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## "STUDY OF TURBULENT BOUNDARY LAYER BEHAVIOUR"

BY

Dr. SAMIR FRANCIS HANNA

A B S T R A C T

The main objective of this paper is to study the behaviour of turbulent boundary layer regarding the surface roughness. The existing theoretical and empirical methods as well as various experimental results have been reviewed and studied. A new simplified methodology is suggested to take into account the different behaviour parameters of boundary layer. The effect of such parameter and velocity profiles are well defined and given in the form of design charts.

Moreover, using these design charts a proposed numerical procedure is introduced to solve the momentum equation of turbulent flow over rough surface with the help of isocline method.

NOMENCLATURE

$c_{\infty}$	free stream velocity,	m/s
$\bar{c}$	velocity at the outer edge of the boundary layer,	m/s
$c_t$	friction velocity,	
$c_f$	local skin friction coefficient,	
$c_x$	velocity of the fluid inside the boundary layer in x-direction,	m/s
$c_y$	velocity component of the fluid inside the boundary layer in y-direction,	m/s
$H_{12}$	boundary layer form parameter	
$I$	boundary layer shape parameter	
$K_s$	roughness height,	m
$F$	velocity profile parameter	
$p_{\infty}$	free stream static pressure,	N/m <sup>2</sup>
$Re_{\delta^{**}}$	Reynolds number based on the momentum thickness	
$w(y/\delta)$	wake function,	
$\delta$	boundary layer thickness,	m
$\delta^{**}$	displacement thickness of the boundary layer,	m
$\delta^{**}$	momentum thickness of the boundary layer,	N/m <sup>2</sup>
$\tau$	shear stress in the boundary layer,	N/m <sup>2</sup>

$\tau_w$	wall shear stress,	$N/m^2$
$\nu$	kinematic viscosity of fluid,	$m^2/s$
$\Lambda$	Euler number	
$\pi$	pressure gradient parameter	
$\rho$	density of fluid,	$kg/m^3$

## 1- INTRODUCTION

This is a study of the behaviour of the turbulent boundary layer of incompressible fluids, regarding the surface roughness. Such study has been done in [1] for smooth surface. In general the calculation of turbulent boundary layer is based on approximate methods. However, an exact solution of the Prandtl boundary layer equations is available only for special relations for  $\bar{c}(x)$ . The basis of such calculations depends upon the momentum equation of boundary layer. This equation has, in general case, two variable coefficients, namely, the form parameter  $H_{12}$  and the dimensionless wall shear stress  $\tau_w/\rho\bar{c}^2$ . These coefficients are dependent functions of Reynolds number  $Re_{\delta}^{**}$  as well as Euler number  $\Lambda$ .

Many methods for calculating turbulent boundary layer regarding pressure gradient were introduced, checked and modified by several authors among them [2], [3], and [4]. In recent years, it is still required, for technical applications, to know more information about these variable coefficients for rough surface.

In this paper the behaviour of turbulent boundary layer for rough surface is theoretically studied and the parameters affecting this behaviour have been investigated.

## 2- BASIC ANALYTICAL RELATIONS

For fully turbulent boundary layer, the surface roughness has the major effect on the velocity profile. It should be noticed that, in this study Coles [5] neglected the effect of surface roughness on the velocity profile. In this case, it is suggested to expand Cole's definitions to take into account the surface roughness in the same manner as it has been considered by Rotta [6]. Accordingly, the proposed expression for the velocity profile will be:

$$\frac{c_x}{c} = \left( \frac{1}{x} \ln \frac{c \tau_w \nu}{\rho} + C \right) + \frac{P(x)}{x} w(y/\delta) + C_1 \left( \frac{c \tau_w k_s}{\rho} \right) \dots (1)$$

It is clear from equation (1) that this representation of the velocity profile consists of the law of the wall, the law of the wake and the roughness effect. These are given by the first, the second and the third terms in the right hand-side of Eq.(1), respectively.

The thickness of the boundary layer is defined by:

$$c_x = \bar{c} \quad \text{for } y = \delta,$$

so that equation (1) can be rewritten in the form of the defect

law as

$$\frac{\bar{c} - c_x}{c_\tau} = -\frac{1}{\alpha} \ln(y/\delta) + \frac{P}{\alpha} \left[ 2 - w(y/\delta) \right] - C_1 \left( \frac{c_\tau k_s}{\nu} \right) \dots (2)$$

Rearranging equation (2), dividing by  $\bar{c}$  and solving for  $c_x/\bar{c}$ , one obtains,

$$\frac{c_x}{\bar{c}} = 1 + \sqrt{\frac{\tau_w}{\rho \bar{c}^2}} \left\{ \left[ \frac{1}{\alpha} \ln(y/\delta) - \frac{2P}{\alpha} \right] + \frac{P}{\alpha} w(y/\delta) + C_1 \left( \frac{c_\tau k_s}{\nu} \right) \right\} \dots (3)$$

Equation (3) is a modified form of the velocity distribution for turbulent flow regarding the effect of surface roughness. Also, it is one of the basic concepts for solving and estimating the different parameters affecting the behaviour of the turbulent flow.

Moreover, equation (1) represents the basis for the determination of the dimensionless wall shear stress  $(\tau_w/\rho \bar{c}^2)$ . At the outer edge of the boundary layer (i.e.,  $y = \delta$ )  $c_x = \bar{c}$ ,  $w = 2$  and  $C_1(c_\tau k_s/\nu) \rightarrow 0$ . The constant  $C$  in Eq.(1)<sup>x</sup> was found experimentally [5] to have the value of 5.1. Therefore, Eq.(1) becomes:

$$\frac{\bar{c}}{c_\tau} = \frac{1}{\alpha} \ln\left(\frac{c_\tau \delta}{\nu}\right) + 5.1 + \frac{2P}{\alpha} \dots (4)$$

The term  $(c_\tau \delta/\nu)$  which is a type of Reynolds number, will be expanded to have the following form:

$$\frac{c_\tau \delta}{\nu} = Re_{\delta^{***}} \frac{\sqrt{\tau_w/\rho \bar{c}^2} \rho}{\delta^{***}/\delta}$$

The last obtained form of Reynolds number is substituted in equation (4) to give:

$$\frac{1}{\sqrt{\tau_w/\rho \bar{c}^2}} = \frac{1}{\alpha} \ln Re_{\delta^{***}} \frac{\sqrt{\tau_w/\rho \bar{c}^2} \rho}{\delta^{***}/\delta} + 5.1 + \frac{2P}{\alpha} \dots (5)$$

This derived equation (5) is used in the computer programme of the present work to give the value of the dimensionless wall shear stress  $\tau_w/\rho \bar{c}^2$ .

### 3- EFFECT OF ROUGHNESS

The distribution of surface roughness and roughness height have a major effect in the development of the turbulent boundary layer. This effect is defined by the function  $C_1(c_\tau k_s/\nu)$  in Eq.(3). Therefore, the local skin friction coefficient  $c_f$  can conveniently be expressed as

$$c_f = c_f(\text{Re}_\sigma^{**}, H_{12}, k_s/\sigma^{**}) \quad \dots (6)$$

For small roughness objects the value of  $C_1$  is constant and is independent of the ratio  $k_s/\sigma^{**}$ . The surface can be regarded as smooth. With increasing heights of roughness, the ratio  $k_s/\sigma^{**}$  gains more and more influence relative to the Reynolds number  $\text{Re}_\sigma^{**}$ , until finally the skin friction coefficient becomes a function of  $k_s/\sigma^{**}$  and  $H_{12}$  only. In other words the dimensionless wall shear stress can be expressed as:

$$\frac{\tau_w}{\rho \bar{c}^2} = f_1(k_s/\sigma^{**}; H_{12}) \quad \dots (7)$$

Also, the dimensionless wall shear stress  $\tau_w/\rho \bar{c}^2$  can be expressed as a function of  $k_s/\sigma^{**}$ , and Euler number  $\Lambda$ , in the form,

$$\frac{\tau_w}{\rho \bar{c}^2} = f_2(k_s/\sigma^{**}, \Lambda) \quad \dots (8)$$

For the numerical solution of the velocity profile, it is necessary to know more information about the function  $C_1(c_\tau k_s/\nu)$ . This is clearly demonstrated by the experimental Nikuradse's [7] sand grain roughness as follows:

A) For values of  $c_\tau k_s/\nu < 5$ ,

$$C_1\left(\frac{c_\tau k_s}{\nu}\right) = 5.5 + 2.5 \ln\left(\frac{c_\tau k_s}{\nu}\right), \quad \dots (9)$$

like completely smooth pipe. This is consistent with the idea that as long as the roughness is well immersed in the laminar sublayer it does not shed eddies and therefore can have no effect on the flow.

B) For values of  $c_\tau k_s/\nu > 70$ ,

$$C_1\left(\frac{c_\tau k_s}{\nu}\right) = \text{constant} \approx 8.5 \quad \dots (10)$$

In this case the flow and surface friction are independent of Reynolds number. The flow is referred to fully developed roughness flow.

C) The intermediate region is one in which the viscous friction and roughness form drag contributions to the surface drag are both significant and change in dominance from one to the other takes place as  $c_\tau k_s/\nu$  increases from 5 to 70. So the representation of the transient regime between hydraulically smooth and completely rough surface is more complex.

#### 4- CHARACTERISTIC PARAMETERS OF THE BOUNDARY LAYER

##### 4.1- FORM PARAMETER $H_{12}$

The displacement thickness, as it is well known, is defined in dimensionless form by:

$$\frac{\sigma^{**}}{\sigma} = \int_0^1 \left(1 - \frac{c}{\bar{c}}\right) d(y/\delta) \quad \dots (11)$$

From equation (3), substituting in equation (11) and integrating by parts, a convenient relation between the dimensionless displacement thickness and velocity profile parameter  $P$ , roughness effect and dimensionless wall shear stress  $\tau_w/\rho\bar{c}^2$ , is obtained as:

$$\frac{\sigma^{**}}{\sigma} = \frac{1}{\kappa} \sqrt{\frac{\tau_w}{\rho\bar{c}^2}} \left[1 - P - \kappa C_1 R_S\right] \quad \dots (12)$$

Also, the momentum thickness can be illustrated in the following shape:

$$\begin{aligned} \frac{\sigma^{**}}{\sigma} = & \frac{1}{\kappa} \sqrt{\frac{\tau_w}{\rho\bar{c}^2}} (1 + P - C_1 R_S) - \\ & \frac{2}{\kappa^2} \cdot \frac{\tau_w}{\rho\bar{c}^2} \left[1 + 1.6P + 0.761P^2 + \kappa C_1 R_S (2 + C_1 R_S - P)\right] \end{aligned} \quad \dots (13),$$

where:  $R_S = \frac{c \epsilon k_s}{\nu}$

To have the conventional form parameter as a function of the velocity profile parameter  $P$ , dividing Eq.(12) by (13) gives

$$\begin{aligned} H_{12} = & \frac{\sigma^{**}/\delta}{\sigma^{**}/\delta} \\ = & \frac{1}{1 - \frac{2}{\kappa} \sqrt{\frac{\tau_w}{\rho\bar{c}^2}} \left( \frac{1 + 1.6P + 0.761P^2 + \kappa C_1 R_S (2 + \kappa C_1 R_S - P)}{1 + P - \kappa C_1 R_S} \right)} \end{aligned} \quad \dots (14)$$

This form of equation (14) is used in FORTRAN IV computer programme.

#### 4.2- EULER NUMBER

The Euler number  $\Lambda$  is one of the most important boundary layer parameters. This importance can be seen from the different representation of the results obtained and given in the form of design charts. The definition of this parameter is given by:

$$\Lambda = - \frac{1}{\bar{c}} \frac{d\bar{c}}{dx} \sigma^{**} \quad \dots (15)$$

#### 4.3- THE SLOPE OF THE MOMENTUM THICKNESS

VON Kármán's momentum integral equation is performed to

have the following form:

$$\frac{d\delta^{**}}{dx} = (H_{12} + 2) \Lambda + \frac{\tau_w}{\rho \bar{c}^2} \quad \dots (16)$$

Equation (17) is the slope of the momentum thickness as a function of the form parameter  $H_{12}$ , Euler number  $\Lambda$ , and the dimensionless wall shear stress.

### 5- REPRESENTATION AND DISCUSSION OF RESULTS

Based on the results given in figures from (1) to (4) the controlling parameters, which affect the behaviour of the turbulent boundary layer over a rough surface, could be found. In order to find the parameters, information to determine Reynolds number  $Re_{\delta^{**}}$ , Euler number  $\Lambda$  and the ratio  $k_s/\delta^{**}$  have to be given. Therefore, the momentum thickness  $\delta^{**}$ , The velocity gradient  $d\bar{c}/dx$  and the height of surface roughness  $k_s$  should be known.

By the aid of the above figures, the form parameter  $H_{12}$  and the dimensionless wall shear stress  $\tau_w/\rho \bar{c}^2$  may then be expressed as follows:

$$\frac{\tau_w}{\rho \bar{c}^2} = f_3(\Lambda, Re_{\delta^{**}}, k_s/\delta^{**}) \quad \dots (17)$$

$$H_{12} = f_4(\Lambda, Re_{\delta^{**}}, k_s/\delta^{**}) \quad \dots (18)$$

By increasing the heights of roughness, the ratio  $k_s/\delta^{**}$  has more influence on  $\tau_w/\rho \bar{c}^2$  and  $H_{12}$  than  $Re_{\delta^{**}}$ ; and by increasing these heights of roughness further more, the local skin friction coefficient  $\tau_w/\rho \bar{c}^2$  and the form parameter  $H_{12}$  become only functions of Euler number  $\Lambda$  and the ratio  $k_s/\delta^{**}$  as given by equation (8).

Fig.(1) shows the relation between the local skin friction coefficient and Euler number  $\Lambda$  for Reynolds number  $Re_{\delta^{**}} = 3 \cdot 10^2, 5 \cdot 10^3, 10^4, 5 \cdot 10^4, 10^5, 5 \cdot 10^5$  and  $10^6$ .

For boundary layers at the same Euler number, the local skin friction coefficient increases as the Reynolds number decreases. Also, as can be seen from the figure, for the low values of Euler number, there is small variation of the local skin friction coefficient versus large variation in Euler No.

Fig.(2) represents the variation of the local skin friction coefficient with Euler number for different values of the ratio  $k_s/\delta^{**}$ . For the same value of the ratio  $k_s/\delta^{**}$  the slope of the curve  $dc_f/d\Lambda$  is too small. This means that, there is a variation in the local skin friction coefficient, corresponding to a large variation of Euler number. By increasing the ratio  $k_s/\delta^{**}$ , the slope  $dc_f/d\Lambda$  increases, i.e., a large variation in the local skin friction coefficient, corresponding to small variations of Euler number occurs.

The form parameter is plotted in Fig.(3): it represents the form parameter  $H_{12}$  versus Euler number  $\Lambda$  with  $k_g/\delta^{**}$  as parameter. The diagram consists of a family of parallel curves and shows that for the same value of the ratio  $k_g/\delta^{**}$ , the form parameter  $H_{12}$  increases as the Euler number increases. This means that, by increasing Euler number, the dimensionless displacement thickness  $\delta^*/\delta$  increases while the dimensionless momentum thickness  $\delta^{**}/\delta$  decreases.

In Fig.(4) the relation between the slope of the momentum thickness  $d\delta^{**}/dx$  and Euler number  $\Lambda$  for different values of  $Re_{\delta^{**}}$  is represented. For constant value of  $\Lambda$ , the slope  $d\delta^{**}/dx$  increases as  $Re_{\delta^{**}}$  decreases. For low values of Euler number ( $\Lambda < 3 \cdot 10^{-3}$ ), the slope of each curve is positive and is approximately constant. This means that, the increase of the slope of the momentum thickness  $d\delta^{**}/dx$  causes an increase of Euler number  $\Lambda$ .

## 6- CONCLUSIONS

The conclusions obtained from the present investigation will now be summarized. In the present work the proposed semi-empirical method for two-dimensional turbulent boundary layer takes into account the wall roughness. The results of computations are classified and organized according to Euler number  $\Lambda$  and the roughness parameter  $k_g/\delta^{**}$ .

The result of computations have showed that the wall roughness  $k_g/\delta^{**}$  has a greater effect than the boundary layer parameters  $H_{12}$  and  $\tau_w/\rho c^2$ . As a result the total resistance increases. This is due to the fact that the boundary layer profile is strongly affected by the development of roughness.

The slope of the momentum thickness  $d\delta^{**}/dx$  increases with the increase of Euler number  $\Lambda$  and decrease with the increase of Reynolds number  $Re_{\delta^{**}}$ .

It should be noted that, as a result of the present work, the momentum integral equation can be simply solved with the aid of the deduced diagrammes using the isocline method.

## REFERENCES

- [1] S. F. HANNA, Et. Al. "An Investigation of some turbulent boundary layer parameters. El-Mansoura University Bulletin, Vol.6, No.1, June 1981.
- [2] F. Clauser "Turbulent boundary layer in adverse pressure gradient". J. Aero.Sci. 21. P.91, 1954.



- [3] E. GRUSCHWITS "The turbulent boundary layer in plane flow with pressure rise and fall". Ing. Arch.V2, No.3, 1931.
- [4] A. BURI "A method of calculation for turbulent boundary layer with accelerated and retarded flow". Thesis No.652, Federal Tech. College, Zurich 1931.
- [5] D. COLES "The law of the wake in turbulent boundary layers". Journal of fluid mechanics, 1, p. 191 - 226. 1956.
- [6] J. C. Rotta "Turbulent boundary layers in incompressible flow". Progress in aeronautical sciences. Vol.2, pp.1 - 220, Pergamon 62.
- [7] J. WIKURADSE "Strömungsgesetze in rauhen Röhren". Forschungsheft 361. Ausgabe B Band 4 Juli/August 1933.

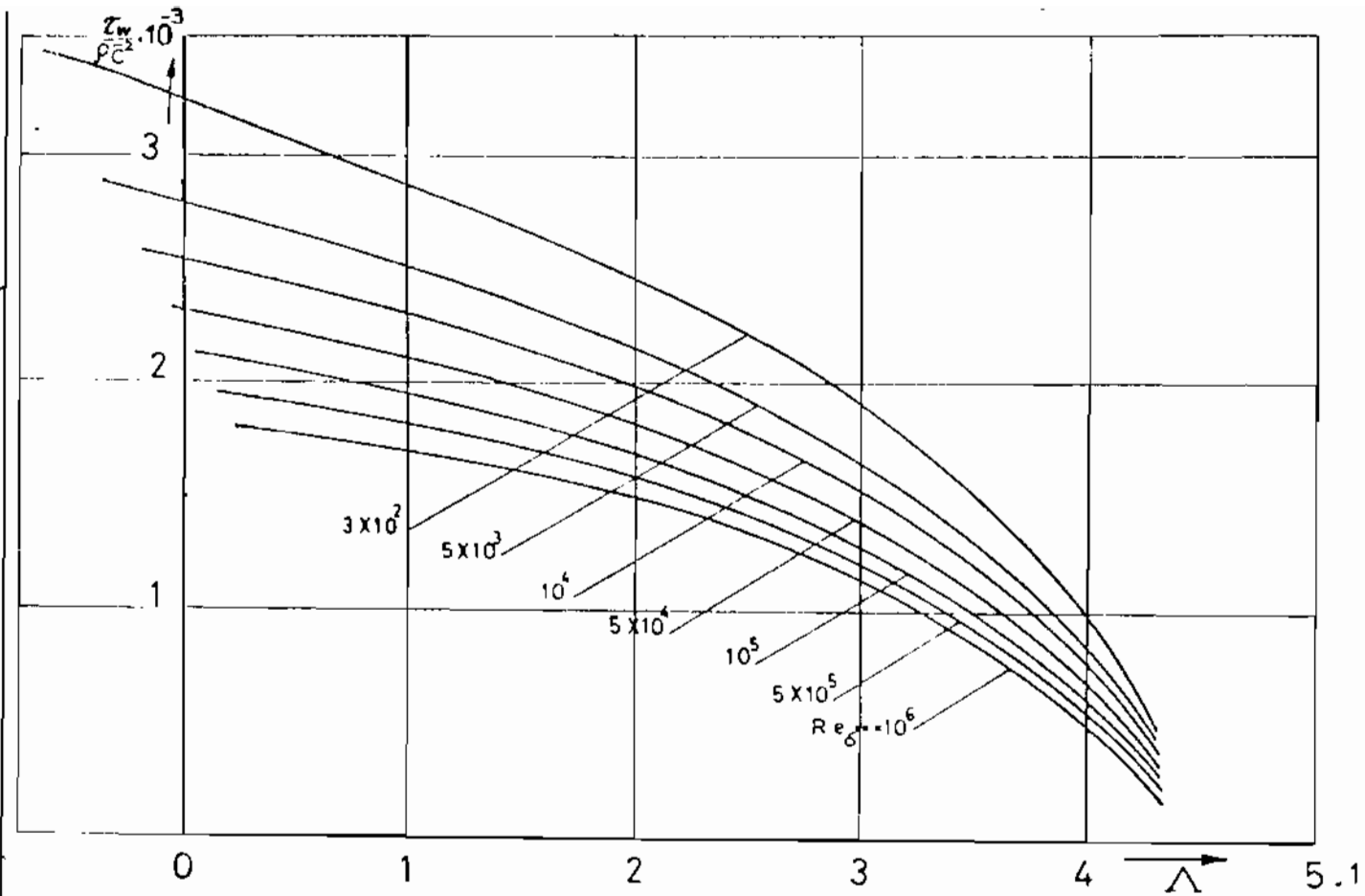


Fig.(1) Local Skin Friction Coefficient Versus Euler Number  $\Lambda$   
With Reynolds Number  $Re_{\delta^{**}}$  As Parameter

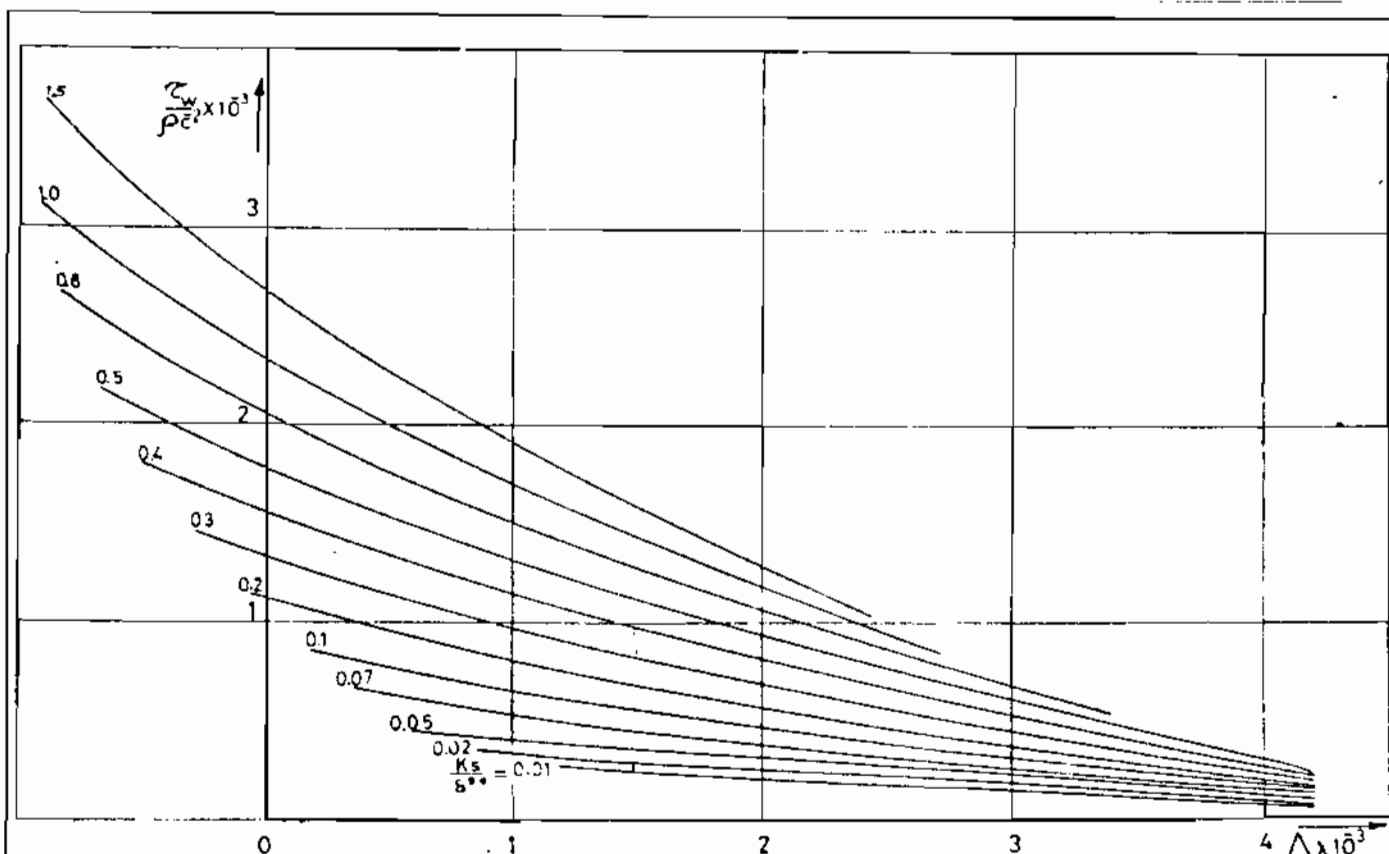


Fig.(2) Local Skin Friction Coefficient As Function of  
Euler Number With  $\frac{K_s}{\delta^{**}}$  As Parameter

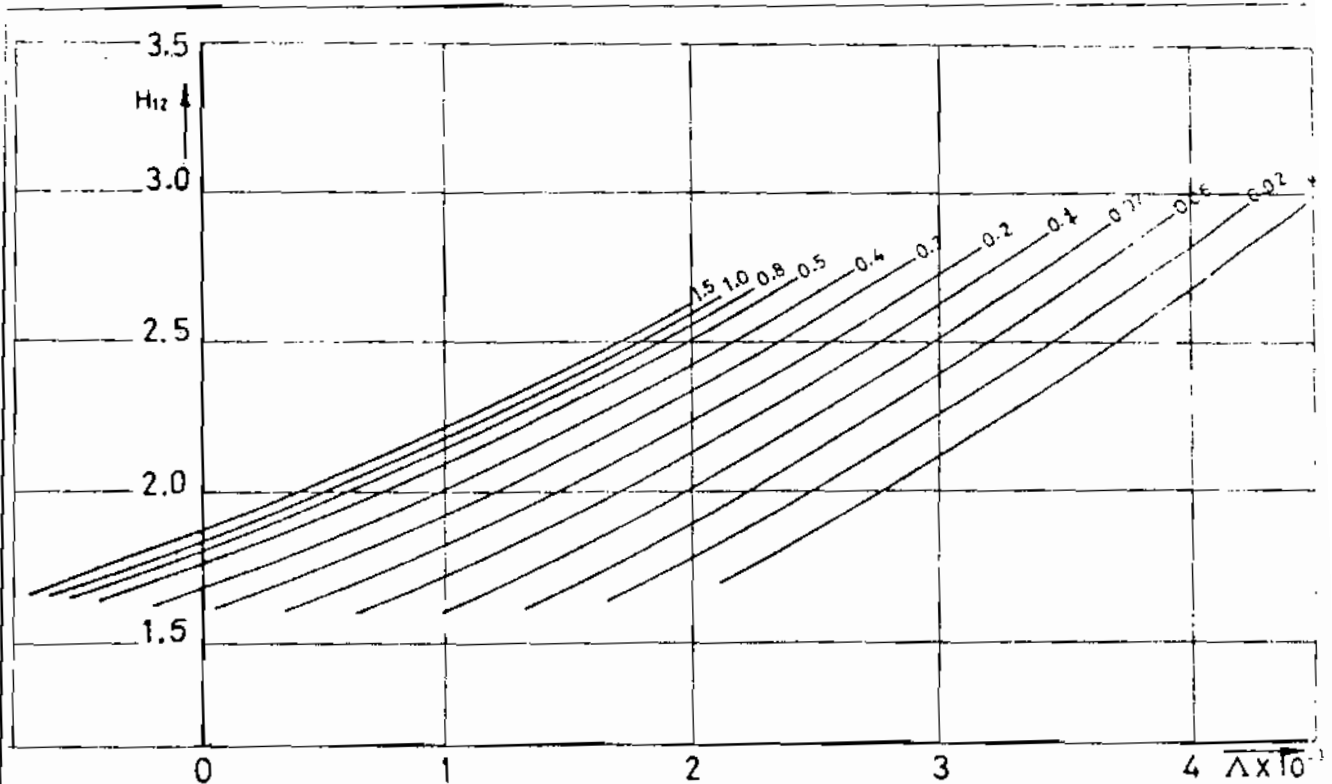


Fig.(3) Form Parameter  $H_{12}$  Versus Euler Number  $\Lambda$  with The Ratio  $Ks/\delta^{**4}$  As Parameter

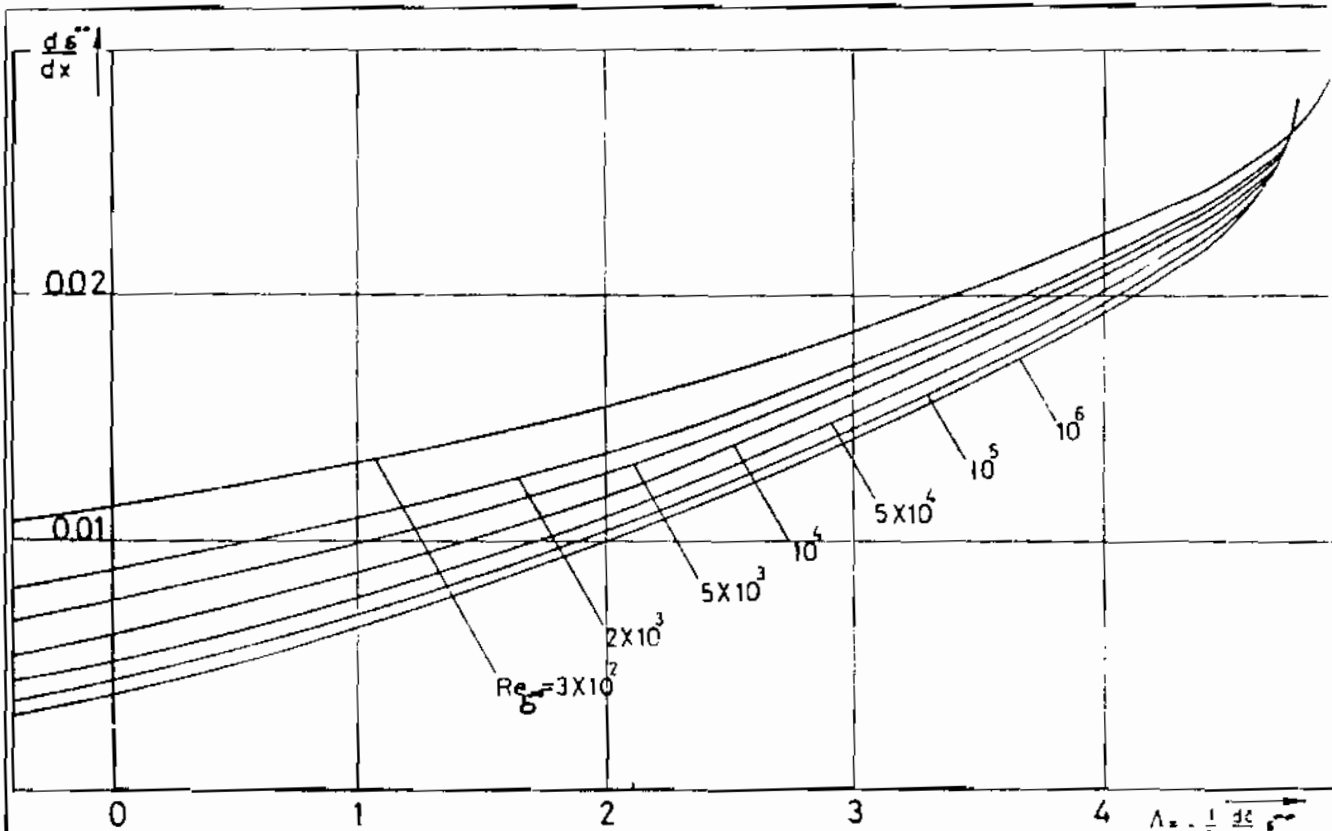


Fig.(4) Slope Of The Momentum Thickness As Function Of Euler Number For Different Values Of Reynolds Number  $Re_{\delta^{**}}$