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A THEORETICAL AND EXPERIMENTAL INVESTIGATION OF ADVANCING SURGE WAVES

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خلاصه – يختص هذا البحث بدراسة الامواج الناشئة نتيجة قفل بوابات الرى قفلا مفاجئا جزئيا أو كاملا ، كذلك الامواج الناشئة فى القنوات المتصلة بمحطات القوى المائية نتيجة الرفض الكامل أو التخفيف المفاجىء للاحمال .

استنبطت بالدراسة مجموعة من العلاقات النظرية التي تربط بين العوامل التى تتحكم فى هذه الظواهر كما شملت اخذ قراءات معملية ومقارنتها بتلك العلاقات التي استنبطت نظريا .

وقد تم تبطين قاع المجرى المائى المعملى بدرجات مختلفة من الخشونة لدراسة تأثير عامل الاحتكاك على خصائص واشكال هذه الامواج .

ABSTRACT- This research work provides analytical solution of the positive surge waves, propagating either upstream or downstream based on the momentum principle. The solution based on the numerical solution of the governing equations of motion (the characteristics method) is also highlighted.

Experimental work was carried out to make a comparative study between the actual measurements and the corresponding theoretical relationships for a horizontal plain bed channel. The effect of bed roughness on the shape and characteristics of such waves was under consideration.

INTRODUCTION

Engineers are interested in the determination of the maximum water level that could be developed as a result of sudden gate closure in an irrigation canal or sudden rejection of load in a power canal for establishing the height of walls necessary to prevent flooding.

If the flow in a channel is decreased or stopped abruptly by a partial or complete closure in the opening of sluice gate, the formed wave travels upstream of the gate. Such waves, having significant heights, are known as surges or surge waves.

Surge waves are classified into two kinds, the positive surge and the negative surge A surge is known as positive when it causes an increase in water level in the direction of its travel, advancing either upstream (rejection surge in power canals) or downstream (surge due to a dam failure) with stable wave front. A negative surge is the one which causes a decrease in water depth retreating either upstream (demand surge in power canals) or downstream (surge due to the closing of head gate in a canal or at the tailrace of a hydroplant) with unstable wave front.

THEORETICAL STUDIES

(A) Momentum Principle

(i) Sudden partial closure, Fig.(1).

The continuity equation (5) is given by,

$$A_1(u_1 + c) = A_2\ (u_2 + c) \qquad .....(1)$$

$$\text{or } u_2 = \frac{A_1u_1 - c(A_2 - A_1)}{A_2} \qquad .....(2)$$

The momentum equation is

$$gA_1\ \bar{y}_1 - gA_2\ \bar{y}_2 + A_1(u_1+c)(u_1-u_2) = 0 \qquad .....(3)$$

where $\bar{y}_1$ and $\bar{y}_2$ are the respective depths of centres of water areas A_1 and A_2.

From equation (2) and equation (3) we have:

$$g(A_2\bar{y}_2 - A_1\bar{y}_1)\ \frac{A_2}{A_1(A_2 - A_1)} = (u_1 + c)^2$$

$$\text{or } c = \quad gA_2\left[\frac{(A_2\bar{y}_2 - A_1\bar{y}_1)}{A_1\ (A_2 - A_1)}\right]^{1/2} - u_1 \qquad .....(4)$$

For a rectangular channel,

$$A = b.y; \quad \bar{y} = y/2$$

Hence equation (4) becomes

$$c = \left[\frac{gy_2}{2}\ \frac{(y_2 + y_1)}{y_1}\right]^{1/2} - u_1 \qquad .....(5)$$

Continuity equation gives

$$c\ (y_2 - y_1) = u_1y_1 - Q_{out} \qquad .....(6)$$

$$Q_{out} = \xi\, Q_{in} \qquad .....(7)$$

$$c(y_2-y_1) = u_1y_1 - \xi u_1y_1 = u_1y_1\ (1 - \xi)$$

From which;

$$c = \frac{(1 - \xi)}{y_2 - y_1}\ u_1y_1 \qquad .....(8)$$

Equating equation (5) and equation (8) we have:

$$\left(\frac{1 - \xi}{y_2 - y_1}\right)u_1y_1 = \left[\frac{gy_2}{2y_1}\ (y_2+y_1)\right]^{1/2} - u_1$$

$$u_1\left(1 + \frac{(1 - \xi)y_1}{y_2 - y_1}\right) = \left[\frac{gy_2}{2y_1}\ (y_2 + y_1)\right]^{1/2}$$

$$\frac{u_1}{\sqrt{g y_1}} = Fr_1 = \frac{y_2 - y_1}{y_2 - \zeta y_1}\left[\frac{y_2}{2y_1}\,(y_2/y_1 + 1)\right]^{1/2} \quad .....(9)$$

(ii) Sudden increase in water discharge, Fig.(2).

Similar to case (i), momentum and continuity equations yield the condition for a positive surge propagating downstream.

$$c = \left[\frac{g y_2}{2y_1}\,(y_2 + y_1)\right]^{1/2} + u_1 \quad .....(10)$$

Continuity equation gives

$$y_1 u_1 + c(y_2 - y_1) = y_2 u_2 \quad .....(11)$$

$$c(y_2 - y_1) = y_2 u_2 - y_1 u_1$$

Assuming the water discharge is increased by a certain ratio ζ, hence

$$c(y_2 - y_1) = \zeta u_1 y_1 \quad .....(12)$$

Equations(10) and (12) yield the condition

$$\frac{\zeta u_1 y_1}{y_2 - y_1} = \left[\frac{g y_2}{2y_1}\,(y_2 + y_1)\right]^{1/2} + u_1$$

$$u_1\left(\frac{\zeta y_1}{y_2 - y_1} - 1\right) = \sqrt{g y_1}\left[\frac{y_2}{2y_1}\,(y_2/y_1 + 1)\right]^{1/2}$$

From which;

$$Fr_1 = \frac{u_1}{\sqrt{g y_1}} = \frac{y_2 - y_1}{\zeta y_1 + y_1 - y_2}\left[\frac{y_2}{2y_1}\,(y_2/y_1 + 1)\right]^{1/2} \quad .....(13)$$

(8) Method of Characteristics

(i) Upstream positive surge

- complete closure

The method of characteristics leads to the following equation for horizontal frictionless channel (1) Fig.(1).

$$u_1 + 2c_1 = u_2 + 2c_2 \quad .....(14)$$

$u_2 = 0$ for a complete closure

$$u_2 + 2\sqrt{g y_1} = 2\sqrt{g y_2} \quad .....(15)$$

Dividing throughout by $g y_1$

$$\frac{u_1}{\sqrt{g y_1}} + 2 = 2\sqrt{\frac{y_2}{y_1}}$$

$$\frac{u_1}{\sqrt{gy_1}} = Fr_1 = 2\left(\sqrt{\frac{y_2}{y_1}} - 1\right) \quad .....(16)$$

- Partial closure

$$Q_{out} = \zeta_1 Q_{in} = \zeta_1 u_1 y_1 = y_2 u_2 \quad .....(17)$$

Equation (14) becomes

$$u_1 + 2\sqrt{gy_1} = \frac{\zeta_1 y_1 u_1}{y_2} + 2\sqrt{gy_2}$$

$$\frac{u_1}{\sqrt{gy_1}}\left(1 - \zeta_1 \frac{y_1}{y_2}\right) = 2\left(\sqrt{\frac{y_2}{y_1}} - 1\right) \quad .....(18)$$

wave celerity

The wave celerity c for an upstream positive surge ranges between

$u_1 - \sqrt{gy_1}$ and $\sqrt{gy_2}$

$$\frac{c}{u_1} = 1 - \frac{1}{Fr_1} \quad .....(19)$$

which is valid for subcritical flow

or $$\frac{c}{u_1} = \frac{\sqrt{gy_2}}{u_1} \cdot \frac{\sqrt{y_1}}{\sqrt{y_1}} = \frac{1}{Fr_1}\sqrt{y_2/y_1} \quad .....(20)$$

(ii) Downstream positive surge, Fig.(2):

The negative characteristics yield

$$u_1 - 2c_1 = u_2 - 2c_2 \quad .....(21)$$

$$u_1 - 2\sqrt{gy_2} = u_2 - 2\sqrt{gy_2}$$

$$\frac{u_1}{gy_1} = 2(1 - \sqrt{y_2/y_1}) + \frac{u_2}{\sqrt{gy_2}} \quad .....(22)$$

$$u_2 y_2 = (1 + \zeta_1)\, u_1 y_1$$

$$u_2 = (1 + \zeta_1)\frac{u_1 y_1}{y_2} \quad .....(23)$$

Equations (22) and (23) yield the condition

$$\frac{u_1}{\sqrt{gy_1}} = 2(1 - \sqrt{y_2/y_1}) + \frac{(1 + \zeta_1)\, u_1}{\sqrt{gy_1}} \cdot \frac{y_1}{y_2}$$

From which;

$$Fr_1 = \frac{u_1}{\sqrt{g y_1}} = \frac{2(\sqrt{y_2/y_1} - 1)}{(1 + \zeta_1)\frac{y_1}{y_2} - 1} \qquad .....(24)$$

wave celerity

Wave celerity ranges between

$$c = u_1 + \sqrt{g y_1}$$

$$\text{i.e. } \frac{c}{u_1} = 1 + \frac{1}{Fr_1} \qquad .....(25)$$

$$\text{and } c = u_2 + \sqrt{g y_2}$$

$$u_2 y_2 = (1 + \zeta_1)\, u_1 y_1$$

$$u_2 = (1 + \zeta_1)\, u_1 y_1 / y_2$$

$$\therefore\ c = (1 + \zeta_1)\frac{u_1 y_1}{y_2} + \sqrt{g y_2}$$

$$\text{i.e. } \frac{c}{u_1} = (1 + \zeta_1)\frac{y_1}{y_2} + \frac{1}{Fr_1}\sqrt{y_2/y_1} \qquad .....(26)$$

UNDULAR SURGE

There are two forms of the positive surge. The first one is a roller, mobile hydraulic jump. In order to create such a roller the free surface of surge above the initial water depth should be more than 20% of the average initial depth (1). When the incident wave height is smaller the second type occurs, the front of the surge remains a smooth and continuous surface as it propagates.

Lamb (1) introduced the following equation for an undulating hydraulic jump, Fig. (4).

$$c = \left(\frac{gL}{2\pi} \tanh \frac{2\pi y_2}{L}\right)^{\frac{1}{2}} \qquad .....(27)$$

in which;

L = wave length;
c = wave celerity; and
y_2 = mean water depth.

On the other hand, these undulations may have a Cnoidal character as in Draison (France) power canal (3), Fig. (4).

The Cnoidal wave celerity can be expressed by:

$$c = \left(\frac{gh}{1 + \frac{a}{h}\left(\frac{1}{k^2} - 1\right)}\right)^{\frac{1}{2}}$$

The parameter k is uniquely related to wave amplitude a, the wave length L and the depth h at the wave trough. Fig.(5) gives k versus Ursell parmeter $\left(\frac{aL^2}{h^3}\right)$ (4).

EXPERIMENTAL WORK

Experiments were conducted in a horizontal rectangular perspex walled flume 5.0 m long 150 mm depth and 75 mm wide. A sluice gate was used to create upstream positive surges. Water depths were measured by using electrical point gauge.

Three different sizes of gravels were used to study the effect of boundary resistance on the characteristics of surge. The mean diameters of gravels used are size (1) of 7 mm, size (2) of 12 mm and size (3) of 15 mm. Gravels of each size were arranged in the channel bed on a line given by Yalin (8).

A step in the channel floor is usually assumed to simulate the effect of bed slope (2). A set of steel plates each of 8 mm height were used at the upstream end of the channel to get supercritical flow conditions.

Photographs were taken for undular surges for measuring wave lengths and wave amplitudes. Mean wave celerities were measured by using stop watches having an accuracy of 1/100 second.

RESULTS AND ANALYSES

Theoretical curves for upstream positive surges under different degrees of gate closure (γ) based on the momentum and continuity equations, are given in Figs. (6,7), Fig.(8) and Fig.(9) for the relationships between y_2/y_1 and Fr_1. y_2/y_1 and c/u_1 and between c/u_1 and Fr_1, respectively. Corresponding graphs for downstream positive surges, flood waves, under different degrees in water discharge (η) are given in Figs. (10, 11), Figs.(12, 13) and Fig. (14) respectively. In Figs.(10,11), the curves are asymptotic to horizontal lines at values of y_2/y_1 equal to$(1+\eta)$.

Figures (15) and (16) exhibit the relationships between y_2/y_1 and Fr_1 and between c/u_1 and Fr_1 respectively, using both the momentum principle and the characteristics method. Corresponding curves for downstream positive surges are given in Fig.(17) and Fig. (18) respectively.

Theoretical analyses show that at the critical flow for an upstream positive surge, $y_2/y_1 = 1.17$ and $c/u_1 = 0.855$ and $y_2/y_1 = 2.0$, $Fr_1 = 0.866$ and $c/u_1^2 = 1.0$. This is for applying the momentum principle, using the characteristics method, $y_2/y_1 = 2.225$, and $c/u_1 = 1.5$ at $Fr_1 = 1.0$, and at $y_2/y_1 = 2.0$, $Fr_1^2 = 0.83$ and c/u_1 values range between 0.21 and 1.71. For surge due to partial closure $\gamma = 0.9$, at critical flow $y_2/y_1 = 1.37$ and $c/u_1 = 0.274$. $y_2/y_1 = 2.0$ occurs at $Fr_1 = 1.575$ and $c/u_1 = 0.0998$. The characteristics method provides $y_2/y_1 = 1.365$ and $c/u_1 = 1.168$ at critical flow, whilst at $y_2/y_1 = 2.0$, $Fr_1 = 1.506$ and $c/u_1 = 0.94$. Considering the downstream positive surge, for $\eta = 0.1$, 1.0 and 2.0 at critical flow, values of y_2/y_1 using the momentum principle, are 1.049, 1.43 and 1.78 respectively. For $\eta = 2.0$, $y_2/y_1 = 2.0$

occurs at Fr_1 = 1.732 and c/u_1 = 2.0. The corresponding values of c/u_1 are 2.037, 2.32 and 2.573 respectively. The characteristics method gives values of 1.049, 1.435 and 1.79 for y_2/y ; and 2.073, 2.592 and 3.014 for c/u_1 respectively. For ξ = 2.0, y_2/y_1 = 2.0 occurs at Fr_1 = 1.657 and c/u_1 = 2.354. However y_2/y_1 ratio depends on the value of ξ , for ξ = 1.0, y_2/y_1 = 2 at $Fr_1 \to \infty$ and for ξ = 1.1, y_2/y_1 = 2 at Fr_1 = 17.32.

For the upstream surge, the momentum principle and the characteristics method provide nearly identical results (y_2/y_1 versus Fr_1) till $Fr_1 < 0.6$. As the Froude number increases, the more divergence between solutions exists. In case of the downstream surge, an increase of discharge of 10% has a negligible difference between solutions using both the momentum principle and the characteristics method. With more increase in ξ . discrepancies exist, which depends also on the value of Fr_1 Fig.(17).

The characteristics equations are the numerical solution of St. Venant equations, the dynamic and continuity equations of motion. These equations are not valid within the surge, since vertical accelerations are not negligible, the pressure distribution is not hydrostatic and certain amount of energy is dissipated in highly turbulent roller. Based on these reasons, the characteristics method could only be used in the analysis of an upstream positive surge for low values of Fr_1 in the region of subcritical flow ($Fr_1 < 0.6$). As the Froude number increases(in the region of supercritical flow) the error in using the characterstics solution will be very pronounced.

The wave front of the downstream surge extends over a longer distance than the corresponding distance of the upstream surge.6). This could decrease the effect of vertical accelerations. the deviation of pressure from being hydrostatic and the dissipation of energy at the wave front. For these reasons error between the two solutions will be less in case of the downstream surge Fig.(17), than the corresponding one in the case of the upstream surge Fig.(15).

As the positive surge travels upstream due to a sudden gate closure, the downstream water level must keep rising if the surge height is to remain constant. This effect is to slow down the motion of the surge (6). Also it is not correct to assume that the water downstream of surge is always stationary with horizontal surface. There will be a slight water slope with a slight downstream movement of the water. Deviations between the actual measurements and theoretical relationships between y_2/y_1 and Fr_1 and between c/u_1 and Fr_1 are given in Fig.(19) and Figs.(20,21) respectively.

Owing to the presence of the channel friction the front of the surge changed from turbulent and broken roller to undular waves. Experimental relations for different degrees of bed roughness, between y_2/y_1 and Fr_1 and between c/u_1 and Fr_1 are given in Fig.(22) and Fig. (23) respectively. It seems that at low values of Fr_1 the bed roughness has, to some extent, a slight effect on the relation between c/u_1 and Fr_1 . However, in very shallow water, viscosity will become significant, but friction forces may be ignored apart from the question of drag on gravels (7).

Measured celerities of the undular surges approach the celerities based on the sinusoidal hypothesis. At low values of Fr_1 the measured wave celerities may approach the celerities of waves having a Cnoidal character. Solitary waves which are a special type of Cnoidal wave give celerities of bigger values than the Cnoidal wave. Table (1) exhibits a sample of calculations for the analysis of wave celerities of undular surge. Because of the effect of friction, the wave front is greatly reduced, on the account of energy losses, in height and celerity.

CONCLUSIONS

Developed charts, based on deduced equations, are presented. These graphs could be used in relating the different parameters of advancing surges.

The momentum principle is the more accurate method in tackling any problem related to advancing surges, than the characteristics method. Undular surge waves, occuring at subcritical flow, can be explained accurately either by characteristics equations or the momentum principle.

In case of downstream positive surges, flood waves, the solution using the characteristics method approaches the corresponding solution using the momentum principle. The difference between solutions using both methods increases with the increase of discharge and the initial Froude number.

The front of surge is undular, even if the bed channel is smooth, at Froude number (Fr_1) $<$ 0.6. When the Froude number increases, the undulation will disappear and the surge will have a sharp and steep front. The surge wave tends to have a sinusoidal character. At values of Fr_1 less than 0.3, it may have a Cnoidal character. Characteristics of the undular surge depend on Froude number, boundary roughness and water discharge.

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NOTATION

The following symbols are used in this paper.

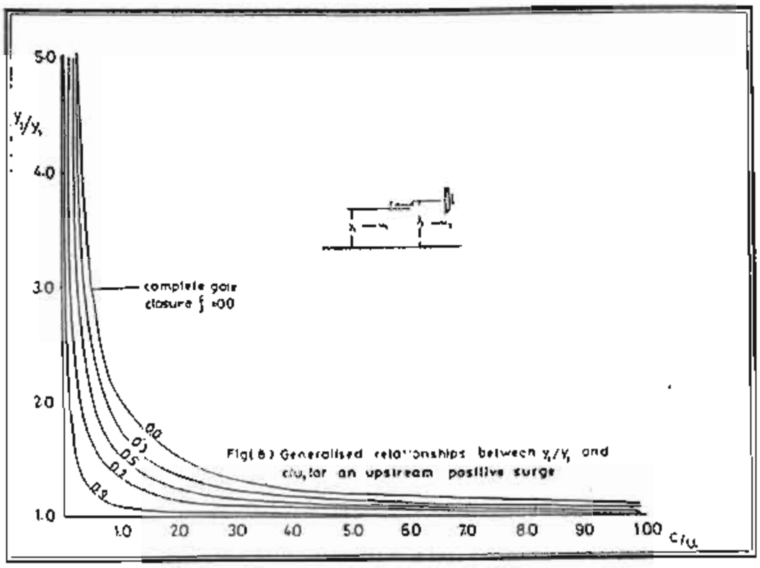
A = water area;
a = wave amplitude;
b = channel width;
c = wave celerity;
Fr = Froude number;
g = acceleration due to gravity;
h = water depth at the wave trough;
k = wave parameter;
L = wave length;
Q = water discharge;
u_r = Ursell parameter;
y = water depth;
$\bar{y}$ = centre of area depth;
Δt = time increment;
ζ = degree of increase in discharge; and
γ = outflow/inflow.

Subscripts

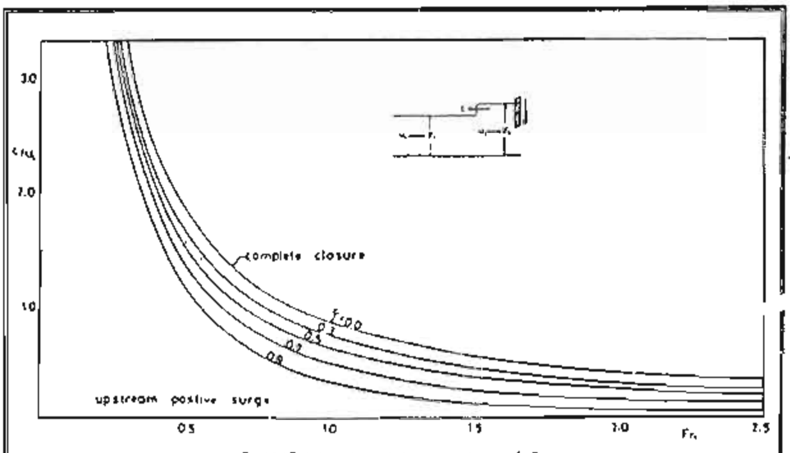
1 = initial; and
2 = final.

Table (1) Sample of undular surge calculations, Cnoidal, Sinusoidal and actual wave celerities.

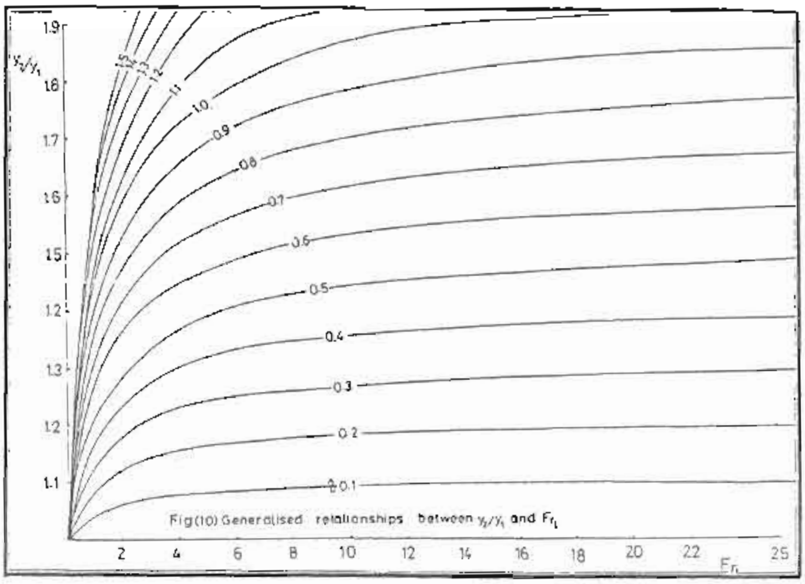
No.		Q cm³/sec	y cm	h cm	Fr	a cm	L cm	Ur	k	Celerity cm/sec. Cnoidal	Sinusoidal	actual	$\frac{a}{h}$	$\frac{a}{y}$
1	gravels size(1) 7.0mm	2647.0	7.9	10.28	0.51	1.71	39.50	2.46	0.43	78.01	76.29	68.63	0.17	0.22
2		2319.6	7.4	9.43	0.49	1.37	37.60	2.31	0.40	75.64	74.06	66.04	0.15	0.19
3		1754.4	6.4	7.71	0.46	1.29	33.85	3.23	0.46	72.07	69.50	61.40	0.17	0.20
4		1132.1	5.2	5.14	0.41	1.29	28.21	7.56	0.65	67.95	61.50	53.85	0.25	0.25
5		804.3	4.3	4.71	0.38	1.03	24.45	5.88	0.60	62.84	57.71	46.67	0.22	0.24
6		549.5	3.7	3.86	0.33	0.86	19.75	5.83	0.59	56.32	52.02	43.75	0.22	0.23
7	gravels size(2) 12.0mm	2586.0	8.9	10.71	0.42	2.14	43.25	3.26	0.47	83.55	79.54	68.63	0.20	0.24
8		2272.7	8.3	9.43	0.41	1.71	39.50	3.18	0.46	78.69	75.61	66.67	0.18	0.21
9		1363.7	6.5	6.86	0.35	1.29	34.80	4.84	0.54	72.82	68.96	57.38	0.19	0.20
10		952.4	5.6	6.00	0.31	1.03	30.10	4.32	0.52	67.50	64.18	53.44	0.17	0.18
11		669.6	4.9	5.14	0.26	0.43	19.25	1.24	0.30	53.50	53.72	50.73	0.08	0.09
12		436.7	4.0	3.86	0.22	0.26	15.10	1.03	0.27	46.01	46.83	42.68	0.07	0.07



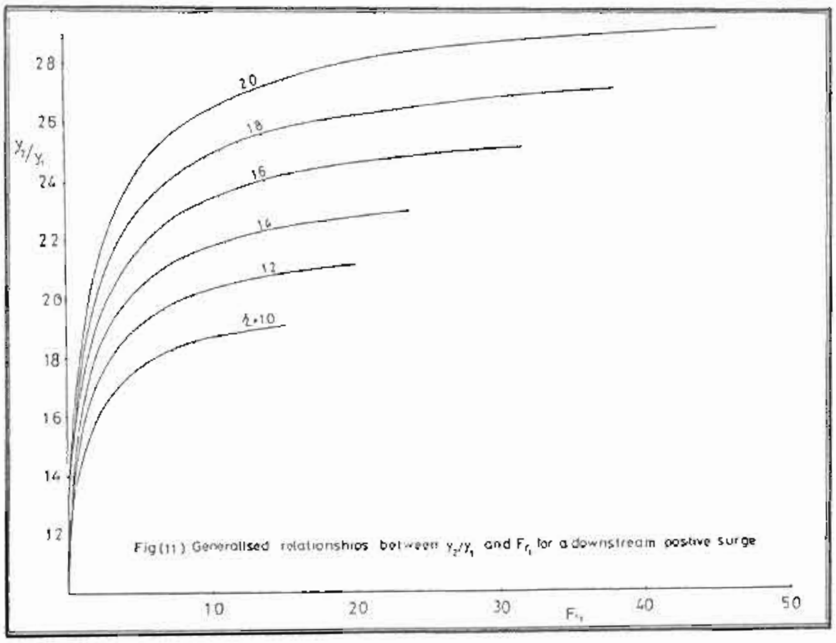
Fig(8) Generalised relationships between y_2/y_1 and c/u_1 for an upstream positive surge



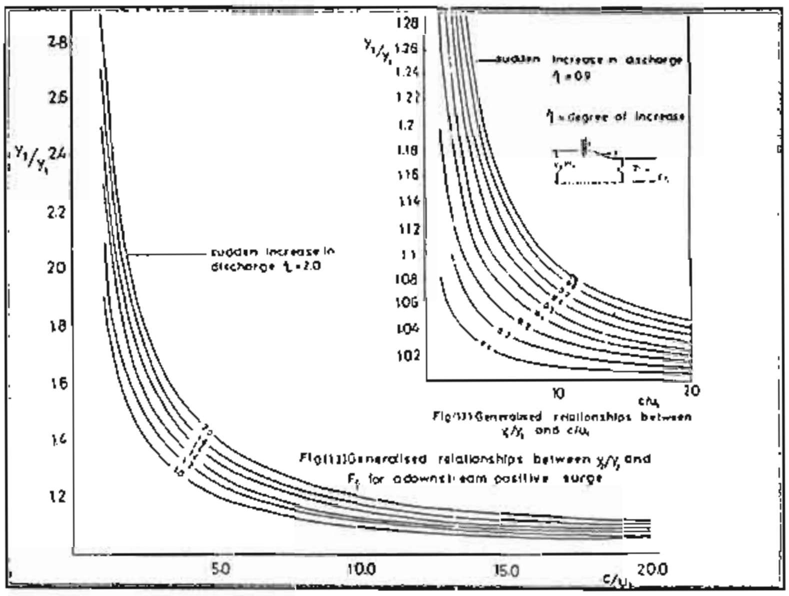
Fig(9) Relationships between c/u_1 and Fr_1



Fig(10) Generalised relationships between y_2/y_1 and Fr_1

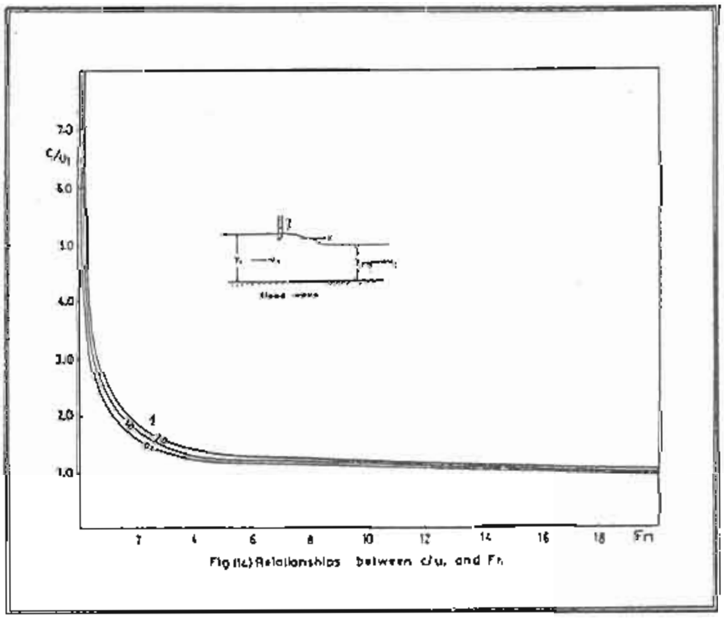


Fig(11) Generalised relationships between y_2/y_1 and Fr_1 for a downstream positive surge



Fig(13) Generalised relationships between Y_2/Y_1 and c/u_1

Fig(12) Generalised relationships between Y_2/Y_1 and F_1 for a downstream positive surge



Fig(14) Relationships between c/u_1 and Fr

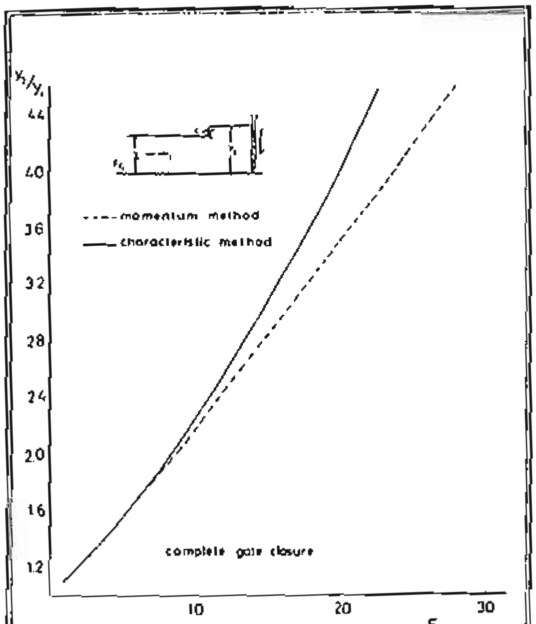
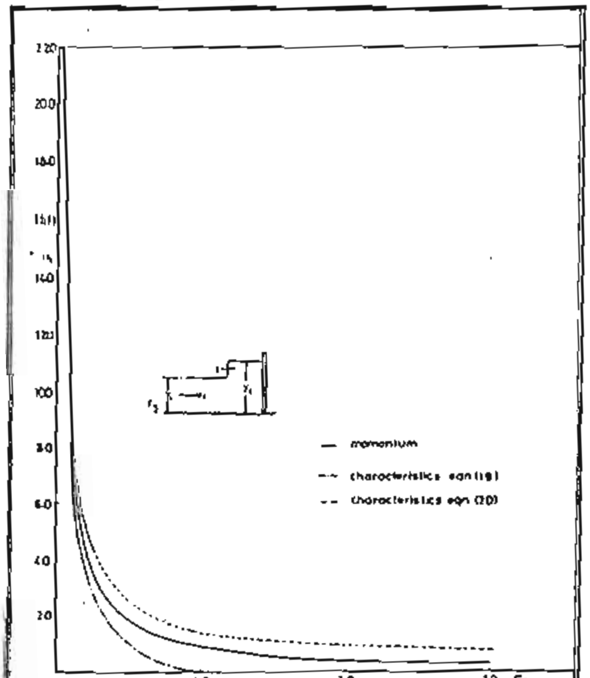


Fig (15) Relationships between y_2/y_1 and F_{r_1}



Fig(16) Analytical relationships between c/u_1 and Fr1

upstream positive surge

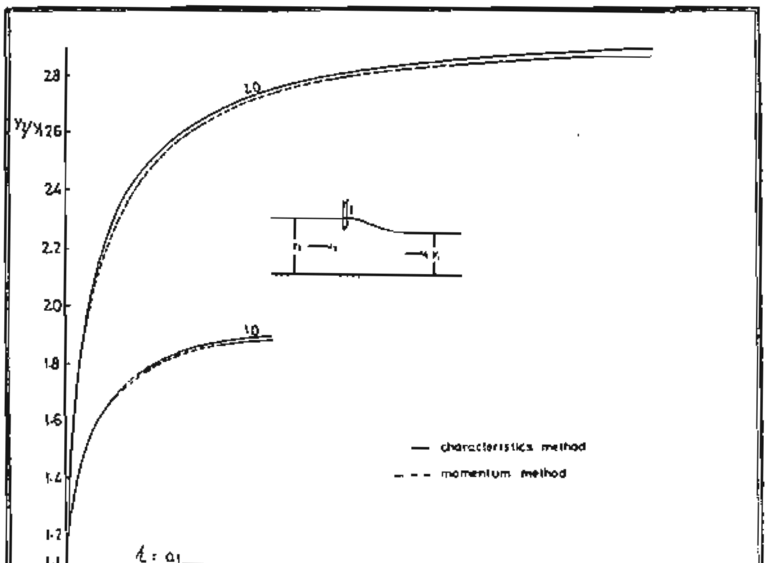
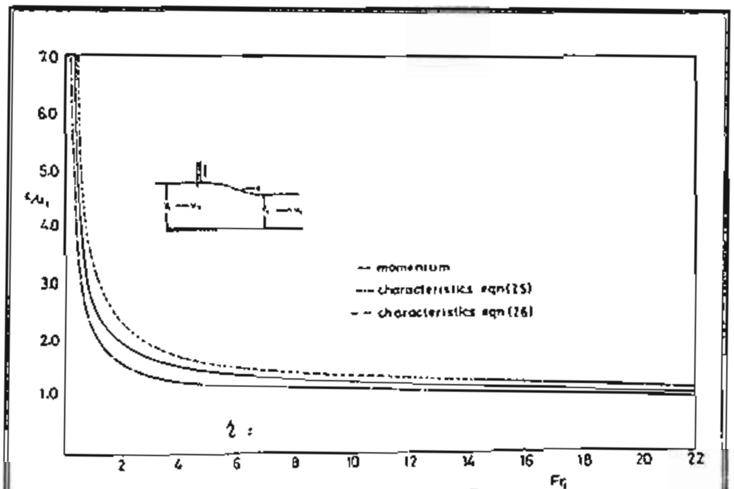
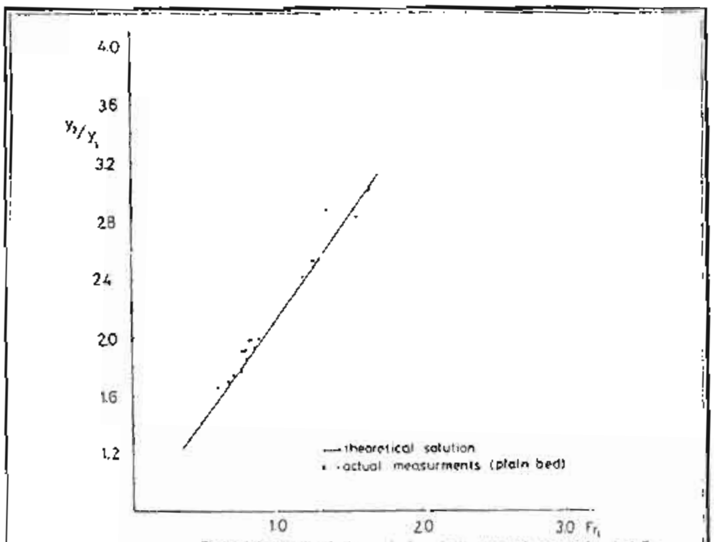


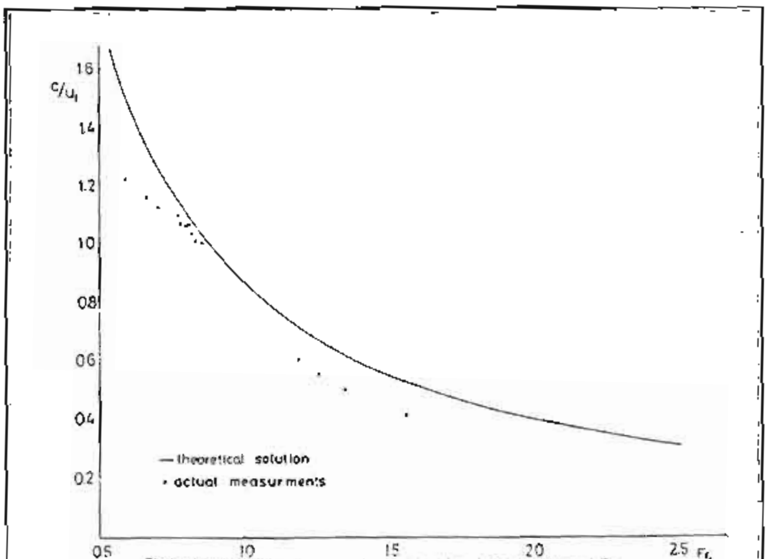
Fig (17) Relationships between y_2/y_1 and Fr_1



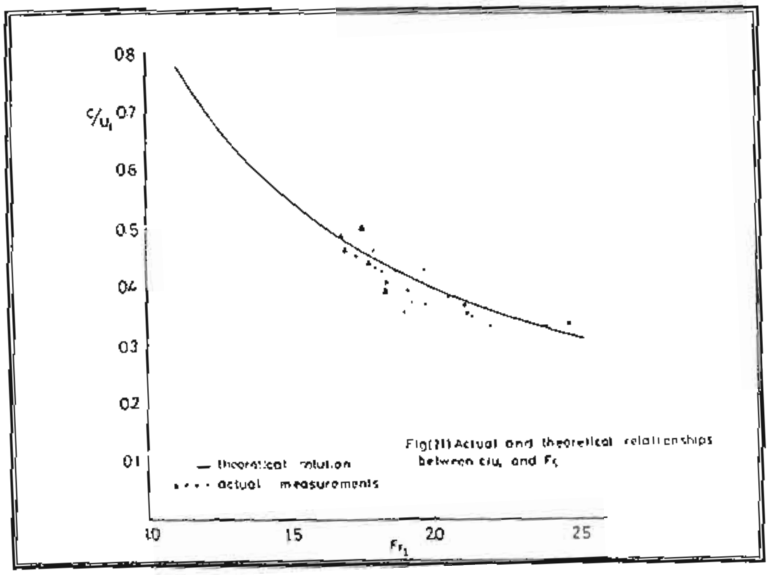
Fig(18) Analytical relationships between c/u_1 and Fr_1 downstream positive surge



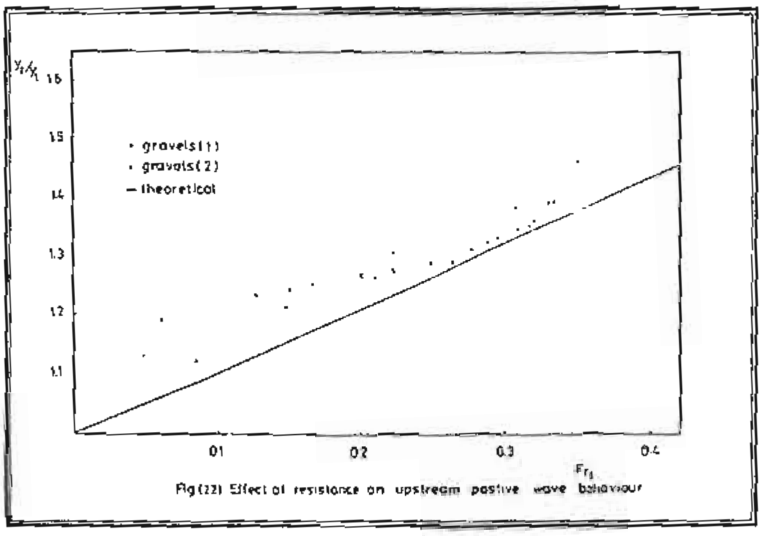
Fig(19) Actual and theoretical relationship between y_2/y_1 and F_{r_1}



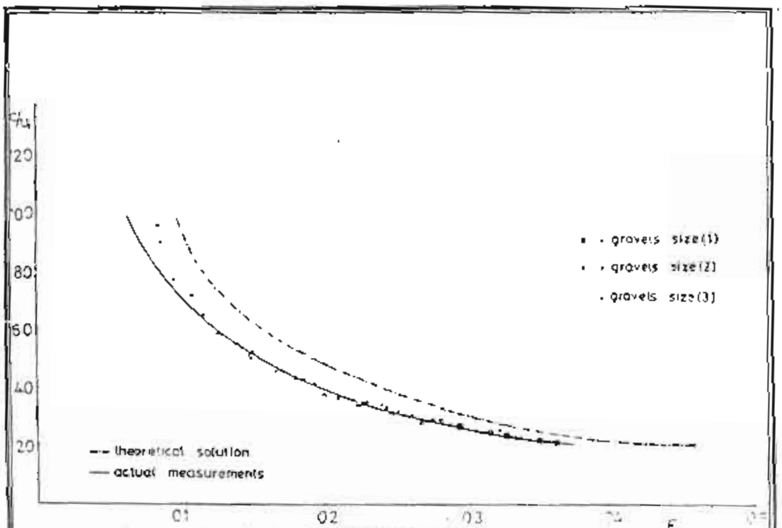
Fig(20) Actual and theoretical relationships between c/u_1 and F_{r_1}



Fig(21) Actual and theoretical relationships between c/u1 and Fr1

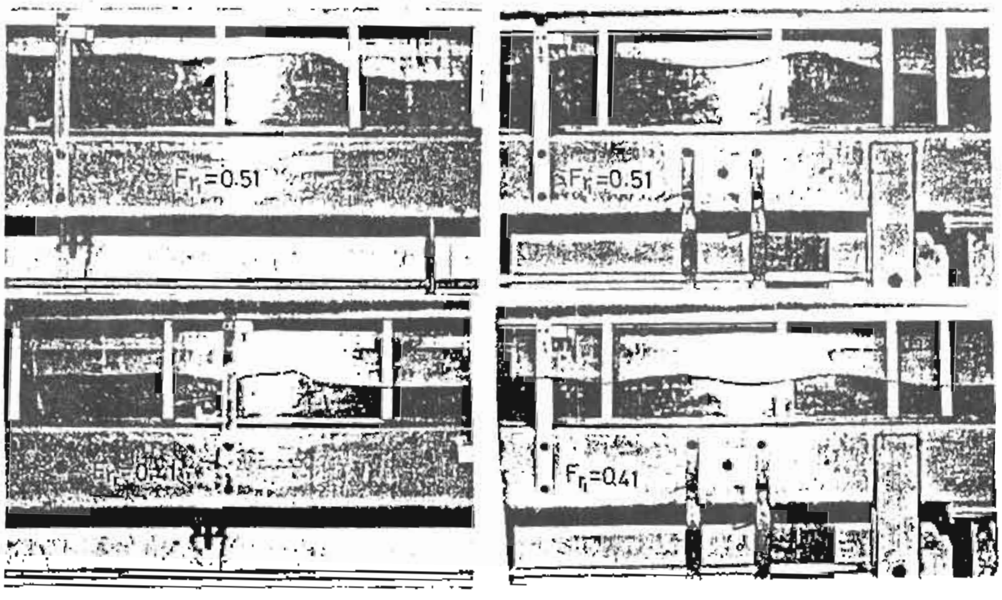


Fig(22) Effect of resistance on upstream postive wave behaviour



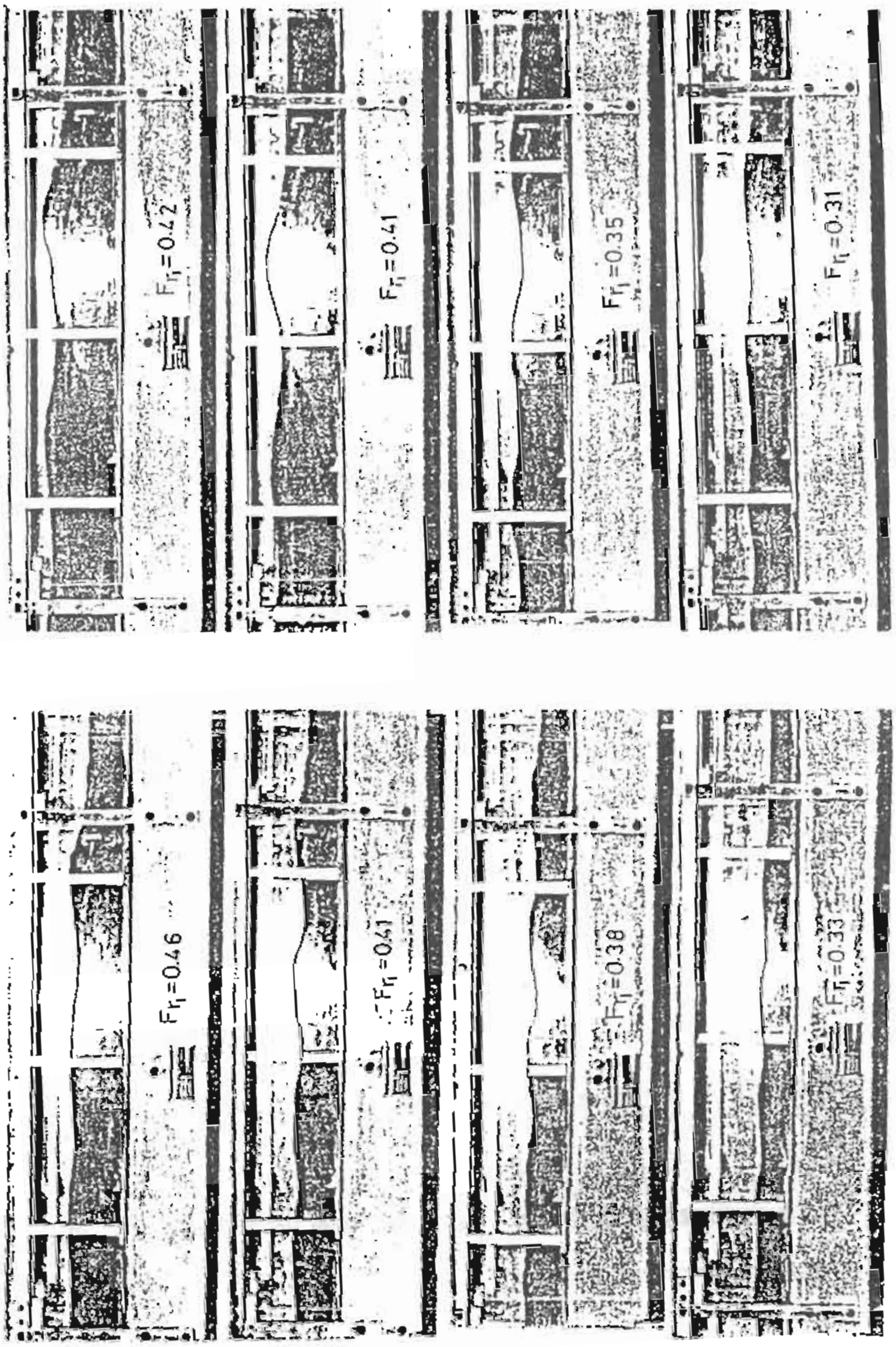
Fig(23) Experimental and analytical relationships between c/u_1 and Fr_1

APPENDIX



Upstream Downstream

Propagation of undular surges



Gravels size (1) 7.0 mm

Gravel size (2) 12.0 mm

Undular surges on rough bed