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DESIGN OF AN OPTIMAL VARIABLE STRUCTURE CONTROLLER FOR ELECTROMECHANICAL SYSTEMS

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ملخص البحث:

يقدم البحث طريقة مقترحه لتصميم جهاز تحكم مثالي على أساس نظرية منظومات الهيكل المتغير • وقد تم تطبيق الطريقة المقترحه على منظومة كهروميكانيكية • ويمثل النموذج الذى يوفر متطلبات التصميم جزءا من المنظومه • وقد تم استخدام طريقة مطورة لاختيار مستويات الانزلاق تعتمد على تقليل معامل الخواص. يحتوى البحث على شرح تفصيلي لطريقة التصميم وكذلك نتائج التطبيق على منظومة كهروميكانيكية لبيان مدى فاعلية طريقه التصميم المقترحه فى التغلب على تأثير الاضطرابات الخارجيه •

Abstract

A control design technique based on the theory of variable structure (VSS) is proposed. The proposed technique is developed for designing an optimal model following adaptive controller, for an electromechanical system. The model that specifies the design objectives is a part of the system. A systematic procedure for the selection of the switching hyperplanes in the design of the optimal variable structure controller is developed, by minimizing a quadratic performance index in the sliding mode operation. The design procedure is described and the results of a simulation study is presented, showing the effectiveness of the designed controller under the effect of disturbances.

1- Introduction

Electromechanical systems, in general, are widely used in industries in various applications. Improvement of the performance of such systems is very important for successful operation of different industrial plants. Generally, in response to an input signal, it is desirable that any electromechanical system should have a fast response, reasonably small overshoots and zero steady state error. Moreover, the system should possess invariance properties under the effect of disturbances, such as, load changes or changes in supply voltage, as well as, less sensitivity to fast or wide range parameter variations. These requirements are the motivation of the development of an effective control scheme, which leads to the optimal performance of an electromechanical system.

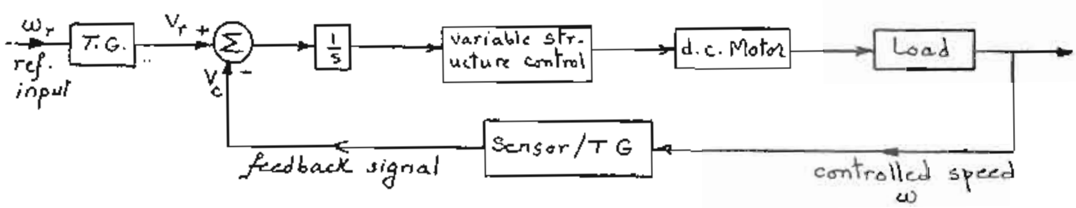
The direct application of linear optimal control theory to the design of the control system encounters a main difficulty in specifying the desired response, in terms of a quadratic performance index. Model following adaptive control scheme is a suitable method to avoid this difficulty. The idea is to use a model, which specifies the design objectives, as a part of the control system. The objective of the controller synthesis is then to minimize the error between the states of the model and those of the controlled plant. There are different design methods of adaptive model following control systems, such as, the methods based on the hyperstability concept [1] and those based on Lyapunov theorems [2]. Adaptive systems can also be designed as a variable structure [3] system, by applying the theory of VSS so that, sliding mode [4] exists in the intersection of the switching hyperplanes.

Once in a control system a sliding mode is realized, the system becomes less sensitive to wide range parameter variations and external disturbances. Research in VSS theory have shown that, adaptive controllers designed on the basis of that theory have many attractive features, than other adaptive mechanisms. The design procedures of these controllers are systematic and their implementation showed great feasibility, leading to improvement of the transient performance of the system, as well as, keeping the dynamic performance less affected by any parameter variations or noise disturbances.

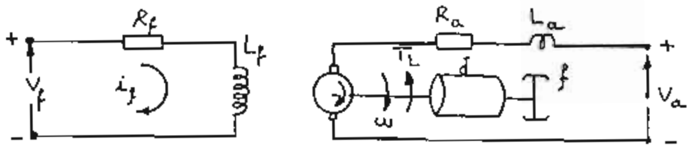
This paper describes a design procedure of an optimal adaptive model following control system, based on the theory of variable structure. For realizing sliding mode in the control system, a new general approach is used [5]. This approach does not need any information about the parameters variations, as well as the level of external disturbances. Only the upper and lower limits of these variations, are to be known. The designed controller is applied for the speed control of a loaded separately excited d.c motor.

2- System Model

The system considered in this paper, as shown in Fig(1), consists of a separately excited d.c motor, with a load connected to the motor's shaft. An integral controller is included for comparison purposes, when used alone.



Fig(1.a) Block diagram of system



Fig(1.b) Schematic diagram of a separately excited d.c motor
-for armature controlled operation $u = v_a$, v_f = constant
-for field controlled operation $u = v_f$, v_a = constant

the equations describing the dynamic behaviour of the motor are as follows:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + K_f i_f w \qquad (1)$$

$$v_f = R_f i_f + L_f \frac{di_f}{dt} \qquad (2)$$

$$\frac{dw}{dt} = \frac{K_f i_f i_a}{J} - \frac{f}{J} w \qquad (3)$$

where K_f is a constant. Linearizing eqns 1-3 about the operating point X_o including the integral controller, we obtain the linearized state equation of the DC motor drive system as

$\dot{x} = Ax + bu + DF$

$x(0) = 0$

where

$x = (x_1 \;\; x_2 \;\; x_3 \;\; x_4)^T$, state vector

$x_1 = \int (\Delta w_{ref} - \Delta w)\, dt$

$x_2 = \Delta w$

$x_3 = \Delta i_a$

$x_4 = \Delta i_f$

u = Δv_a, for armature-controlled DC motors. v_f = constant
Δv_f, for field-controlled DC motors. v_a = constant

F = Δw_{ref}, command signal, equivalent to load variations

b = $(0 \;\; 0 \;\; 1/La \;\; 0)^T$, for armature-controlled DC motors
$(0 \;\; 0 \;\; 0 \;\; 1/Lf)^T$, for field-controlled DC motors

$$D = [1 \quad 0 \quad 0 \quad 0]^T$$

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -f/J & k_f I_{fo}/J & k_f I_{ao}/J \\ 0 & -k_f I_{fo}/L_a & -R_a/L_a & -K_f w_o/L_a \\ 0 & 0 & 0 & -R_f/L_f \end{bmatrix}$$

In fig(1), the variable structure controller block, is replaced by a constant (K_a) for conventional control scheme.

3- Statement of the problem

It is required to design an optimal model following controller, by utilizing design methods of VSS. The designed controller is used for the speed control of a seperately excited DC motor. The objective of the controller is to force all the states of the system, to follow those of the model to insure high speed adaptation. This means that the effect of the optimal VSS controller must result in fast response of the system and zero steady state error relative to the states of the chosen model. Also, under the effect of external distrubances such as, load variations, changes in reference set point or supply voltage variations, the dynamic response must show invariance performance. A comparison study with the conventional controller is made.

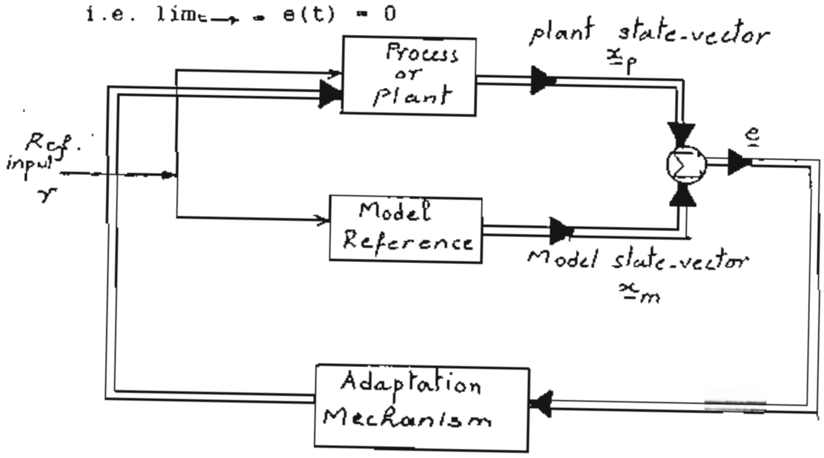
However, in developing the design of our control system four objectives are considered :

(1) simple control laws
(2) Strong stability characteristics
(3) High speed adaptation
(4) Systematic design methods

4- Model-following control system

In model-following systems, fig(2), the plant is controlled in such a way that its dynamic behaviour approximates that of a specified model. The model is part of the system and it specifies the design objectives. The adaptive controller should force the error between the model and the plant to zero as time tends to infinity,

i.e. $\lim_{t \to \infty} e(t) = 0$



fig(2) General scheme of model-following adaptive systems

The plant and model are described by the equations:

$$\dot{x}_p(t) = A_p(t)x_p(t) + B_p(t)u(t) + D(t)F(t) \quad (4)$$

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \quad (5)$$

where x_p, $x_m \in R^m$ and $r \in R^l$. r is the input and u is the control. The error vector is $e = x_m - x_p$. We assume that, the pairs (A_p, B_p) and (A_m, B_m) are stabilisable and furthermore Am is a stable matrix. The plant matrices A_p and B_p may be uncertain and time varying. The upper and lower bounds of the elements of these matrices are assumed to be known to the designer.
It can be easily shown that

$$\dot{e}(t) = A_m e(t) + (A_m - A_p(t))x_p + B_m r(t) - B_p(t)u(t) - D(t)F(t) \quad (6)$$

Perfect model following occurs if, for zero initial conditions, the error vector e is null for any input r belonging to the class of piecewise-continuous vector functions. The necessary conditions (6.7) are:

$$\text{rank } B_p = \text{rank } (B_p, B_m) = \text{rank } (B_p, A_m - A_p) \quad (7)$$

We shall assume throughout this work that, perfect model-following conditions hold. Adaptive model-following design allows the system parameters to vary and to be uncertain, but not the structure of the plant, for eqn.7 to remain satisfied.

5- Variable-structure systems

Variable structure systems are characterized by discontinuous control, which changes structure on reaching a set of switching surfaces. The control has the form

$$u_i = \begin{cases} u_i^+ (x_p, e, r) & S_i(e) > 0 \\ u_i^- (x_p, e, r) & S_i(e) < 0 \end{cases}$$

where u_i is the i^{th} component of u and $S_i(e) = 0$ is the i^{th} component of the m switching hyperplanes in the error state vector.

$$S(e) = Ce = 0 \quad (8)$$

The above system with discontinuous control is termed a variable-structure system (VSS), since the effect of the switching hyperplanes is to alter the feedback structure of the system. The VSS design of the model-following control system (eqn.6) is the same as that of the general system

$$\dot{x} = Ax + Bu + DF$$

5-1. Sliding motion

Variable structure systems possess a very important property, called sliding mode, in which the performance of the system is less sensitive to changes of the plant parameters within wide ranges.

This property was first investigated by Emelyanov [8], who discovered that a sliding mode may exist is a VSS. The transient response can be improved by a sliding-mode. For multivariable linear VSS, suitable choice of switching plane can yield different control laws. However, the choice of a suitable control law is dictated by practical implementation. A detailed description of the properties of sliding motion appear in Reference [8]. A necessary condition for sliding motion to occur on the i^{th} hyperplane is

$$\lim_{\sigma_i \to 0} \dot{S}_i \leq 0 \quad \text{and} \quad \lim_{\sigma_i \to 0} -\dot{S}_i \geq 0$$

or equivalently

$$S_i \dot{S}_i \leq 0$$

in the neighbourhood of $S_i(e)=0$. In the sliding mode, the system satisfies the equations

$$S_i(e)=0 \quad \text{and} \quad \dot{S}_i(e)=0$$

and the system has invariance properties, yielding motion which is independent of certain system parameters and disturbances. Thus variable-structure systems are usefully employed in systems with uncertain and time varying parameters.

[5-2] A new algorithm for realizing a sliding mode in a model following control system

The vectorial control problem could be divided into m-scalar problems. From the state eqn.(6) we can write

$$\dot{e}(t)=A_m e(t)+(A_m-A_p(t))x_p(t)+B_m r(t)$$
$$-b^1_p(t)u_1-b^2_p(t)u_2-\ldots\ldots-b^m_p(t)u_m$$
$$-D(t)F(t) \qquad (9)$$

where

$b^1_p(t), b^2_p(t), \ldots, b^m_p(t)$ are the columns of matrix $B_p(t)$

Hence a sliding mode could be organized simultaneuosly on the m-hyperplanes

$S_1 = C^{1T}e$; C^1 - n vector column: $e \in R^n$

$S_2 = C^{2T}e^1$; C^2 -(n-1) vector column: $e^1 \in R^{n-1}$

.... ;

$S_m = C^{mT}e^{m-1}$; C^m -(n-m+1) vector column: $e^{m-1} \in R^{n-m-1}$

The necessary and sufficient condition, for realizing a sliding mode on the plane S_1 is $S_1 \dot{S}_1 \leq 0$.

To achieve this condition, we shall require that the following conditions are realized :

$$C^{1T}b^1(t) = 0 \quad (10\text{-a})$$

$$C^{1T}b^1(t).\phi_1(S_1) < 0 \text{ when } S_1 > 0 \quad (10\text{-b})$$

$$C^{1T}b^1(t).\phi_1(S_1) > 0 \text{ when } S_1 < 0 \quad (10\text{-c})$$

$$|C^{1T}b^1(t).\phi_1(S_1)| > |C^{1T}A_m e(t) + C^{1T}(A_m - A_p(t))x_p(t) + C^{1T}B_m r(t) - b_p^2(t)u_2 \ldots - C^{1T}b^m(t)u_m - C^{1T}D(t)F(t)| \quad (10\text{-d})$$

Condition (10-a) could be realized if the following conditions are satisfied:

(i) the element c_{n-m-1} in the vector row C^{1T} has a nonzero value. i.e. C^{1T} has the form

$$C^{1T} = (C_1, C_2, \ldots, C_{n-m-1}, 0, 0, \ldots, 0) \quad (11\text{-a})$$

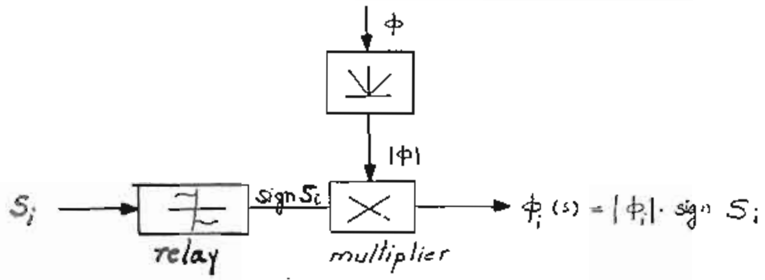
(ii) The vector column has the form

$$b^1(t) = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ b_m \\ b_{m-1} \\ b_1 \end{bmatrix} \quad (11\text{-b})$$

Conditions (10-b), (10-c) and (10-d) could be realized if the function $\phi_1(S_1)$ was chosen as a nonlinear multi-valued function, which have the following properties [9]

(i) multi-valued
(ii) closed at $S_1 = 0$ as a set and limited
(iii) semi-continuous at $S_1 = 0$
(iv) values of $\phi_1, \phi_2 \ldots \phi_m$ are in the neighbourhood of $\phi(S_0, t) \in \Phi(S_0)$

The previous mentioned multi-valued function is shown in fig(3)



fig(3) Generation of multivalued function

After satisfying conditon (10-a), (10-b), (10-c) and (10-d) we get

$$S_1 = C^{1T} e \tag{12-a}$$

$$\begin{aligned} \dot{S_1} &= C^{1T} \dot{e} \\ &= C^{1T} [A_m e(t) + (A_m - A_p(t)) x_p(t) \\ &\quad + B_m r(t) - b^1_p(t) \phi_1(S_1) - b^2_p(t) u_2 \\ &\quad - \ldots\ldots - b^m_p(t) u_m - D(t) F(t))] \end{aligned} \tag{12-b}$$

from (12-a) and (12-b) we have

$$\begin{aligned} S_1 \dot{S_1} = S_1 C^{1T} & [A_m e(t) + (A_m - A_p(t)) x_p(t) + B_m r(t) \\ & - b^2_p(t) u_2 - \ldots\ldots - b^m(t) u_m - D(t) F(t)] \\ & - S_1 C^{1T} b^1_p(t) \phi_1(S_1) \end{aligned} \tag{13}$$

from (13) it is easy to show that the inequality

$$S_1 \dot{S_1} \leq 0$$

will be always satisfied; i.e. there will be a permanent sliding motion on the hyperplane S_1.

Existence of a sliding mode on the plane S_1 as well as achievement of perfect model following conditions means that the motion of system (9) can be described by the following equations:

$$\dot{e}=[A_m-A_p(t)]x_p(t)+B_m r(t)-b^1{}_p(t)\xi_1 -b^2{}_p(t)u_2$$

$$-\ldots\ldots\ldots -b^m{}_p(t)u_m-D(t)F(t) \qquad (14\text{-a})$$

$$c^{1T}e=0 \qquad (14\text{-b})$$

where

ξ_1 is a single valued (scalar) function. or the first component of the nonlinear predetermined vector function $\emptyset$. it represent the additional controlling input and is given by

$$\xi_1=(C^{1T}b^1{}_p)^{-1}C^{1T}[A_m-A_p(t))x_p(t)+B_m r(t)$$

$$-b^2{}_p(t)u_2-\ldots\ldots\ldots-b^m{}_p(t)-D(t)F(t)] \qquad (15)$$

The control is given by

$$u_1 - u_1 - \xi_1 sign(S_1)$$

Similarly. using the same technique, it is possible to establish sliding modes on the hyperplans S_2, S_3, ... S_m. This illustrates the systematic design of the suggested algorithm.

5-3 Procedure for selection of the switching hyperplanes

To find the switching vector C, we define the nonsingular transformation [10]

$$y = Mx$$

such that

$$Mb= \begin{bmatrix} 0 \\ b_2 \end{bmatrix}$$

where b_2 is a nonzero scalar. Then the state equation becomes

$$\dot{y}=MAM^{-1}y + Mbu = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_2 \end{bmatrix} u \qquad (16)$$

then, by minimizing the quadratic performance index in the sliding mode,

$$J =\frac{1}{2}\int_{t_m}^{\infty} y^T Q\, y\, dt$$

$$=\frac{1}{2}\int_{t_m}^{\infty}(y^T Q_{11}y_1+2y_1 Q_{22}y_2+y_2 Q_{22}y_2)dt \qquad (17)$$

where t_s is the initial time of the sliding mode and Q_{11}, Q_{12} and Q_{22} are submatrices of Q, the switching vector C is given by [10]

$$C = M^T [\ C_{11} \quad 1\ ]^T \tag{18}$$

where

$$C_{11} = Q_{22}^{-1} A_{12}^T P + Q_{22}^{-1} Q_{12}^T$$

and p is the solution of the algebraic matrix Riccati equation

$$PA' + (A')^T P - PB'(R')^{-1}(B')^T P + Q' = 0 \tag{19}$$

with

$$A' = A_{11} - A_{12}Q_{22}^{-1}Q_{12}^T$$

$$B' = A_{12}$$

$$R' = Q_{22}$$

$$Q' = Q_{11} - Q_{12}Q_{22}^{-1}Q_{12}^T$$

For a. discussion of how to choose the weighting matrix Q in equation 18, see references [11-12]

6- Illustrative example

6-2 System parameters

Consider the DC motor shown in fig(2), the corresponding system matrices are given by

$$A_p = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -0.355 & 6.127 & 27.268 \\ 0 & -138.88 & -133.33 & -6666.66 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$B_p = [\ 0 \quad 0 \quad 111.11 \quad 0]$ for armature controlled operation $v_t = 240v$

$[\ 0 \quad 0 \quad 0 \quad 0.016]$ for field-controlled operation $v_m = 240v$

The model matrices are given by

$$A_m = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -0.5336 & 5.768 & 24.46 \\ 0 & -120 & -120 & -5497.8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$B_m = \begin{matrix} [0 & 0 & 100 & 0] \\ [0 & 0 & 0 & 0.016] \end{matrix}$$

The armature controlled case is taken as an example.

6-2 Armature-controlled case

consider the folllowing two control schemes :

(i) For conventional control, we use the control law

$$u = -5x_1$$

(ii) For VSS control, we choose

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

thus

$$MAM^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -0.5336 & 24.46 & 5.769 \\ 0 & 0 & -1 & 0 \\ 0 & -120 & -5497.8 & -120 \end{bmatrix}$$

The equivalent weighting matrices are given by

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $R = 1$

Then from eqn.18 we obtain the switching vector

$$C = [\, -1 \quad 1.0717 \quad 1 \quad 3.7577 \,]^T$$

6-3 Simulation results

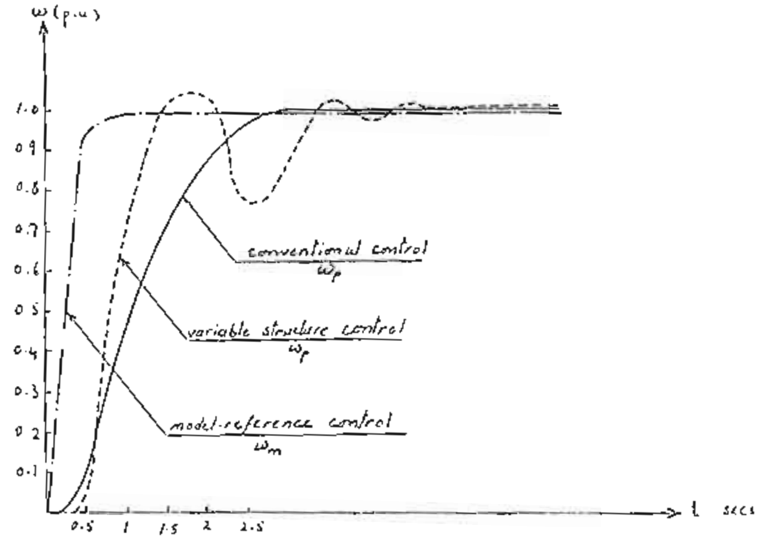
In this section the performance of the proposed adaptive scheme is shown by means of simulation results.

Fig (4a) shows the response of the system for a reference input $r(t) = 1u(t)$ p.u, from zero initial conditions to steady state.

Fig (4b) shows the dynamic response of Δw_p when the system is subject to $\pm$ 0.2 pu (or $\pm$ 36 rad/sec) step change in the reference speed Δw_{ref} which is equivalent to $\mp$ 2NM change in load torque i.e about 10% of nominal value.

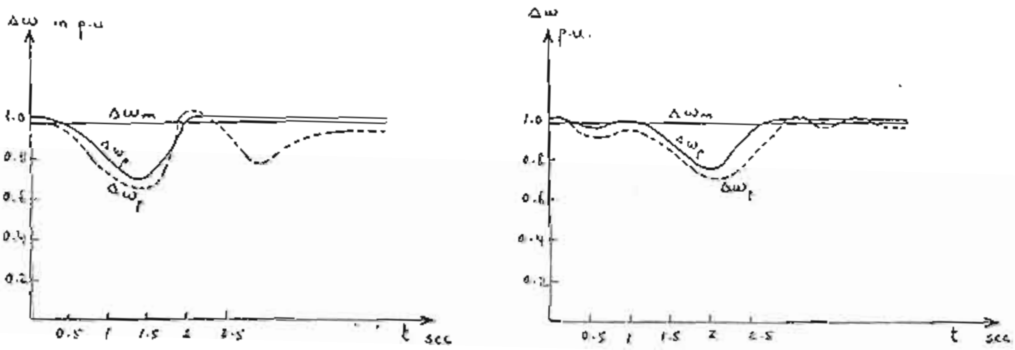
Results using conventional controller are also included for comparison purposes.

From the simulation results shown in fig (4a) it is clear that for VSS control the system shows fast response with no overshoots leading to high speed adaptation. The



— VSC
---- Conventional control $K_A = -5$
—·— Model-reference response

Fig.(4a)



— VSC
--- Conventional control $K_A = -5$
$\Delta\omega_{ref.} = -0.2$ p.u.

— VSC
--- Conventional control $K_A = -5$
$\Delta\omega_{ref.} = +0.2$ p.u.

Fig.(4.b)

dynamics of the system in Fig. (4b) also proves the optimality and adaptive properties of the designed controller. Comparing with the results obtained for the conventional control, the VSS control shows superior system performance.

Conclusion

An adaptive model following control algorithm, based on the sliding mode property of VSS, is developed for an electromechanical system. An optimal variable structure controller is designed on the basis of the proposed algorithm. This controller insures the optimal adaptive control of the system and its invariance to the external disturbances. The effectivness of the controller is verified when used for the speed control of a loaded separately excited d.c. motor. The travel of the system to steady state acroos sliding curve is faster than any other trajectory on the plane. This insures the strong stability characteristics of the chosen system.

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