An Improved Algorithm for a High Speed Distance Relay Using Walsh Functions.

Gabr Abdel-Sallam
Assistant Professor, Electrical Power & Machines Department, Faculty of Engineering, Mansoura University, Mansoura, Egypt.

Fayez Areed
Assistant Professor, Electrical Power & Machines Department, Faculty of Engineering, Mansoura University, Mansoura, Egypt.

AN IMPROVED ALGORITHM FOR A HIGH SPEED DISTANCE
RELAY USING WALSH FUNCTIONS

BY
G.M. Abdel-Salam   &   F.F.G. Areed[*]
B.Sc. M.Sc; Ph.D   &   B.Sc; M.Sc; Ph.D

ABSTRACT

An algorithm for calculating the apparent transmission
line impedance to the point of fault is presented. This
algorithm is based on a half-cycle Window using Walsh Funct-
ion technique. The real and the imaginary components of the
fundamental frequency voltage and current of the faulty
phases are evaluated and used to calculate the impedance, as
seen from the relay location. The sampling rate used to test
this algorithm is 16 samples/cycle. The algorithm has been
tested using digital computer simulation of the sample model.

A comparison between this algorithm and four other algo-
rithms, suitable for fault impedance measurement was carried
on here with particular reference to their accuracy and speed
of calculations.

List of Symbols

$Z(jw_e)$ = fault loop impedance at extraction angular frequency $v_r(jw_e)$, $i_r(jw_e)$: finite fourier transform components
of voltage and current

T = Time constant of the line in seconds i.e. (L/R of line)

W_k = Walsh coefficient

Wal (k,t) = Walsh function of integral K

F_1, F_2 = Fourier coefficients

$v(I)$, $i(I)$ = Sample values of voltage and current for ith sample

R = The evaluated resistance of the faulty line

X = The evaluated reactance of the faulty line

N = The sampling frequency per cycle

T_w = The processing data window.

1. INTRODUCTION

Protection of power system elements by programmable
digital computers has been an active area of research since
late 1960. Protection of the transmission lines has received
a special attention from the researchers using digital comp-
uter, a single minicomputer or a microprocessor, to perform
its protection function. The main problem associated with
this technique is, how to get within an acceptable time an
accurate estimate of the apparent impedance of the line from
the relay location to the faulted point using digitized
voltage and current samples. Various algorithms have been
proposed to eliminate errors due to the d.c. offset, harmon-
ics and noise associated with the incoming signals to the
relay under fault conditions. These algorithms can be

[*]   Authors are lecturers in electrical power & machines
      department, Faculty of Engineering, El-Mansoura Univer-
      sity.

categorized in five groups.

The first group [2] was developed to calculate the line impedance by predicting the peak value of current and voltage signals using sample and derivative approach. The second group Fourier or Walsh analysis [3,4,5]. This is based on the calculations of the real and the imajinary components of the fundamental frequency currents and voltages to determine the resistance and the reactance of the faulty line as seen from the relay location. The third group [6] uses digital filters to extract the fundamental frequency information from the faulty signal. The fourth group [7] depends on exploits the differential equation, describing the behaviour of the trans- mission line, as a series R-L model to calculate the Relaying impedance. The last group [8] proposes a least-square fit to a waveform containing sinusoidal and exponentially decaying components.

In this paper, an attempt has been made to reduce the computational time of the finite Fourier transform algorithm[4] using the Walsh Functions technique. The Walsh technique has successfully reduced the computational time to about half its value compared with the finite Fourier technique. The modified algorithm has been tested and compared with other algorithms. Results of this study are included.

2. FINITE WALSH TRANSFORM ALGORITHM

The finite Fourier transform algorithm[4] provides good accuracy of the impedance calculation, but requires many multiplications with corresponding expensive hardware and or extra processing time. The apparent impedance of the trans- mission line at any particular frequency, (w_e) is given by

$$Z(jw_e) = \frac{v_r(jw_e)}{i_r(jw_e) - \frac{\tau \cdot \mu}{1 + jw_e \tau}}$$

where

$$\mu = i_r(T_1).\exp(-jw_e T_1) - i_r(T_2).\exp(-jw_e T_2);$$

$v_r(jw_e)$ finite Fourier transform components of voltage which can be given by

$$v_{r_n}(jw_e) = \frac{n.\Delta T + T}{n.\int_{\Delta T}} v_r.\exp(-jw_e t).dt \quad \ldots\ldots\ldots\ldots(2)$$

$$\simeq \Delta T \left[0.5 v_{r_n}.\exp(-jw_e. nT) + v_{r_{n-1}} \exp(-jw_e(n-1).\Delta T + \ldots \right.$$

$$+ v_{r(n-(N-1))}. \exp(-jw_e(n-(N-1)\Delta T)$$

$$\left. + 0.5\, v_{r(n-N)}. \exp(-j\, w_e(n-N). \Delta T) \right]\ldots\ldots\ldots(3)$$

where, T_w is the processing data window time;

$$N = \frac{T_w}{\Delta T} \, , \quad \Delta T \text{ being the interval time}$$

Similar equations can be written for $i_n(j w_e)$.
Table 1 shows the computational requirements for the algorithm. The impedance of the transmission line is evaluated using 16 samples per cycle, applied over half cycle window. It is clear that the algorithm requires a large number of multiplication operations which must be reduced for reducing the processing time. Therefore, this algorithm is modified here to get the same accuracy by using the Walsh functions which reduce the calculation time of the impedance. This modification may be helpful to use the microprocessors to act as a high speed distance relay with an acceptable operating time.

Table - 1

The calclated element	Arithmatic Operation		
	Multiplication	Division	Addition and subtraction and shift
$v_r(jw_e)$ and $i_r(jw_e)$	36	--	36
$\frac{\tau \cdot \mu}{1 + jw_e \tau}$	8	--	6
The resistance and reactance R, X	6	2	3
Total	50	2	45

3. THE MODIFIED ALGORITHM - FINITE WALSH ALGORITHM

It is clear from equation (3) that the real and the imaginary components of the voltage and current signals at the fundamental frequency can be evaluated by correlating the sample values with sinusoidal orthogonal functions. Therefore the signals can be represented in Fourier series in the interval (0,T) as follows,

$$v(t) = F_o + \sum_{k=1}^{\infty} \sqrt{2} \left[F_{k_s} (\sin \frac{2\pi k}{T} \cdot t + F_{k_c} \cos \frac{2\pi k}{T} \cdot t) \right]$$

$$\ldots\ldots (4)$$

At the same time the Walsh expansion of the same wave can be written as,

$$v(t) = \sum_{k=0}^{\infty} W_k \cdot wal (k, t/T) \qquad \ldots\ldots\ldots(5)$$

where

$$F_c = -\frac{1}{T} \int_0^T v(t).dt;$$

$$F_{1,s} = \frac{\sqrt{2}}{T} \int_0^T Img\left[v(t).exp - j\,\frac{2\pi t}{T}\, dt\right]; \quad ....(6)$$

$$F_{1,c} = \frac{\sqrt{2}}{T} \int_0^T Real\left[v(t).exp(-j\,\frac{2\pi t}{T})\right]; \quad ....(7)$$

&

$$W_k = -\frac{1}{T} \int_0^T v(t).\,wal(k,t/T).dt. \quad ....(8)$$

Here the set of components F_k represents a vector in the Hilbert space, implying thereby that the set W_k also forms a vector in Hilbert space. These two sets are related by the relation

$$W = A.F. \quad ....(9a)$$

where A is given as [5] is an orthogonal matrix [5]
Equation (9a) can be rewritten as,

$$F = A^t . W \quad ....(9b)$$

From the above equation the imaginary and the real components of the signal at the fundamental frequency can be obtained as follows:

$$F_{1s} = F_1 = 0.9\ W_1 - 0.373\ W_5 - 0.074\ W_9 \quad ....(10)$$

and

$$F_{1c} = F_1 = 0.9\ W_2 + 0.373\ W_6 - 0.074\ W_{10} \quad ....(11)$$

To obtain the particular degree of accuracy, equations (10) and (11) can be rearranged accordingly by retaining or eliminating the last term. Using equations (6 and 10), equation (6) can be rewritten as,

$$Img\ F(j\ w_e)) = -\frac{m}{\sqrt{2}} \left[0.9\ W_1 - 0.373\ W_5 - 0.074\ W_9 \right] \quad ....(12)$$

While using equations (7 and 11), equation (7) can be given as

$$Real\ F(j\ w_e) = -\frac{T}{\sqrt{2}} \left[0.9\ W_2 + 0.373\ W_6 - 0.074\ W_{10} \right] \quad ....(13)$$

Now a faster algorithm can be devised to calculate the Walsh coefficients W_k using the trapozoidal rule. This algorithm requires simple operations addition, subtraction and shift instructions which are convenient for microprocessors. Consequently, the total mathematical computational time required to evaluate the fundamental frequency components can almost be reduced to half its value. This is explained below.

3.1 The Walsh Coefficient Algorithm

Here one cycle is divided into 16 equal parts and the half cycle window is considered for the application of this algorithm. The walsh functions which are represented in Fig. 1 are continuous functions. The window samples of the processing wave and the corresponding window samples of the walsh functions are used to evaluate numerically the walsh coefficient W_k. It should be noted that the walsh functions and the processing signal are referred to the same base time. For example, at any sample (I), the walsh function coefficient

$W_k(I)$ can be given using the trapozoidal rule for the numerical integration of equation (8) with the help of Figs.(1,2 and 3) as follows:

$$W_1(I) = -\frac{1}{8} \left[0.5 \ v(I-8) + v(I-7) + v(I-6) + v(I-4) + v(I-3) - v(I-1) - 0.5 \ v(I) \right]$$

and

$$W_2(I) = -\frac{1}{8} \left[0.5 \ v(I-8) + v(I-7) - v(I-5) - v(I-4) - v(I-3) - v(I-2) - v(I-1) - 0.5 \ v(I) \right] \quad \dots\dots\dots(14)$$

by employing Figs. 1, 2:,

$$W_5(I) = -\frac{1}{8} \left[-0.5 \ v(I-8) - v(I-7) - v(I-5) + v(I-3) - v(I-1) - 0.5 \ v(I) \right]$$

and

$$W_5(I) = \frac{1}{8} \left[-0.5 \ v(I-8) - v(I-7) + v(I-5) - v(I-3) - v(I-2) - v(I-1) - 0.5 \ v(I) \right] \quad \dots\dots(15)$$

where
$v(I)$ is the latest voltage sample in the window;
$v(I-8)$ is the oldest voltage sample in the window.
it is very important to note that for each sample, the equations of the walsh coefficients change according to the propagation of the walsh waves which require more memory.

The imaginary and real components of the signals at the fundamental frequency can be evaluated using equations (12 and 13). In addition, the scalar constant in these equations can be produced by shift instructions to aviod the multiplication requirements. This is given below:

$$0.9 \simeq 1 - \frac{1}{2^4} - \frac{1}{2^5} \qquad \dots\dots\dots\dots(16a)$$

and

$$0.373 \simeq \frac{1}{2^2} + \frac{1}{2^3} \qquad \dots\dots\dots\dots(16b)$$

From the above derivation, it can be seen that the use of the walsh function technique has successfully reduced the computational time. Table 2 shows the computational requirements of the finite walsh and finite Fourier algorithms. From table 2, it can be noted that the number of multiplications has been reduced to about half its value with respect to the finite Fourier transform techniaue.

4.   TESTING THE ALGORITHM

The algorithm presented here was tested to calculate the real and imaginary components of the fundamental frequency phasors of voltage and current. These components were then used to calculate the apparent resistance and reactance of the faulty line using equation (1), which can be written as,

$$Z(j \ w_e) = R(n) + j \ X(n) = \frac{VR(n) + j \ VI(n)}{CR(n) + jCI(n)} \quad \dots\dots\dots(17)$$

where
$VR(n) + j \ VI(n) = v(jw_e)$,

$CR(n) + j \ CI(n) = i(jw_e) - \dfrac{\tau \cdot \mu}{1 + j \ w_e \ \tau}$,

$R(n)$ the evaluated resistance at sample number (n) &
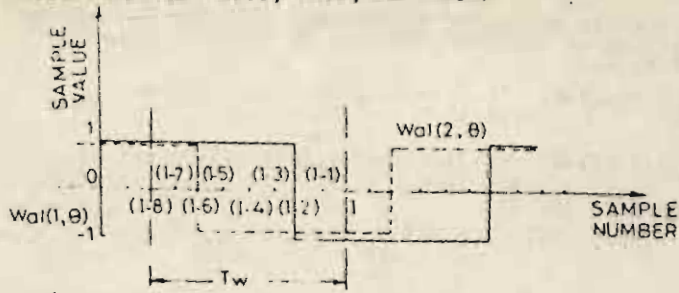
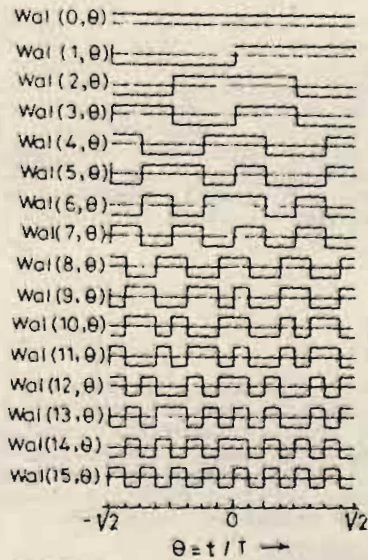FIG·1 THE CORRELATION OF THE WINDOW SAMPLES WITH THE FIRST AND SECOND WALSH FUNCTIONS.
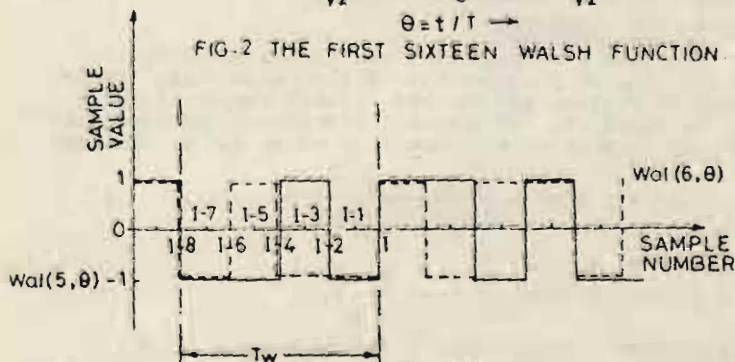

FIG.2 THE FIRST SIXTEEN WALSH FUNCTION

FIG·3 THE CORRELATION OF THE WINDOW SAMPLES WITH THE FIFTH AND SIX WALSH FUNCTIONS,

given as

$$R(n) = \frac{CR(n) \cdot VR(n) + CI(n) \cdot VI(n)}{SC(n)} \qquad \ldots\ldots\ldots(18)$$

$X(n)$ is the evaluated reactance at the same sample, given as

$$X \quad X(n) = \frac{CR(n) \cdot VI(n) - CI(n) \cdot VR(n)}{SC(n)} \qquad \ldots\ldots\ldots(19)$$

The evaluated functions.	Mathematical Operations					
	Multiplication		Finite	Modified algrith	Finite	Modified algrith
	Finite	Modified algorithm				
$v(j\,w_e)$	18	4 Can be eliminated as shown	–	–	18	30+4
$i(j\,w_e)$	18	4 Can be eliminated as shown	–	–	18	30+4
$\dfrac{\sqrt{2}}{\Delta T} \dfrac{\tau}{(1+jw_e\,\tau)} = A$	Constant		Can be stored		--	--
μ	8	4	–	–	4	2
$A.\mu$		4	–	–		2
$i(j\,w_e)-A$	–	–	–	–	2	2
The impedance calculation, resistance R, and Reactance X.	6	6	2	2	3	3
The total mathematical computations	50	22, or 14	2	2	45	77

Table 2, The computational requirements of the algorithms
to evaluate the impedance using 16 sample/cycle.

and

$$SC(n) = \left[CR(n)\right]^2 + \left[CI(n)\right]^2 \qquad \ldots\ldots(20)$$

Using equations (12) and (13) it can be given,

$$VR(n) = \left[0.9\ F_{2,v}(n) + 0.373\ F_{6,v}(n)\right] \qquad \ldots\ldots(21)$$

$$VI(n) = -\left[0.9\ F_{1,v}(n) - 0.373\ F_{5,v}(n)\right] \qquad \ldots\ldots(22)$$

$$CR(n) = \left[0.9\ F_{2,c}(n) + 0.373\ F_{6,c}(n)\right] - \text{Real}\,\mu.A \quad \ldots(23)$$

and

$$CI(n) = -\left[-0.9\ F_{1,c}(n) - 0.373\ F_{5,c}(n)\right] - \text{Img}\,\mu.A \qquad \ldots(24)$$

where

$$\mu.A = \frac{\sqrt{2}}{\Delta T} \cdot \frac{\tau}{1+jw_e} \left[i_r(T_1).\exp(-jw_e T_1) - i_r(T_2).\exp(jw_e T_2)\right]$$

4.1   The Test Model

The algrithm is tested using the data generated from a digital model   of a transmission line as given in Fig.4. [1] This test has been performed under the following considerations:

i)   The samples of voltage and current signals are obtained by an off-line digital computer program.

ii)   The voltage and the current signals are sampled at the rate of 16 samples cycle continuously for four cycles.

iii) The fault inception angle is varied from $0°$ to $360°$ with an interval of $22.5°$.

iv)   A half cycle window is considered here to evaluate the resistance and the reactance of the transmission line model as seen from a relay location (Fig. 4), using equations (18, 19).

v)   Fault location on the line is considered at 10 %, 50 % and 100 % of the transmission line from the relay location.

vi)   Since the line is modelled by a series combination of resistance and reactance, the voltage and current signals include only the decaying d.c. and fundamental frequency components. Therefore, the voltage and the current signals are modified by including, i) third harmonics with peak value equal to 25 % of fundamental peak and ii) third and fifth harmonic components having peak v values of 25 % and 10 % of the fundamental peak respectively. The modified data samples are used to test the algorithm.

5.   DIGITAL SIMULATION RESULTS

Results obtained for R and X as seen from the relay location can be divided as follows:

i)   The pre-fault values represent the normal conditions of the system (steady state condition).

ii)   The post-fault results show a tracking of the evaluated resistance and reactance of the faulty line for the various predefined fault locations as explained before.

In the following sections, results of the evaluated parameters are discussed for the inception of the fault at two cases; (1) when the voltage signal is passing through its peak value and (2) when the voltage signal is passing by the
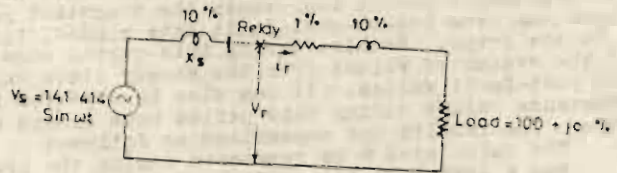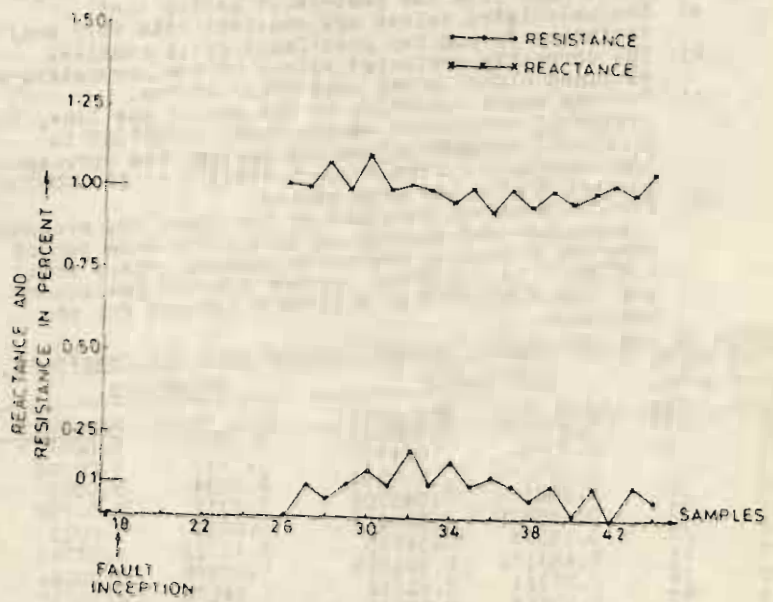
FIG 4   TEST SAMPLE

FIG. 7. CALCULATED RESISTANCE AND REACTANCE WITH THIRD AND
FIFTH HARMONICS FOR FAULT AT 10 PERCENT OF THE LINE,
WITH INCEPTION ANGLE 22·5°

zero positive value.

5.1 Fault occurred at the Maximum voltage without Including Harmonics

Figure 5, shows the trajectory of the evaluated values for resistance and reactance of the line as seen by the relay for different fault locations. Each of these curves includes pre-fault, pre-post-post-fault and post-fault periods. The pre-post-fault period represents the transient period of the algorithm from pre-to-post-fault data window, i.e. from b to c in the graph. Results during this period show the tracking of the evaluated values from the steady state values towards the post-fault values. It can also be observed that the reactance values during this period have always a positive sign. The results are summerised as follows:

i)   The calculated % ge reactance during the prefault period has a deviation sector from, 9.9998620 to 9.9995860, wh while the actual value is 10 percent. For the resistanc, it was, 100.9960000 to 100.9954000, while the actual value is 101 percent including the load (i.e. 100 percent)

ii) It can be seen from the post-fault period that,
   a) The calculated values are constant with very small deviation during two post-fault cycle camples.
   b) The correctly evaluated values of the parameters are obtained almost after half cycle window.
   o) For the fault occurring at the end of the line, the evaluated resistance varies from, 0.9957089 to 1.1231000 percent during two cycles. The corresponding reactance varies from, 0.8996950 to 10.1209700 percent during the same period.
   d) For fault at the midpoint of the line, the evaluated resistances and reactances during the same period have the following deviation sectors, viz., from 0.4798253 to 0.5497053 percent for the resistancy and from 4.9623700 to 5.0175890 percent for the reactance.

Table 3: The calculated values of resistance and reactance

Post fault sample No.		RESISTANCE		REACTANCE	
Zero.	Max.	Zero volt	Max.volt	Zero volt.	Max.volt
41	29	0.497789	0.498916	4.99002	5.000229
42	30	0.4359	.522797	5.1417	5.04902
43	31	0.492514	0.499392	4.9934	5.0009
44	32	0.288378	0.549705	5.4311	5.01758
45	33	0.489879	0.500405	5.0014	5.0012
46	34	0.69204	0.54595	5.46213	4.97786
47	35	0.493174	0.501055	5.00798	5.00064
48	36	0.96324	0.51632	5.14176	4.96237
49	37	0.50249	0.50123	5.01040	4.99969
50	38	0.87717	0.47982	4.7738	4.96698
51	39	0.50858	0.50078	5.0057	4.99909
52	40	0.60081	0.47976	4.61719	4.98398
53	41	0.50923	0.4999	4.99721	4.99892
54	42	0.33325	0.48806	4.671149	5.01875
55	43	0.50505	0.49933	4.99275	4.99932
56	44	0.16566	0.48454	4.86459	4.98214

5.2   Fault occurred at Zero Voltage Without Including Harmonics

Figure (6) shows the evaluated values of the resistance and reactance against the sample numbers. The results obtained during the pre-fault period are the same as given before, while at the pre-post-fault period, the tracking values are smoother than the earlier results. The post-fault samples produce the stable values as in the previous case with small difference in the deviation sector of the estimated values of the resistance. The same is true for the reactance values. Table 3, shows the calculated values of resistance and reactance for both the cases (i.e. Sections 5.1 and 5.2) during the post-fault period when the fault occurred at the midpoint of the line. From this table, it can be observed that the presence of the maximum d.c. offset component has a small effect in the evaluated resistance, while its effect on the reactance is negligible.

5.3   Fault at Zero Voltage Including Harmonics

In this test, the voltage and current signals during the fault are modified by third and fifth harmonics of peak values 25% and 10% of the peak respectively. Figure 7, shows the effect of harmonics on the evaluated parameters of the line, when the fault occurred at the voltage angle 22.5°. From graph, it is seen that the presence of harmonics in the incoming data samples does not affect the accuracy of the evaluated components, indicating that this algorithm has a good transient components, indicating that this algorithm has a good transient immunity.

6.   COMPARATIVE STUDY

Here four algorithms have been selected to evaluate the parameters of the transmission line of the same model using the digital computer. This study has been carried out to compare the capability of the present algorithm with those of the selected algorithms. These algorithms are:
1.   Full cycle window walsh function developed by Horton[5].
2.   McInns and Morrison[6] algorithm.
3.   The algorithm propoged by Gilbert and Shovline[9].
4.   Prodar 70 algorithm[3] which has been tested in the actual system at Tesla substation for about six years.

These algorithms have been selected because they may be implemented in a microprocessor based distance relaying se scheme. The test has been carried out under the same considerations as before. Results proved that all the algorithms have evaluated the parameters of the line correctly during the steady state period (pre-fault period), while the calculated parameters during the post-fault period have been affected according to the algorithms stated. Figure 8 shows the tracking of the evaluated resistance and reactance when the incoming data consists of the fundamental and the d.c. offset components only. It may be ovserved that:
1.   McInns and Morrison algorithm and Prodar 70 algorithm have given a faster response and the best tracking values than all others including the present algorithm.
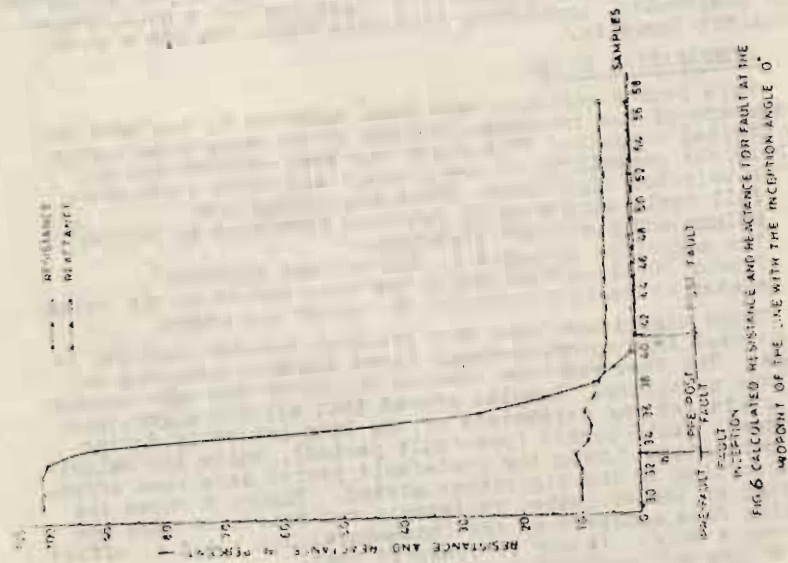2.   Gilbert algorithm results suffer from the presence of the d.o. offset component.
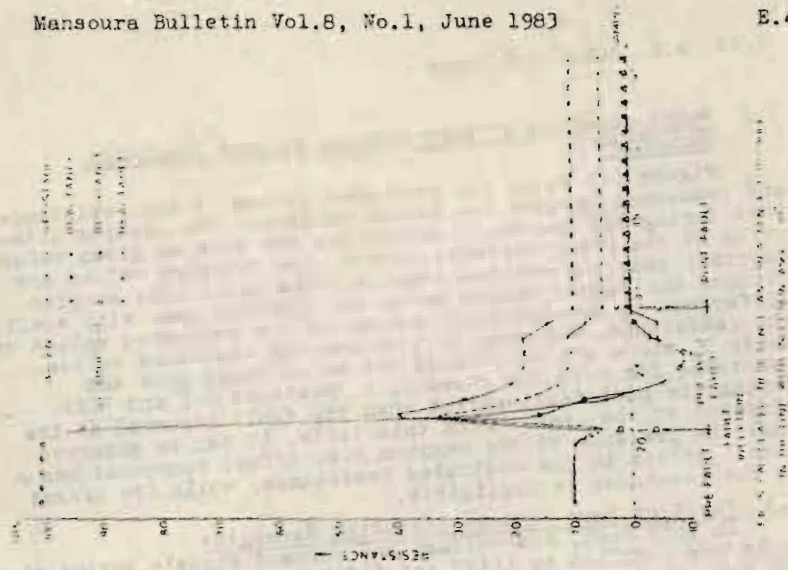
FIG. 6 CALCULATED RESISTANCE AND REACTANCE FOR FAULT AT THE MIDPOINT OF THE LINE WITH THE INCEPTION ANGLE 0°
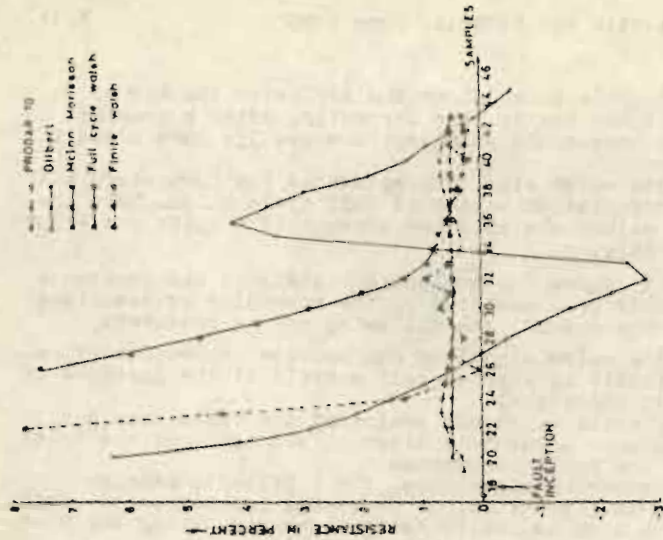
FIG.8   CALCULATE RESISTANCE OF SELECTED ALGORITHMS FOR
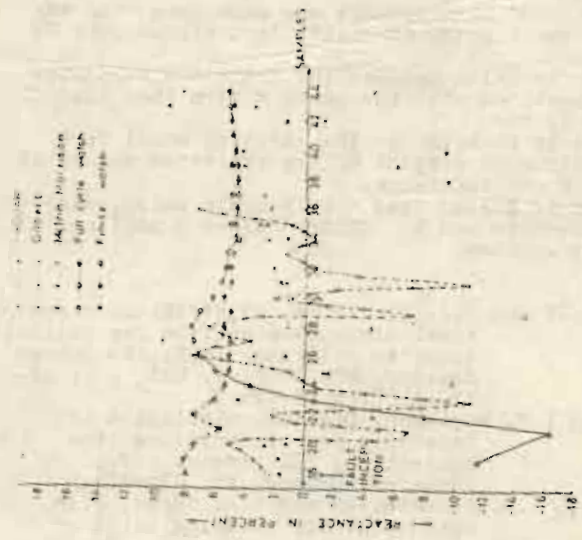   FAULT AT MIDPOINT OF THE LINE WITH INCEPTION ANGLE=27.5°

FIG.9   CALCULATED REACTANCE OF SELECTED ALGORITHMS
        FOR FAULT AT MIDPOINT OF THE LINE WITH INCEPTION
        ANGLE=27.5° (SIGNALS ARE MODIFIED BY THIRD AND
        FIFTH HARMONICS)

3.   The full cycle walsh algorithm evaluates the actual
     values of the faulty line correctly, after a complete
     cycle of post-fault samples. Here results have a small
     deviation.
4.   The finite walsh algorithm estimates the parameters with
     the same deviation sector as full cycle walsh. Here the
     correct values are obtained after half a cycle post-fault
     samples only.

     Figure (9) shows the evaluated resistance and reactance
when the signals were modified by the harmonics as described
before. From the graphs, the following may be observed:

1.   The finite walsh algorithm can provide proper detection
     of the faulty line after half a cycle of the instance of
     the fault inception.
2.   The full cycle walsh can evaluated the resistance and
     the reactance accurately after a full cycle of the inst-
     ance of the fault occurrence.
3.   In the remaining algorithms, i.e., Gilbert, McInns-
     Morrison and Prodar 70, it is rather impossible to decide
     whether a line is faulty as the tracking during the post-
     fault period continues to be unstable.

7.   CONCLUSION

     In the present paper, a new algorithm has been presented
for the evaluation of resistance and reactance of a system,
which will be useful for computerised relaying, particularly
using the micro-processors. The features of the new algorithm
are as follows:
1.   It's computational requirements are much less than any
     one based on the Fourier analysis. It involves only 22
     multiplications.
2.   It has a good immunity against the transient harmonics
     which are associated with the power system than the
     existing algorithms.
3.   It appears to be superior to the existing small data
     window algorithms in respect of the estimated values of
     the resistance and reactance.
4.   It's response is faster than a full cycle walsh, since
     the correct results can be obtained after a half cycle
     of post-fault samples.

REFERENCES

1   E.O. Schweitzer and A.J. Flechsing, An efficient direct-
                         ional distance algorithm for digital
                         computer relaying, IEEE, PES Summer
                         Meeting, MEXICO City, MEX, July 17-
                         22, 1977; Q 77-725-5.
2   B.J. Mann and I.F. Morrison, Digital calculation of
                         impedance for transmission line
                         protection, IEEE Trans. PAS.,
                         January/February, PP. 270-278.
3   G.B. Gilcrest, G.D. Rockefeller and E.A. Udren, High
                         speed distance relaying using a
                         digital computer, part I - System
                         Description, IEEE Trans. PAS. vol.
                         91, No.3, May/ June, 1972, PP.
                         1235-1243.

4   A.T. Johns, Fundamental digital approqch to the distance
                  protection of e.h.v. transmissi on
                  lines, IEEE Proc. vol, 125, No. 5,
                  May 1978, PP. 377-384.
5   J.W. Horton, Walsh functions for digital impedance relay-
                  ing of power lines, Journal of resea-
                  rch and development, IBM, Jr. R. and
                  D., U.S.A, Vol. 20, No. 6, Nov. 1976,
                  PP. 530-541.
6   A.D.McInnes and I.F. Morrison, Real time calculation of
                  resistance and reactance for transm-
                  ision line protection by digital
                  computer, IEEE Transaction, Institu-
                  tion of Engineers, Australia, EET-No.
                  1, 1971, PP. 16-23.
7   A.M. Ranjbar and B.J. Cory, Filters for digital protect-
                  ion of long transmission lines, IEEE,
                  PES, Summer meeting, Vahcouver,
                  British Columbia, Canda, July 15-20,
                  1979, A 79 416-9.
8   M.S. Sachdev and M.A. Baribeau, A new algorithm for
                  digital impedance relays, IEEE Trans
                  ., Vol, PAS-98, No. 6, Nov/Dec.1979,
                  PP. 2232-2240.
9   J.G. Gilbert and R.J. Shovlin, High speed transmission
                  line fault impedance calculation
                  using a dedicated minicomputer, IEEE
                  Trans., Vol. PAS-94, No. 3, May/
                  June 1975, PP. 872-883.