Mansoura Engineering Journal

Volume 13 | Issue 1 Article 3

5-27-2021

Optimal Location and Size of Fixed Capacitors on Single Radial Feeder.

Kamal Shebl
Electrical Power Engineering Department., Faculty of Engineering., El-Mansoura University., Mansoura., Egypt.

M. El-Said
Electrical Power Engineering Department, Faculty of Engineering, Mansoura University, Mansoura, Egypt.

Recommended Citation
Shebl, Kamal and El-Said, M. (2021) "Optimal Location and Size of Fixed Capacitors on Single Radial Feeder.," *Mansoura Engineering Journal*: Vol. 13 : Iss. 1 , Article 3.
Available at: https://doi.org/10.21608/bfemu.2021.172662

Mansoura Engineering Journal (MEJ) Vol. 13, No. 1, June 1988

OPTIMAL LOCATION AND SIZE OF FIXED CAPACITORS ON SINGLE RADIAL FEEDER

الموضع والتقسيم الأمثل للمكثفات الثابتة على موزع شعاعي مفرد

Kamal M. Shebl M.E. Elsaid

Faculty of Engineering
Mansoura University
Egypt

الخلاصة ـ يقدم هذا البحث طريقة تكرارية جديدة لتحديد الموضع والقيمة المثلى لمكثفات القوى الثابتة عند تركيبها على المغذيات الاشعاعية في نظم التوزيع في حالة التحميل الاستاتيكي على نقط محددة على طول الموزع . يتعرض البحث للموديل الرياضي لتمثيل كل من الوفر في التكلفة الناتج عن خفض الطاقة والقدرة المفقودة في الموزع وذلك بأخذ بعض العوامل في الاعتبار مثل معدل النمو في الأحمال ، معدل النمو في معامل الحمل ، ومعدل الزيادة في تكلفة وحدة الطاقة ، كما يقدم البحث استنباطا للعلاقة بين كل من الوفر في تكلفة القدرة المتوفرة وتكلفة المكثفات والقيد على انخفاض الجهد عند الأحمال والقيد على الارتفاع في الجهد عند انخفاض الأحمال مع تواجد المكثفات وبين التيار السعوي المار في الموزع . ويوضح البحث تأثير قيمة وحدة المكثفات المستخدمة على الحل الأمثل وكذلك تأثير عوامل النمو في الأحمال ومعامل الأحمال وتكلفة وحدة الطاقة على الحل الأمثل . ويمتاز هذه الطريقة عن معظم الطرق المستخدمة في هذا المجال ببساطتها وسهولة البرنامج المستخدم وسهولة التطبيق وأنها تفترض من الأول استخدام مكثفات قياسية وذلك عكس الطرق الأخرى والتي تعطي نتائج غير قياسية للمكثفات ثم يتم تقريبها بعد ذلك الى القيم القياسية وكذلك استخدام قيد على الزيادة في الجهد عند الأحمال المنخفضة في تواجد المكثفات .

Abstract: This paper presents an optimal method for locating and sizing of fixed shunt capacitor banks in case of static load on single radial distribution feeder. Mathematical models to represent cost saving due to power and energy loss reduction are presented considering growth in load, growth in load factor and increase in cost of energy. The cost saving due to release in system capacity, capacitor cost, voltage drop and voltage rise constraints as a function of capacitive current flows in feeder segments have been formulated. The cost functions have been performed for optimizing the choice of fixed shunt capacitors. This proposed method has a special advantage that the optimal location of capacitors is limited by the lean period voltage rise constraint and thus avoiding over-voltage problems during the off-peak hours. The effect of unit-capacitor value on optimal solution is introduced.

1. INTRODUCTION

The continuous increase in consumption of electric energy tends to increase power and energy losses in primary distribution feeders which reduce the available capacity of feeder and increase voltage drop along the feeder which results an increase in electric energy cost for the consumer. There are several methods to reduce these losses and improving the voltage profile at the consumer. One of the simplest methods is the use of shunt capacitors on the primary distribution feeders to reduce the feeder currents, reduce power and energy losses, improve voltage profile along the feeder and cause an appreciable release in feeder capacity that can be used to feed extended additional loads on the feeder. The principle problem in installing power capacitors on primary distribution feeders is determining the optimal location and size of these capacitors on the feeder to gain maximum cost savings and, at the same time, the system constraint are achieved.

The optimal application of shunt capacitors on distribution circuits has always been an important subject for distribution engineers [1]. Duran [2] presented an optimization technique known as the dynamic programming approach. He determines the optimum number, location and size of capacitors to be connected on the radial feeder with discrete lumped loads so as to maximize overall savings. However, his method considers the fixed banks only, put no constraints on the feeder voltage and disregards the released capacity due to capacitor installation.

Bae [3] presented an analytical method to determine the best location of capacitor banks, optimum reactive compensation level and maximum yearly loss reduction. However, this method assumed the feeder and loads to be uniform, did not consider the growth in load, put no constraint on feeder voltage and the cost of fixed capacitor banks is considered only.

Grainger and Lee [4] defined the reactive current distribution method and maximizing the benefit from power and energy lose reduction. This method gives generalized procedures for shunt capacitor placement without any constraint on load or feeder. Also, the fixed term of capacitor cost is not considered. Later on, they defined a new voltage dependent methodology for shunt-capacitor compensation of primary feeders [5] . They neglected the feeder energy loss and the fixed cost of capacitor banks and no voltage constraint was considered.

Fawzi and Elsobki [6] introduced a simple analytical technique for the selection of capacitor location and size depending on the location of the additional loads that can be served with the existing capacitors. The minimization of objective cost function is subjected to voltage drop constraint while the voltage rise constraint was ignored. They assumed that the feeder has a constant cross section and the load in uniformally distributed along the feeder length.

This paper presents a method for optimally choosing fixed shunt capacitors in case of static load on a single radial distribution feeder. Mathematical models to represent cost saving due to power and energy loss reduction considering the growth in load, growth in load factor and the increase in energy cost. The cost savings due to release in system capacities, capacitor cost, voltage-drop and voltage-rise constraints as a function of capacitive current flows in the feeder segments have been formulated. The effect of unit- capacitor value on the optimal solution is introduced.

2. PROBLEM FORMULATION

The objective in solving the optimal combination of capacitor bank size and location is considered here as maximizing the present worth of revenue requirements savings due to the application of shunt capacitor banks to the radial feeder. The optimal process is reached through maximizing the objective function while achieving the system constraints.

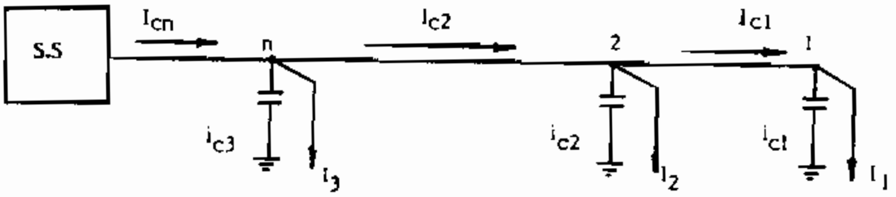

Fig.1 Single line radial feeder representation

2.1. Cost of System Components

The components of objective cost function to be maximized are power and energy loss reduction in a feeder, release in feeder capacity, release in system components against construction cost of involved capacitor banks. A single line representation of a single distribution feeder is shown in Fig.1 where i_n is the load current at point n, I_{n-1} is the segment current from node n to node n-1, and i_{cn} is the capacitive current due to capacitor installation at point n.

2.1.1 Cost saving due to power loss reduction

The power loss reduction due to installation of power capacitors (L_p) is determined as the difference between the power loss with and without capacitors, i.e.;

$$L_p = 3 \sum_{i=1}^{n} R_i \, (2 \, I_i \, . \, i_{ci} - i^2_{ci}) \qquad (1)$$

Defining $L_f = I_i / I_{1i}$ (2)

where L_f is the inductive load factor, and I_{1i} is the maximum inductive loed current in segment i. Then the net cost saving due to power loss reduction (C_p) is given as;

$$C_p = K_p \, . \, L_p$$

$$= 3 \, K_p \sum_{i=1}^{n} R_i \, (2 \, i_{ci} \, . \, L_f \, . \, I_{1i} - i^2_{ci}) \, . \, 10^{-3} \quad \text{L.E./year} \qquad (3)$$

where K_p is the cost of power loss reduction in L.E./Kw/Year.
If N is the capacitor bank life period in years, and r is the rate of interest, the sum of the present worth will be given as;

$$C_p = 3 \, K_p \sum_{i=1}^{n} R_i \, (2 i_{ci} \, . \, L_f \, . \, I_{1i} - i^2_{ci}) \, . 10^{-3} \sum_{k=1}^{N} (1/(1+r)^k) \qquad (4)$$

2.1.2 Cost saving due to energy loss reduction

The present worth of the income through cost saving (C_E) in the life period of capacitors for a working 8760 hours/year is obtained as;

$$C_E = 26.28 \, K_E \sum_{i=1}^{n} R_i \, (2 \, i_{ci} \, . \, L_f \, . \, I_{1i} - i^2_{ci}) \sum_{k=1}^{N} (1 \, / \, (1+r)^k) \qquad (5)$$

where K_E is the cost of energy in L.E./Kwh.

The growth in feeder load may be due to the incremental additions to the existing loads or due to the addition of new loads to the feeder. The effect of load growth on cost saving can be introduced by multiplying both equ. (4) & (5) by a factor $(1+g)^{2j}$, $j=1,2,\ldots,M$
where M is the plan period up to which the feeder can take load growth, subject to a minimum period limited by the capacitor life period [7].

The system experiences a contiuous growth in load factor with time due to various reasons such as increase in load diversity, increase in the energy consumption per Kw connected load. The new load factor L_{fk} at any year k is given in [7] as;

$$L_{fk} = L_{fu} - Y_k \, (L_{fu} - L_{fp}) \qquad (6)$$

where $Y_k = (0.5)^{k/36}$
L_{fu} = ultimate load factor
L_{fp} = present load factor

The cost of unit energy undergoes a contious increase with time. This means that k_E is a time-dependent varible as;

$$K_E = K_{Ek} \qquad k = 1,2,\ldots\ldots,N$$

Then, the present worth of cost savings due to power and energy loss, C_{po} and C_{Eo} are given, considering the effect of growth factors, respectively as;

$$C_{po} = 0.003\,K_p \sum_{i=1}^{n} \sum_{k=1}^{N} \sum_{j=1}^{M} R_i\ (2.i_{ci}\ .\ L_{fk}\ .\ I_{1i}(1+g)^{2j} - i^2_{ci})(1/[1+r]^k) \quad (7)$$

and;

$$C_{Eo} = 26.28 \sum_{i=1}^{n} \sum_{k=1}^{N} \sum_{j=1}^{M} R_i . K_{Ek}(2i_{ci} . L_{fk} . I_{1i}(1+g)^{2j} - i^2_{ci})(1/[1+r]^k) \quad (8)$$

2.1.3 System capacity release in feeder sections

By installing capacitors, the net current flow in feeder sections is reduced as the capacitor current, flowing towards supply, compensates the inductive-load currents. This reduction in current flow in feeder segments, transformer, transmission lines and generators results in an appreciable quantim of release in their capacities.

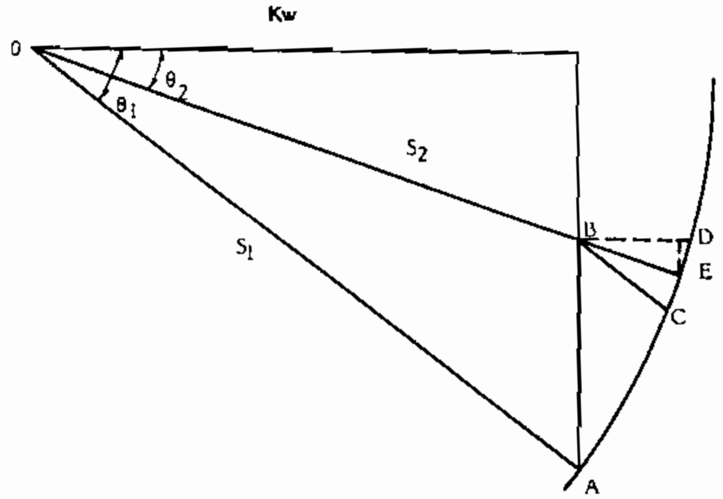

Fig.2 Power phase diagram

Fig.2 shows the phasor representation of active and reactive power flows in a feeder segments, where S_1 & S_2 are the segment Kva before and after adding a capacitor bank respectively. θ_1 & θ_2 are segment-load power factor angles before and after adding a capacitor bank respectively. The releasad Kva depends on the power factor of the extended load. Assuming that the extended load has a power factor angle as θ_2, then the releasad Kva, S_r is given as described in [8];

$$S_r = S_1 - S_2 = OE - OB \quad (9)$$

By the use of the correlations from Fig.2

$$C_r = \sum_{i=1}^{n} C_{ri} . S_i \; [1-(1+(V.i_{ci})/S_i)^2-(2V \, . \, i_{ci} \, . \, \sin \theta_i \, / \, S_i \,)^{1/2} \,] \qquad (10)$$

where C_{ri} is the marginal cost required to supply every additional Kva demand for the i_{th} feeder segment, and V is the system line voltage.

2.1.4 Capacity release in system components

The cost saving due to capacity release in substations, transformers, transmission lines and generators C_{ro} can be written by the use of equation (10), as;

$$C_{ro} = C_o \, . \, S_m \; [1- \; (1+(V.i_{ct}/S_m)^2 -2V.i_{ct}.\sin \theta_i \, / \, S_m \,)^{1/2} \,] \qquad (11)$$

where C_o is the marginal cost required to invest in the system components capacitors to meet every additional Kva demand on the system, and S_m is the maximum Kva demand in feeder.

2.1.5 Cost of capacitors

The capital cost of capacitor bank C_c bears a linear relationships to the capacitor current. This involves fixed and variable cost for the capacitor as; [8]that can be written

$$C_c = \sum_{i=1}^{n} \; e \, . \, i_{ci} + a \, . \, d$$

where;

i_{ci} = capacitor current at node i,
e = variable cost component of capacitor bank,
d = fixed cost component of capacitor bank,
a = decision factor for capacitor.

If r is the annual discount rate within the life period of the capacitor, the general cost model for capacitor bank is written as;

$$C_c = \sum_{i=1}^{n} \sum_{k=1}^{N} \; (\, e \, . \, i_{ci} + a \, . \, d \,) \, / \, (\, 1 + r \,)^k \qquad (12)$$

2.2 Objective Function

The objective function to be maximized here is a combination of cost components of power and energy loss reduction in the feeder, release in feeder capacity, release in system components against capacitor cost. Then, the objective function is to maximize the following cost function;

$$F = \sum_{i=1}^{n} [C_{po} \; (i_{ci}) \; + \; C_{Eo} \; (i_{ci}) \; + \; C_r \; (i_{ci}) \; + \; C_{rc} \; - \; C_c \; (i_{ci})] \qquad (13)$$

2.3 System Constraints

The distribution system must regulate the voltage at which power is delivered to users within certain prescribed limits as load demand varies. The voltage at any capacitor along the feeder depends not only on the capacitor location, but also on the status of the other capacitors on the same feeder and on the substation voltage. The voltage-rise and voltage-drop constraints are the two important voltage-dependent constraints to be considered.

2.3.1 Voltage-rise constraint

As the power factor during off-peak hours is normally high, heavy capacitor compensation may lead to over-voltage problems during off-peak hours. Therefore, it is essential to consider the voltage rise constraint in the optimization model during lean hours. The voltage rise is given as;

$$G = I'_o \,.\, X \,.\, \sin\alpha - I'_o \,.\, R \,.\, \cos\alpha$$

where I'_o is the new segment current after adding capacitor, and $\cos\alpha$ is the new power factor of feeder segment.

$$G = \sum_{i=1}^{n} (i_{ci} - I_{oi} . \sin\theta_o) X_i - I'_{oi} \,.\, R_i [1 - ((i_{ci} - I_{oi} . \sin\theta_o)/I'_{oi})^2]^{1/2} \tag{14}$$

where, I_{oi} and θ_o is the i_{th} segment current and power factor before adding capacitor, respectively.

The voltage is to be kept at zero level or less. Then, the above constraint can be stated as:

$$G (i_{ci}) \leqslant 0 \tag{15}$$

2.3.2 Voltage-drop constraint

The voltage drop (V_o) along the distribution feeder is the main constraint in system planning. It should not exceed a specified value (E) which is suitably save for equipment operation, i.e.;

$$V_o \leqslant E \tag{16}$$

where;

$$V_o = \sum_{i=1}^{n} I_i \,.\, R_i \cdot \cos\theta_i + (I_i \,.\, \sin\theta_i - i_{ci}) X_i \tag{17}$$

3. SOLUTION TECHNIQUE

The present solution of the problem to choose fixed capacitors along a feeder depends on determining capacitive current flows in the different feeder-segments to maximize the net cost saving by varying the flow of the capacitive current in descrete steps, decided by the minimum size of the capacitor bank available subject to voltage constraint. Thus, it is a descrete variational problem with a separable objective function and constraint equation. The proposed technique to solve this problem has certain well established edvantages over the dynamic programming approach in terms of computational requirements, in addition to the special feature that at any stage in the solution process, there is always a feasible solution that can be considered as a suboptimal.

Fig.3 shows the flow chart of the proposed solution technique. This is started by setting a minimum capacitor-size at all feeder nodes. This results a current flow i_{ci} in all segments that would satify the constraints (15) & (16). As a first step, add one capacitor unit to the i_{th} node, keeping all other states values unchanged. This would result change in segment current by i_{ci}. Checks are then performed to see whether this change satifies the voltage constraints and if the objective function value becomes higher than its old value at the i_{th} stage. If the answer is affirmative, the state value at the i_{th} stage is set at the new value. If the answer is negative, one capacitor unit must be removed from the i_{th} node. In this way the proposed technique is applied successively up to the$(n+1)_{th}$ stage to complete one iteration. The resulting trajectory at the end of one iteration forms the nominal trajectory for the next iteration. This process is to be continued as long as it ensures an increase in the value of the objective function in each iteration. The iterative proposed process is to be terminated when a satisfactory convergence to an optimum value has been achieved.

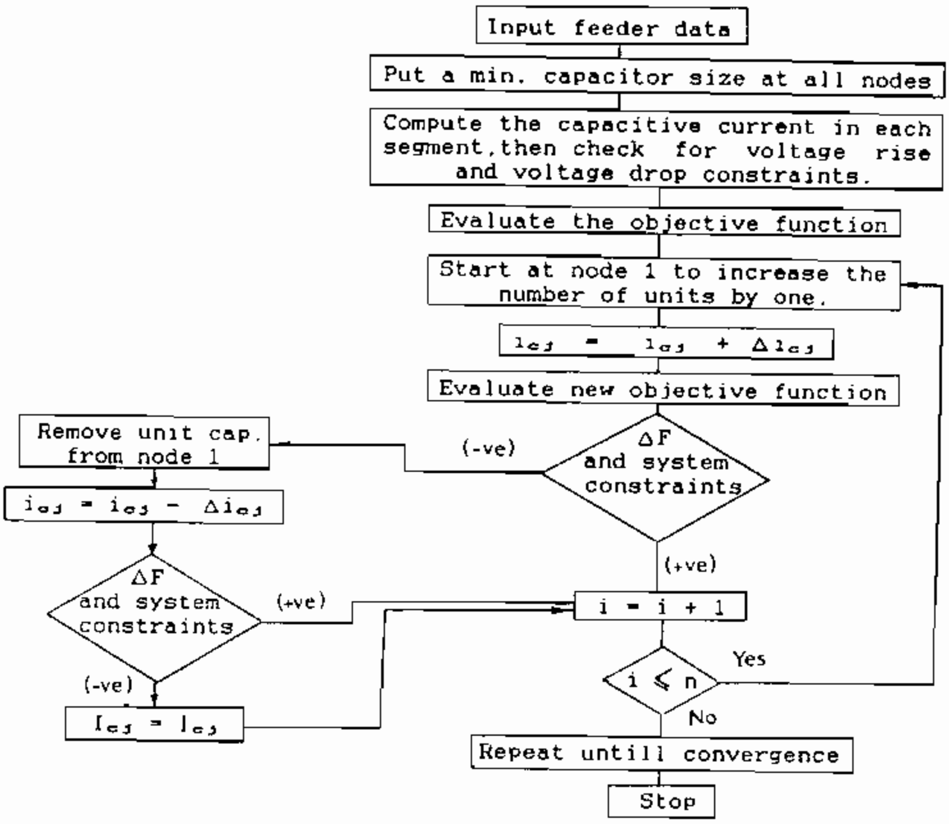

Fig.3 Flow chart of the proposed technique

4. APPLICATIONS

4.1 Test System

The proposed technique has been tested on an 11-Kv, ten-section feeder with five wire-sizes. Table 1 illustratee the main data of the test system where the source substation is located at node 11.

Table 1. Main data of the test system

Feeder segments	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11
Wire size (mm^2)	10	10	10	30	30	50	50	75	75	75
Segment length (km)	2.5	1.6	2.5	1.0	2.3	1.9	0.8	3.0	1.8	2.0
Resistance (Ohm/km)	2.96	2.96	2.96	1.09	1.09	0.66	0.66	0.42	0.42	0.34
Inductance (Ohm/km)	0.11	0.11	0.11	0.09	0.09	0.083	0.083	0.083	0.083	0.081
Load (Kw)	90	80	95	80	70	75	65	80	60	100

Other data of the test system are listed below;

Variable cost of capacitor banks	30 L.E./Kvar
Fixed cost per location of capacitor installation	150 L.E.
Annual charge on capacitor	0.05
Cost of substation capacity	20 L.E./Kva
Cost of power	180 L.E./Kw/Yeer
Cost of energy	0.07 L.E./Kwh
Size of unit capacitor (otherwise stated)	50 Kvar
Rate of growth of energy cost	1 %
Discount rate	5 %
Power factor	0.74
Present load factor	0.35
Ultimate load factor	0.45
Rate of load growth	10 %
Load growth period	7 Yeers
Life period of the feeder	15 Years

4.2 Results and Discussions

The proposed technique was applied to the test system with a unit-capacitor banks of 50-Kvar each as a base case. Table 2 illustrates the results such as the released Kva capacity, the capacitor Kva and the saving percentage. The released capacity obtainad was assumed to be available at the substation node.

Table 2. Optimal results obtained by the proposed technique.

Node No.	Bank size (Kvar)	Released capacity (%)	Cost saving (%)
2	1 x 50	0.04	9.5
4	1 x 100		
6	1 x 50		
8	1 x 50		

To highlight the effect of unit capacitor size on the optimal solution, different unit-capacitor sizes are used where the results are tabulated in Table 3.

Table 3. Capacitor Kvar at each node for different values of unit-capacitor value

Unit capacitor size (Kvar)	Node capacitor Kvar										Total Ckvar	No.of cap.	Saving (%)
	1	2	3	4	5	6	7	8	9	10			
10	0	20	20	30	40	20	20	20	20	0	190	19	3.9
20	0	40	0	20	60	20	40	20	20	0	220	11	6.2
30	0	30	0	90	0	30	60	0	30	0	240	8	8.4
50	0	50	0	100	0	50	0	50	0	0	250	5	9.5
75	0	75	0	0	150	0	0	75	0	0	300	4	11.6

Fig.4 shows the variation of both the capacitor compensation level and the percantage of annual cost saving for differant values of unit-capacitor size. In casa of using small unit-capacitor value, the percentage annual saving is low due to the higher cost of large number of units and higher installation cost . and vice verse. Fig.5 shows the discrete nature of compensation level of capacitors according to installing different values of unit capacitors. It can be concluded that the choice of unit-capacitor size is a trade-off between cost saving, system opereting reliability and the smoothing of voltage profila along the feeder.

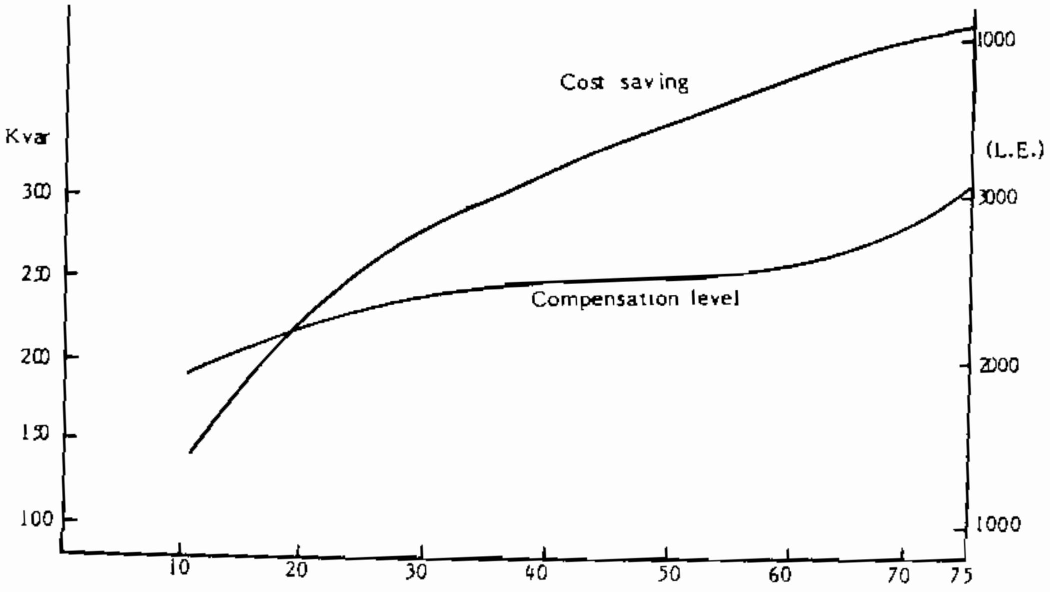

Fig.4 Variation of Ckvar and cost saving with unit-bank size

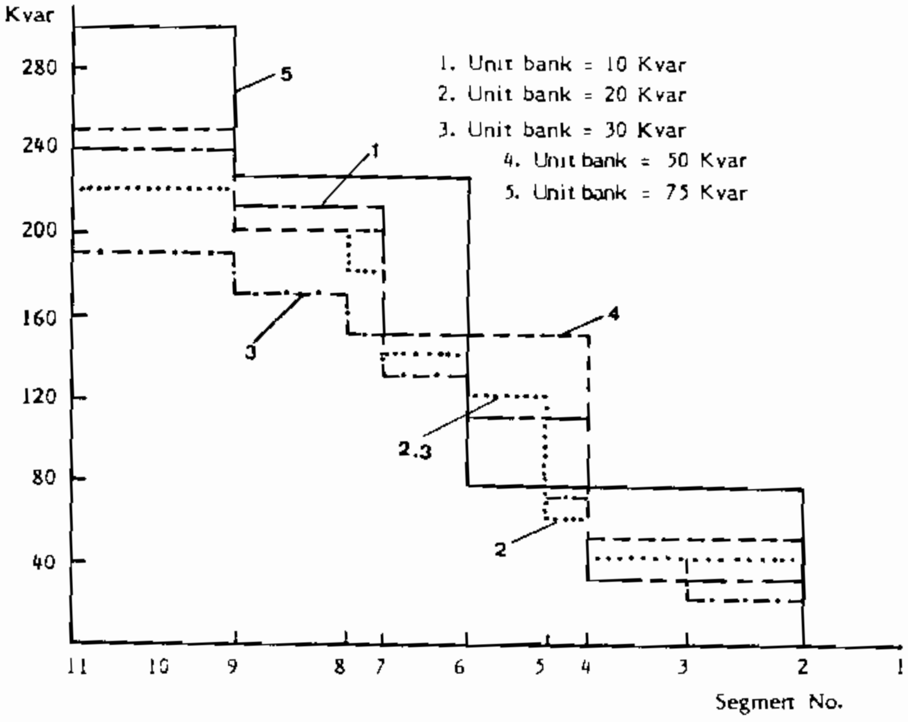

Fig.5 Discrete variation of Ckvar with unit-bank size

Tables 4, 5 and 6 illustrate the effect of change in annual rate of load growth, effect of change in annual rate of energy cost and the effect of cost of system released capacity respectively on the optimal solution.

Table 4. Effect of load growth on optimal solution

Rate of load growth (%)	Nodes Ckvar 1	2	3	4	5	6	7	8	9	10	Total Ckvar	No. of cap.	Saving (%)
0	0	50	0	0	0	0	0	50	0	0	100	2	4.4
5	0	50	0	0	0	50	0	50	0	0	150	3	6.5
10	0	50	0	100	0	50	0	50	0	0	250	5	9.5
15	0	50	0	0	150	100	0	50	0	0	350	7	12.4
20	0	50	0	150	0	200	0	0	100	0	500	10	15.2
25	0	100	0	200	0	0	250	100	0	0	650	13	18.1

Table 5. Effect of rate of increase in energy cost on optimal solution

Rate of Energy cost (%)	Nodes Ckvar 1	2	3	4	5	6	7	8	9	10	Total Ckvar	No. of cap.	Saving (%)
0	0	50	0	0	150	0	0	0	50	0	250	5	5.8
1	0	50	0	100	0	50	0	50	0	0	250	5	9.5
2	0	50	0	100	0	50	0	50	0	0	250	5	9.53
3	0	50	0	100	0	50	0	50	0	0	250	5	9.6
4	0	50	0	100	0	150	0	0	50	0	350	7	11.9
5	0	50	0	100	0	0	150	0	50	0	350	7	12.1

Table 6. Effect of change in cost of system released capacity on the optimal solution.

Change in cost of released Cap.	Nodes Ckvar 1	2	3	4	5	6	7	8	9	10	Total Ckvar	No. of cap.	Saving (%)
0 %	0	50	0	100	0	50	0	0	0	0	200	4	1.4
20 %	0	50	0	100	0	50	0	50	0	0	250	5	9.5
40 %	0	50	0	100	0	50	0	50	0	0	250	5	17.7
60 %	0	50	0	100	0	50	0	50	150	0	400	8	36.6
80 %	0	50	0	100	0	50	0	50	250	0	500	10	56.0
100 %	0	50	0	100	0	50	0	50	250	100	600	12	73.9

It can be noticed from Table 4 that an increase in compensation level of 150 Ckvar is required to meet the increase in load along the feeder from 0 to 10%. Also, it can be noticed from Table 6 that the effect of increase in the cost of system released capacity is more pronounced than the other factors.

5. CONCLUSIONS

A new method of optimal choosing fixed capacitors for single feeder is introduced. The effect of load growth, growth in load factor and increase in energy cost on the optimal solution is discussed. The problem of choosing the fixed capacitors is formulated as an optimally one where the cost of power loss reduction, cost of energy loss reduction, system released capacity cost and capacitor cost are taken as the main objective function. This objective cost is maximized while the system is subjected to two constraints (voltage-rise at off-peak hours and voltage drop).

The method is applied on a single radial feeder with static load to optimally locate and size fixed capacitors. The effect of change in growth factors on the optimal choice of number, location and size is well established. The effect of load growth shows to be more pronounced than the other factors. As per the results, the cost of energy must be taken as a variable in the formulation of mathematical model of the problem.This may be helpful for the utility authorities to decide their planning on rate of change in energy cost in future. This method has the advantage that, it permits to choose any available bank size which has a standard value, in which this is a very important contribution in the proposed method over all the other methods, as the size of capacitor bank obtained from the other methods is not standard size which is to be approximated to the nearest standard value, and that will affect the optimall solution.The proposed method is simple, easy to programm and more efficient.

5. REFERENCES

[1] M. Kaplan,"Optimization of number, location, size, control type, and control setting of shunt capacitors on radial distribution feeders", IEEE Trans., Vol. PAS-103, No. 9, pp. 2659-2664, 1984.

[2] H. Duran, "Optimum number, location and size of shunt capacitors in radial distribution feeders. A dynamic programming approach", IEEE Trans., Vol. PAS-87, No. 9, pp. 1769-1774.

[3] Y.C. Bae, "Analytical method of capacitor allocation on distribution primary feeders", TEEE Trans., Vol. PAS-97, No. 4, pp. 1232-1238, 1978.

[4] J.J. Grainger and S.H. Lee, "ptimum size and location of shunt capacitors for reduction of losses on distribution feeders", EEE Trans., Vol. PAS-100, No. 3, pp. 1105-1118, 1981.

[5] J.J. Grainger and S.H. Lee, "Capacity release by shunt capacitor placement on distribution feeders", IEEE Trans., Vol. PAS-101, No. 5, pp. 1236-1244, 1982.

[6] T.H. Fawzi and S. El Sobki, "New approach for the application of shunt capacitors to the primary distribution feeder", IEEE Trans., Vol. PAS-102, No. 1, pp. 10-13, 1983.

[7] M. Ponnavaikko and K.S. Prakasa Rao, "Optimal choice of fixed and switched shunt capacitors on radial distributors by the method of local variations", IEEE Trans., Vol. PAS-102, No. 6, pp. 1607-1614, 1983.