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ON THE INTEGRATION OF MOMENTUM EQUATION WITH BOUNDARY LAYER SUCTION

BY

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ABSTRACT

Boundary layer suction is one of the effective methods for the prevention of flow separtion. In this paper, a method is introduced to integrate the momentum equation, in which the suction velocity is assumed constant, and the velocity gradient varies. This particular type of flow can be used to form an estimate of the quantities of air that must be sucked on an aerofoil, so that separation of flow does not take place. In this case, although the boundary layer is very thin before separation, the velocity gradient may be very large, and it is obvious that only small quantites of air withdrawn are sufficient to delay the stall.

NOMENCLATURE

c	free stream velocity,	m/s
$\bar{c}$	velocity at the outer edge of the boundary layer,	m/s
c_x	velocity of the fluid inside the boundary layer in x-direction,	m/s
c_y	velocity component inside the boundary layer in y-direction,	m/s
c_{yo}	suction velocity, normal to the surface,	m/s
H_{12}	boundary layer form parameter	
p	free stream static pressure,	N/m^2
x	streamwise coordinate	
y	coordinate perpendicular to surface	
δ	boundary layer thickness,	m
δ^{**}	momentum thickness,	m
τ_w	wall shear stress,	N/m^2
ν	kinematic viscosity of fluid,	m^2/s
ρ	density of fluid,	kg/m^3

1- INTRODUCTION

With the rapid development in the field of aeronautical engineering in recent years, the need arose for a broader treatment, especially of controlling the behaviour of boundary layer. In particular, it is often to prevent separation in actual application. Suction, is one of the effective methods which have been developed for controlling the separation. The application of suction is successfuly used in the design of aircraft wings [1] in order to reduce drag and to attain high lift.

By the use of suitable arrangements of suction slits, it is possible to remove the decelerated fluid particles from the boundary layer before they are given chance to cause separation. This enables to shift the point of transition in the boundary layer in the downstream direction.

2- MATHEMATICAL MODEL OF THE GENERAL CASE

Owing to the importance of the method of boundary layer control by suction, various mathematical methods, for the calculation of the influence of suction on boundary layer flow have been developed. Approximate methods, which deal with the boundary layer on the surface of an arbitrary shape and with arbitrary suction distribution, were developed by Schlichting [1] and Torda [2]. These methods, like those for the case of no suction, are based on the momentum integral equation. Therefore, the effect of the surface suction on the momentum equation may now be considered:

$$\frac{d\delta^{**}}{dx} + (2 + H_{12})\frac{\delta^{**}}{\bar{c}}\frac{d\bar{c}}{dx} - \frac{c_{y_o}}{\bar{c}} = \frac{\tau_w}{\rho\bar{c}^2} \qquad (1)$$

The derivation of the momentum equation (1) , where the velocity normal to the surface is c_{y_o} , is given in [3].

Equation (1) can be written in the following form:

$$\bar{c}\frac{d\delta^{**}}{dx} + (2 + H_{12})\delta^{**}\frac{d\bar{c}}{dx} - c_{y_o} = \frac{\nu}{\bar{c}}\left(\frac{\partial c_x}{\partial y}\right)_{y=0} \qquad (2)$$

In conjunction with this, the condition holding at the boundary is used, which can be obtained by putting y = O in the following equation of motion:

$$c_x\frac{\partial c_x}{\partial x} + c_y\frac{\partial c_x}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\partial^2 c_x}{\partial y^2} \qquad (3)$$

Thus :

$$c_{y_o}\left(\frac{\partial c_x}{\partial y}\right)_{y=o} = \bar{c}\frac{d\bar{c}}{dx} + \left(\frac{\partial^2 c_x}{\partial y^2}\right)_{y=O} \qquad (4)$$

A qualitative picture of flow under the influence of suction can be obtained by means of equation (4).

Let

$$\left(\frac{\partial c_x}{\partial y}\right)_{y=O} = \frac{\bar{c}}{\delta^{**}} Y_1,$$

$$\left(\frac{\partial^2 c_x}{\partial y^2}\right) = \frac{\bar{c}}{\delta^{**2}} Y_2,$$

Y_1 , Y_2 being numbers, where Y_1 may be of any value or zero. It may be assumed that Y_2 = O which denotes the Blasius profile , and hence in this case, Y_1 = 0.2205.

Therefore, equation (4) becomes :

$$- \frac{d\bar{c}}{dx} \frac{\delta^{**2}}{\nu} + c_{y_o} \frac{\delta^{**}}{\nu} Y_1 = Y_2$$

$$\frac{\delta^{**}}{\nu} \left(- \frac{d\bar{c}}{dx} \delta^{**} + c_{y_o} \cdot Y_1 \right) = Y_2 \tag{5}$$

From the previous equation, it is possible to deduce the qualitative nature of several types of flow. Now, a special type of flow, will be considered and one can easily estimate the amount of suction necessary to delay the stall of serofoils.

3- FLOW WITH VARYING VELOCITY GRADIENT AND CONSTANT SUCTION

The following method may be used to integrate the momentum equation, in which constant suction velocity is assumed, and the velocity gradient varies.

Suppose that, a velocity distribution is maintained, which approximated closely to the Blasius [4] distribution (i.e., $Y_1 = 0.2205$) , thus:

$$\left(\frac{\partial c_x}{\partial y} \right)_{y=o} = 0.2205 \frac{\bar{c}}{\delta^{**}} ,$$

$$\left(\frac{\partial^2 c_x}{\partial y^2} \right)_{y=0} = 0 ,$$

and assume

$$\frac{d\bar{c}}{dx} = 0.2205 \frac{c_{y_o}}{\delta^{**}} \tag{6}$$

Then, the boundary condition, in equation (4), is satisfied. The momentum equation therefore becomes:

$$\bar{c} \frac{d\delta^{**}}{dx} = - (H_{12} + 2) \delta^{**} \frac{d\bar{c}}{dx} + c_{yo} + 0.2205 \frac{\nu}{\delta^{**}}$$

Therefore ,

$$- 0.2205\, c_{yo} \frac{d^2\bar{c}}{dx^2} \frac{\bar{c}}{(d\bar{c}/dx)^2} = [\, 0.2205\, (H_{12} + 2) + 1]\, c_{yo} + \frac{\nu}{c_{y_o}} \frac{d\bar{c}}{dx} \tag{7}$$

Equation (7) would be difficult to solved, however for the case of $H_{12} = 2.5345$, the previous equation may havs the following form :

$$\frac{d^2\bar{c}}{dx^2} \frac{1}{(d\bar{c}/dx)^2} + 4.5345 \frac{\nu}{c_{y_o}^2 \cdot \bar{c}} \frac{d\bar{c}}{dx} = 0 \tag{8}$$

That equation can be integrated to give

$$-\frac{1}{d\bar{c}/dx} + 4.5345\ \frac{\nu}{c_{y_o}^2} \log \frac{\bar{c}}{c_\infty} = 0 \qquad (9)$$

The integration constant has been adjusted so that when $d\bar{c}/dx = -\infty$, i.e., at the begining of boundary layer where $\delta^{**} = 0$ in equation (6), $\bar{c} = c_\infty$.
The equation (9) can be rewritten as

$$-1 + 4.534\ \frac{\nu}{c_{y_o}^2}\ \frac{d\bar{c}}{dx} \log \frac{\bar{c}}{c_\infty} = 0 \qquad (10)$$

By integrating the previous equation, the following form is obtained :

$$\frac{x.c_{yo}^2}{c_\infty \nu} = 4.5345 \left(\frac{\bar{c}}{c_\infty} \log \frac{\bar{c}}{c_\infty} - \frac{\bar{c}}{c_\infty} + 1 \right) \qquad (11)$$

The integration constant may be adjusted using the following condition :

at $x = 0 \quad \bar{c} = c_\infty$

From equation (10), $d\bar{c}/dx$ is found in terms of $\bar{c}$, and by substituting $d\bar{c}/dx$ in equation (6), δ^{**} can be obtained.

The calculations of the above equations give the results which are given in table 1. Moreover, these results are illustrated in Fig. 1, which shows the flow with varying velocity gradient and constant suction.

4- CONCLUSION

The previous method may be used to integrate the momentum equation, in which constant suction velocity is assumed, and the velocity gradient varies. The particular type of flow can be used to form an estimate of the quantities of air that must be sucked on an aerofoil at high incidence so that separation of flow does not take place. In this case, although the boundary layer is very thin before separation, the velocity gradient may be very large, and it is obvious that only small quantities of air withdrawn are sufficient to delay the stall.

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$\bar{c}/c_\infty$	$-\delta^{**} c y_o/\nu$	$x c y_o^2/c_\infty \nu$
1.0	0	0
0.9	0.105	0.024
0.8	0.223	0.098
0.7	0.357	0.228
0.6	0.511	0.424
0.5	0.693	0.696
0.4	0.916	1.059
0.35	1.050	1.281
0.3	1.204	1.514
0.25	1.386	1.829
0.2	1.609	2.172
0.15	1.895	2.564

Table (1) Results

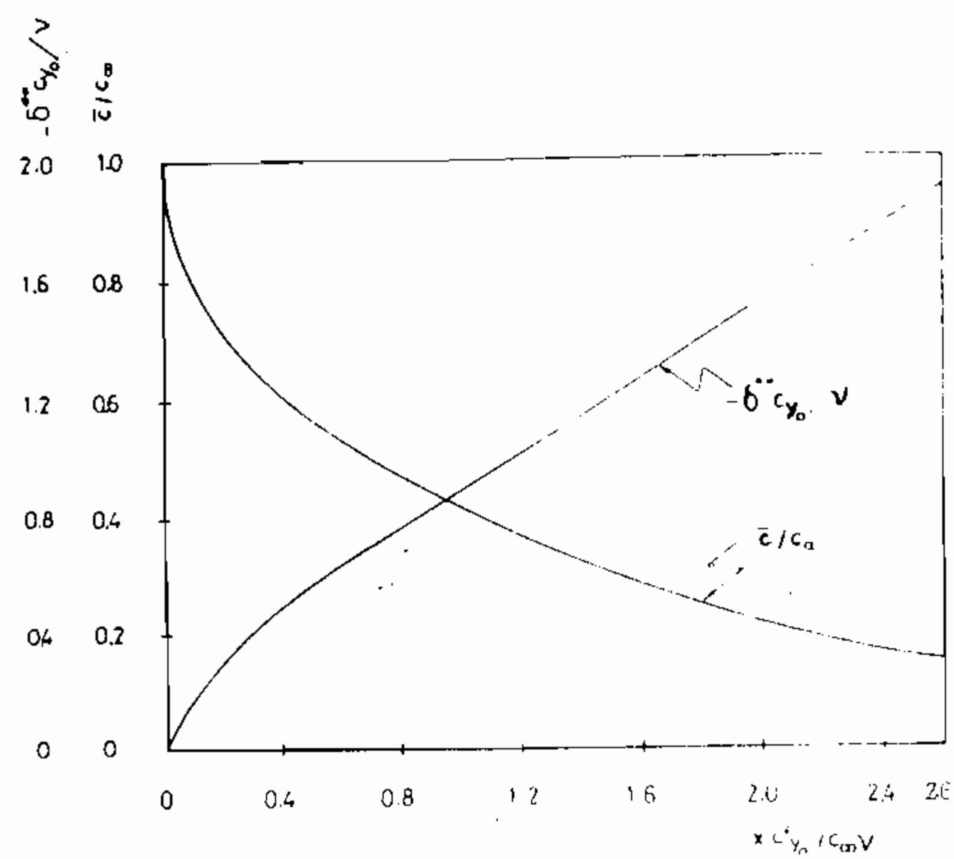

Fig.(1) Flow with varying velocity gradient and constant suction