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AN ANALYTICAL STUDY BETWEEN SOME TYPES OF TRANSITION CURVES

BY

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Transition or easement curves are curves introduced between the straight tangent and circular curve on which the radius of curvature decreases gradually from infinity at tangent-spiral interesection to the radius of the circular curve at the spiral-circular curve intersection.

Due to the fact that the speed of vehicles have been greatly increased in the last few years, particular attention must be paid to the design, alignment and setting-out works of the transition curves in order to eliminat speed restrictions on highways. However for these reasons an investigation on transitions is a necessity.

The subject of this paper deals entirely with the study of the different types of transition which commonly are used in highway Engineering practice. It gives a survey for the different types of transition from all aspects. It also treats the analytical and mathematical part in sufficient depth. The paper is devoted to represent the principles and characteristics of transition, typical cases of their insertion as well as objectives of their provosion in practice.

The paper contains a detailed study for different types of transitions and a comparative analysis between the different types which are: the clothoid, the cubic parabola and lemniscate.

The fundamental equation of each curve is discussed. Moreover the setting-out elements and data are thoroughly presented.

The study also includes a comparative study between the different approaches to find the lengths of each type of transition and the setting-out works for each curve.

The results and conclusions as well as practical recommendations are derived, adopted and represented.

SUPERELEVATION

Superelevation is provided in order to counteract the effect of the centrifugal force and to reduce the tendency of cars to overturn. The amount of super-elevation depends mainly on the speed of vehicles and the radius of curves. The amount of superelevation varies from zero at the beginning of the transition curve to its maximum value at the beginning of the circular curve.

The advantages of providing superelevation can be summarised as; (1) obtaining high speeds without any danger of overturning which means increasing volume of traffic.(2) keeping the vehicles at their correct side and lessens the danger of skidding at bends. (3) economy in maintenance, because it keeps the pressure on the wheels equally distributed and it results in less wear and tear of wheel tyres and springs, (4) allowing the water to drain off doing away with the gutter on inner adge of road. (5) enabling vehicle to move at high speeds; (6) making the change from the tangent section to the curved section, to the tangent section of a track in a safe and comfortable operation. and (7) transition curve provides gradual increase of superelevation to the outer edge above the inner edge and thus totally eliminate or partially shocks or severe jerks on the moving vehicles.

TYPICAL CASES FOR INSERTING TRANSITION CURVES

The following are the main typical cases for inserting transition curves:-

Case 1:

Transition between straight and curved portion as shown in Fig. (1).

Case 2:

Two transition curves between two tangents and a simple circular curve as shown in Fig. (2).

Case 3:

Wholly transitional curves

In case the circular curve is very small, it may be eliminated and this case will be reduced to a case of two successive transitions between two tangents as shown in Fig. (3).

Wholly transitional curves have the advantage that there is only one point at which the radial force is a maximum and, therefore, the safety is increased.

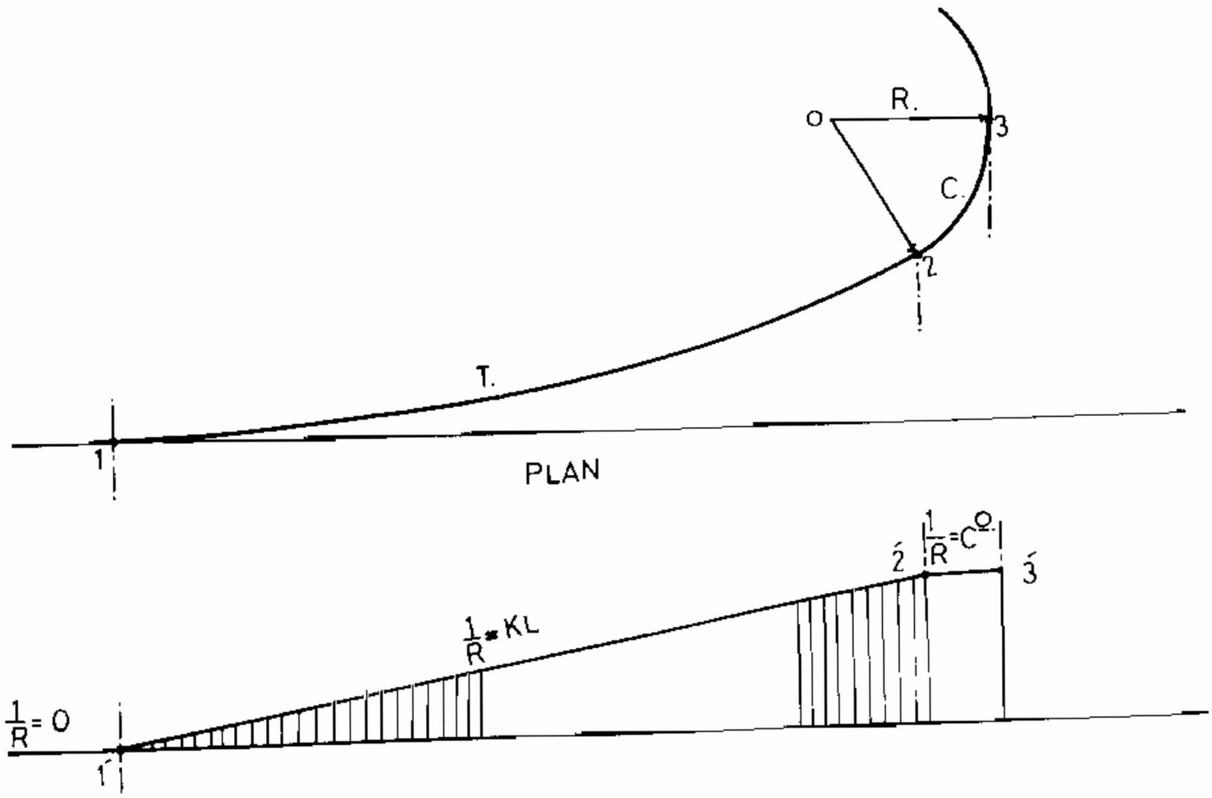


Fig.(1) Superelevation and curvature digram.

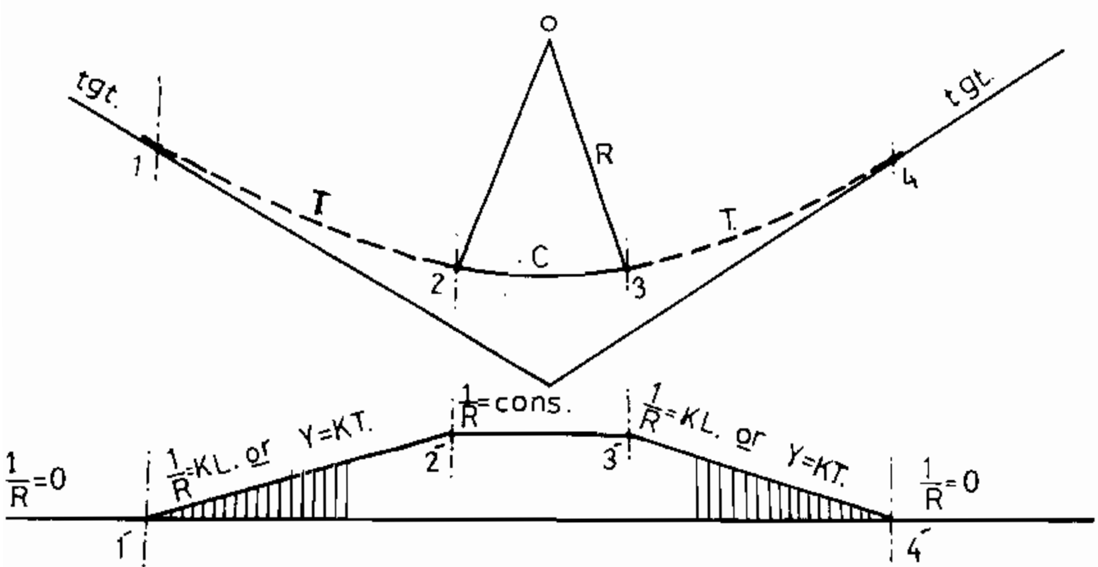
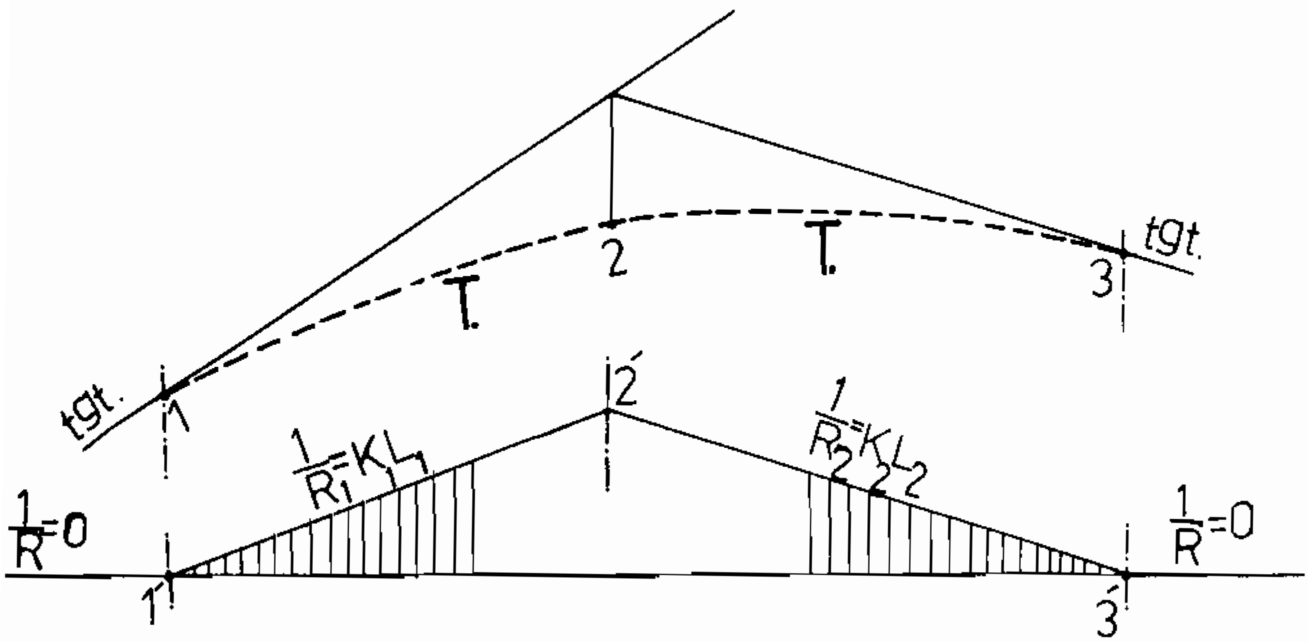


Fig.(2) Superelevation and curvature diagram.



Superelevation and curvature diagram.

Fig.(3) Wholy transitional curves.

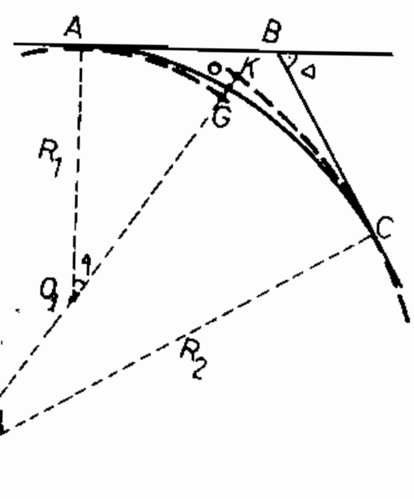


Fig.(4) Combining spiral.

Case 4:

Combining spiral

In Fig. (4), the combining spiral AC is tangent to the curves having radii R_1 and R_2, and it has the same radius of curvature as the circular arc at each point of tangency. That is, AC is a portion of a simple spiral cut to fit as a transition between curves of degree D_1 and D_2.

Only one combining spiral will satisfy the given condition.

Case 5:

Transition curves applied to a compound curves:

In this case, as shown in Fig. (5a) the amount of superelevation can not be designed to conform both circular arcs, and the design speed must relate to the smaller radius. If the two curves are to be connected by a transition curve of length (L), the shortest distance (S) Fig. (5b) between the two circular curves is given by glover,

$$S = \frac{L^2}{24} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

The length of the transition portion (L) between the two circular portions can be deduced as follows,

$$L = \sqrt{\frac{R_1 L_2^2 - R_2 L_1^2}{R_1 - R_2}}$$

Case 6:

Transition applied to reverse curves

If transition are applied to reverse curves, the radii must be reduced to allow the transition curves to be introduced, Fig. (6).

Practically, a straight portion must be laid down between the two reversed curves and length must not be less than 1/10th of speed in meters. (5)

Case 7:

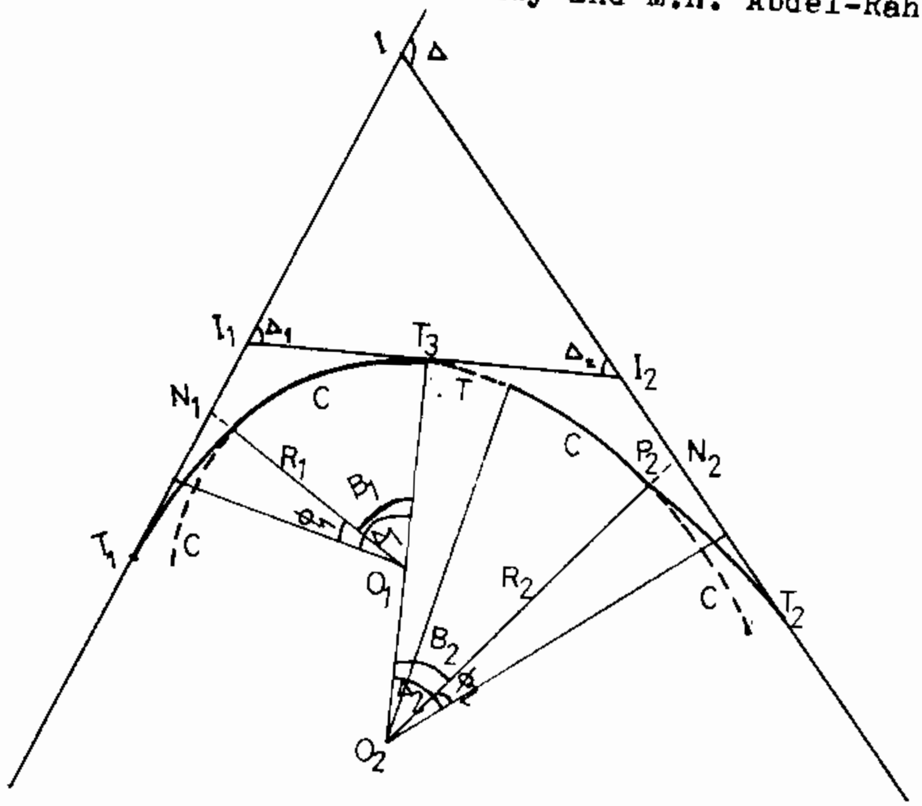
Series of transitions

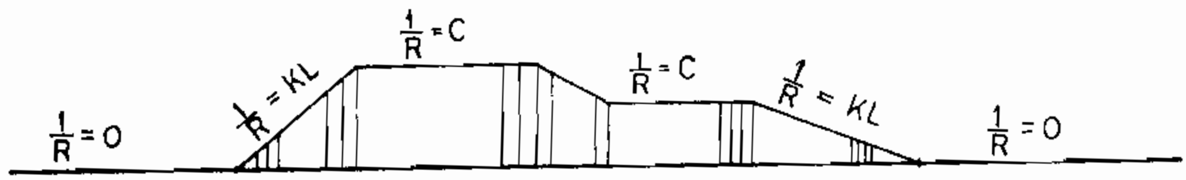
As shown in Fig. (7), to join different and levels.

Case 8:

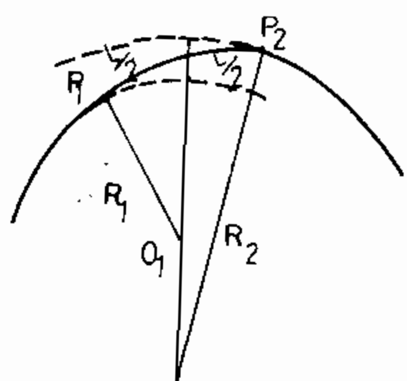
Transition between two separate circular curves

Fig. (8), illustrates a method of connecting two circles by means of a reverse transition curve, the size of the transitions being proportional to the radii of the circules thay touch.





(a) Superelevation and curvature diagram.



(b) Shift between the two circular curves.

Fig.(5) Case of transation curves applied to compound curve.

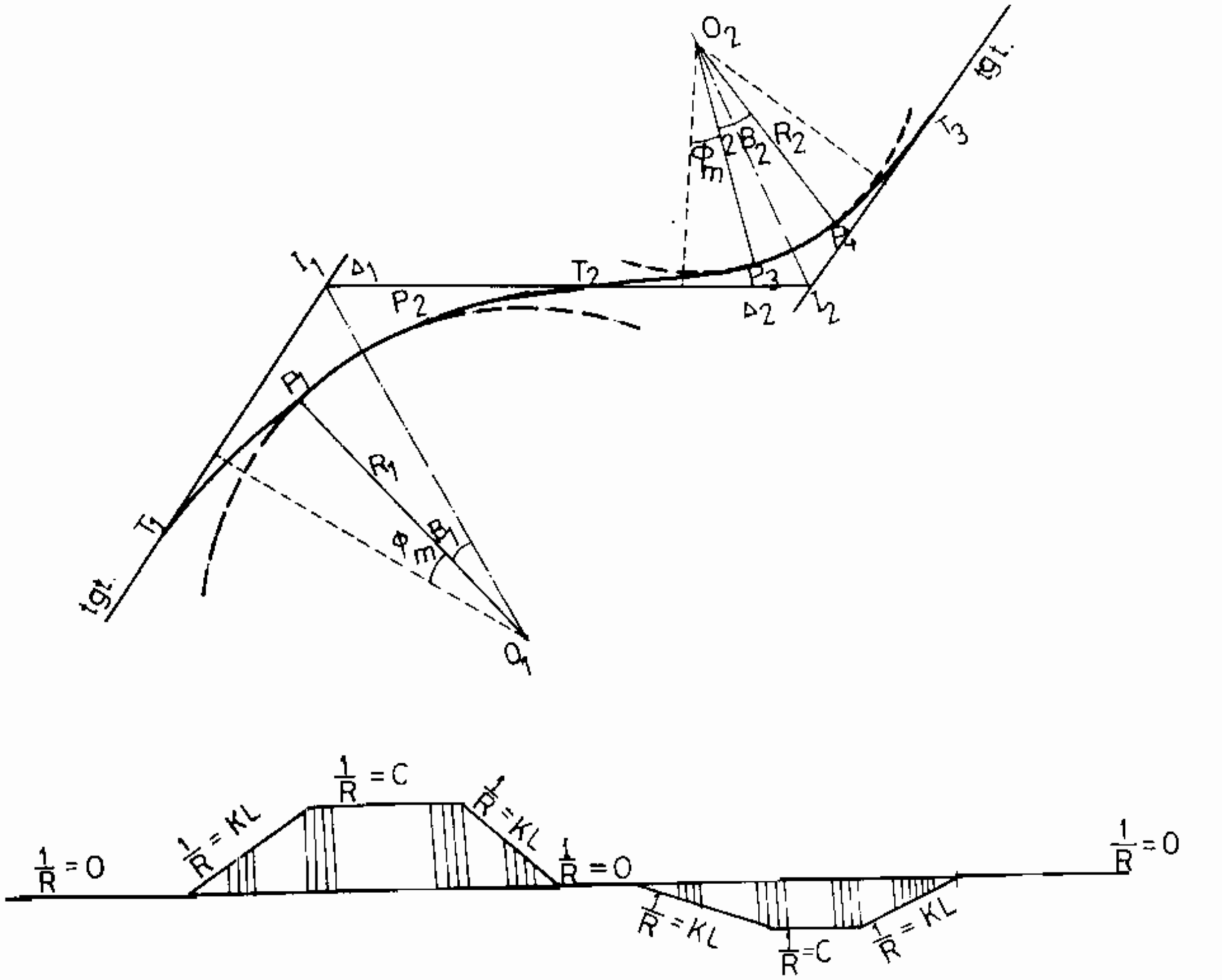


Fig.(6-a) Superelevation and curvature diagram.

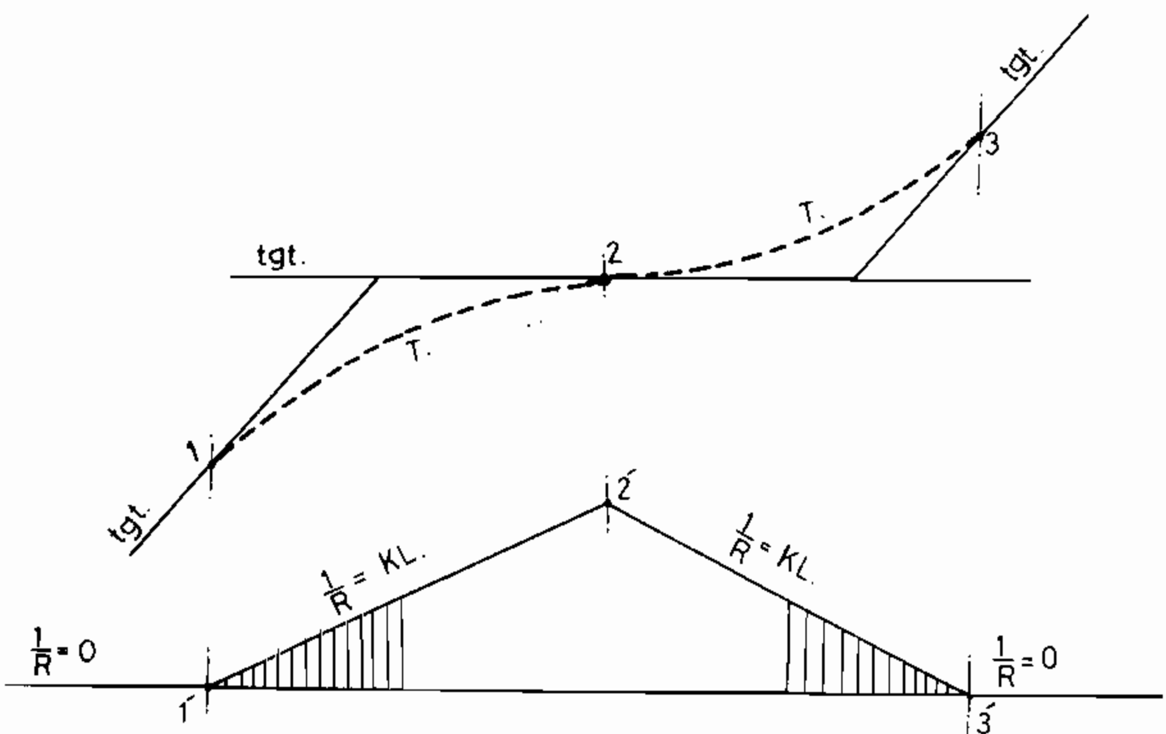


Fig.(6-b) Transition between three tangents.

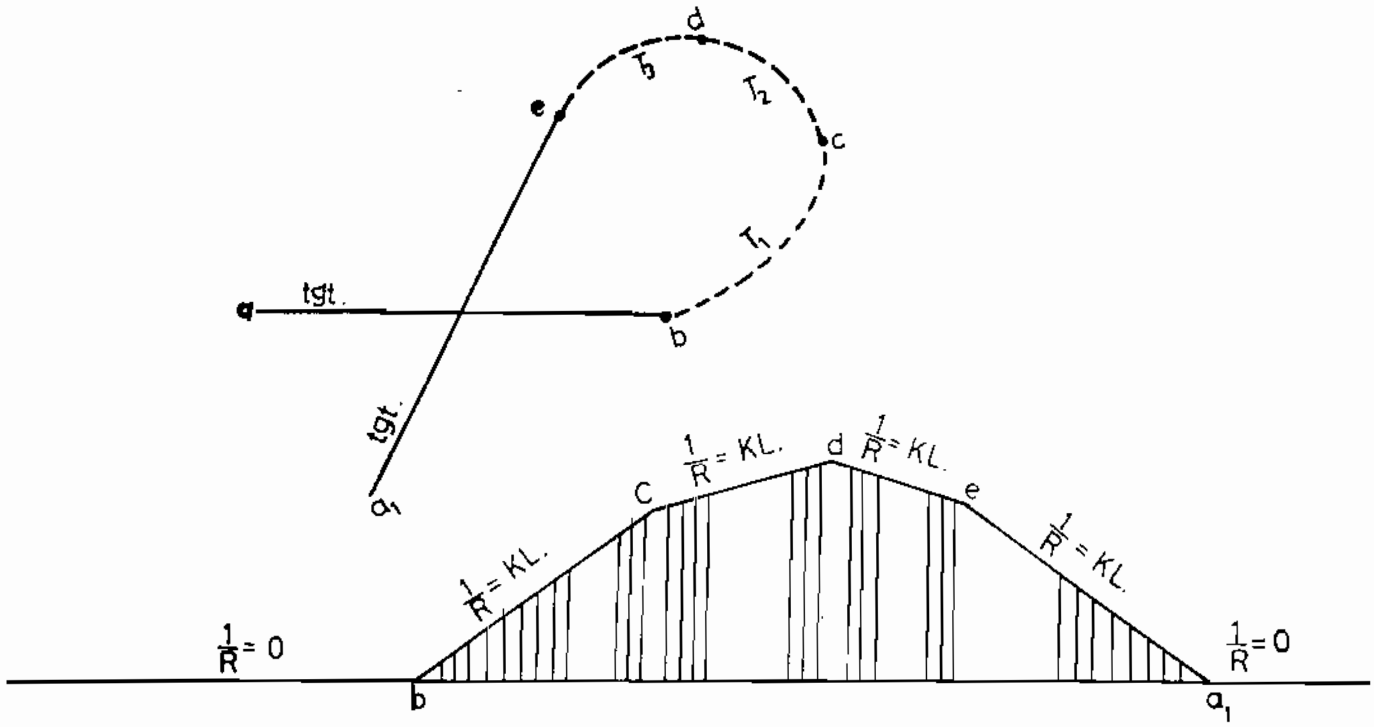


Fig.(7) Series of transitions.

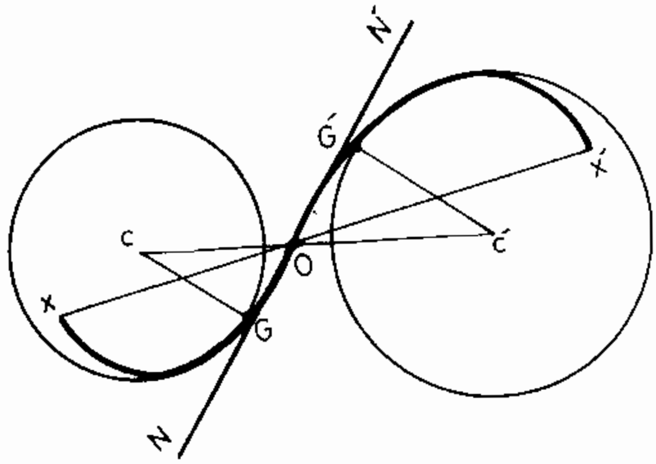


Fig.(8) Transition between two separate

circular portions.

THE SHIFT IN TRANSITION CURVE:

The circular curve may be located at the position it would assume without transitions if the two tangents are shifted outwards from the curve by an amount which will permit the introduction of the transitions. The amount of shift is given by the following equation.

$$S = \frac{L^2}{24R}$$

where,

S = Shift in meters.

L = Length of the transition in meters.

R = Radius of the circular curve in meters.

THE INSERTION OF TRANSITION CURVES INTO THE EXISTING ALIGNMENT

Designing the new lines it is preferable to choose the transitions to permit the predict future widening by running-out the shift of the circular and transition lengths according to the maximum Permissible speed on the actual radius not according to the actual speed to avoid any required future shift which could result in increasing costs and delaying vehicles. The insertion of transition curves into the existing alignment of tangents is done by one of the following methods:-

1- The radius of the existing circular curve is reduced by the value of shift, Fig. (9a).

$$\text{Shift} \quad S = R_1 - R_2 = \frac{L^2}{24R}$$

2- The radius and centre O are retained and the tangents are moved outwards to allow transition, Fig. (9b).

$$\text{Shift} \quad S = I_1 \quad I_2 \quad \cos\frac{\Delta}{2}$$

3- The radius of the curve is retained, but the centre O is moved away from the intersection point. Fig. (9c).

$$\text{Shift} \quad S = O_1 \quad O_2 \quad \cos\frac{\Delta}{2}$$

4- Tangent, radius and part of the existing curve are retained but a compound circular curve is introduced to allow shift, Fig. (9d).

5- A combination of any of these cases.

TYPES OF TRANSITTON CURVES

In ths following study three different transitions are considered namely the clothoid, the cubic parabols and the lemniscate.

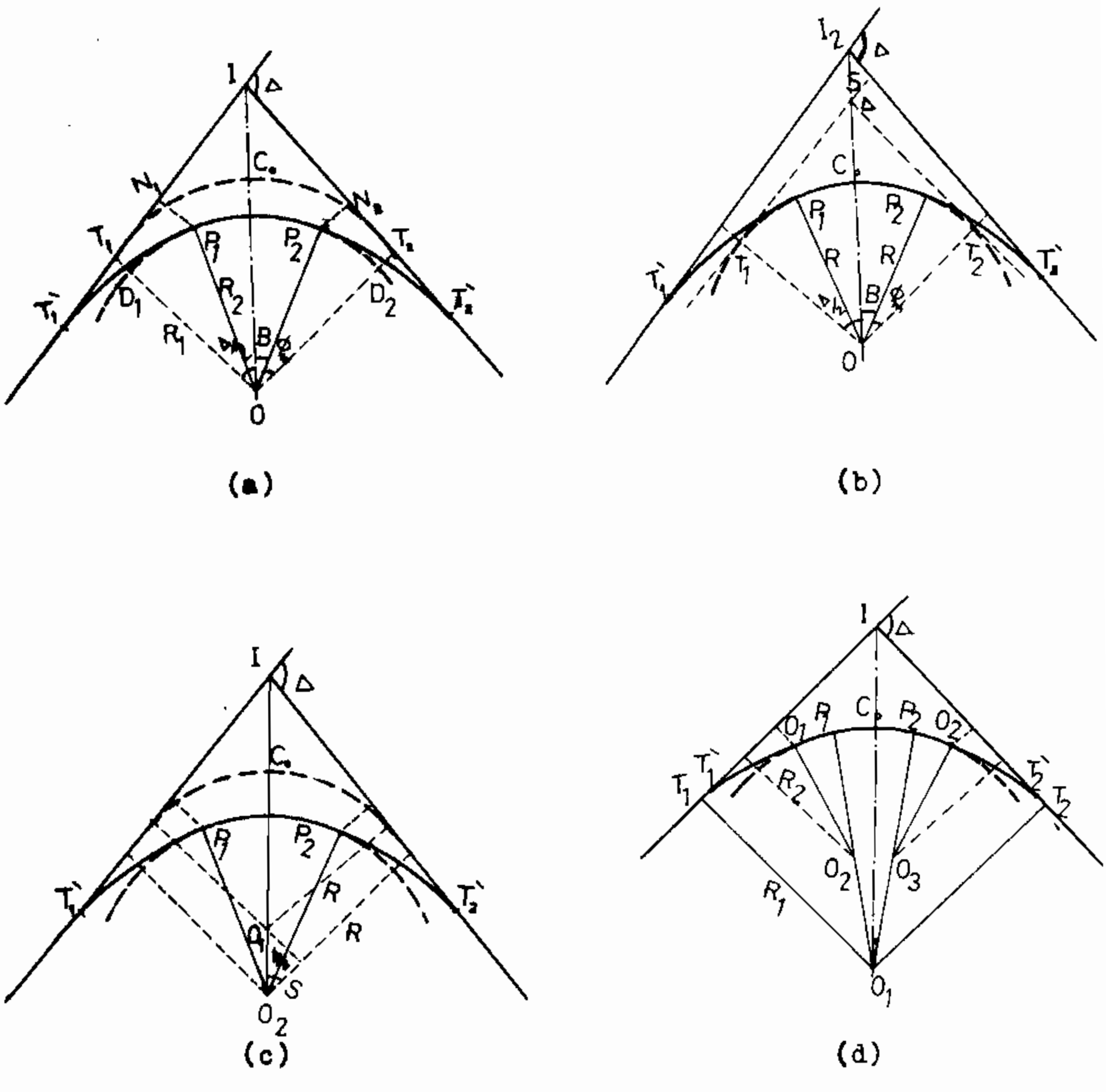


Fig.(9) The insertion of transition curves into the existing alignment.

THE CLOTHOID

Fundamental equation of the clothoid

The centrifugal force acting on a vehicle at a certain point on the curve is given by,

$$P = \frac{WV^2}{gr}$$

where,

r is the radius of curvature at this point.

For a constant speed, the distance L along the transition curve measured from the tangent point will vary with time. Hence, we have

$$P \propto L \propto \frac{WV^2}{gr}$$

putting W, V and g as constant values. Hence $L \propto \frac{1}{r}$

or $r\,L$ = constant = $R\,L$.

which is the physical fundamental formule of the clothoid. Thus the fundamental requirement of a transition curve is that its radius of curvature r at any point shall vary inversely as the distance L from the beginning of the curve. As R & L are lengths, the fundamental equation of the clothoid will be in the form

$$R\,L = A^2$$

where

A is a parameter.

CARTISIAN CO-ORDINATES FORM

From Fig. (10a), we have

$$\Delta l = r \;\Delta \phi$$

$$d\phi = 1/r \;dl$$

but $rl = RL = A^2$

$$d\phi = L/A^2 \;dl$$

By integrating:

$$\phi = \frac{L^2}{2A^2} + C$$

$$C = 0 \text{ at } L = 0, \; \phi = 0$$

Thus, $\phi = \frac{L^2}{2A^2} = \frac{L^2}{2RL}$

and $L = \sqrt{2RL} \;.\; \sqrt{\phi}$

The length of the clothoid is given by

$$L = 2\,R\phi$$

As the variable angle Ø can not be measured or plotted form one position, it is difficult to set it out in this form, therefore, the use of the cartision co-ordinates form is a must.

From Fig. (10b), we have,

$$\frac{dx}{dl} = \cos \emptyset$$

by expanding Cos Ø

$$\frac{dx}{dl} = 1 - \frac{\emptyset^2}{2\,\dot{1}} + \frac{\emptyset^4}{4\,\dot{1}} - \frac{\emptyset^6}{6\,\dot{1}} + \ldots$$

but we have, $\emptyset = \frac{L^2}{2RL}$

Thus,

$$\frac{dx}{dl} = 1 - \frac{L^4}{2\,\dot{1}(2RL)^2} + \frac{L^8}{4\,\dot{1}(2RL)^4} - \frac{L^{12}}{6\,\dot{1}(2RL)^6} + \ldots$$

by integrating,

$$X = L - \frac{L^2}{5x2\,\dot{1}(2RL)^2} + \frac{L^9}{9x4\,\dot{1}(2RL)^4} - \frac{L^{13}}{13x6\,\dot{1}(2RL)^6} + \ldots$$

Also,

$$X = L\,(1 - \frac{\emptyset^2}{5x2\,\dot{1}} + \frac{\emptyset^4}{9x4\,\dot{1}} - \frac{\emptyset^6}{13x6\,\dot{1}} + \ldots.)$$

For Ø max.

L = L total, then

$$X = L\,(1 - \frac{L^2}{40\,R^2}) \qquad \ldots(1)$$

Simolarly we have,

$$Y = \frac{L^2}{6R}\,(1 - \frac{L^2}{56\,R^2}) \qquad \ldots(2)$$

THE BASIC ELEMENTS OF THE CLOTHOID

As illustrating in Fig. (10b), the main elements of the clothoid are:

1. Co-ordinates of the center of curvature

$$Y_0 = Y + R \cos \emptyset$$

$$X_0 = X - R \sin \emptyset$$

2. Length of the two tangents

$$T_s = GP = \frac{Y}{\sin \emptyset}$$

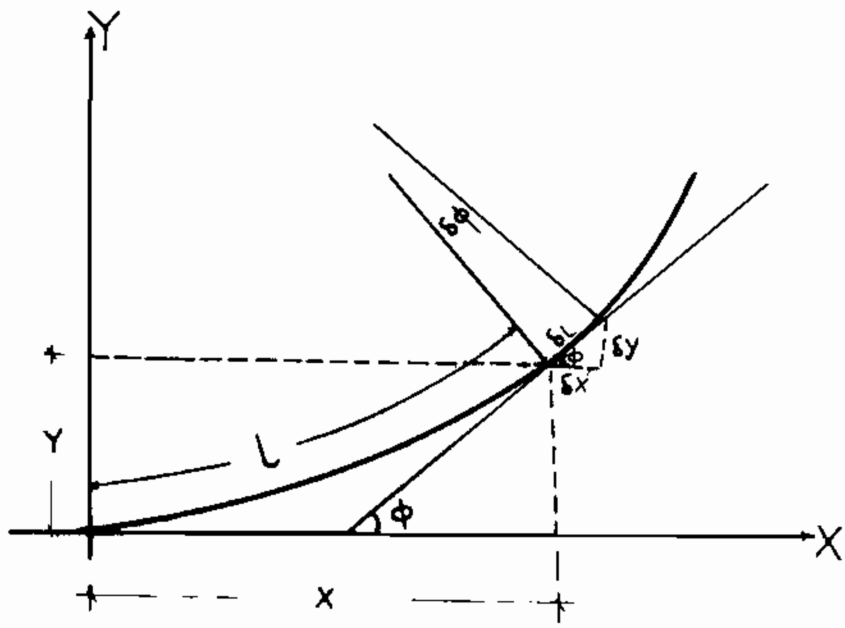


Fig. (10-a) Equation of the clothoid.

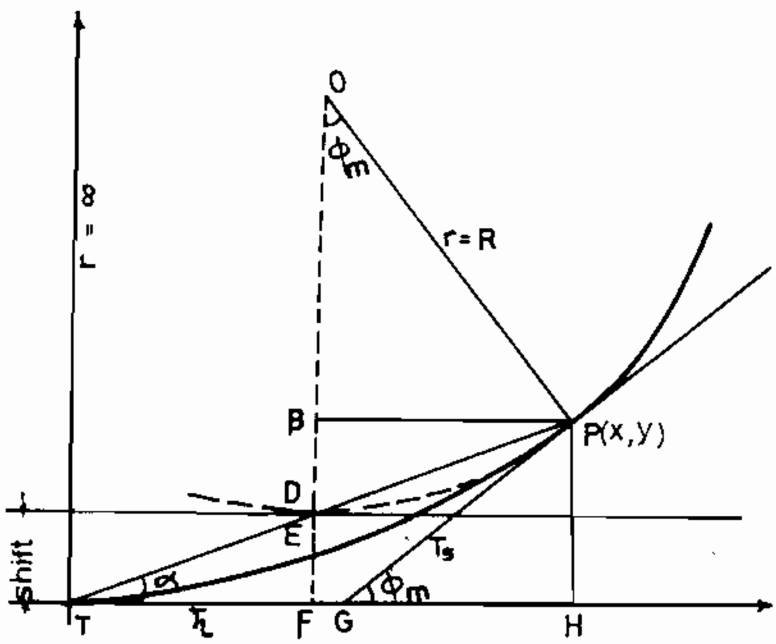


Fig.(10) The basic elements of the clothoid.

$$T_1 = GT = X - Y \text{ cost } \phi$$

3. The chord length (TP)

$$C = \sqrt{X^2 + Y^2}$$

4. Length of the transition clothoid (L).
The transition length can be obtained in terms of the parameter and velocity through the relation,

$$A = 0.207 \sqrt{V^3}$$

and $L = A^2/R$

Then $L = 0.0428 \times \frac{V^3}{R}$

where, V in Km/hr.

5. The shift (S),

$$S = Y_0 - R$$

6. The tangential angle (α)
To find the tangential angle (α).

$$\tan(\alpha) = \frac{Y}{X} = \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{26\ \phi^5}{155925} + \ldots$$

Since, $(\alpha) = \tan(\alpha) - \frac{\tan^3\alpha}{3} + \frac{\tan^5\alpha}{5} \ldots$

It can be shown that

$$\alpha = \frac{\phi}{3} - \frac{8\ \phi^3}{2835} + \frac{32\ \phi^5}{467775} - \ldots$$

and so, $\alpha = \frac{\phi}{3} - K$

where,

$$K = 3.095 \times 10^{-3}\phi^3 + 2.285 \times 10^{-8}\phi^5$$

ϕ being in degrees and K in seconds.

Values of K are given in the Table (1), showing the values of K for different values of ϕ. Thus, if ϕ is smell,

$$\alpha = \phi/3$$

Jenkins (2), shows that if $\phi < 6°$, no correction is required, and if $\phi < 20°$ no correction -> 20" is required. According to Prof. Royal Dawson (2), up to a value $\phi = 20°$ in the case fo sporal, the deflection angle is equal to $\phi/3$, but for $\phi > 20°$, the following relationship holds food,

$$\alpha = \phi/3 - N_s$$

The value of N_s can be read out from Royal Dawson curve given in Fig. (11).
This value is sufficiently accurate for all practical purposes.

ϕ^o	K ′	K ″	ϕ	K ′	K ″	ϕ^o	K ′	K ″
1	0	0	16	0	13	31	1	32
2	0	0	17	0	15	32	1	41
3	0	0	18	0	18	33	1	51
4	0	0	19	0	21	34	2	2
5	0	0	20	0	25	35	2	13
6	0	0	21	0	29	36	2	24
7	0	1	22	0	33	37	2	37
8	0	2	23	0	38	38	2	50
9	0	2	24	0	43	39	3	4
10	0	3	25	0	48	40	3	18
11	0	4	26	0	54	41	3	33
12	0	5	27	1	1	42	3	49
13	0	7	28	1	8	43	4	6
14	0	8	29	1	16	44	4	24
15	0	10	30	1	25	45	4	42

Table (1) The values of K for different values of ϕ .

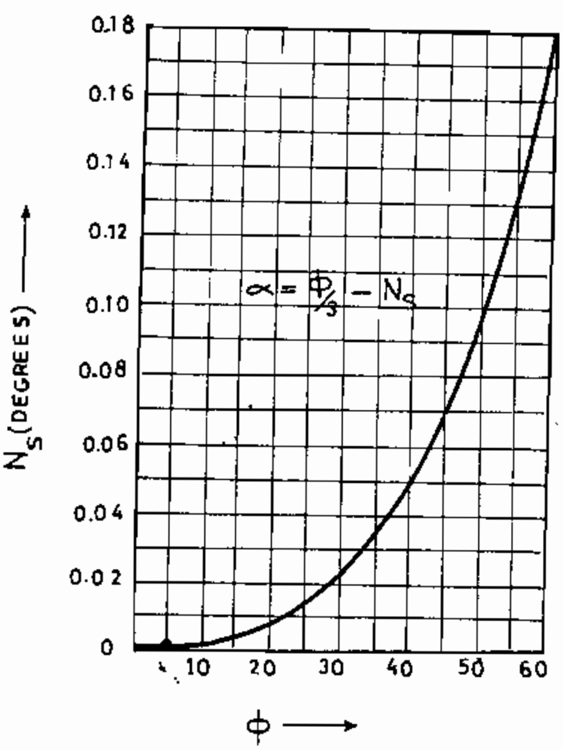


Fig.(11) The values of N_s for different values of ϕ.

THE LEMNISCATE

The lemniscate mostly is used in modern road constrction in preference over the clothoid for the following reasons: It is desired for a transition curve, that the rate of decrease of the radius of curvature should be less towards the end of the curve than at the beginning, the lemniscate of Bernoulli is a curve which satisfies that condition. The lemniscate and the clothoid are very similar up to a deviation angle of 60° but after that, the radius of curvature of the lemniscate is greater than that of the clothoid. Thus the rate of increase of the radius of curvature, of the lemniscate is more gradualey than that of the clothoid.

At deviation angle of 135°, the radius of curvature of the lemniscate is a minimum and at a greater deviation angle it begins to increase again. Furthermore, the rate of change of the radial acceleration for lemniscate is a maximum at the beginninl and decreases very slowly to a minimum at the end of the major axis.

Lemniscate is a symmetrical curve with large deflection angles which can be better used than the clothoid. Lemniscate has a valuable property, from the settir -out point of view, that the exterior deflection angle is ays exactly three times the polar deflection angle.

EQUATION OF THE LEMNISCATE

Referring to Fig. (13). The polar equation of the lemniscate is $C^2 = a^2 \sin 2\alpha$ Differentiating this equation, with respect to the deviation angle (α).

$$2C \frac{dc}{d\alpha} = 2\, a^2 \cos 2\alpha$$

and deviding by $2C^2$

$$\frac{1}{C} \frac{dc}{d\alpha} = \frac{a^2 \cos 2\alpha}{C^2}$$

$$= \frac{a^2 \cos 2\alpha}{a^2 \sin 2\alpha} = \cot 2\alpha$$

From Fig. (13), we have,

$$\cot \theta = \frac{P_1M}{P_2M} = \frac{Sc}{CS} = \frac{1}{C} \frac{dc}{d\alpha}$$

and so, $\theta = 2\alpha$

but, $\phi = \theta + \alpha$

hence $\phi = 3\alpha$

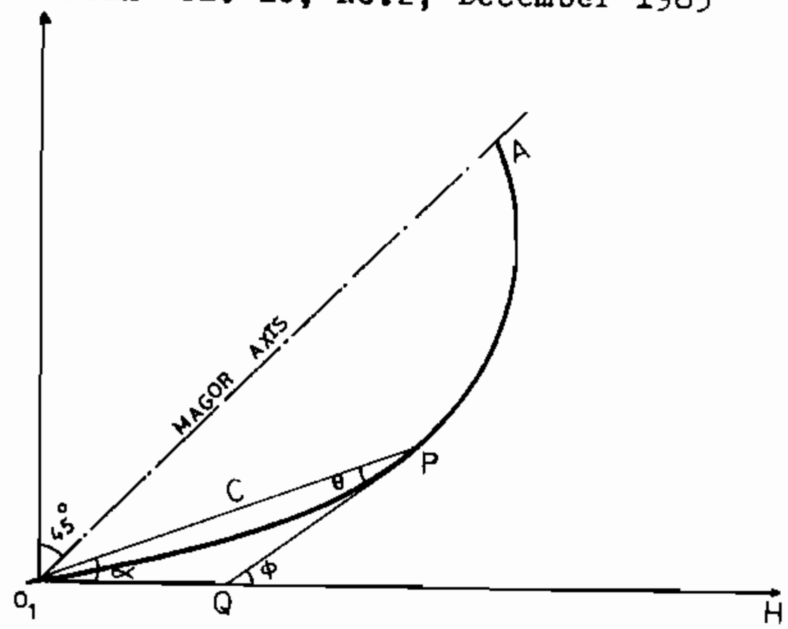


Fig. (12) elements of the lemniscate

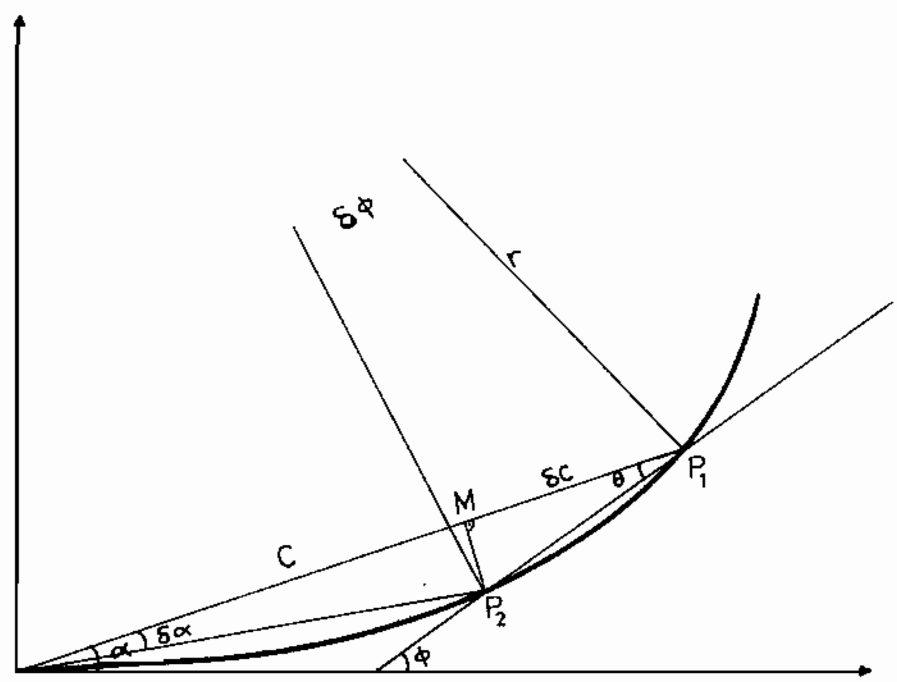


Fig. (13) Equation of the lemniscate

That is, the deviation angle is exactly three times the polar deflection angle, a most important property of the curve.
In Fig. (13), we have,

$$P_1P_2 = CL = \frac{P_2M}{\sin\theta} = \frac{CS}{\sin\theta} = \frac{CS}{\sin 2\alpha}$$

$$\text{hence} \quad \frac{dl}{d\alpha} = \frac{C}{\sin 2\alpha} = \frac{C}{C^2/a^2} = \frac{a^2}{C}$$

Now, $D\& = rd\emptyset = 3r \; d\alpha$

and so, $\frac{a^2}{C} = 3r$ or $a^2 = 3rc$.

Thus, $C^2 = 3rc \sin 2\alpha$

$C = 3r \sin 2\alpha$

If the lemniscate approximates to a circle of radius R, then, $C = 3R \sin 2\alpha$ which is the practical form of the lemniscate curve.

THE CUBIC PARABOLA

The cubic parabola is most widely used in practice especially for railroads because of its simplicity in both calculations and setting-out. It is having the advantage that no special tables are required for setting-it out. It is almost identical with the clothoid and lemniscate for deviation angles up to 12°. The equation for cubic parabola $Y = X^3/6RL$ is the first approximation of the true intrinsic equation of the spiral clothoid. This approximation is valid only for small deviation angles with negligable errors. The radius of curvature of the curve decreases from value of infinity when $\emptyset = 0$ to a minimum value of $r = 1.39 \sqrt{RL}$ when the deviation angle $\emptyset$ is (24° 05' 41"), and it begins to increase again, and consequently the curve becomes useless as a transition.

THE EQUATION OF THE CUBIC PARABOLA

If the radius of curvature of the curve is (r), then

$$\frac{1}{r} = \frac{d^2y:dx^2}{[1 + (dy/dx)^2]^{3/2}}$$

It is assumed that the deviation angle is small, therefore, $(\frac{dy}{dx})^2$ can be neglected and $\frac{1}{r} = \frac{d^2y}{dx^2}$

It has been outlined that in the ideal transition curve, R L = conctant, and the length of the cubic parabola is considered equal to the length of the abscissa (X=L) then $r.X = \text{constant} = R.X$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{1}{r} = \frac{X}{RX}$$

$$\frac{dy}{dx} = \frac{X^2}{2RX}$$

$$Y = \frac{X^3}{6RX}$$

which is the fundamental basic equation of the cubic parabola. The constant of integration disappear since Y and dy/dx are zero when X is zero.

Comparison between spiral clothoid and cubic parabola:

1. Clothoid

$$X = L\left(1 - \frac{L^2}{40\ R^2}\right)$$

$$Y = \frac{L^2}{6R}\left(1 - \frac{L^2}{56\ R^2}\right)$$

L	14.45	28.90	43.35	5[illegible]0	72.25
X	14.45	28.89	43.33	57.77	72.19
Y	0.087	0.35	0.78	1.39	2.174

2. Cubic parabola

$X = L$ (The length being measured along the X-axis).

$$Y = \frac{X^3}{6RL}$$

L	14.45	28.90	43.35	57.80	72.25
X	14.45	28.90	43.35	57.80	72.25
Y	0.017	0.14	0.47	1.11	2.175

Toleeance ΔX & ΔY.

ΔX	0.00	0.01	0.02	0.03	0.06
ΔY	0.07	0.21	0.31	0.28	0.001

DETERMINATION OF THE LENGTH OF TRANSITION CURVES

Safe operation at high speeds requires that curves are designed to fit natural driver-vehicle behaviour. It is obviously impossible, when travelling at any appreciable speed to change instant gradually from a straight to a circular path at the T.C. of an untransitional curve. On such alignment the driver makes his own transition as a matter of necessity, usually by starting to stear towards the curve in advance of the T.C. In doings, there is bound to be some deviation from the traffic lane or railway track. If the curve is sharp or if the speed is high, the deviation may result in dangerous encroachment on the shoulder or on the adjacent traffic lane or railway track. There are two conditions must be fulfilled to determine the length of transition curves namely:

1. Rate of change of centrifugal acceleration to be gradway developed.
2. Distribution of the designed superelevation at a reasonable rate.

LENGTH OF TRANSITION CURVES FOR HIGHWAYS

The different approaches may be followed for determining the length of transition curves for highways:

1. Length of transition in terms of rate of change of centrifugal acceleration (SHORTT'S formula)

$$L_s = \frac{V^3}{46.6\ CR}$$

for speeds up to 32 and exceeding 96 Km/hr the value of C will be 0.76 and 0.46 m/sec^3. respectively. For speeds between 32 and 96 Km/hr, the value of C is given by:

$$C = \frac{73}{V+64}\ m/sec^3.$$

2. Length of transition in terms of the centrifugal ratio: By Combining SHORTT'S formula with a centrifugal ratio of 1/4 we get,

$$L_s = \frac{3.84}{C} \sqrt{R}$$

(R & L_s in m, C in m/sec^3).

3. Length of transition in terms of rate of introduction of superelevation:
For the case in which the pavement is rotating about centerline.

L_s = ½ (superelevation X width of highway X lateral gradient).

= ½ S.W.e

For the case in which pavement is rotating about inner edge

$$L_s = S.W.e$$

4. Typical formula applied on clothoid
The parameter of clothoid is given by:

$$A = 0.207 \sqrt{V^3}$$

from which the length of transition can be determined from the relation

$$LR = A^2$$

so the length of the transition is

$$L_s = \frac{0.0428 V^3}{R}$$

In fact there are different emperical formulas which are in use by different countries, namely

1. U.S.A $L_s = 8.7 \ Ea \ V$
   $L_s = 12.1 \ Eu \ V$
   $L_s = 475.0 \ Ea$
2. England $L_s = 4.83 \ Ea \ V$
   $L_s = 4.83 \ Eu \ V$
   $L_s = 300.0 \ Ea$
3. Japan $L_s = 6.2 \ Ea \ V$
   $L_s = 7.5 \ Eu \ V$
4. Germany $L_s = 8.0 \ Ea \ V$
   $L_s = 400.0 \ Ea$

where:

L_s in meters, V in Kmphr., Ea is the actual superelevation and Eu is the unbalanced one.

It must be noticed that the largest value of the above mentioned forms to be used for each category.

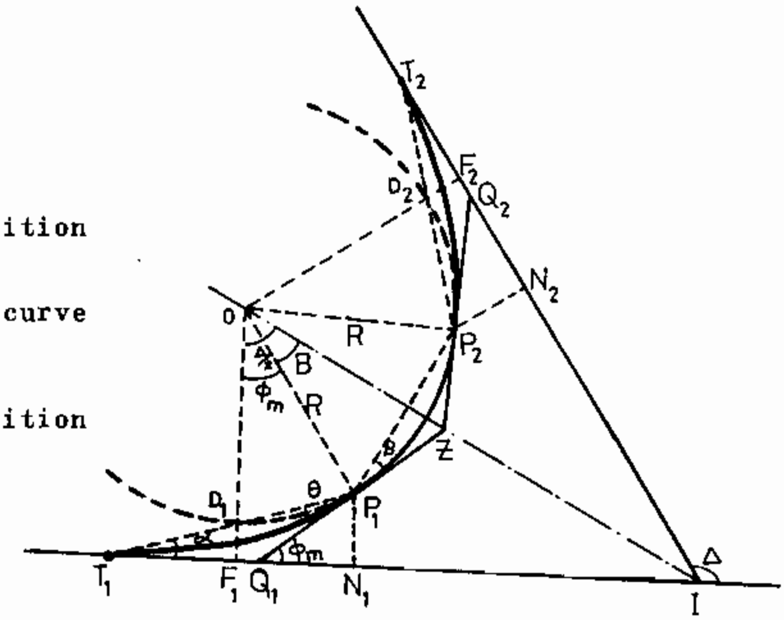
SETTING-OUT OF TRANSITION CURVES:

The field procedure is to locate the tangent points on the straights, and set-out the exit transition by deflection angles or by offsets from the first tangent point. The circular arc is set-out from the last point of the entry transition, B, Fig. (14).

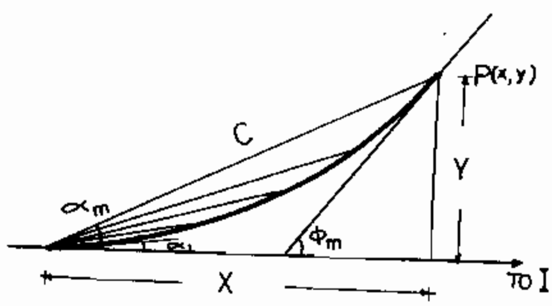
T_1P_1 1st transition

P_1P_2 circular curve

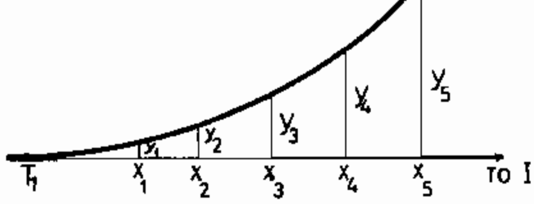
P_2T_2 2nd transition



(a)



(b) Deflection angles from the tangent



(c) Offsets from the tangent

Fig. (15)

In order to determine the direction of the common tangent FB, the theodolite at B is sighted on A and the horizontal angle.

$$180^\circ - (\phi_L - \theta_L)$$
$$\text{or } 180^\circ - 2\,\theta_L$$

is turned off. The circular arc is then set-out in the usual way, with initial and final sub-chords as necessary. A closing check is provided by the peg at the beginning of the exit transition. Though, transition usually is laid out with equal chords, the number commonly being 10, such a process does not serve all purposes. For example, on location prior to grading, earthwork estimates are made more rapidly if cross sections are taken at regular full stations and possibly half station. Furthermore, important "breaks" requiring cross sectioning way fall between regularly spaced points.

During construction it may be necessary to set points on a transition at trestle bents or on bridge piers. For these reasons it is convenient to have a simple formulas for determining the deflection angle to any point at a distance L ft beyond the beginning of a transition. Generally, the steps of setting-out transition curves are:
Fig. (14)

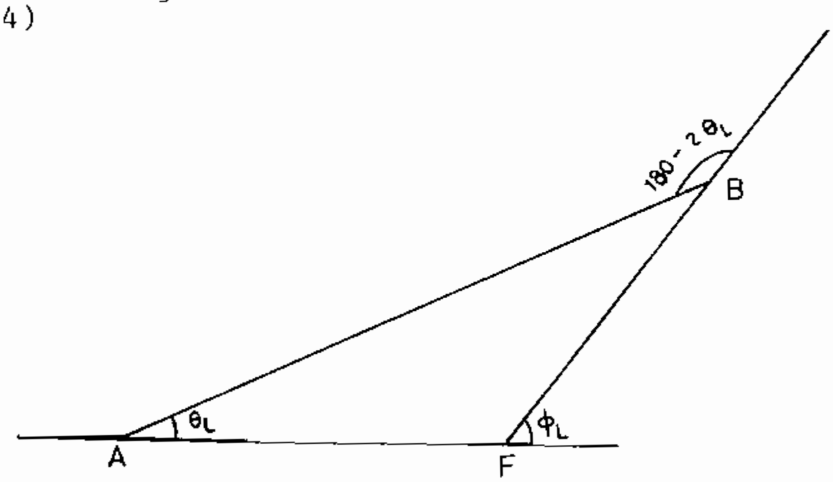


Fig. (14) Setting out of transition curves.

1. Locate the point of intersection of the two tangents (I).
2. Measure the twotangent lengths from the point of intersection (1) to locate the two tangent points T_1 & T_2

$$\text{Shift } (S) = FD = L^2/24\,R$$
$$= FI = (R+S) \tan \Delta/2$$
$$FT = L/2$$

tangent length $T_1 I = (R+S) \tan \Delta/2 + L/2$

3. Measure the value of intersection angle (Δ)

4. Set-out the 1st transition curve T_1P_1 from T_1 using one of the following two methods:

   a) Offsets from the tangent by taking different values for (X) on the tangent length and determining the corresponding values of (Y) from equation

   $$Y = \frac{X^3}{6\ R\ L}$$

   b) Tangential angles for different chosen chord length using theodolite in setting-out.
   The values of tangential angle must be calculated from the following relations:

   for clothoid, $$\alpha'' = \frac{L^2}{6\ R\ L} \cdot \rho''$$

   $$\alpha^{\circ} = \frac{9.55\ L^2}{RL}$$

   for lemniscate, $$\alpha = \tfrac{1}{2} \sin^{-1} \frac{C}{3R}$$

   for cubic parabols, $$\alpha = \tan^{-1} \frac{X^2}{6\ R\ L}$$

   The method of tangential angles is more accurate than the method of offsets especially when a theodolite of high accuracy is used, even assuming chord (C) equals arc (L) equals (X)

5. Locate and check the joining point (P_1) of transition to circular vurve

   $$N_1\ P_1 = \frac{L^2}{6R}$$

6. Move the theodolite to (P_1) and set-out the shifted circular curve by offsets or deflection angles from the new tangent $Q_1\ P_1\ Z$.

7. Move the theodolite to T_2 and set-out the second transition curve towards (P_2) using the previous procedures.

Comparative study between the different types of transition curves from setting-out point of viex:

The following sample example is just a mean for comparing the different types of transition curves regarding tangential angle method and offsets method adopted for setting-out in this examples.

Assuming R= 120 m & L = 150 m. The curve is devided each 15m; and chainage of the (T.S.) is 30. By calculating

Point	Curve length (l)	Chain-age	Tangential angles (α)			Offsets (y)		
			Clothoid	Lemnis-cate	Cubic parabola	Clothoid	Lemnis-cate	Cubic parabola
T.S	0	30.00	00 00 00	00 00 00	00 00 00	0.000	0.000	0.000
1	15	30.75	00 06 05	00 07 10	00 07 09	0.031	0.031	0.031
2	30	31.50	00 28 30	00 28 39	00 28 39	0.250	0.250	0.250
3	45	32.25	01 04 18	01 04 28	01 04 27	0.844	0.844	0.844
4	60	33.00	01 54 35	01 54 41	01 54 33	1.999	1.999	2.000
5	75	33.75	02 59 01	02 59 22	02 58 53	3.900	3.911	3.906
6	90	34.50	04 17 18	04 18 48	04 17 21	6.709	6.769	6.750
7	105	35.25	05 57 33	05 53 25	05 49 43	10.648	10.775	10.719
8	120	36.00	07 37 45	07 43 59	07 35 41	15.818	15.147	16.000
9	135	36.75	09 44 36	09 51 44	09 34 42	22.368	23.123	22.781
S.C.	150	37.50	11 53 46	12 18 44	11 46 06	30.389	31.996	31.250

Table (2) Giving the different values of tangential angles and offsets required for setting-out the different types of transition curves.

L = 150 ms. R = 120 m.

tangential angles and off'ets, as given before, we reach to the following values shown in Table (2).

It is clear from the given example, that the cubic parabola is almost identical with the clothoid up to a distance of 90m with difference not exceeding 9 seconds, in the case of tangential angles, and 4 cin, in the case of offsets. After this distance, while in the case of co-ordinates the values of offsets for the clothoid is less than that of the cubic parabola and this is due to the assumption that curve length is measured on the abacissa.

In the case of the cubic parabola, hence the actual curve length will be greater than its designed length.

In the case of leminscate, vales of deflection angles and offsets are always greater than that of both clothoid and cubic parabola.

The use of the tangential angles method for setting-out is resulting in fewer less error than offsets method, and this is obvisous in the case of cubic parabola as the offsets calculated by tangential angles are less than that calculated from equation,

$$Y = \frac{X^3}{6\ RX}$$

As a partial conclusion, it si clear that the cubic parabola is most identical with the clothoid, this means that the cubic parabola can be set-out instead of the clothoid.

CONCLUSION:

The types of transition curves discussed are the spiral clothoid, the cubic parabola and the lemniscate of Bernoulli. The calculations for setting-out these curves involve the use of special tables. Moreover, the principles and fundamental formulae in respect of superelevation must be followed in order to obtain a reasonable transitional path. From this study it is clear that for deflection angles up to 9°, there is no practical difference between the different types of transitions. In such a case the formula for both the transition spiral and the cubic parabola when calculating ordinates are closely identically. Moreover the cubic parabola is almost identical with the clothoid for the deviation angles up to 12°. However the assumptions made in the derivation of the cubic parabola formula begin to break doan beyond 12° and further terms must be included to maintain the accuracy. Hence, if greater accuracy is required, the clothoid should be used.