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HEAT TRANSFER TO LAMINAR FLOW ACROSS A FLAT PLATE SUBJECTED TO A PERIODIC CHANGE OF SURFACE TEMPERATURE

BY

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Abstract:

This paper presents an analytical procedure for estimation of the unsteady local heat flux to a steady laminar boundary layer over a flat plate with a periodic change of surface temperature. The fluid flow is parallel to the plate and the fluid physical properties are taken to be constant, and the plate surface temperature is spatially uniform and changes with time according to a sinusoidal function. An expression for instantaneous local Nusselt number is drived.

It is found that the instantaneous local value of Nusselt number differs from the quasi-steady value of Nusselt number. The maximum percentange difference between the two values increases by increasing Prandtl number, amplitude and frequency of plate surface temperature oscillation, and distance from the plate leading edge. On the other hand it decreases by increasing the free stream velocity. It is found also that the time mean value of Nusselt number is the same as that in the case of steady state. Some results of the present work are in agreement with that of previous works.

Nomenclature:

A	relative amplitude of plate surface temperature oscillation, $(a/T_{w,m})$,	——— ;
a	amplitude of plate surface temperature oscillation,	K ;
f	frequency of plate surface temperature oscillation,	Hz;
k	thermal conductivity,	W/m.k;
Nu	Nusselt number, $(q.x/k.T)$	——— ;
P_r	Prandtl number, (ν/α),	——— ;
q	heat transfer rate,	W/m^2;
Re	Reynold number	———;
T	temperature difference, $(t - t_\infty)$	K ;
t	temperature,	K ;
u	velocity component in direction x,	m/sec. ;
v	velocity component in direction y,	m/sec. ;
W	nondimentional angular velocity of plate surface temperature oscillation, $(w\, x/u_\infty) = (2\pi f\, x/u_\infty)$,	——— ;
x	linear coordinate calculated along the plate from the leading edge,	m ;
Y	nondimensional coordinate ; (y/δ_t),	——— ;
y	linear coordinate calculated along the normal to the plate,	m ;
α	thermal diffusivity,	m^2/sec.;
δ	boundary layer thickness,	m ;
λ	dimensionless time, $(\tau u_\infty/x)$,	——— ;
ν	kinematic viscosity,	m^2/sec.;
τ	time,	sec.;
w	angular velocity of plate surface temperature oscillation, $(2\pi f)$,	rad/sec.;

Subscripts:

h , hydrodynamic boundary layer;

m , mean value;

t , thermal boundary layer;

w , plate surface;

∞ , main stream.

Introduction:

The case of unsteady heat transfer to a fluid flowing over a flat plate is widely encountered in engineering applications , and its calculation presents many difficults. Due to the unsteady temperature profile in the boundary layer there should be some differences between the instantaneous heat transfer rate in the unsteady state and that in the steady state. The size of such difference would be depentent of the response charateerestic of the boundary layer.

Moore and Ostrach [1] were among the first researchers who studied in details the behaviour of a laminar boundary layer over isothermal flat plate moving with a harmonic variable velocity. Yang and Huang [2] studied the unsteady characteristics of laminar boundary layer on a flat plate moving with arbitrary time-dependent velocity. Kotake and Isao [3] studied experimentally the heat transfer characteristics of a flat plate located in a sinusoidally oscillating flow with large amplitude. They found that the velocity oscillation could not make any difference between the time mean Nusselt number and the steady state Nusselt number.

Mohamed [4] studied analytically the same problem, and found that the time mean Nusselt number was not affected by neither the frequency nor the amplitude of flow velocity oscillation in the case of flat plate with constant heat flux. In the case of plate of constant temperature he found that the time mean Nusselt number is slightly affected by the frequency, where as, the amplitude of flow velocity affected it.

Sparrow [5], and Adams and Gebhart [6] studied by integral method the response to a unit step of time variation in a flat plate surface temperature. Cess [7], and Pavlovskii and Awad [8] studied the unsteady heat transfer to laminar boundary layer over a flat plate with a stepwise and arbitrary time variation of surface temperature. It was assumed that plate surface temperature is spatially constant and initially equal to the free stream temperature. It found that for small times the process was that of transient heat conduction. The heat flux was inversely proportional to the square root of the time from the step change in surface temperature. For large times the heat flux returned to steady state.

The present work is devoted to study the unsteady heat transfer to laminar boundary layer over a plate. An analytical procedure is proposed to estimate the instantaneous heat transfer from a flat plate with a sinusoidally oscillating surface temperature. This case can be encountered in internal combustion engines.

Analysis:

Fig. (1) shows schematically the physical model and the system of coordinates. Assume that the flat plate is subjected to steady state laminar flow. The plate surface temperature is a spatially uniform and is periodically changed and discribed by sinusoidal function. This case can be encountered, for example, in the reciprocating internal combustion engines, where the wall is heated periodically from inside, and considering the wall has zero heat capacity.

The plate surface temperature can be described by the following relation

$$t_w = t(x,o,\tau) = t_{w,m} + a \sin w\tau \quad , \qquad (1,a)$$

or

$$T_w = T(x, o,\tau) = T_{w,m} + a \sin w\tau$$

$$= T_{w,m}(1 + A \sin w\tau) . \qquad (,b)$$

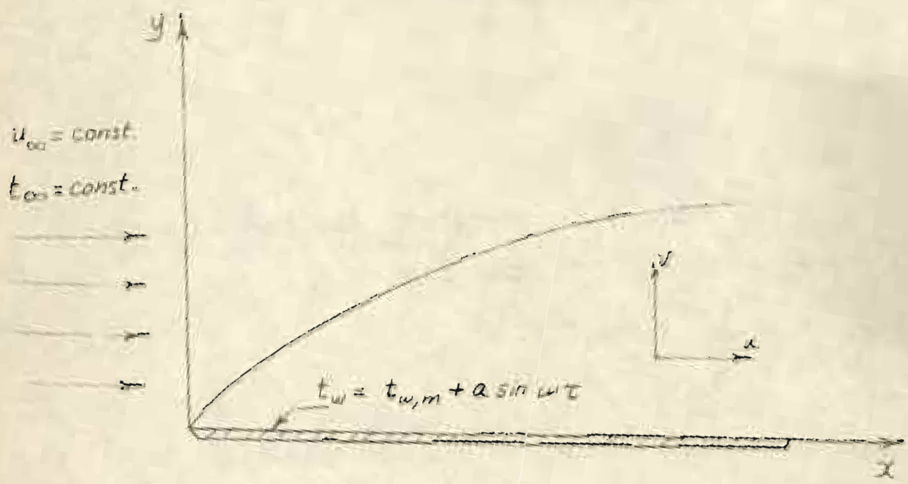

Fig.(1). Physical Model.

The fluid free stream velocity u_∞ and temperature t_∞ are taken to be constants, and it is assumed that fluid is incompressible, its physical properties do not depend on temperatur and the flow is parallel to the plate. Neglecting the heat generated within the fluid flow due to friction the energy equatio which describes the temperature field $T(x,y,\tau)$ in the fluid ca be written in the following form:

$$\frac{\partial T}{\partial \tau} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} . \qquad (2)$$

Using Eqn. (1) the energy Eq n(2) for the fluid on the plate surface takes the following form:

$$\frac{\partial^2 T}{\partial y^2}(x,0,\tau) = \frac{1}{\alpha}\ \frac{\partial T}{\partial \tau}(x,0,\tau)$$
$$= \frac{1}{\alpha}\ T_{w,m}(Aw \cos w\tau) \qquad (3)$$

From the specifications of the thermal boundary layer at the outer edge of the boundary layer we write:

$$T(x, \delta_t, \tau) = 0 \qquad (4)$$

and

$$\frac{\partial T}{\partial y}(x, \delta_t, \tau) = 0. \qquad (5)$$

Assume that the temperature profile in the thermal boundary layer is described by the following form

$$T(x,y, \ \ ) = \sum_{n=0}^{3} f_n(x,\tau)_* \, y^n \qquad (6)$$

The periodic change of the plate surface temperature has no effect on the velocity profile in the hydrodynamic boundary laye because the fluid physical properties are assumed to be constant The velocity profile in laminar boundary layer over a flat plate an be given by a cubic parabola as reported in [9].

The thickness of the hydrodynamic boundary layer in this case is given by the following relation.

$$\delta_h = 4.64 \; (\nu x/u_\infty)^{1/2}$$

Considering that in the present case the ratio of the thickness of the thermal boundary layer to the thickness of the hydrodynamic boundary layer equals to that in the case of steady state, i.e.

$$(\delta_t/\delta_h) = 1/(1.026 \; P_r^{-1/3})$$

or

$$\delta_t = 4.52 \; (\nu x/u_\infty)^{1/2} \cdot P_r^{1/3} \qquad (8)$$

Using Eqns. (1, 3, 4, 5, 6 and 8) we can get the functions $f_n(x,\tau)$ as functions of distance from leading edge, plate surface temperature, and flow properties. Introducing these functions in Eqn. (6) gives the temperature field in the following nondimeusional form

$$\frac{T(x,Y,\lambda)}{T(x,0,\lambda)} = 1 - 1.5\,Y + 0.5\,Y^3 - 5.11\,P_r^{1/3} \cdot \left[\frac{AW \cos W}{1 + A \sin W}\right]\left[Y - 2\,Y^2 + Y^3\right] \qquad (9)$$

The time histories of temperature in boundary layer are plotted in Fig. (2), assuming that the flowing fluid is air with free stream t_∞ = 300 k. The rate of temperature change with time in boundary layer increases by increasing the relative amplitude A, and by increasing the nondimensional angular velocity of plate surface temperature oscillation $W(=2\pi fx/u_\infty)$, i.e. by increasing frequency f, distance from plate leading edge x, and by decreasing free stream velocity.

The time histories of Nusselt number (heat flux from the plat surface) are plotted in Fig. (3) for air stream at t_{∞}=300 K. It i clear that the deviation of the instantaneous value of Nusselt number in case of periodic change of plate surface temperature fro that in quasi-steady state increases as the rate of plate surface temperature change with time increases. This result is in agreemen with that obtained experimentally [10] , where it is found that th initial value of heat transfer coefficient is 3.8 times the mea value as the surface temperature changes at a rate of 600 K/sec. T instantaneous value of Nusselt number is greater than the quasi-stationary value when the temperature difference $T_w=(t_w - t_{\infty})$ incr ses with time and vice-versa. This result is in agreement with th theoretically obtained and reported in [8]. The rate of the increasing of instantaneous value of Nusselt number is greater tha the rate of its decreasing.

The time mean Nusselt number can be obtained by integration of Eqn. (11) within a period of complete cycle of plate surface tempe ature variation and dividing by the value of this period. The time mean Nusselt number takes the following form.

$$Nu_m = \frac{q_m \cdot x}{k \cdot T_{w,m}} = 0.332 \ Re^{1/2} \ Pr^{1/3} \qquad (12)$$

Equation (12) is the same Nusselt relation for the case of steady heat transfer from flat plate to laminar boundary [9] . This resul is simillar to that obtained experimentally [3] and theoretically [4] for constant plate surface temperature and periodic change of free stream velocity.

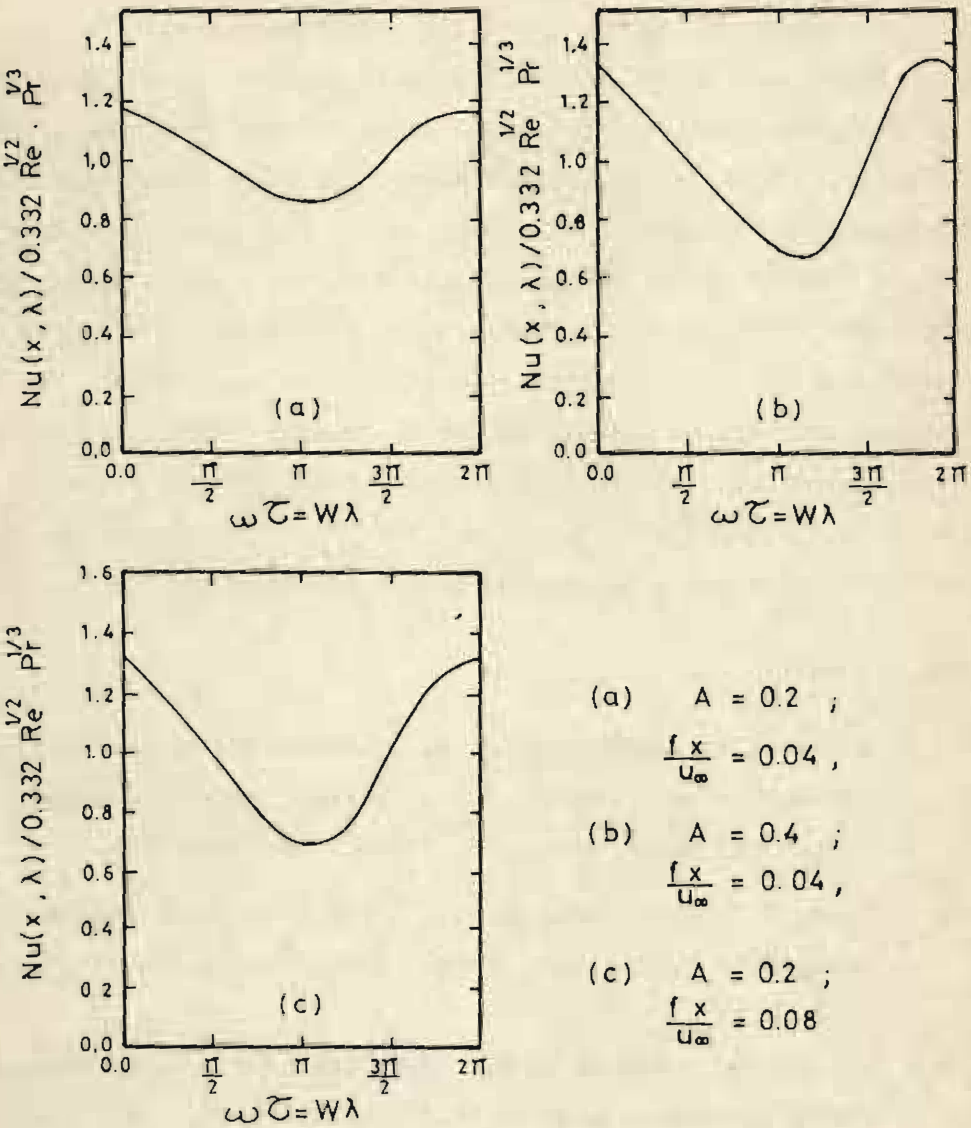

FIG.(3) INSTANTANEOUS NUSSELT NUMBER

Conclusion:

The presented analysis and the performed calculations show that the instantaneous value of the Nusselt number in the case of periodic change of plate surface temperature differs from that of quasi-stationary plate surface temperature. The percentage difference increases by increasing Prandtl number, amplitude, frequency of plate surface oscillation, distance from the plate leading edge, and rate of plate surface temperature change with time, and it decreases by increasing the velocity of the free steam. The instantaneous value of Nusselt number is greater than the quasi-steady value when the plate surface temperature-difference increases with time, and vice-versa. It is found also that time mean value of Nusselt number is the same as that in the steady state.

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