_2 = velocity head at sec. 2; end h_f^2 = friction loss.

Other losses include eddy loss, bed and bridge pier loss etc.

(i) The Standard Step Method:

This method could be classified into two techniques for computing the water surface profile; method (\land) which takes the velocity head into consideration and method (\lor) without the effect of velocity head.

The friction loss can be determined by applying Manning's formula interms of the friction slope $\mathbf{S}_{\mathbf{f}}$

$$s_f = \frac{n^2 v^2}{R^{4/3}}$$
(3)

$$h_f = L.\bar{s}_f = \frac{L}{2} (s_{f1} + s_{f2})$$
(4)

in which; $z_{\rm fl}$ and $s_{\rm f2}$ ere the friction slopes at the end of section (1) and section (2) respectively.

In method (A), the computed flow profile depends on the position of water surface with respect to a horizontal datum. It can be represented by step from section to another

$$Z_2 = Z_2 + Y_2$$

$$z_1 = z_1 + y_1$$
(6)

Introducing \mathbf{Z}_1 and \mathbf{Z}_2 into equation (2).

$$z_2 + hv_2 = z_1 + hv_1 + h_f + other losses(7)$$

Also introducing H₂ and H₃ instead of Z_2 + hv₂ and Z_1 + hv₁ respectively in the above equation.

$$H_2 = H_1 + h_f + Other losses$$
(8

In method (B), the computed flow profile is a function of the water surface at the upstream section where the water surface at the downstream is known

$$z_2 = z_1 - (hv_2 - hv_1) + h_f + losses$$
 (9)

The friction loss h, can be determined in the following way (10). Assuming the friction slope is equal to 0.0001 then

$$Q_{0.01} = \frac{1}{0} \wedge R^{2/3} (0.0001)^{\frac{1}{2}} \dots (10)$$

For a given discharge Q in the specified reach:

$$Q = \frac{1}{n} A R^{2/3} S_f^{X}$$
 (11)

Equations (10) and (11) yield the condition;

$$S_f = (\frac{0.01 \ 0}{0.0.01})^2$$
(12)

Knowing the initial elevation of the reach and using equations (8) and (9) for the above mentioned two methods respectively, the water surface at the upstream of the same reach could be determined. This procedure is repeated from section to another for the computation of flow profile.

(11) The Ezra Method:

The Ezra method is a graphical version of the standard step method and it is particularly suitable when a large number of gradually flow profiles have to be computed for a given discharge with different starting points. $Z_2+\alpha_2'\frac{V_1^2}{2g}=Z_1+\alpha_1'\frac{V_1^2}{2g}$

$$Z_{2} + \alpha_{2} \frac{v_{1}}{2g} = Z_{1} + \alpha_{1} \frac{v_{1}}{2g}$$

$$+ (S_{f1} + S_{f2}) \cdot 1/2 + K(\alpha_{2} \frac{v_{2}^{2}}{2g} - \alpha_{1} \frac{v_{1}^{2}}{2g})$$
Substituting $V = Q/AS$; and $S_{f} = \frac{n^{2} V^{2}}{R^{4}/3}$

$$Z_{2} + \left[\alpha_{2}^{2} \frac{Q^{2}}{2g A_{2}^{2}} - \frac{L}{2} \frac{n^{2} Q^{2}}{A_{2}^{2} R_{2}^{4}/3}\right]$$

$$= Z_{1} + \left[\alpha_{1}^{2} \frac{Q^{2}}{2g A_{1}^{2}} - \frac{L}{2} \frac{n^{2} Q^{2}}{A_{2}^{2} R_{1}^{4}/3}\right]$$

$$+ K(\alpha_{2} \frac{v_{2}^{2}}{2g} - \alpha_{1} \frac{v_{1}^{2}}{2g}) \qquad(14)$$

There is no rational method for estimating the eddy loss but it is usually expressed by the last term of the equation where K is a coefficient depends on the channel shape. It varies between 0.0 for regular prismatic channel to 0.5 for abrupt change in channel cross section (1).

Equation (14) could be written in the following form

$$z_2 + F(z_2) = z_1 + F(z_1) + K(\alpha_2 V_2^2/2g - \alpha_1 V_1^2/2g)..(15)$$

Neglecting the last term of the equation and this Could be compensated by slight increase in the value of Manning's n

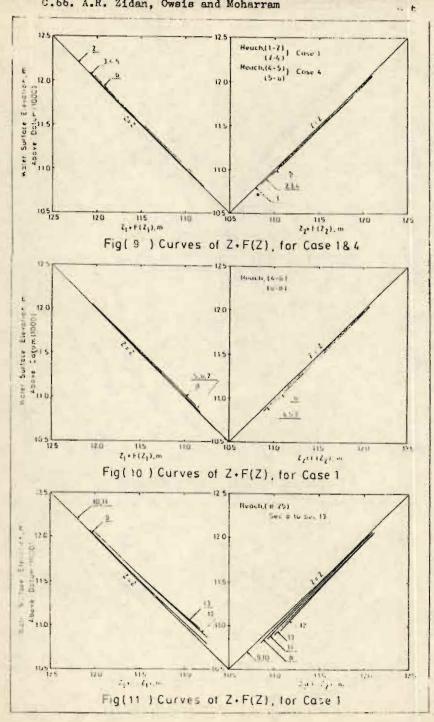
$$Z_2 + F(Z_1) = Z_1 + F(Z_1)$$
(16)

The two values of $F(Z_1)$ and $F(Z_2)$ should be computed for each section Figs.(9) to Fig.(14) exhibit the values of $Z_1+F(Z_1)$ and $Z_2+F(Z_2)$ for case (1) of max. deily discharge. In computing $F(Z_1)$ of the section the length of reach L is the value measured downstream, whils for $F(Z_2)$. L is measured upstream. The resulting value of $Z_2+F(Z_2)$ for the downstream section is therefore equal to $Z_1+F(Z_1)$ for the next cross section end so on.

For a given initial water surface elevation at the downstream section of the reach, the value of $Z_2+F(Z_2)$ can be obtained from appropriate $Z_2+F(Z_2)$ curve by using the curve $Z_1+F(Z_1)$ for the next upetreem etation. With this value, the corresponding water surface elevation could be determined. This procedure is repeated from section to another for tracing the flow profile Table (1) provides the steps of calculation for case (1) of maximum daily discharge.

Yabla ()): Computation of the Flow Profile For Case
[1). by Erra Mathod.

Sec.	K.M.	(c.s.e.)		Z ₂ +F(Z ₂)	M.S.E. (2),s. Above Ostus (10.00)		
1	222,293		-	11.32	11.32		
2	221.500	105.21	11.32		11.32		
2	221,500	-	-	11.34	11.32		
3	212,800	123,73	11.34	11.37	11.35		
4	209,000		11.37		11.30		
4	209.000		-	11.40	11.30		
5	204.000	126.93	11.40	11.43	11.41		
6	201,000		11.43	-	11,44		
	201,000		-	11.47	11.44		
7	191.500	136.46	11.47	11.50	11,48		
8	161.800		11.50	-	11.53		
	181.500			11.54	11.53		
	172.500		11.54	11.55	11.55		
10	171,450		11.55	11.55	11.55		
11	170.860		11.55	11.56	11.55		
12	160,000		11.59	11.70	11.65		
23	150,000		11.70	11,80	11.76		
14	143,500		11.60	11.84	11.63		
15	140,000	147 (14)	11.84	11.92	11.87		
16	135,000	132.52	11.92	12.07			
17	. 130,000	1	12.07	15.30			
18	125.000		12.30	12,60	12.45		
19	120.000		12.60	12.91	12,74		
20	THE SHOOL		12.91	11.34	13.11		
51	A COLUMN TO SERVICE AND A COLU		11.34	1 (7)	11.46		
22	I Charles III		13.60	(3.45.6)	13.83		
23	17777755		14.12	300000			
24			14.72	10000	14.95		
25	89,000		15.28	-	15.48		



References (1) and (5) deal with different modifications of the basic method.

(iii) The Stage Fall Discharge Method:

This method is based on records of observed stages and their corresponding discharges (5)

$$Q = \frac{1}{n} A R^{2/3} \left(\frac{H}{L}\right)^{\frac{N}{2}}$$
(17)

For a single stage the value $AR^{2/3}/nL^{\frac{1}{2}}$ is constant denoted by K, thus equation (17) becomes

At any discharge \mathbb{Q}_{χ} and the corresponding fall H_{χ} , equation (18) ie

$$Q = K H_{\lambda}^{X}$$
(19)

Equations (18) and (19) yield the condition $Q_{x} = Q \left(\frac{H_{x}}{H}\right)^{\frac{1}{2}}$

$$Q_{x} = Q \left(\frac{H_{x}}{H}\right)^{\frac{1}{2}} \qquad \dots (20)$$

If H, = 1 m the corresponding discharge

$$Q_{\underline{i},\underline{m}} = \frac{Q}{\mu_{\underline{i}}^{\underline{M}}} \qquad (21)$$

Therefore, when discharge of one meter is known at any mean water surface elevetion

$$Q_{x} = Q_{1m} \cdot H_{x}^{\chi}$$
 (22)

A one meter discharge curve is prepared, from available observed stages and discharges. The discharge of one meter fell (Q_{lm}) is computed from equation (21). The relationship between the Q_{lm} and the mean water surface elevation is given in Fig.(15) for daily records by using Table (2).

The discharge values at the selected meen water surface elevations are taken from one meter fall discharge curve Fig.(15) for the computation of points, for plotting the discharge curves. These values are shown in columns (1) and (2) of Table (3) columns (3) to (13) of the same table contains the water surface elevations and corresponding discharges $Q_{\rm X}$ at variable falls which calculated by equation (22). From Table (3) the discharge diagram can be plotted as the relation between the discharge $(Q_{\rm X})$ and the water surface elevation at downstream of the reach Fig.(16).

Table (2): Daily Date and Computations for Curve of One-meter-Fall Vs. Discharge. (Cross-Section 14 to 25).

		urface E.		Fall. (a.)	Olecharge, (c.s.s.)		
Date	Sac. 14	5ec. 25	Pean	H observed	Q observed	Q (1 m.)	
	11.92	14,46	13.19	2.54	52.08	32,68	
	11.78	14.92	13.35	3.14	96.64	54.54	
	11.85	15.22	13,54	3.37	132.52	72.19	
	12.04	14.70	13.37	2.66	66.29	41.87	
	11.74	14.70	13,22	2.96	103.00	59.87	
	11.88	14.82	13.35	2.94	119.79	69.86	
	11.98	14.76	13.37	2.78	115.74	69.42	
	11.92	14.50	13.21	2.58	105.32	65.57	
	11.60	13.65	12.63	2.05	38.77	27.08	
	11.78	14.38	13.08	2.60	97.22	60.29	
	11.73	14.15	12.94	2.42	69.44	44.64	
	11.64	14,01	12.83	2.37	60,18	39.09	
	11.76	14.35	13.05	2.59	104.17	64.73	
	11.71	14,30	13.00	2.59	100.69	62.57	
	11.63	14.10	12.84	2.47	72.92	46.40	
	11.52	13.83	12.68	2.31	37.62	24.75	
	11.43	13.52	12,28	2.09	20.83	14.41	
	11.39	13.41	12,40	2.02	16.20	11.40	
	11.57	13,86	12.72	2.29	55,55	36.71	
	11.62	13,72	13.67	2.10	42.25	29.16	
					100000000000000000000000000000000000000		

Q(1 =.) = Q/HX

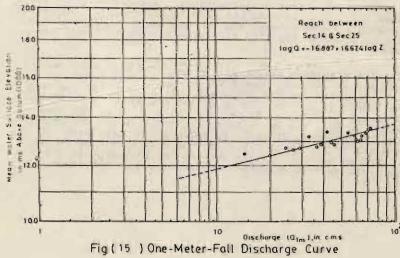


Table (3): Computation of Discharge Diagram.
(Cross-Section 14 to 25).

W.S.E.	Fall H = 1.0m.		H _x = 2.00		H _x = 2,5		H _X = 3.0		H _x = 3.5		H _x = 4.0	
	_			The second		7 1112131		Elev.at	_ ^	1	-	100000000000000000000000000000000000000
1	2	3	4	5	6	7	0	9	10	11	12	13
13.50	80,1	13.00	113.3	12.50	126.6	12,25	130.7	12.00	149.9	11.75	160.2	11.50
13.00	42,8	12,50	60.5	12,00	67.6	11.75	74.1	11.50	B0.0	11.25	85,5	11.00
12.50	22.3	12.00	31.5	11.50	35.2	11.25	38.6	11.00	41.7	10,75	44,5	10,50
12.00	11.3	11,50	16.0	11.00	17,9	10.75	19.6	10.50	21.1	10.25	22.6	10.00

Qx = Q(1 =.) HX

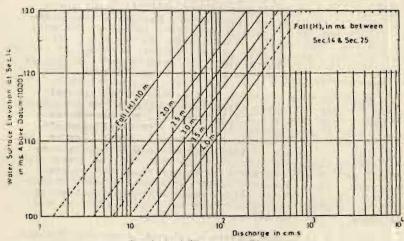


Fig (16) Discharge Diagram

Knowing the initial elevation at the downstreem of the reach, from Fig.(16), the fall can be obtained which in turne added to the initial elevation given the water surface at the upstream of the same reach. The upstream water surface elevation will be the initial elevation of the next reach. This procedure will be treated for each reach till the end of the study through.

RESULTS AND ANALYSES

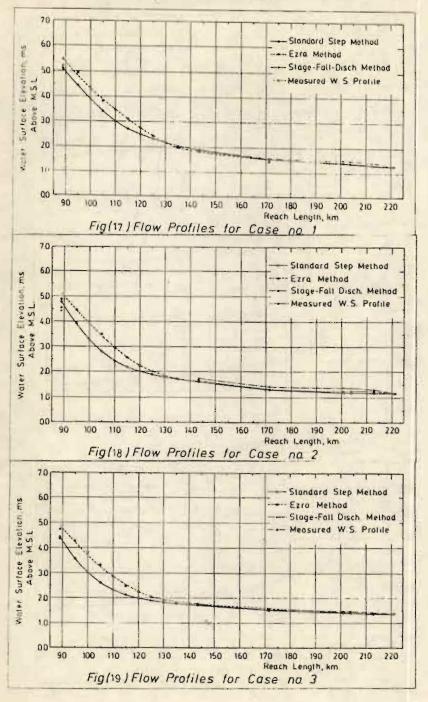
In the condition of mex discharge, case (1), Fig.(17), the computed profile exhibite a decreese in lavel by average values of 15.5% and 8.2% for both the standard step method and Ezra method respectively. This prevails from the begining of the reach K.M. 89.00 till K.M. 143.0. Using the stege fall discharge method, the computed values of water levels are lower than the measured by a value of 3.4% in average. Both the stendard step method and Ezra method provide identical results from K.M. 143.50 till the end of the reach. The computed elevations by the fall discharge method are higher than the measured by an average value of 4.9%.

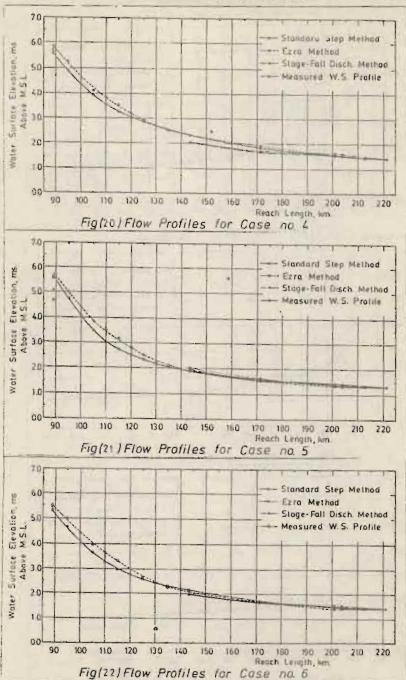
The Ezra method ie more accurate then the stendard step method, as the effect of velocity head and eddy losses is included. The eddy losses could be compensated by slight increase in the value of Manning'sn. On the contrary the standard etep method considers the average slope for the section, in addition it is besed on the trial and error technique. It provides satisfactory results in the downstream of the reach where the velocity is small. In this method the computation can be carried in the wrong direction without resulting in serious errore, although it is addisable to carry the computation upstream of the flow is subcritical end downstream if it is supercritical. subcritical and downstream if it is supercritical

When the flow profiles of a stream in its natural state without backwater affect are available for a number of discharges, the stage fall discharge method has the advantages of simplicity and economy, it is independent on the boundary resistance and hydraulic properties of the section. Inaccuracy of this method could occur, because the affect of velocity head is ignored. At the downstream of the main reach the error is bigger than that at the upstream. This is mainly due to existence of meny hydraulic structuras. Satisfactory results could be gained for lic structures. Satisfectory results could be gained for problem in which the velocity is wall below critical and decrease in the downstream direction.

For the ten days average discherge, case (2), Fig.(18), the standard atap method and Ezra method provide differences between the measured end computed water lavele of 17.7% and 8.54% respectively for the first part of the reach, while the fall discharge method exhibits identical results as the

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velocity head is smaller than in case (1). From K.M. 143.5 till the end of the reach, both the standard step method and Ezra method provides an error in the range of 2.1% higher than the measured while the fall discharge method gives an error of 6.8%.

For case (3) normal dealy discharge Fig.(19), the computed water surface profiles by the standard step method and Ezra method are less than the measured by values of 18.7% and 7.2% respectively.

The fall discharge method provides identical results. This is mainly due to the smaller value of velocity head than the corresponding head of case (1) and case (2). In the second part of the reach the standard step method and E2ra method exhibit water surface profile less than the measured by values 2.9% and 0.7% respectively. For the etage fall discharge method the computed elevations have nearly the measured values till the end of the reach.

The increase of discharge, due to the increase of water demand of El-Salam canal will result in smaller boundary resistance per unit area resulting in relative bigger depths, longer profile and smaller difference between the computed and measured water profile.

The three corresponding predicted cases, after the operation of El-Salam canal, are given in Fig.(20), Fig. (21) and Fig.(22) for the maximum discharge ten days average discharge and the normal daily discharge respectively.

CONCLUSIONS :

The following points could be concluded from this study:

- The Ears method seems to be the most convenient method among the three different techniques used for the computation of water surface elevation in natural irregular non prismatic channel.
- 2) The predicted water surface elevation upstream the head regulator of El-Salam is expected to be 1.53 m. The three different techniques have nearly provided the same value.
- 3) After the construction of El-Salem canel, it seems from calculations that some of the regions between Zefts barrages and Mansours lock could be flooded, for that reason it is convenient to avoid this situation before hand.
- 4) The predicted water surface elevation after the operation of El-Salam, according to Ezra method as follows:

downstream Zefta barrage (K.M. 89.00) could be 5.83 m, 0.61 meter higher then the existing elevation. At Mansoura lock (K.M. 143.5) would equal to 2.3m by an increase of 0.45 m of the present elevation, at Bosat station (K.M. 170) the water level could be 1.92 m with an increase of 0.46 m; upstream Demiette dem could be 1.42 m. Increasing the water current is also demonstrated. The percentage of increase of the velocity could reach the value 43% at Zefta barragee, 72% at Mansoura lock, and 73% at Bosat station.

- 5) From Bosat etation till the end of the reach the water velocity exhibite emaller values than that of station upstream Bosat. This is mainly due the increase of cross sectional area, and the growth of squatic plants. Its value ranges from 0.08 to 0.26 m/sec.
- 6) The backwater curves terminates in staps and ends on a lake at Damietta dam. At that site most of the water energy is dissipated although a surplus of water flowing over the apillway on the left side of the dam. It is difficult to specify accurately the shape and levels of backwater curve at that location. This is mainly due to smaller discharge, the increase of water depth and the various obstructions include the channel.

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