

6-1-2021

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Recommended Citation

Amin, Abd El-Rahman (2021) "Evaluation of the Air-Gap MMF Distribution and Harmonic Contents of the Six-Phase Stator Windings.," *Mansoura Engineering Journal*: Vol. 10 : Iss. 1 , Article 7.

Available at: <https://doi.org/10.21608/bfemu.2021.176995>

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EVALUATION OF THE AIR-GAP MMF DISTRIBUTION AND HARMONIC
CONTENTS OF THE SIX-PHASE STATOR WINDINGS

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Abstract:

The paper aims at giving the general procedure for calculating the magnitudes of all the spatial harmonic magnetomotive force waves; MMF. The analytical and graphical methods have been employed. These methods are applicable to any polyphase winding with uniform coils and slot pitches, which is excited by a balanced polyphase supply.

Two types of six-phase stator winding configuration are investigated. Each of these configuration consists of two types of three-phase winding with displacement angle, either equal to 30 or 60 electrical degrees. The scope of the investigation of double layer windings cover the unity and chording pitch, for q either equal to one or two slots.

Analytical investigation shows that, the MMF space harmonic orders of the six-phase winding with δ equal to 30 electrical degrees, being $h = 12k + 1$ compared with $h = 6k + 1$ for the conventional three-phase winding. On the other hand, the six-phase winding with δ equal to 60 electrical degrees shows the same MMF space harmonic contents of the conventional three-phase winding.

Goerge's vector diagram has been used as a graphical method for plotting the MMF produced by the six-phase windings. From which, the instantaneous values of the MMF over the different teeth could be found. Accordingly, a computer program is implemented to give the fundamental and higher space harmonics for each case discussed above.

In order to verify the computer program results, previously published [5] space harmonic analysis for a three-phase winding with $q = 2$, confirms the accuracy of the computer program used for the analysis presented in this paper.

1. Introduction:

Because of ever-increasing power supply requirements, increased transmission capability and efficient utilization of rights of way are some of the most common concerns of present day electric utilities. A considerable amount of research has been carried out and several alternative for that purpose were considered. The concept of multiphase transmission [1] in place of conventional three-phase system, offers an appealing and unique solution to the problem. At present, six-phase transmission appears to be the most promising among multiphase systems for possible realization in near future [2].

Therefore, because of the growing interest of six-phase transmission and distribution, the paper aims at giving the analytical and graphical investigation of the MMF produced by a six-phase stator winding. The scope of investigation covers two types of six-phase double layer windings, with displacement angle, δ , either equal to 30 or 60 electrical degrees. The first arrangement with δ equal to 30 electrical degrees is considered for numerous winding pitch with the number of slots per pole per phase, q , either equal to one or two slots. The second arrangement with δ equal to

60 electrical degrees is considered for unity pitch with q equal to three slots.

2. The Principle Theory Of The Six-Phase Windings:

It is well known that, increasing the number of phases in a winding reducing the phase spread angle, and the fundamental spread factor is increased.

Therefore, greater output can be obtained from a machine with a given frame size. This explains why the majority of industrial conventional three-phase windings are, actually, six-phase windings; in which each phase belt is halved. These windings have a phase spread of 60-electrical degree and a spread factor of 0.955, compared with 0.827 for a 120-electrical degree spread. The 60-spread winding has the additional advantage that the number of parallel circuits that can be obtained in each phase is, in general, twice the number possible in a 120-spread winding. This is particularly important in relation to large low voltage machines, because it permits a low effective value of conductors in series per phase.

Generally, an improved MMF waveform can be obtained by reducing the phase spread, as the fundamental spread factor will be increased. Accordingly, from practical point of view, a study is made on the six-phase winding. Such a winding can be obtained from two three-phase windings displaced in phase by δ electrical degrees. Numerous six-phase windings configuration can be obtained by varying the displacement angle δ . However, the investigation is limited for two values of δ , which could be found in practical application. These two values of the displacement angles are 30 and 60 electrical degrees.

Strictly speaking, the space harmonic analysis is carried out for the winding as a whole, which is made up of two winding groups, having their effective number of turns and displaced in space by an angle either equal to 30 or 60 electrical degrees. This winding is supplied from two three-phase voltage sources of the same phase voltage and a time displacement equal to the displacement angle δ . The winding phases will draw currents of very nearly the same magnitude, but displaced in time phase by δ electrical degrees.

3. The Resultant MMF Of A Symmetrical Six-Phase Windings:

The analytical method has been employed to deduce the general equations giving at any instant of time. The only assumption being made that, the currents carried thereby are of sinusoidal waveforms.

All phases of a symmetrical six-phase winding are identical and forming two three-phase groups displaced from each other by δ electrical degrees. The windings of each three-phase group are uniformly distributed and have axes that are displaced 120 electrical degrees apart. Figure 1 shows the representation of the stator winding MMF axes. The six stator phases are labelled A, B, C, D, E, and F. The magnitudes of the fundamental and harmonic components of the MMF waveform of the completed winding when supplied with a balanced six-phase current, and their directions of rotation, can be simply determined from the magnitude of the fundamental and harmonic components of the MMF waveform of one phase of the winding.

Burbidge 3 has shown that the MMF distribution for a phase given by;

$$F(\theta) = \frac{2 i_p}{a\Pi} \sum_{c=1}^C \sum_{h=1}^{\infty} N_c \frac{\sin h \alpha_c}{h} \cos h(\theta - B_c) \quad (1)$$

The h-th harmonic component of MMF is therefore;

$$F_h(\theta) = \frac{2 i_p}{a\Pi} \sum_{c=1}^C N_c \frac{\sin h \alpha_c}{h} \cosh(\theta - B_c) \quad (2)$$

The distribution of magnetizing turns, N, due to the phases A, B, C, D, E, and F, respectively, acting at any angular position θ around the air-gap, may be represented by the expressions;

$$\sum_{m=1}^6 N = \sum_{h=1}^{\infty} N_h \left[\begin{aligned} &\cos h \theta + \cos h(\theta - 2\Pi/3) + \cos h(\theta - 4\Pi/3) + \\ &\cos h(\theta - \delta) + \cos h(\theta - 2\Pi/3 - \delta) + \cos h(\theta - 4\Pi/3 - \delta) + \\ &\dots \dots \dots \end{aligned} \right] \quad (3)$$

The current in each phase may be represented by

$$\begin{aligned} i_A &= I_0 \sin \omega t, \quad i_B = I_0 \sin(\omega t - 2\Pi/3), \quad i_C = I_0 \sin(\omega t - 4\Pi/3), \\ i_D &= I_0 \sin(\omega t - \delta), \quad i_E = I_0 \sin(\omega t - 2\Pi/3 - \delta), \\ i_F &= I_0 \sin(\omega t - 4\Pi/3 - \delta) \end{aligned} \quad (4)$$

The resultant MMF, F(θ, t), is found by summing the six separate phase MMF's as follows;

$$F(\theta, t) = i_A N_A + i_B N_B + i_C N_C + i_D N_D + i_E N_E + i_F N_F \quad (5)$$

Therefore,

$$\begin{aligned} F(\theta, t) = I_0/2 \sum_{h=1}^{\infty} N_h \left[&\sin \alpha + \sin(\alpha - (1+h) 2\Pi/3) + \\ &+ \sin(\alpha - (1+h) 4\Pi/3) + \sin(\alpha - (1+h) \delta) + \\ &+ \sin(\alpha - (1+h) 2\Pi/3 - (1+h) \delta) + \\ &+ \sin(\alpha - (1+h) 4\Pi/3 - (1+h) \delta) + \\ &+ \sin B + \sin(B - (1-h) 2\Pi/3) + \sin(B - (1-h) 4\Pi/3) + \\ &+ \sin(B - (1-h) \delta) + \sin(B - (1-h) 2\Pi/3 - (1-h) \delta) + \\ &+ \sin(B - (1-h) 4\Pi/3 - (1-h) \delta) \right] \quad (6) \end{aligned}$$

where, $\alpha = h\theta + \omega t$, $B = h\theta - \omega t$

The fundamental MMF, of the six-phase winding with $\delta = \Pi/6$, can be determined from equation 6 by substituting $h = 1$ and $\delta = \Pi/6$; as follows:

$$F_1(\theta, t) = 6/2 (I_0 N_1) \sin(\theta - \omega t) \quad (7)$$

As well as the fundamental MMF of the six-phase winding with $\delta = \Pi/3$, can be determined in the same way, which gives the same fundamental MMF derived in equation 7.

4. The Space Harmonics Of The Six-Phase Windings:

It is well known that, when a balanced six-phase winding is excited from a six-phase voltage source of sinusoidal waveform, rotating fields associated with the winding MMF harmonics are established. The harmonic

field of order h , has a speed of rotation $1/h$ of that of the fundamental, its direction of rotation depends on the number of phases and the type of winding. Equation 6 shows the magnitude of the h -th harmonic MMF produced by a symmetrical six-phase winding. Therefore, the MMF's of all orders can most easily be found for each of the six-phase winding with δ either equal to 30 or 60 electrical degrees.

The harmonic components of the resultant six-phase MMF with δ equal to 30 electrical degrees can be found from equation 6, which gives;

$$\begin{aligned} F_5(\theta, t) &= 0, & F_7(\theta, t) &= 0 \\ F_{11}(\theta, t) &= 6/2 (I_0 N_{11}) \sin(11\theta + \omega t) \\ F_{13}(\theta, t) &= 6/2 (I_0 N_{13}) \sin(13\theta - \omega t) \end{aligned} \quad (8)$$

On the other hand, the harmonic components of the resultant six-phase MMF with δ equal to 60 electrical degrees, can be found from equation 6, which gives;

$$\begin{aligned} F_5(\theta, t) &= 6/2 (I_0 N_5) \sin(5\theta + \omega t) \\ F_7(\theta, t) &= 6/2 (I_0 N_7) \sin(7\theta - \omega t) \\ F_{11}(\theta, t) &= 6/2 (I_0 N_{11}) \sin(11\theta + \omega t) \\ F_{13}(\theta, t) &= 6/2 (I_0 N_{13}) \sin(13\theta - \omega t) \end{aligned} \quad (9)$$

Thus all even harmonics are commonly absent; but in the uncommon cases when they are present, they are reproduced in the resultant MMF waveforms to the same scale. Using the result already obtained in equation 8 for the harmonic components of a six-phase winding with $\delta = 30$, it follows from this, the only MMF harmonics are those of orders, $h = 12k + 1$, where $k = 0, 1, 2, 3, \dots$ etc. Therefore, fifth and seventh harmonics, which they are often particularly troublesome, are completely eliminated. On the other hand, using the result obtained in equation 9 for the harmonic components of a six-phase winding with $\delta = 60$, it follows from this, the MMF harmonics are those of orders, $h = 6k + 1$, where $k = 0, 1, 2, 3, \dots$ etc. Therefore, it produces the same harmonic orders of the conventional three-phase windings.

5. The Graphical Determination Of The MMF Space Waveform Of A Six-Phase Windings:

Although of great practical use of analytical method in MMF harmonic analysis, it is a laborious process and as a rule is of the nature of an approximation. However, the graphical method is simple and quick. It has been employed in making the diagrams, since it gives the MMF shape at once and avoids the troublesome of analytical method.

The value of the MMF over the different teeth can be found easily by using Goerge's vector diagram of the MMF. It offers the advantage of giving a graphical representation of the MMF over the different teeth, from which the magnitude of the harmonics, the influence of chording on the harmonics, and so forth, can be visualized.

The Goerge's diagram is based upon the following consideration. If $A(\theta, t)$ denotes the ampere-conductor per unit circumference, then the MMF at a point θ is ;

$$F(\theta, t) = \int A(\theta, t) d\theta + C \quad (10)$$

where $A(\theta, t)$ is a sinusoidal function of time, $F(\theta, t)$ can be represented as a vector [3]. On the other hand, if the ampere-conductor is concentrated into the slots, the instantaneous value of the MMF above the r -th slot pitch is ;

$$F(\theta, t) = \sum A_r(\theta, t) + C \quad (11)$$

Figure 2 shows the MMF space waveform and Goerge's diagram for a two pole, 12 slots, three-phase winding. Figures 3 to 8 show the MMF space waveforms and Goerge's vector diagrams for a two pole, six-phase winding with δ equal to 30 electrical degrees, with q either equal one or two slots. However, Figures 9 and 10 show the MMF space waveforms and Goerge's vector diagrams for a two pole, six-phase winding with δ equal to 60 electrical degrees, and the three-phase winding, respectively. In order to explain the procedure, Fig. 3 could be considered as an example. Figure 3 shows this procedure for a double pole pitch, six-phase winding with δ equal to 30, $q=1$, and unity pitch winding. In this case the upper and the lower layers lie in the same slots. Figure 3a shows the distribution of the 12-slot between the layers and phases. Figure 3b shows the directions of the currents, at an instant of time, in different slot-groups, and the Goerge's vector diagram. The polygon closes after each two poles, when the number of slots per pole per phase, q , is an integer. Figure 3c shows the space MMF waveform.

The polygon represents the integral, $\int A(\theta, t) d\theta$. In order to find the MMF over the different teeth 1, 2, 3, ..., it is necessary to determine the constant C , that is, to find the pole of the polygon. If this pole has been found to be the point P ; then the distances P_1, P_2, P_3, \dots , are the amplitudes of the MMF at the tooth centers 1, 2, 3, It is assumed that, the centers of the slots coincide with the centers of the distances 1-2, 2-3, ... and that, at these points, the MMF increases step by step, each being equal to the maximum ampere-conductor per slot, $I_0 N_c$; the points 1, 2, 3, ... are then the centers of the teeth. The instantaneous values of the MMF over the different teeth can be found by projecting the vectors P_1, P_2, P_3, \dots on the time line. The shape of the MMF of a six-phase system with $\delta = 30$, changes each one-twelfth of a period and repeats itself every one-sixth of a period. This condition can be satisfied by the vector diagram. The position of the time line can be chosen arbitrarily, since the magnetic energy in the air-gap is constant. The time line was chosen vertically in Figure 3b. If we make the unit of length in the vector diagram, $q = 1$, equal to $I_0 N_c$, the amplitudes of the MMF's of the different teeth, as well as, the instantaneous values of the MMF's, will be expressed per unit. On the other hand, for the winding with $q = 2$, the maximum ampere-conductor per slot $I_0 N_c$, assumed equal to half the unit length, in order to keep the algebraic sum of the ampere-conductor per phase constant. The instantaneous values of the MMF waveform were recorded every five electrical degrees and supplied to the computer program as a data.

The MMF waveforms of Figures 2 to 8 were analysed by the computer program, in order to determine the belt and slot harmonic components, and to facilitate the comparison between three- and six-phase with $\delta = 30$ harmonic contents, as well as, to investigate the effect of increasing q from one to two slots, for unity and chording pitch. The results are listed in Tables 1 to 7.

On the other hand, Goerge's vector diagrams are established for the

six-phase winding with $\delta = 60$, and for the three-phase winding, respectively. The resultant MMF space distribution for $q = 3$, in Figures 9 and 10 show the same pattern, which confirms the analytical investigation presented in the foregoing section. Although, there is no difference between the six-phase winding with $\delta = 60$, and the conventional three-phase winding, the six-phase winding gives more reliability in practical operation.

6. Discussion Of The Computer Results:

By means of a computer program using the step by step technique, some of six-phase windings have been investigated. The results are contained in Tables 1 to 7. Table 1 can serve as a check on the computer program validity. It contains the harmonic analysis for a unity pitch, double layer, three-phase winding with $q = 2$. The computer results of the harmonic contents showing consistent with the previously published results [5]. The coil arrangement for the three-phase winding, 12 slots, unity pitch, and 2-pole is shown in Figure 2a. The minus sign indicates a reversed coil current. The winding connection is described by the listing of stator-coil currents. The body of the Table gives the relative values of the various space harmonic waveforms produced by the winding in terms of the maximum height of the stepped waveform; which denoted by YN_1 , and also as a percentage of the fundamental harmonic; YN_2 .

The 12-slot winding of double pole pitch is wound for a three-phase and six-phase with $\delta = 30$, and unity pitch, as shown in Figures 2a and 3a. The results are given in Tables 1 and 2, respectively. The computed fundamental harmonic of the six-phase winding with $\delta = 30$ is found equal to 103.231% of 3.73 p.u. maximum height of the stepped waveform, while the corresponding three-phase winding fundamental harmonic is equal to 92.69% of 4.0 p.u., which confirm the advantage of increasing the number of phases. However, a greater output equal to 3.85% can be obtained from a three-phase machine, if it is rewound as a six-phase winding with $\delta = 30$. In Table 1, the higher space harmonics of orders 5-th, 7-th, 11-th, 13-th, 17-th, 19-th, 23-th, and 25-th harmonic waves need especial attention. However, the comparison of these with the results in Table 2, for six-phase winding with $\delta = 30$, which show a very small 5-th and 7-th harmonic values. On the other hand, it was noticed that, 11-th, 13-th, 17-th, 19-th, 23-th, and 25-th show the same corresponding values. These harmonics are known as the slot harmonics, which were shown to be the same for all different winding arrangements, since their strength are solely a function of slots per pole and are in no way dependent upon the winding distribution.

The effect of chording the winding can be investigated by chording the winding with various slot pitches. Figures 4 to 6 show the Goerge's vector diagrams and the MMF distribution for the same winding discussed above, with chording the pitch by one slot pitch, two slot pitch, and three slot pitch, respectively. Tables 3 to 5 give the space MMF harmonics contents. From which, it can be concluded that, the effect of chording the winding is to reduce the relative importance of the belt harmonics, while the slot harmonic magnitudes, YN_2 , is the same as with unity pitch results in Tables 1 and 2.

The effect of increasing the number of slots per pole per phase, $q = 2$, for six-phase winding with $\delta = 30$, unity pitch, and chording are investigated. Figures 7 and 8 show the Goerge's vector diagram and the space MMF stepped waveforms of particular instant of time.

The results given in Table 7 correspond exactly to those given in Table 6, for the case of unity pitch coils, and $q = 2$, with the exception that the winding is taken to be chorded by one slot pitch. The belt harmonic magnitudes have shown to be considerably reduced.

In general, the results of six-phase winding with $\delta = 30$, show that, the belt space harmonic contents in the MMF are rarely of importance in comparison with the conventional three-phase winding. Accordingly, a six-phase winding with $\delta = 30$ has been suggested for use, in order to suppress the lower belt harmonic of orders 5-th, and 7-th, which they are often troublesome.

7. Conclusion:

A complete investigation of the MMF produced by a six-phase stator winding are carried out by the analytical and graphical methods. Firstly, the analytical examination shows that, the MMF produced by the six-phase winding with $\delta = 30$ are greater because of the higher winding factors. However, the space belt harmonic orders being, $h = 12k + 1$, compared with $h = 6k + 1$ of three-phase winding. Thus fifth and seventh harmonics are completely eliminated. Secondly, Goerge's diagram for six-phase winding with $\delta = 30$ was established for double pole pitch with unity and chording pitch, for $q = 1$ or 2 slots. From which a six-phase winding with $q = 2$, and chording by one slot pitch has a vector polygon near to the circle, as shown in Figure 8. Such winding provide less harmonic contents as given in Table 7.

In order to verify the above investigation a computer program was implemented to give the fundamental and higher harmonics. Tables 1 to 7 contains the results. Previously published studies on a conventional three-phase winding with $q = 2$, has been compared with the results in Table 1. This comparison shows the validity of the computer program used, and confirms the analysis of the six-phase winding presented in this paper.

On the other hand, the space MMF for six-phase winding with $\delta = 60$ show no difference with the conventional three-phase winding. However, it provides more reliable operation in practical application.

In general, an improved MMF waveform can be obtained by reducing the phase spread from 60 to 30, and supplying the winding from a quasi six-phase supply.

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9. List Of Symbols:

- a Number of similar parallel paths into which each phase is divided.
- C Number of coils per phase.
- $F(\theta, t)$ MMF distribution as a function of position, θ , and time t .
- h Space harmonic order.
- i_p Instantaneous phase current.
- I_0 The maximum value of no-load currents.
- $2\alpha_c$ Span of coil C.
- B_c Angular displacement of axis of coil C from arbitrary datum.
- m The number of phases.
- q The number of slot per pole per phase.

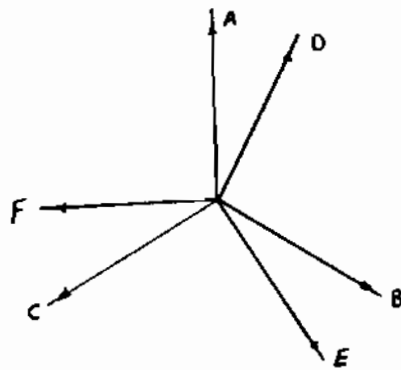


Fig. (1) The representation of the stator winding MMF axes.

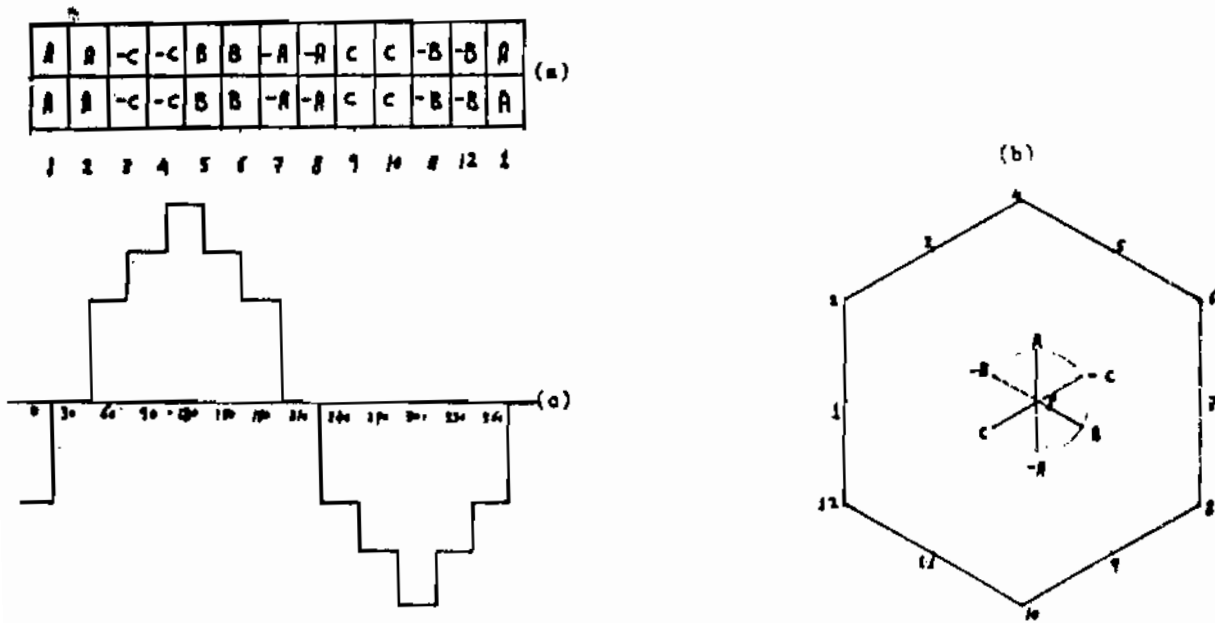


Fig. (2) Derivation of space MMF for, a 3-phase winding, unity-pitch, $q = 2$.
 (a) Stator-coil currents for double pole pitch.
 (b) Goerge's vector diagram.
 (c) The space MMF waveform.

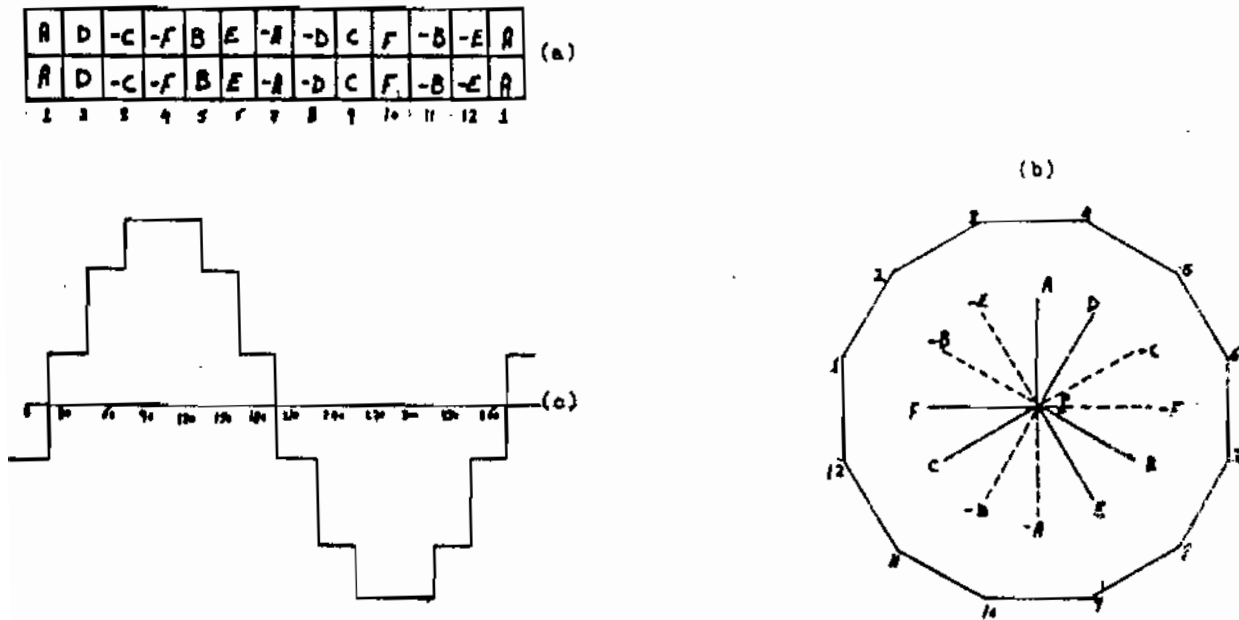
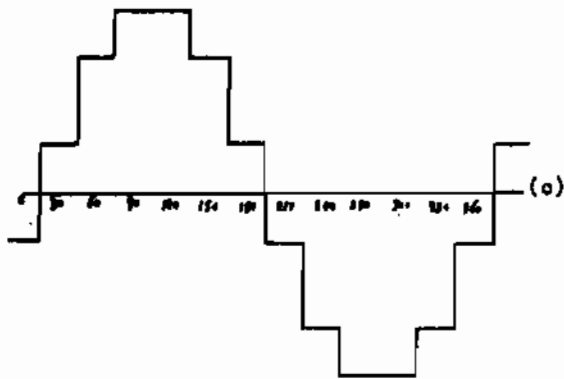


Fig. (3) Derivation of space MMF for, a 6-phase winding with $\delta = 30^\circ$, unity-pitch, $q=1$.
 (a) Stator-coil currents for double pole pitch.
 (b) Goerge's vector diagram.
 (c) The space MMF waveform.

A	D	-C	-F	B	E	-A	-D	C	F	-B	-E	A
D	-C	-F	-B	E	-A	-D	C	F	-B	-E	A	D
1	2	3	4	5	6	7	8	9	10	11	12	1

(a)



(c)

(b)

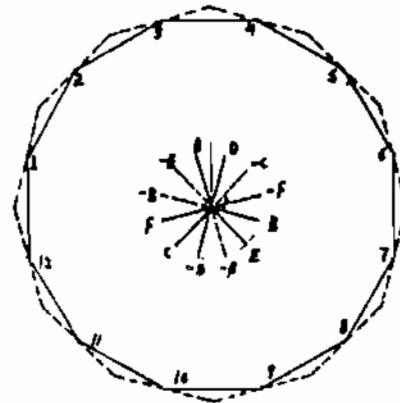
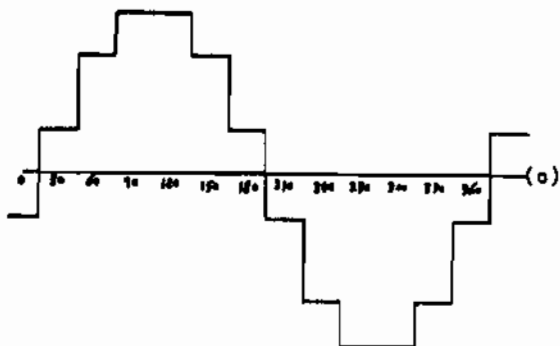


Fig. (4) Derivation of space MMF for, a 6-phase winding with $\delta = 30^\circ$, chording-pitch by one slot, $q=1$.

- (a) Stator-coil currents for double pole pitch.
- (b) Georg's vector diagram.
- (c) The space MMF waveform.

A	D	-C	-F	B	E	-A	-D	C	F	-B	-E	A
-C	-F	B	E	-A	-D	C	F	-B	-E	A	D	-C
1	2	3	4	5	6	7	8	9	10	11	12	1

(a)



(c)

(b)

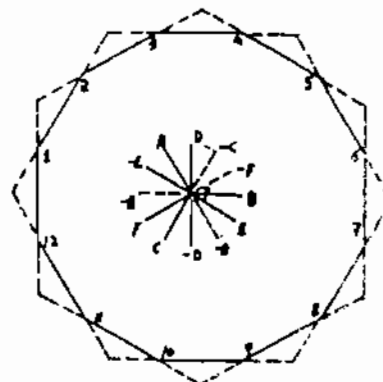


Fig. (5) Derivation of space MMF for, a 6-phase winding with $\delta = 30^\circ$, chording-pitch by two slots, $q=1$.

- (a) Stator-coil currents for double pole pitch.
- (b) Georg's vector diagram.
- (c) The space MMF waveform.

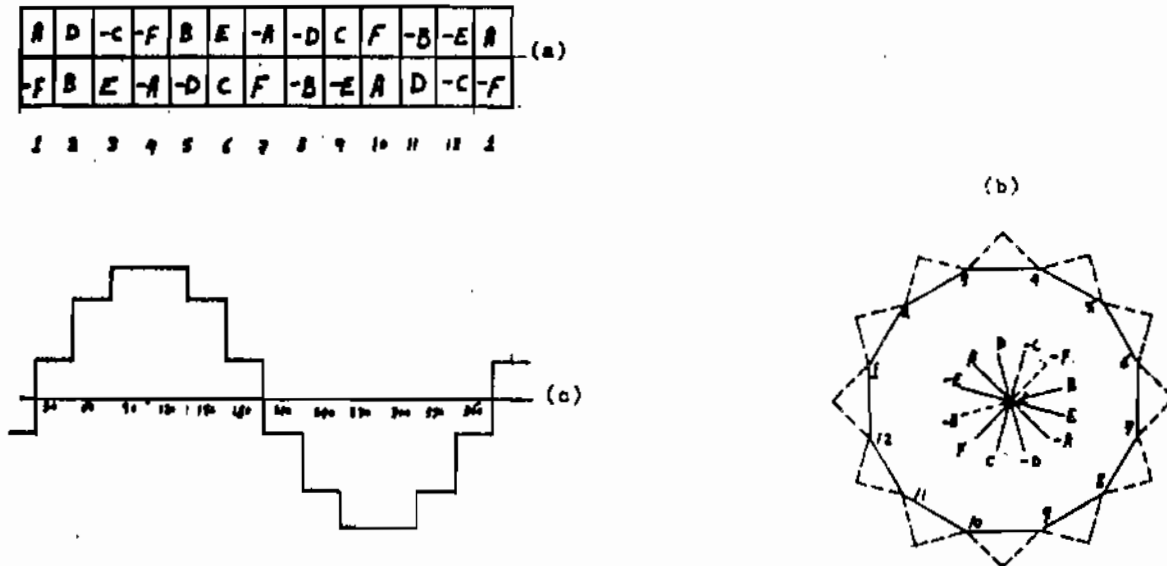


Fig. (6) Derivation of space MMF for, a 6-phase winding with $\delta = 30^\circ$, chording-pitch by three slots, $q = 1$.
 (a) Stator-coil currents for double pole pitch.
 (b) George's vector diagram.
 (c) The space MMF waveform.

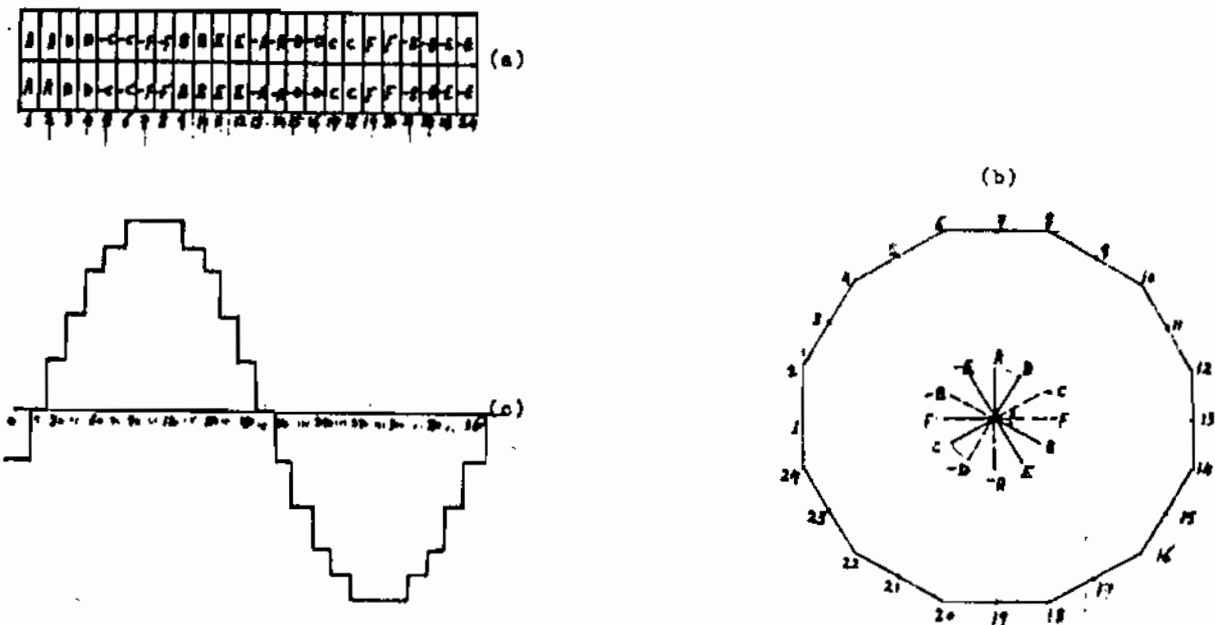


Fig. (7) Derivation of space MMF for, a 6-phase winding with $\delta = 30^\circ$, unity-pitch, $q = 2$.
 (a) Stator-coil currents for double pole pitch.
 (b) George's vector diagram.
 (c) The space MMF waveform.

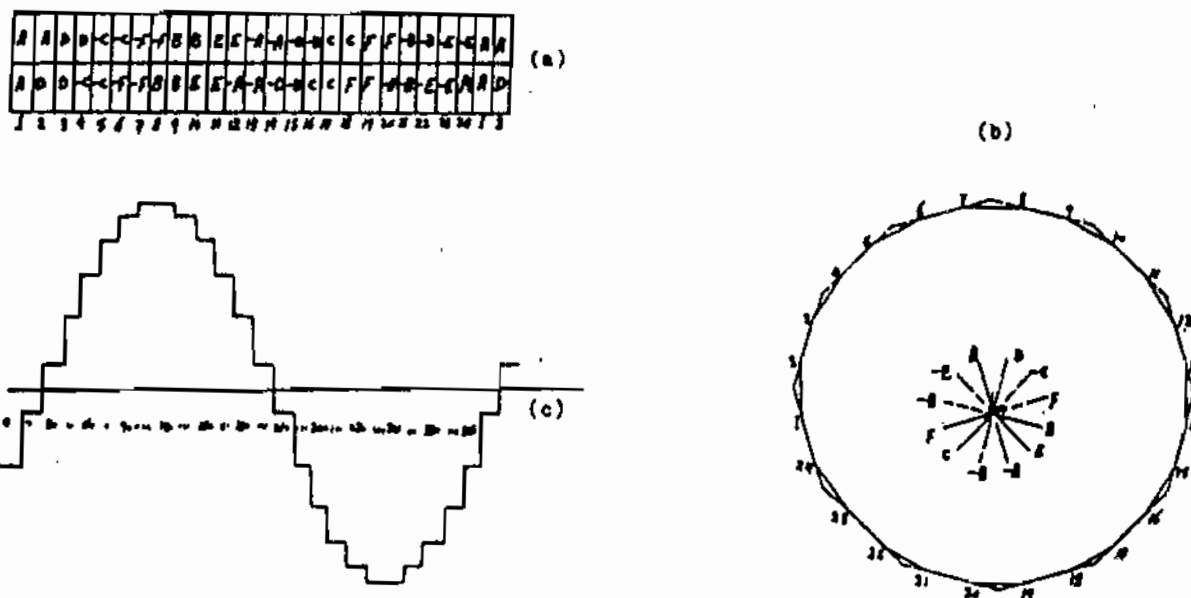


Fig. (8) Derivation of space MMF for, a 6-phase winding with $\delta = 30^\circ$, chording-pitch by one slot, $q=2$
 (a) Stator-coil currents for double pole pitch.
 (b) George's vector diagram.
 (c) The space MMF waveform.

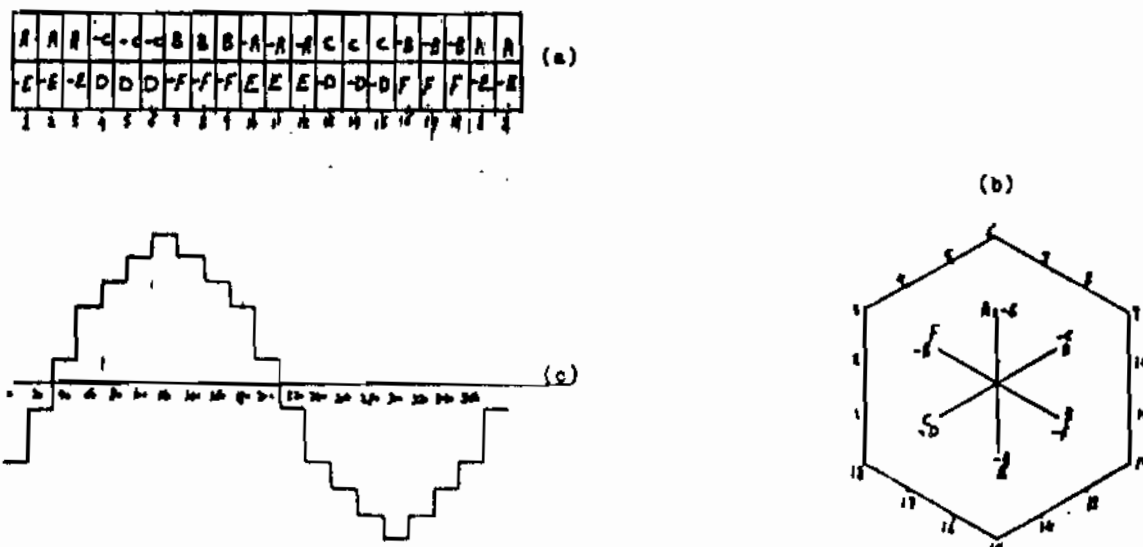


Fig. (9) Derivation of space MMF for, a 6-phase winding with $\delta = 60^\circ$, unity-pitch, $q = 3$.
 (a) Stator-coil currents for double pole pitch.
 (b) George's vector diagram.
 (c) The space MMF waveform.

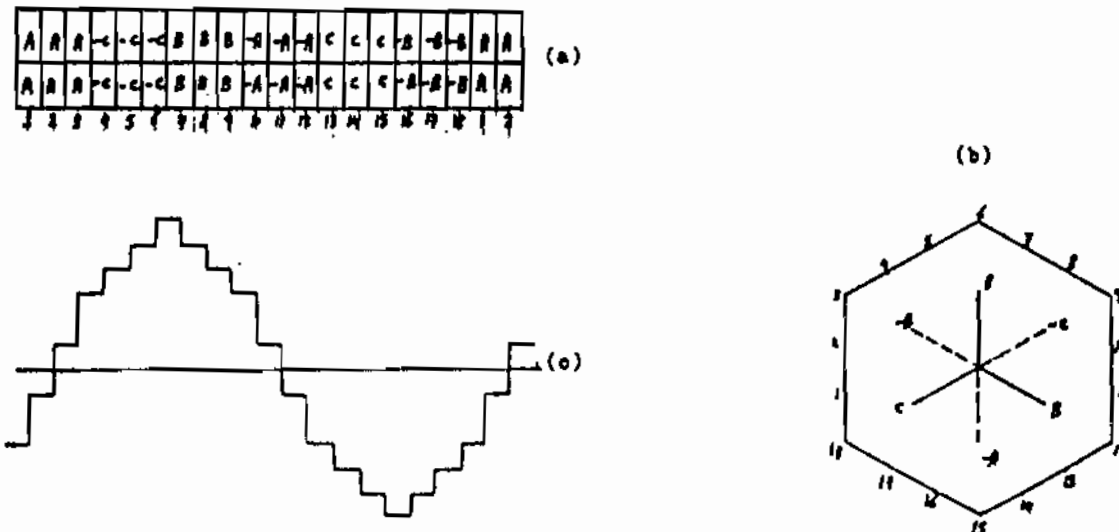


Fig. (10) Derivation of space MMF for, a 3-phase winding, unity-pitch, $q = 3$.
 (a) Stator-coil currents for double pole pitch.
 (b) George's vector diagram.
 (c) The space MMF waveform.

TABLE 1

Space MMF harmonic analysis for 3-ph, unity-pitch, $q = 2$.

PHASE= 1.0 P = 2.0 SLOTT=12.0 COE RESING = .0 YMAX =4.00

HARMONIC ORDER	ALPHA	YX1	YX2
1	-12.300	72.477	100.000
2	86.184	.000	.000
3	31.647	.000	.000
4	-12.263	.000	.000
5	17.300	4.662	2.028
6	-53.291	.000	.000
7	-47.300	3.154	3.417
8	81.727	.000	.000
9	81.234	.000	.000
10	-4.240	.000	.000
11	2.300	8.727	9.447
12	-36.138	.000	.000
13	-47.300	7.325	8.118
14	81.474	.000	.000
15	-1.034	.000	.000
16	-7.053	.000	.000
17	-12.477	1.411	1.411
18	-84.499	.000	.000
19	-77.499	1.342	1.474
20	-78.353	.000	.000
21	19.302	.000	.000
22	-4.700	.000	.000
23	-27.479	4.727	5.172
24	-77.494	.000	.000
25	87.300	4.334	4.918
26	-31.850	.000	.000
27	-28.222	.000	.000
28	30.474	.000	.000
29	-12.263	1.058	1.141
30	-30.804	.000	.000

TABLE 2

Space MMF harmonic analysis for 6-ph, $\delta = 30^\circ$, unity-pitch, $q = 1$.

PHASE= 4.0 P = 1.0 SLOTT=12.0 COE RESING = .0 YMAX =3.72

HARMONIC ORDER	ALPHA	YX1	YX2
1	-17.300	102.251	100.000
2	.000	.000	.000
3	-32.300	1.141	1.102
4	-17.928	.000	.000
5	-87.477	.184	.178
6	-79.743	.000	.000
7	57.307	.130	.129
8	87.374	.000	.000
9	72.477	.389	.377
10	-1.344	.000	.000
11	-12.300	9.732	9.447
12	-12.304	.000	.000
13	-47.300	8.381	8.118
14	77.243	.000	.000
15	-82.517	.243	.237
16	-4.131	.000	.000
17	42.300	.037	.037
18	-88.727	.000	.000
19	77.514	.034	.032
20	-77.207	.000	.000
21	-7.489	.188	.182
22	-13.233	.000	.000
23	-42.499	5.209	5.172
24	-78.013	.000	.000
25	-77.300	3.874	4.918
26	-44.309	.000	.000
27	67.491	.134	.134
28	34.222	.000	.000
29	12.309	.042	.040
30	-25.838	.000	.000

TABLE 3

Space MMF harmonic analysis for 6-ph, $\delta = 30^\circ$, chording pitch by one slot, $q = 1$.

PHASE= 4.0 P = 1.0 SLOTT=12.0 COE RESING = .0 YMAX =3.34

HARMONIC ORDER	ALPHA	YX1	YX2
1	-17.300	102.257	100.000
2	-84.344	.000	.000
3	78.514	.000	.000
4	31.722	.000	.000
5	-87.300	1.224	1.221
6	-87.917	.000	.000
7	57.499	.104	.079
8	85.980	.000	.000
9	-79.229	.000	.000
10	-2.426	.000	.000
11	-12.300	9.714	9.447
12	-10.294	.000	.000
13	-47.300	8.344	8.118
14	80.084	.000	.000
15	-14.478	.000	.000
16	-8.432	.000	.000
17	62.300	.042	.031
18	-84.144	.000	.000
19	27.498	.345	.359
20	-77.240	.000	.000
21	17.933	.000	.000
22	-12.981	.000	.000
23	-42.499	5.211	5.172
24	-77.297	.000	.000
25	-77.300	3.857	4.918
26	-45.921	.000	.000
27	-44.243	.000	.000
28	34.900	.000	.000
29	12.494	.185	.177
30	-25.733	.000	.000

TABLE 4

Space MMF harmonic analysis for 6-ph, $\delta = 30^\circ$, chording-pitch by two-slots, $q = 1$.

PHASE= 4.0 P = 1.0 SLOTT=12.0 COE RESING = 2.0 YMAX =3.25

HARMONIC ORDER	ALPHA	YX1	YX2
1	-17.300	102.442	100.000
2	38.244	.000	.000
3	34.722	.000	.000
4	-1.122	.000	.000
5	-87.300	.787	.747
6	-77.104	.000	.000
7	57.498	.567	.532
8	85.184	.000	.000
9	-82.394	.000	.000
10	-2.772	.000	.000
11	-12.300	9.478	9.047
12	-11.679	.000	.000
13	-47.300	8.334	8.118
14	77.773	.000	.000
15	-17.179	.000	.000
16	-8.447	.000	.000
17	42.300	.232	.244
18	-88.102	.000	.000
19	27.497	.231	.223
20	-77.299	.000	.000
21	18.194	.000	.000
22	-13.729	.000	.000
23	-42.499	5.209	5.172
24	-77.227	.000	.000
25	-77.300	3.848	4.918
26	-44.392	.000	.000
27	-49.701	.000	.000
28	34.729	.000	.000
29	12.498	.179	.174
30	-25.733	.000	.000

TABLE 5

Space MMF harmonic analysis for 6-phase,
 $\delta = 30^\circ$, chording-pitch by three-slots,
 $q = 1$.

PHASE= 4.0 $\theta = 1.0$ SLOTT=12.0 CHO BONDG = 3.0 YWIG = 2.70

HARMONIC ORDER	ALPHA	YH11	YH21	YH31	YH41	YH51	YH61
1	-17.500	102.638	100.000				
2	-80.582	.000	.000				
3	59.274	.000	.000				
4	.000	.000	.000				
5	-87.500	.711	.702				
6	-82.104	.000	.000				
7	57.498	.515	.504				
8	87.442	.000	.000				
9	-85.752	.000	.000				
10	-2.000	.000	.000				
11	-12.500	5.676	9.447				
12	-11.945	.000	.000				
13	-47.500	8.332	8.118				
14	80.247	.000	.000				
15	-16.751	.000	.000				
16	-8.516	.000	.000				
17	62.500	.711	.722				
18	-88.103	.000	.000				
19	27.476	.712	.706				
20	-77.668	.000	.000				
21	16.175	.000	.000				
22	13.382	.000	.000				
23	-42.499	5.306	5.172				
24	-78.226	.000	.000				
25	-77.500	5.047	4.918				
26	-47.049	.000	.000				
27	-47.157	.000	.000				
28	74.509	.000	.000				
29	22.476	1.64	1.57				
30	-34.847	.000	.000				

TABLE 6

Space MMF harmonic analysis for 6-phase,
 $\delta = 30^\circ$, unity-pitch, $q = 2$.

PHASE= 4.0 $\theta = 2.0$ SLOTT=24.0 CHO BONDG = .0 YWIG = 3.80

HARMONIC ORDER	ALPHA	YH11	YH21	YH31	YH41	YH51	YH61
1	-15.000	101.552	100.000				
2	-82.047	.000	.000				
3	-44.979	.313	.305				
4	-4.129	.000	.000				
5	-75.000	.849	.853				
6	-77.444	.000	.000				
7	74.975	.847	.834				
8	75.741	.000	.000				
9	45.002	.541	.532				
10	-4.130	.000	.000				
11	15.000	1.071	1.055				
12	-31.000	.000	.000				
13	-15.000	.921	.907				
14	76.932	.000	.000				
15	-45.006	.340	.325				
16	-10.500	.000	.000				
17	-75.004	.377	.371				
18	-42.125	.300	.300				
19	75.010	.253	.251				
20	-80.443	.000	.000				
21	44.945	.052	.051				
22	-4.497	.000	.000				
23	14.977	3.252	3.172				
24	80.819	.000	.000				
25	-15.000	4.994	4.918				
26	-77.142	.000	.000				
27	-44.917	.044	.041				
28	38.461	.000	.000				
29	-74.987	.177	.174				
30	-54.617	.000	.000				

TABLE 7

Space MMF harmonic analysis for 6-phase,
 $\delta = 30^\circ$, chording-pitch by one-slot,
 $q = 2$.

PHASE= 6.0 $\theta = 2.0$ SLOTT=24.0 CHO BONDG = 1.0 YWIG = 3.78

HARMONIC ORDER	ALPHA	YH1	YH2	YH3	YH4	YH5	YH6
1	-22.500	100.053	100.000				
2	-59.470	.000	.000				
3	-32.509	.404	.404				
4	36.449	.000	.000				
5	67.442	.012	.012				
6	44.819	.000	.000				
7	22.500	.344	.344				
8	76.521	.000	.000				
9	-22.476	.246	.246				
10	-5.793	.000	.000				
11	-67.494	.310	.309				
12	-37.626	.000	.000				
13	67.489	.264	.264				
14	77.177	.000	.000				
15	22.519	.152	.152				
16	-11.024	.000	.000				
17	-22.531	.153	.153				
18	-82.175	.000	.000				
19	-44.712	.004	.004				
20	-79.370	.000	.000				
21	67.443	.064	.064				
22	-1.158	.000	.000				
23	22.499	5.175	5.172				
24	-79.173	.000	.000				
25	-22.500	4.920	4.918				
26	-48.871	.000	.000				
27	-67.505	.057	.057				
28	36.717	.000	.000				
29	68.197	.005	.005				
30	-34.259	.000	.000				