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DECENTRALIZED CONTROL OF LINEAR SYSTEMS VIA MODIFIED BELLMAN-LYAPUNOFF EQUATION

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ABSTRACT:

The paper presents a new approach for designing Decentralized Controllers for linear dynamic systems. In this approach, sufficient conditions for suboptimal decentralized control are decided by the aid of using Modified Bellman-Lyapunoff Equation, with quadratic form for the performance index under optimization.

INTRODUCTION:

In last few years, much attention has been directed in literatures to decentralized control of large scale systems [1-8]. This paper tries to present a more general method, which is applicable for wider classes of systems, specially that, when those systems are represented by linear dynamic models for the different subsystems [4,6]. The basic idea of the design procedure depends on the insurance of the stability for each isolated subsystem with fulfilling a minimum value for the constructed performance index for the global interconnected system. For this purpose, the objective function under optimization will be constructed as the form of Modified Bellman-Lyapunoff Equation [3,6,7,8], with a special quadratic construction for Lyapunoff Functions of the linear isolated subsystems. In future, by some added modifications for the subsystems Lyapunoff Functions, the corresponding objective function and the optimization steps, this way of design can be generalized to be applicable for nonlinear systems which represent the more complicated condition of large scale dynamic control systems.

The first part of the paper contains the mathematical description and the problem formulation. The second part presents the suggested suboptimal decentralized controllers design. Finally the paper contains a numerical example of two different interconnected subsystems as application [9].

STATEMENT OF THE PROBLEM:

Let us consider the general form of the large scale system which can be described by the following linear time invariant state model:

$$\mathring{x}_i = A_{ii}\, x_i + B_i\, u_i + \sum_{\substack{j=1 \\ j\neq i}}^{s} A_{ij}\, x_j \qquad , \quad i=1,2,\ldots,s \qquad (1)$$

where: A_{ii} , A_{ij} and B_i are coefficient matrices of constants of appropriate sizes.

Supposing that the full controllability conditions of the system affected by decentralized control are fulfilled, then the main task of the problem is to find the control vector of:

$$u_i = K_i\, x_i \qquad i=1,2,\ldots,s \qquad (2)$$

which satisfies the following conditions:

1- Asymptotic stability for all the isolated subsystems.
2- Stability for the global interconnected system.
3- Minimum value for the performance index:

$$J = \sum_{i=1}^{s} \int_{0}^{\infty} (x_i^T\, Q_i\, x_i + u_i^T\, R_i\, u_i)\, dt \qquad (3)$$

where: Q_i , R_i are positive definite - at least semidefinite - energy matrices [3,6] , which can be constructed according to practical considerations for the real system under study.

SUBOPTIMAL DECENTRALIZED CONTROLLERS DESIGN:

For solving the previous stated problem, the objective function under optimization can be constructed according to the basic idea of Bellman-Lyapunoff concept [2,3,6,7,8]in the form:

$$B(x,u) = \min_{u_i} \left[\frac{dV(x,u)}{dt} + \sum_{j=1}^{s} (x_j^T\, Q_j\, x_j + u_j^T\, R_j\, u_j) \right] \qquad (4)$$

where: $V(x,u)$ is the global system Lyapunoff Function,which can be chosen in the form:

$$V(x,u) = \sum_{i=1}^{s} a_i\, V_i \qquad (5)$$

where: a_i is a suggested positive modification factor [3,7,8] for the subsystem number i.

V_i is the Lyapunoff Function of the isolated subsystem number i, which can be chosen in the following quadratic form:

$$V_i = x_i^T x_i \qquad , \quad i=1,2,\ldots,s \qquad (6)$$

Substituting equations (5),(6), into (4), the Bellman-Lyapunoff equation can be constructed in the following modified form:

$$B = \underset{u_i}{\text{Min}} \left\{ \sum_{i=1}^{s} a_i \left\{ x_i^T (A_{ii}^T + A_{ii})\, x_i + 2x_i^T \sum_{\substack{j=1 \\ j\neq i}}^{s} A_{ij}\, x_j + 2u_i^T B_i^T x_i + x_i^T Q_i x_i + u_i^T R_i u_i \right\} \right\} \leqslant 0 \qquad (7)$$

Applying the sufficient conditions of optimality [3,6,7], sub-optimal expression for the control vector of subsystem number i, can be deduced to the form:

$$u_i = - a_i R_i^{-1} B_i^T x_i \qquad , \quad i=1,2,\ldots,s \qquad (8)$$

Back substitution leads to the following form of the Modified Bellman-Lyapunoff Equation:

$$B = \sum_{i=1}^{s} \left\{ x_i^T \left\{ Q_i + a_i (A_{ii}^T + A_{ii}) - a_i^2 B_i R_i^{-1} B_i^T \right\} x_i + 2a_i x_i^T \sum_{\substack{j=1 \\ j\neq i}}^{s} A_{ij} x_j \right\} \leqslant 0 \qquad (9)$$

Optimal design for the suggested modification factors (a_i) can be expected in such a way that, the global system Lyapunoff Function (equation(5)), must be close as possible to the optimal Lyapunoff Function for this system, i.e. by applying the condition :

$$\underset{a_i}{\text{Max}} \left\{ B (x, a_i) \right\} \leqslant 0 \qquad (10)$$

Substituting the positive - semi- definite energy matrices Q_i and R_i by some chosen values from practical experience, and applying the sufficient conditions of maximization for eqn. (10), the expected values for the suggested modification factors a_i can be deduced as it is clear in the model example.

REMARKS ON THE DESIGNED CONTROLLERS:

1- If Rank $[B_i R_i^{-1} B_i^T]$, i=1,2,...,s is similar to the size of x_i, the Decentralized Controllers given by equation (8) will exist always.

2- If Rank $[B_i R_i^{-1} B_i^T]$, i=1,2,...,s is smaller than the size of x_i, and subsystems A_{ii} are stable, the solution of the decentralized control:

   a- always exists when the dynamic equation (1) is stable via Vector Lyapunoff-Function.

   b- may exist when the system (1) is unstable, via Vector Lyapunoff-Function.

NUMERICAL EXAMPLE:

Let us consider an example of a global system, which consists of the two interconnected subsystems [9] :

$$\mathring{x}_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} L_{12}\, x_2 \qquad (11)$$

$$\mathring{x}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} L_{21}\, x_1 \qquad (12)$$

where: $\|L_{12}\| \leqslant 1$, $\|L_{21}\| \leqslant 1$

subsystem (11) is stable
subsystem (12) is unstable
global system is unstable too.

The main task is to design decentralized controllers:

$$u_1 = k_1\, x_1 \quad , \quad u_2 = k_2\, x_2 \qquad (13)$$

which insure stability for both the isolated subsystems, and for the global system. According to remark (2), let us consider the local control for the second subsystem in the form :

$$u_2 = u_2' + u_2'' \quad \text{where} \quad u_2'' = [-2 \quad -5 \quad -3]\; x_2 \tag{14}$$

and local control u_2'' provides stability of second subsystem. After diagonalization of subsystems with equation (14), it can be obtained that:

$$\dot{x}_1' = \begin{bmatrix} -.5 & -.866 \\ .866 & -.5 \end{bmatrix} x_1' + \begin{bmatrix} .577 \\ -.577 \end{bmatrix} u_1 + .577 \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} L_{12} x_2'$$

$$x_1 = T_1\, x_1' = \begin{bmatrix} 1 & 1 \\ .366 & -1.366 \end{bmatrix} x_1'$$

$$\dot{x}_2' = \begin{bmatrix} -1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} x_2' + \begin{bmatrix} -.5 \\ -.5 \\ 1 \end{bmatrix} u_2' + \begin{bmatrix} -.683 & .183 \\ -.683 & .183 \\ 1.366 & -.366 \end{bmatrix} L_{21}\, x_1'$$

$$x_2 = T_2\, x_2' = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 0 & -1 \\ 2 & -2 & 1 \end{bmatrix} x_2'$$

According to equation (8), the suggested decentralized controllers will be:

$$u_1 = -\, a_1\, b_1'^T x_1' = -\, a_1\, b_1'^T\, T_1\, x_1 = -\, a_1 \left[.333 \quad .666\right] x_1$$

$$u_2' = -\, a_2\, b_2'^T x_2' = -\, a_2\, b_2'^T\, T_2\, x_2 = -\, a_2 \left[1.5 \quad 3 \quad 1.5\right] x_2$$

<u>Case I:</u> Substituting $Q_i = [0]$, $R_i = [I]$, $i=1,2,\ldots,s$, a_1 and a_2 can be calculated from equation (10) to be:

$a_1 = 2.9$, $a_2 = 1.25$

and the decentralized controllers will be:

$$u_1 = \left[-.9657 \quad -1.9314\right] x_1$$

$$u_2 = \begin{bmatrix} -3.875 & -8.75 & -4.875 \end{bmatrix} x_2$$

and the eigenvalues of the global system, affected by the controllers will be:

$$\beta_1 = -0.443 + j\,0.594$$
$$\beta_2 = -0.443 - j\,0.594$$
$$\beta_3 = -2.366 + j\,2.940$$
$$\beta_4 = -2.366 - j\,2.940$$
$$\beta_5 = -1$$

Case II: substituting $Q_i = I_i$, $R_i = I_i$, $i=1,2,..,s$ the corresponding results will be:

$$a_1 = 3.1$$
$$a_2 = 2.2$$
$$u_1 = \begin{bmatrix} -1.0326 & -2.0646 \end{bmatrix} x_1$$
$$u_2 = \begin{bmatrix} -5.3 & -11.6 & -6.3 \end{bmatrix} x_2$$

$$\beta_1 = -0.748$$
$$\beta_2 = -0.907$$
$$\beta_3 = -4.524$$
$$\beta_4 = -1.565 + j\,0.832$$
$$\beta_5 = -1.565 - j\,0.832$$

CONCLUSION:

The suggested way of designing Suboptimal Decentralized Controllers, for the large scale systems fulfils the stability conditions of the global system, with confirming the minimum conditions of the optimized performance index. The obtained results can be considered to the level of the best results of that technique presented in reference [9] .

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