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Effect of Applying Time-Varying Load on the Optimal Location and Size of Fixed and Switched Capacitors on Radial Feeder with Laterals.

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EFFECT OF APPLYING TIME-VARYING LOAD ON THE OPTIMAL LOCATION AND SIZE OF FIXED AND SWITCHED CAPACITORS ON RADIAL FEEDER WITH LATERALS

تأثير الأحمال المتغيرة زمنيا على الوضع والتقسيم الأمثل لمكثفات القوى الثابتة والمتغيرة على موزع شعاعي ذو تفريعات

Kamal M. Shebl

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الخلاصة - يقدم البحث في الجزء الأول تحليل جديد لكيفية التعامل مع موزع شعاعي ذو تفريعات ابتدائية وأخرى ثانوية وذلك بالتعريف بها رياضيا لادخالها في ايجاد الوضع والتقسيم الأمثل لمكثفات القوى الثابتة على هذا النوع من الموزعات. كما يوضح البحث في هذا الجزء تأثير قيمة وحدة المكثفات المستخدمة وكذلك تأثير تغيير قيمة الخفض المسموح به في أحمال المشتركين على الحل الأمثل لموضع وقيم المكثفات المطلوب تركيبها عند الأحمال. ويتعرض البحث في الجزء الثاني للتعامل مع مشكلة الوضع والتقسيم الأمثل لمكثفات القوى الثابتة وكذلك المتغيرة والتي يجب تواجدها عندما يكون الحمل متغيرا مع الزمن حيث يتم حل المسألة عند كل مستوى من القيم المتساوية في القدره غير الفعالة وبحث يتم تحديد المكثفات الثابتة فقط من أقل قيمه للقدرة غير الفعالة وبحيث يتم تحديد المكثفات الثابتة فقط من أقل قيمة للقدرة غير الفعالة في منحنى التحميل اليومي. وفي كل من الجزئين السابقين كانت دالة الهدف هي الحصول على أكبر وفر ممكن في التكلفة التي تشمل ثمن المكثفات والوفر في القدرة والطاقة المستهلكة والوفر في تكلفة القدرة الموفرة مع تواجد قيود على كل من الهبوط في الجهد عند الأحمال أو الارتفاع في الجهد عند الأحمال والناتج عن تواجد المكثفات مع انخفاض الأحمال، كما تم دراسة تأثير قيمة الوحده المستخدمه من المكثفات على الحل الأمثل. والطريقة المقدمة تعتبر طريقة عامة بسيطة ويمكن استخدامها لأى نوع من الموزعات ومع أى شكل من أشكال التحميل.

Abstract: A preceding paper (1) deals only with the problem of optimal location and size of fixed capacitors on the primary feeder when it is loaded with static load only. This paper deals with the same problem when the primary feeder has lateral and sublateral branches and loaded with time-varying loads at the nodes. An analysis for the notation of currents and voltages for the main feeder with lateral and sublateral branches is introduced. Also, the analysis of the time-varying load to be introduced in the problem is discussed to obtain the optimal solution of locating and sizing fixed and switchable capacitors on a feeder with lateral and sublateral branches. The time-varying load is also applied on a single radial feeder. The proposed technique proves to be a suiable for the general distribution system with laterals problem to obtain the optimality of fixed capacitors. The effect of unit-capacitor size and growth of cost factors on the optimal solution are discussed. The objective is to maximize the overall savings while satisfying voltage-drop and voltage-rise constraints.

1. INTRODUCTION

The actual configuration of distribution system is not so ea
case of single radial feeders. The complexity of the distribu
arises as it does contain not only main feeder but also latera
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branches which have not been considered in almost all previous studies. The need to reduce the enumeration problems relating to location and sizing of capacitors is of an obvious interest.

Several authors have addressed the capacitor problem but only a few of them has considered the actual configuration of the distribution systems. The first attempt to formulate the problem for radial feeders were made by Neagle and Sampson, Cook, Chang and others [2-5]. These methodologies suffer from lack of generality and consider an oversimplified model for the problem. Grainger and Lee [6-7] have formulated the capacitor problem for the more general case where nonuniform wire sizes, any number of fixed and switched capacitors and a weighted cost function are considered. Cuttino proposed schemes using a combination of discrete tapped of Kvar-sourses and voltage regulators to achieve an effective control over power losses and voltage [8-9].

This paper presents a new general mehodology of optimal location and size of power capacitors on a distribution feeder with lateral and sub-lateral branches. This is accompanied by an analysis for the lateral and sublateral branches that can be incorporated with ease comparable to that of a single radial feeder presented in a preceding paper [1]. Also, an analysis for the time-varying load with laterals and sublaterals or single radial feeders is presented to obtain location and size of fixed and switched capacitors as well. The effect of unit-capacitor value, load growth, growth in load factor and increase in energy cost on the optimal solution is discussed with applications.

2. DISTRIBUTION SYSTEM WITH LATERAL AND SUBLATERAL BRANCHES

2.1 Mathematical Model Formulation and Notation

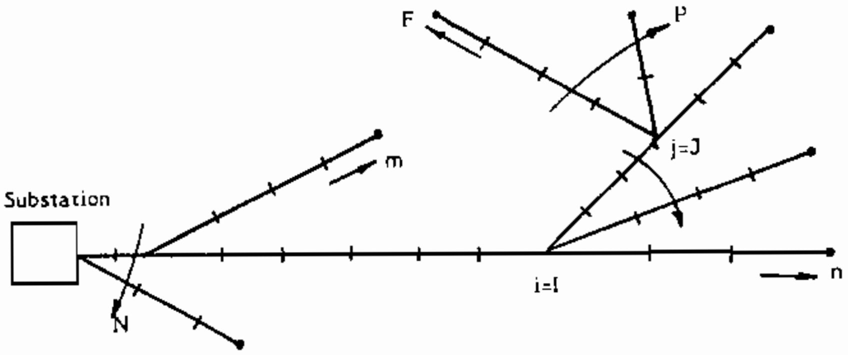

Fig.1 Notation system for general distribution feeder

Fig.1 shows a single-line representation of a general distribution system contains main feeder with lateral and sublateral branches. For such a system there is a clear need to represent the lateral and sublateral branches according to a certain notation system. The followings are the notations presented and used here to define the general system;

n - total number of segments in the main feeder,
N - number of main feeders radiating from the main substation,
- totel number of segments in a lateral branch,
- number of lateral branches radiating from node i=I on the main feeder,
total number of segments in sublateral branch,

$\bar{P}$ = number of sublateral branches radiating from node j=J on a lateral branch,
Ni = notation for the i_{th} segment in the N_{th} main feeder i=1,2, ... ,n),
IMj = notation for the j_{th} segment in the M_{th} lateral branch radiating from node i=I,
JPk = notation for the k_{th} segment in the P_{th} sublateral branch radiating from node j=J.

In order to study the optimal size and location of shunt capacitor banks, some variables must be mathematically modelled such as the cost of power loss reduction, cost of energy loss reduction, system capacity release, cost of capacitor and system constraints according to the system of notation mentioned above.

2.1.1 Cost of powar loss reduction

For the main radial feeder, the present worth of cost saving due to power loss reductioncan be obtained as;

$$C_{po} = 0.003\ K_p \sum_{i=1}^{n} \sum_{k=1}^{N} \sum_{j=1}^{M} R_i\ (2.i_{oi}.L_{ft}.l_{ii}\ (1+g)^{2q} - i^2_{oi}).(1/(1+r)^t) \quad (1)$$

where; t=1,2, ... ,T where T is the life period of capacitors in years.
q=1,2, ... ,Q where Q is the plan periodup to which the feeder can take loadgrowth, . subject to a maximum period limited by the capacitor life period.

In case of general system containing E main, lateral and sublateral branches, the total cost due to power loss reduction is to be given as;

$$C_{poe} = \sum_{e=1}^{E} C_{po} \quad (2)$$

2.1.2 Cost of energy loss reduction

The present worth of the cost saving due to energy loss reduction within the life period of capacitors is obtained as;

$$C_{Eo} = 26.28 \sum_{i=1}^{n} \sum_{k=1}^{N} \sum_{j=i}^{M} R_i.K_{et}(2.i_{oi}.L_{ft}.I_{ii}(1+g)^{2q} - i^2_{oi}).(1/(1+r)^t) \quad (3)$$

In case of general system containing E main, lateral and sublateral branches, the total cost saving due to energy loss reduction is obtained as:

$$C_{Eoe} = \sum_{e=1}^{E} C_{Eo} \quad (4)$$

2.1.3 Cost of capacity release in feeder sections

The cost saving due to capacity release is obtained as;

$$C_r = \sum_{i=1}^{n} C_{ri}\ .S_i\ [1 - (1+t.i_{oi}\ /\ S_i - (2\ t.i_{oi}\ /\ S_i).\sin\theta_i)^{0.5}] \quad (5)$$

where C_{ri} is the marginal cost required to supply every additional Kva demand.

For a general system with E main, lateral and sublateral branches, the total cost saving due to capacity release in system segments is obtained as;

$$C_{re} = \sum_{e=1}^{E} C_{r} \qquad (6)$$

2.1.4 Cost of capacity ralease in system components

The cost saving due to capacity release in substation, transformers, transmission lines and in the generators ib obtained as;

$$C_{ro} = C_{c} \; . \; S_{m} \; [1 - (1+t.I_{ct} \; / \; S_{m})^{2} - (2t.I_{ct}.\sin \theta_{1} \; / \; S_{m})^{0.5}] \qquad (7)$$

The total cost saving for a general system is given as;

$$C_{roe} = \sum_{e=1}^{E} C_{ro} \qquad (8)$$

2.1.5 Cost of capacitors

The capital cost of capacitor bank can be written as;

$$C_{c} = \sum_{j=1}^{n} \sum_{t=1}^{T} (s \; . \; i_{cq} + a \; . \; d) \; / \; (1 + r)^{t} \qquad (9)$$

2.2 Objective Function

The objective function in our problem is to maximize the total present worth cost saving including the power loss reduction cost (3), energy loss reduction cost (4), capacity release cost in feeder segments (6) and cost of capacity release in system components (8) against cost of capacitor banks (9). This total combined cost is obtained as;

$$C_{T} = C_{POe} + C_{EOe} + C_{re} + C_{roe} + - C_{c} \qquad (10)$$

2.3 System Constraints

As mentioned in a preceding paper [1], the voltage rise and voltage drop must be decided in a prior as constraints on maximizing the objective function.

2.3.1 Voltage-drop constraint

In case of a distribution system with laterels and sublaterals, there will be more than one end point depending upon number of laterals and sub-laterals. In such a system, the number of end points (E) is given as;

$$E = N + L + S \qquad (11)$$

where L & S are the total number of lateral and sublateral branches in the system.

The voltage drop between the substation node and an end point on a sub-lateral branch radiating from node j=J on a lateral branch which is radiating from node i=I on the main feeder is obtained as;

$$V_{JPK} = V_{IMJ} + \sum_{k=1}^{P} I_{K} \; . \; R_{K} \; . \; \cos \theta_{K} + (I_{K} \; . \; \sin \theta_{K} - I_{cK}) \; . \; X_{K} \qquad (12)$$

where V_{JPK} is the voltage drop between the substation node and an end point on a lateral branch radiating from node i=I on the main feeder which is given as;

$$V_{IMJ} = V_{Ni} + \sum_{j=1}^{m} I_j \,.\, R_i \,.\, \cos\theta_j + (I_j \,.\, \sin\theta_j - I_{oj}) \,.\, X_j \qquad (13)$$

where V_{Ni} is the voltage drop across the main feeder and is obtained as;

$$V_{Ni} = \sum_{i=1}^{n} I_i \,.\, R_i \,.\, \cos\theta_i + (I_i \,.\, \sin\theta_i - I_{oi}) \,.\, X_i \qquad (14)$$

Then, the voltage drop along the system branches should not exceed a specified limit given as

$$V_{Ni} \leqslant V_{max} \,.\quad V_{IMJ} < V_{max} \,.\quad \& \quad V_{JPK} < V_{max} \qquad (15)$$

2.3.2 Voltage-rise constraint

The voltage rise between the substation and an end point on a sub-lateral branch radiating from node j=J on a lateral branch that radiates from node i=I on the main feeder is obtained as;

$$G_{JPK} = G_{IMJ} + \sum_{k=1}^{F} (I_{cK} - I_{oK}.\sin\theta').X_K - I'_{oK}.R_K[1-(I_{oK} - I_{oK}.\sin\theta')/I'_{oK})^2]^{0.5} \qquad (16)$$

where G_{IMJ} is the voltage rise between the substation and an end point on a lateral branch radiating from node i=I on the main feeder is obtained as;

$$G_{IMJ} = G_{Ni} + \sum_{j=1}^{m} (I_{cj} - I_{oj}.\sin\theta').X_j - I'_{oj}.R_j[1-((I_{oj} - I_{oj}.\sin\theta')/I'_{oj})^2]^{0.5} \qquad (17)$$

where G_{Ni} is the voltage rise across the main feeder and is obtained as;

$$G_{Ni} = \sum_{i=1}^{n} (I_{ci} - I_{oi}.\sin\theta').X_i - I'_{oi}.R_i\,[1-((I_{oi} - I_{oi}.\sin\theta')/I'_{oi})^2]^{0.5} \qquad (18)$$

It is important to make the voltage rise along the system segments less than or equal to specified limit as:

$$G_{Ni} \leqslant 0 \quad . \quad G_{IMJ} \leqslant 0 \quad \& \quad G_{JPK} \leqslant 0 \qquad (19)$$

3. SOLUTION TECHNIQUE

The same concepts of the proposed technique presented in a preceding paper [1] is used after modifying the objective function as well as the system constraints to suite the incorporation of lateral and sublateral branches in the system. The optimal solution of size and location of fixed capacitors is obtained with static loadings.

4. APPLICATIONS

4.1 Test System

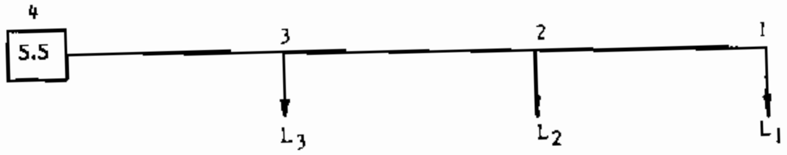

Fig.2 Single-line diagram of the test system

Fig.2 shows the 11-Kv test system that has one main feeder, two lateral branches and one sublateral branch. All loade are aesumed to have the same maximum demand and power factor. The system is balanced under steady state operating conditions having no losses or no voltage drop in the neutral wire. Table 1 illustrates the length of segments, load values, resistance and reactance of each segment and the permissible voltage regulation at the end points. The main data of the test system are given in a preceding paper [1]. The power factor and the present load factor are given as 0.71 and 0.35 respectively.

Table.1 Data of the test system

Segment No.	Load (Kw)	Wire size (Sq. mm)	Length (km)	Resistance (Ohm/km)	Reactance (Ohm/km)	Voltage reg. (%)
node 1	90					
1 - 2	80	10	2.5	2.960	0.1110	7.5 (node 1)
2 - 3	95	10	1.6	2.960	0.1110	
3 - 4	80	10	2.5	2.960	0.1110	
4 - 5	00	10	1.0	2.960	0.1110	
5 - 6	70	120	2.3	0.280	0.0798	
6 - 7	75	120	1.9	0.280	0.0798	
7 - 8	00	150	0.8	0.226	0.0778	
8 - 9	80	240	3.0	0.143	0.0738	
9 - 10	100	240	1.8	0.143	0.0738	
10 - 11	00	300	2.0	0.117	0.0725	
5 - 12	60	30	2.0	0.662	0.0885	
12 - 13	80	40	1.5	0.837	0.0917	
13 - 14	00	40	2.5	2.960	0.0917	
14 - 15	70	10	1.5	1.790	0.1111	
15 - 16	90	10	3.0	2.270	0.1110	7.38 (node 16)
8 - 17	90	18	2.0	2.270	0.1020	
17 - 18	70	14	1.8	2.270	0.1060	
18 - 19	60	14	2.7	2.270	0.1060	
19 - 20	80	14	2.3	2.270	0.1060	7.10 (node 20)
14 - 21	80	14	1.3	2.270	0.1060	
21 - 22	60	14	1.8	2.270	0.1060	
22 - 23	70	14	2.0	2.270	0.1060	7.40 (node 23)

4.2 Results and Discussions

To study the effect of the value of unit-capacitors, different values are used.Table 2 and Table 3 show the optimal solution through the application of the proposed technique on a general test system with lateral and sublateral branches. Table 2 illustrates the results for application of 75 Kvar unit-capacitors, while Table 3 illustrates the results for application of 20 Kvar unit capacitors. It is noticed that in case of small value of unit capacitors, much nodes posses capacitor banks, but less value of total capacitor value for compensation, less released capacity and less annual cost saving, and vice versa. It can be concluded that for the same total value af capacitors available, the optimal solution can not be achieved in case of large unit-capacitor value.

Table 2. Optimal solution using 75-Kvar unit-capacitor

Node No.	4	6	8	13
Capacitor (Kvar)	150	150	300	225
Number of unit capaacitors	2	2	4	3
Released capacity (%)	0.755			
Annual cost saving (%)	11.83			
Total capacitors (Kvar)	11 x 75 = 825			

Table 3. Optimal solution using 20-Kvar unit-capacitor

Node No.	2	4	6	9	12	21	17	19
Capacitor (Kvar)	40	80	120	100	140	60	80	40
Number of unit capacitors	2	4	6	5	7	3	4	2
Released capacity (%)	0.651							
Annual cost saving (%)	7.73							
Total capacitors (Kvar)	20 x 34 = 680							

Tables 4, 5 and 6 illustrate the effect of voltage constraint on the optimal solution. It can be noticed that for a decreased value of the voltage constraint, the total compensation value of capacitor Kvar increases and the locations of capacitors move nearer to the substation. Also, no feasible solution was obtained when the voltage constraint has a value less than 6.2%. The number of nodes that has capacitors becomes less in case of high voltage constraint than that with small value of voltage constraint.

Table 4. Optimal resulta with a voltage drop constraint as 7.0 %

Node No.	2	4	15	13	7	18
Capacitor (Kvar)	100	150	100	200	200	100
Number of unit capacitors	2	3	2	4	4	2
Released capacity (%)	0.813					
Annual cost saving (%)	12.61					
Total capacitors (Kvar)	18 x 50 = 900					

Table 5. Optimal results with a voltage drop constraint as 6.2 %

Node No.	2	4	14	21	12	6	8	9	18
Capacitor (Kvar)	50	100	100	100	150	200	200	150	150
Number of unit capacitors	1	2	2	2	3	4	4	3	3
Releases capacity (%)	0.946								
Annual cost saving (%)	12.37								
Total capacitors (Kvar)	24 x 50 = 1200								

Table 6. Effect of voltage constraint on the optimal solution

Votaltage drop (%)	Total capacitors (Kvar)	Released capacity (%)	Annual cost saving (%)
7.0	900	0.813	12.61
6.5	1000	0.875	13.25
6.2	1200	0.946	12.37
>6.2	No feasible solution		

5. INCORPORATION OF TIME-VARYING LOAD

In case of time-varying incorporation with radial distribution feeder, the changing in reactive requirements are generally satisfied by switched capacitor banks supplemented with load tap-changing transformers and feeder voltage regulators for voltage control. Switched shunt capacitor banks are commonly used for system voltage control, and their ability to accomplish this is a function of the size of switching steps. This part of the present paper deals actually with the procedures to account for switched banks. A new approach to the incorporation of the switched capacitors in optimal capacitor problem is given. This incorporation provides to the electric utility industry new tools for application of fixed and switched capacitor banks on a radial feeder so as to maximize the net cost savings. Since the switched capacitors will not be in service during the off-peak hours, the voltage rise constraint in not needed to be considered in this case.

6. SOLUTION TECHNIQUE

Fig.3 shows an illustratory daily reactive load profile where T_1 and T_2 are duration times over which the switched capacitor banks are in service.

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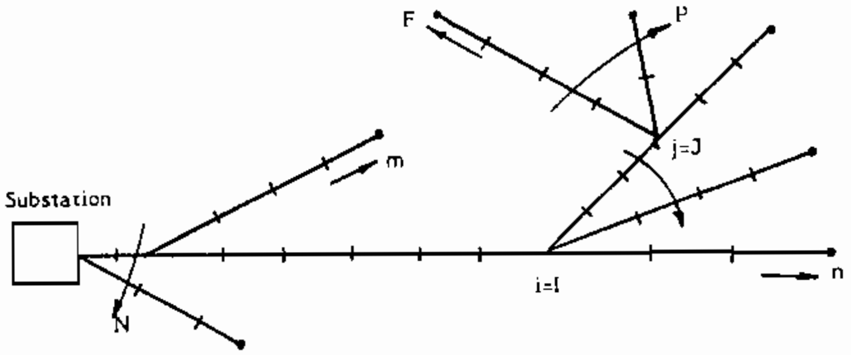

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= number of lateral branches radiating from node i=I on the main feeder.
total number of segments in sublateral branch,

$\bar{P}$ = number of sublateral branches radiating from node j=J on a lateral branch,
Ni = notation for the i_{th} segment in the N_{th} main feeder i=1,2, ... ,n),
IMj = notation for the j_{th} segment in the M_{th} lateral branch radiating from node i=I,
JPk = notation for the k_{th} segment in the P_{th} sublateral branch radiating from node j=J.

In order to study the optimal size and location of shunt capacitor banks, some variables must be mathematically modelled such as the cost of power loss reduction, cost of energy loss reduction, system capacity release, cost of capacitor and system constraints according to the system of notation mentioned above.

2.1.1 Cost of powar loss reduction

For the main radial feeder, the present worth of cost saving due to power loss reductioncan be obtained as;

$$C_{po} = 0.003\ K_p \sum_{i=1}^{n} \sum_{k=1}^{N} \sum_{j=1}^{M} R_i\ (2.i_{oi}.L_{ft}.I_{ii}\ (1+g)^{2q} - i^2_{oi}).(1/(1+r)^t) \quad (1)$$

where; t=1,2, ... ,T where T is the life period of capacitors in years,
q=1,2, ... ,Q where Q is the plan periodup to which the feeder can take loadgrowth, . subject to a maximum period limited by the capacitor life period.

In case of general system containing E main, lateral and sublateral branches, the total cost due to power loss reduction is to be given as;

$$C_{poe} = \sum_{e=1}^{E} C_{po} \quad (2)$$

2.1.2 Cost of energy loss reduction

The present worth of the cost saving due to energy loss reduction within the life period of capacitors is obtained as;

$$C_{Eo} = 26.28 \sum_{i=1}^{n} \sum_{k=1}^{N} \sum_{j=i}^{M} R_i.K_{et}(2.i_{oi}.L_{ft}.I_{ii}(1+g)^{2q} - i^2_{oi}).(1/(1+r)^t) \quad (3)$$

In case of general system containing E main, lateral and sublateral branches, the total cost saving due to energy loss reduction is obtained as:

$$C_{Eoe} = \sum_{e=1}^{E} C_{Eo} \quad (4)$$

2.1.3 Cost of capacity release in feeder sections

The cost saving due to capacity release is obtained as;

$$C_r = \sum_{i=1}^{n} C_{ri}\ .S_i\ [1 - (1+t.i_{oi}\ /\ S_i - (2\ t.i_{oi}\ /\ S_i).\sin\theta_i)^{0.5}] \quad (5)$$

where C_{ri} is the marginal cost required to supply every additional Kva demand.

For a general system with E main, lateral and sublateral branches, the total cost saving due to capacity release in system segments is obtained as;

$$C_{re} = \sum_{e=1}^{E} C_r \qquad (6)$$

2.1.4 Cost of capacity release in system components

The cost saving due to capacity release in substation, transformers, transmission lines and in the generators ib obtained as;

$$C_{ro} = C_o \,.\, S_m \,[1 - (1+t.I_{ct} / S_m)^2 - (2t.I_{ct}.\sin\theta_1 / S_m)^{0.5}] \qquad (7)$$

The total cost saving for a general system is given as;

$$C_{roe} = \sum_{e=1}^{E} C_{ro} \qquad (8)$$

2.1.5 Cost of capacitors

The capital cost of capacitor bank can be written as;

$$C_c = \sum_{j=1}^{n} \sum_{t=1}^{T} (\, s \,.\, i_{cq} + a \,.\, d \,) \,/\, (\, 1 + r \,)^t \qquad (9)$$

2.2 Objective Function

The objective function in our problem is to maximize the total present worth cost saving including the power loss reduction cost (3), energy loss reduction cost (4), capacity release cost in feeder segments (6) and cost of capacity release in system components (8) against cost of capacitor banks (9). This total combined cost is obtained as;

$$C_T = C_{poe} + C_{Eoe} + C_{re} + C_{roe} + - C_c \qquad (10)$$

2.3 System Constraints

As mentioned in a preceding paper [1], the voltage rise and voltage drop must be decided in a prior as constraints on maximizing the objective function.

2.3.1 Voltage-drop constraint

In case of a distribution system with laterels and sublaterals, there will be more than one end point depending upon number of laterals and sublaterals. In such a system, the number of end points (E) is given as;

$$E = N + L + S \qquad (11)$$

where L & S are the total number of lateral and sublateral branches in the system.

The voltage drop between the substation node and an end point on a sublateral branch radiating from node j=J on a lateral branch which is radiating from node i=I on the main feeder is obtained as;

$$V_{JPK} = V_{IMJ} + \sum_{k=1}^{P} I_k \,.\, R_k \,.\, \cos\theta_k + (I_k \,.\, \sin\theta_k - I_{ck}) \,.\, X_k \qquad (12)$$

where V_{JPK} is the voltage drop between the substation node and an end point on a lateral branch radiating from node i=I on the main feeder which is given as;

$$V_{IMJ} = V_{Ni} + \sum_{j=1}^{m} I_j \,.\, R_i \,.\, \cos\theta_j + (I_j \,.\, \sin\theta_j - I_{oj}) \,.\, X_j \qquad (13)$$

where V_{Ni} is the voltage drop across the main feeder and is obtained as:

$$V_{Ni} = \sum_{i=1}^{n} I_i \,.\, R_i \,.\, \cos\theta_i + (I_i \,.\, \sin\theta_i - I_{oi}) \,.\, X_i \qquad (14)$$

Then, the voltage drop along the system branches should not exceed a specified limit given as

$$V_{Ni} \leqslant V_{max.} \quad . \quad V_{IMj} < V_{max.} \quad \& \quad V_{JPK} < V_{max} \qquad (15)$$

2.3.2 Voltage-rise constraint

The voltage rise between the substation and an end point on a sub-lateral branch radiating from node j=J on a lateral branch that radiates from node i=I on the main feeder is obtained as:

$$G_{JPK} = G_{IMj} + \sum_{k=1}^{F} (I_{ck} - I_{ok}.\sin\theta').X_k - I'_{ok}.R_k[1-(I_{ok} - I_{ok}.\sin\theta')/I'_{ok})^2]^{0.5} \qquad (16)$$

where G_{IMj} is the voltage rise between the substation and an end point on a lateral branch radiating from node i=I on the main feeder is obtained as;

$$G_{IMj} = G_{Ni} + \sum_{j=1}^{m} (I_{cj} - I_{oj}.\sin\theta').X_j - I'_{oj}.R_j[1-((I_{oj} - I_{oj}.\sin\theta')/I'_{oj})^2]^{0.5} \qquad (17)$$

where G_{Ni} is the voltage rise across the main feeder and is obtained as;

$$G_{Ni} = \sum_{i=1}^{n} (I_{oi} - I_{oi}.\sin\theta').X_i - I'_{oi}.R_i\ [1-((I_{oi} - I_{oi}.\sin\theta')/I'_{oi})^2]^{0.5} \qquad (18)$$

It is important to make the voltage rise along the system segments less than or equal to specified limit as:

$$G_{Ni} \leqslant 0 \quad . \quad G_{IMj} \leqslant 0 \quad \& \quad G_{JPK} \leqslant 0 \qquad (19)$$

3. SOLUTION TECHNIQUE

The same concepts of the proposed technique presented in a preceding paper [1] is used after modifying the objective function as well as the system constraints to suite the incorporation of lateral and sublateral branches in the system. The optimal solution of size and location of fixed capacitors is obtained with static loadings.

4. APPLICATIONS

4.1 Test System

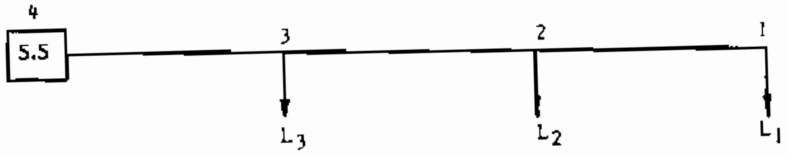

Fig.2 Single-line diagram of the test system

Fig.2 shows the 11-Kv test system that has one main feeder, two lateral branches and one sublateral branch. All loads are assumed to have the same maximum demand and power factor. The system is balanced under steady state operating conditions having no losses or no voltage drop in the neutral wire. Table 1 illustrates the length of segments, load values, resistance and reactance of each segment and the permissible voltage regulation at the end points. The main data of the test system are given in a preceding paper [1]. The power factor and the present load factor are given as 0.71 and 0.35 respectively.

Table.1 Data of the test system

Segment No.	Load (Kw)	Wire size (Sq. mm)	Length (km)	Resistance (Ohm/km)	Reactance (Ohm/km)	Voltage reg. (%)
node 1	90					
1 - 2	80	10	2.5	2.960	0.1110	7.5 (node 1)
2 - 3	95	10	1.6	2.960	0.1110	
3 - 4	80	10	2.5	2.960	0.1110	
4 - 5	00	10	1.0	2.960	0.1110	
5 - 6	70	120	2.3	0.280	0.0798	
6 - 7	75	120	1.9	0.280	0.0798	
7 - 8	00	150	0.8	0.226	0.0778	
8 - 9	80	240	3.0	0.143	0.0738	
9 - 10	100	240	1.8	0.143	0.0738	
10 - 11	00	300	2.0	0.117	0.0725	
5 - 12	60	30	2.0	0.662	0.0885	
12 - 13	80	40	1.5	0.837	0.0917	
13 - 14	00	40	2.5	2.960	0.0917	
14 - 15	70	10	1.5	1.790	0.1111	
15 - 16	90	10	3.0	2.270	0.1110	7.38 (node 16)
8 - 17	90	18	2.0	2.270	0.1020	
17 - 18	70	14	1.8	2.270	0.1060	
18 - 19	60	14	2.7	2.270	0.1060	
19 - 20	80	14	2.3	2.270	0.1060	7.10 (node 20)
14 - 21	80	14	1.3	2.270	0.1060	
21 - 22	60	14	1.8	2.270	0.1060	
22 - 23	70	14	2.0	2.270	0.1060	7.40 (node 23)

4.2 Results and Discussions

To study the effect of the value of unit-capacitors, different values are used. Table 2 and Table 3 show the optimal solution through the application of the proposed technique on a general test system with lateral and sublateral branches. Table 2 illustrates the results for application of 75 Kvar unit-capacitors, while Table 3 illustrates the results for application of 20 Kvar unit capacitors. It is noticed that in case of small value of unit capacitors, much nodes posses capacitor banks, but less value of total capacitor value for compensation, less released capacity and less annual cost saving, and vice versa. It can be concluded that for the same total value af capacitors available, the optimal solution can not be achieved in case of large unit-capacitor value.

Table 2. Optimal solution using 75-Kvar unit-capacitor

Node No.	4	6	8	13
Capacitor (Kvar)	150	150	300	225
Number of unit capaacitors	2	2	4	3
Released capacity (%)	0.755			
Annual cost saving (%)	11.83			
Total capacitors (Kvar)	11 x 75 = 825			

Table 3. Optimal solution using 20-Kvar unit-capacitor

Node No.	2	4	6	9	12	21	17	19
Capacitor (Kvar)	40	80	120	100	140	60	80	40
Number of unit capacitors	2	4	6	5	7	3	4	2
Released capacity (%)	0.651							
Annual cost saving (%)	7.73							
Total capacitors (Kvar)	20 x 34 = 680							

Tables 4. 5 and 6 illustrate the effect of voltage constraint on the optimal solution. It can be noticed that for a decreased value of the voltage constraint, the total compensation value of capacitor Kvar increases and the locations of capacitors move nearer to the substation. Also, no feasible solution was obtained when the voltage constraint has a value less than 6.2%. The number of nodes that has capacitors becomes less in case of high voltage constraint than that with small value of voltage constraint.

Table 4. Optimal resulta with a voltage drop constraint as 7.0 %

Node No.	2	4	15	13	7	18
Capacitor (Kvar)	100	150	100	200	200	100
Number of unit capacitors	2	3	2	4	4	2
Released capacity (%)	0.813					
Annual cost saving (%)	12.61					
Total capacitors (Kvar)	18 x 50 = 900					

Table 5. Optimal results with a voltage drop constraint as 6.2 %

Node No.	2	4	14	21	12	6	8	9	18
Capacitor (Kvar)	50	100	100	100	150	200	200	150	150
Number of unit capacitors	1	2	2	2	3	4	4	3	3
Releases capacity (%)	0.946								
Annual cost saving (%)	12.37								
Total capacitors (Kvar)	24 x 50 = 1200								

Table 6. Effect of voltage constraint on the optimal solution

Votaltage drop (%)	Total capacitors (Kvar)	Released capacity (%)	Annual cost saving (%)
7.0	900	0.813	12.61
6.5	1000	0.875	13.25
6.2	1200	0.946	12.37
>6.2	No feasible solution		

5. INCORPORATION OF TIME-VARYING LOAD

In case of time-varying incorporation with radial distribution feeder, the changing in reactive requirements are generally satisfied by switched capacitor banks supplemented with load tap-changing transformers and feeder voltage regulators for voltage control. Switched shunt capacitor banks are commonly used for system voltage control, and their ability to accomplish this is a function of the size of switching steps. This part of the present paper deals actually with the procedures to account for switched banks. A new approach to the incorporation of the switched capacitors in optimal capacitor problem is given. This incorporation provides to the electric utility industry new tools for application of fixed and switched capacitor banks on a radial feeder so as to maximize the net cost savings. Since the switched capacitors will not be in service during the off-peak hours, the voltage rise constraint in not needed to be considered in this case.

6. SOLUTION TECHNIQUE

Fig.3 shows an illustratory daily reactive load profile where T_1 and T_2 are duration times over which the switched capacitor banks are in service.

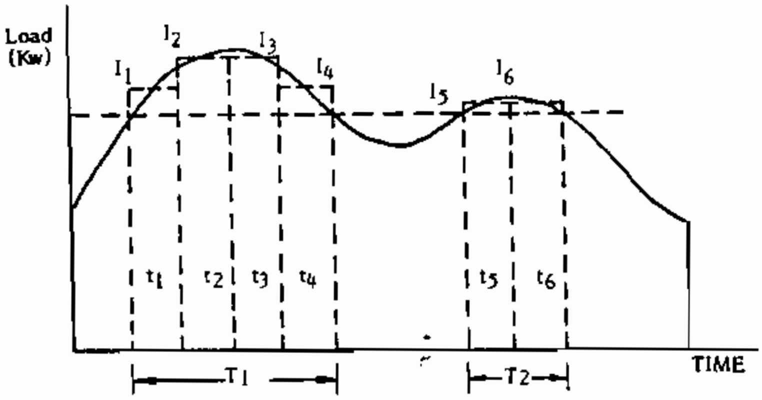

Fig.3 Illustrative daily load-curve

In this case, we must begin with determining the minimum reactive load level, in which the fixed capacitor banks are to be decided according to the minimum level of reactive load, then the reactive load level is increased in steps until the peak value of reactive load is reached. In each step of load increase from minimum load level we determine the location and size of switched capacitor banks taking into consideration the existing of fixed banks when choosing the switched banks. The size of switched capacitors output from the presented technique depends on the time duration at which the switched capacitor will be in service and the level of load power. After obtaining the optimal solution of fixed and switched capacitor we have to move to the next time and the procedure is repeated until convergence is obtained for the objective function at each time all over the whole period.

7. APPLICATION OF TIME-VARYING LOAD

7.1 Test System

The prpoeed technique has been tested on 11-Kv, 3-segments radial distribution feeder shown in Fig.4. Table 7 illustrates thesystem data with a variable cost of switched capacitor bank is taken as 45 L.E./Kvar. Table 8 illustrates the values and power factors of the time-varying loads.

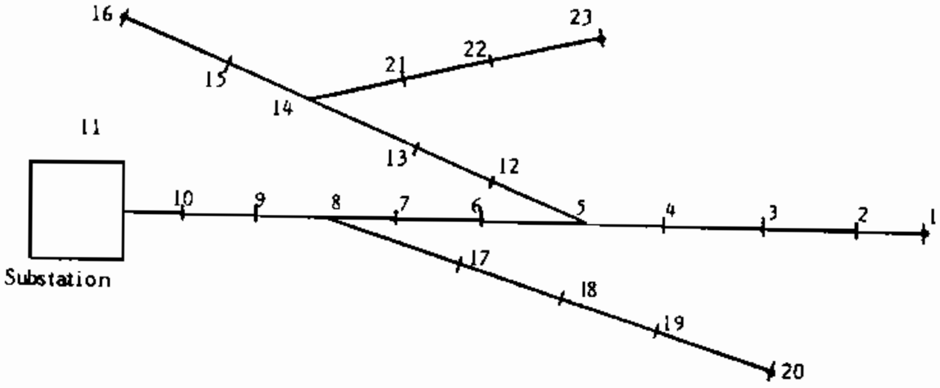

Fig.4. Test system radial distribution feeder.

Table. 7 Feeder data of the test system

Segment No.	Wire size (mm2)	Length (km)	Res. (Ohm/km)	React. (Ohm/km)
1 - 2	10	4.0	2.960	0.1111
2 - 3	30	3.0	1.090	0.0951
3 - 4	50	2.0	0.662	0.0885

Table 8. Data of time-varying loads

Hour	Load at node 1		Load at node 2		Load at node 3	
	P (kw)	P.F.	P (kw)	P.F.	P (kw)	P.F.
1	300	0.681	125	0.757	208	0.680
2	298	0.680	92	0.719	145	0.665
3	263	0.669	93	0.711	180	0.660
4	250	0.630	108	0.710	180	0.660
5	298	0.660	175	0.751	200	0.690
6	244	0.620	194	0.720	204	0.700
7	320	0.720	317	0.740	204	0.700
8	343	0.700	290	0.694	220	0.710
9	225	0.698	269	0.670	252	0.690
10	340	0.704	278	0.725	304	0.720
11	342	0.710	205	0.680	303	0.720
12	400	0.750	218	0.740	268	0.710
13	356	0.720	143	0.660	308	0.730
14	340	0.740	140	0.650	250	0.690
15	322	0.689	137	0.650	252	0.690
16	303	0.670	144	0.690	227	0.670
17	303	0.680	125	0.680	203	0.650
18	322	0.679	113	0.670	255	0.690
19	302	0.670	105	0.660	250	0.710
20	297	0.650	116	0.680	223	0.680
21	295	0.660	126	0.710	245	0.700
22	263	0.592	99	0.700	250	0.710
23	302	0.690	124	0.710	204	0.680
24	320	0.690	84	0.720	206	0.680

7.2 Results and Discussions

Table 9 illustrates the optimal solution of fixed and switched capacitors using 30-Kvar unit-capacitor values. A total of 300-KVAR fixed capacitors ara obtained and sized as 150, 90 and 60 Kvar located at nodes 1. 2 and 3 respectively. The nodes away from the substation has higher values than that near the substation. The maximum value of switched capacitors over the day was obtained at 10 Oclock and located and sizad as 120 Kvar at node 1, 120 Kvar at node 2 and 60 Kvar at node 3. The released power capacity available at the substation node due to the application of shunt capacitors which depends on power level, compensation level and power factor was obtained as 176 Kva, where tha integration of annual cost saving was obtained as 4492 L.E.

Table 10 illustrates similar results obtained using another unit-capacitors of 10-Kvar each to see the effect of unit-capacitor size on the optimal solution. The total value of fixed capacitors decreased to 270 Kvar located and sized as 130 Kvar at node 1. 90 Kvar at node 2 and 50 Kvar at node 3. The maximum value of total switched capacitors was obtained as 260 Kvar. The integrated released power capacity and annual cost saving are obtained as 208 Kva and 2338 L.E. respectively. It was noticed that the compensation level as well as the annual cost saving are reduced in case of using smaller unit-capacitor value.

Table 9. Optimal results using 30-Kvar unit-capacitors

Hour	Switched banks			Released power (Kvar)	Annual saving (L.E.)
	Node 1	Node 2	Node3		
1	30	30	00	208.68	4560
2	00	00	00	176.37	4396
3	00	00	00	176.70	4414
4	00	00	00	177.36	4440
5	60	60	30	221.86	4320
6	00	30	30	211.64	4620
7	90	90	60	277.25	3740
8	90	90	60	281.20	8320
9	90	90	60	282.19	3860
10	120	120	60	303.95	5160
11	90	90	60	263.11	4955
12	120	90	60	298.00	5260
13	90	60	60	266.00	5015
14	90	60	30	240.65	4500
15	60	60	30	234.39	4360
16	60	60	30	222.19	4357
17	30	30	00	208.00	4538
18	60	60	30	227.47	4480
19	30	30	00	216.59	4740
20	30	30	00	209.67	4585
21	60	60	30	219.56	4320
22	00	30	30	201.75	4420
23	30	30	00	207.69	4568
24	30	30	00	201.00	4398

Table 10. Optimal results using 10-Kvar unit-capacitor

Hour	Switched capacitors			Released Power (Kva)	Annual saving (L.E.)
	Node 1	Node 2	Node 3		
1	20	20	10	185.41	2841
2	00	00	00	158.14	2260
3	00	00	00	159.00	2280
4	00	00	00	159.61	2288
5	60	50	10	192.21	1940
6	10	30	10	188.23	2781
7	80	70	50	240.64	2000
8	90	90	40	254.42	2080
9	90	90	40	255.36	2096
10	110	100	50	275.33	2213
11	90	90	40	253.83	2079
12	110	70	50	269.11	2192
13	70	60	50	241.37	2290
14	70	60	20	218.00	2148
15	60	40	20	212.23	2340
16	60	40	20	201.28	2157
17	20	20	10	188.88	2781
18	60	40	20	206.80	2057
19	20	20	10	196.19	2061
20	20	20	10	189.92	2921
21	60	40	20	198.88	1897
22	10	30	10	179.33	2723
23	20	20	10	184.61	2826
24	20	20	10	186.35	2869

Table 11 illustrates a summary of the effect of unit-capacitor value on the total fixed and switched capacitor values, released power capacity and net annual saving. It was observed that the released power capacity increases with the increase on unit-capacitor value.

Table 11. Effect of unit-capacitor value on the optimal solution

Unit cap. (Kvar)	Fixed cap. (Kvar)	Max. switched cap. (Kvar)	Max. number of units	Released power (Kva)	Annual saving (L.E.)
10	270	260	53	158.14	2380
20	280	260	27	164.26	4003
30	300	300	20	176.37	4492

Fig.5 shows the change in both fixed and switched compensation level at each node in case of using 10 Kvar unit capacitor banks. The total fixed-compensation level is 270 Kvar located and sized as 130 Kvar at node 1, 90 Kvar at node 2 and 50 Kvar at node 3. The maximum switched banks are 110 Kvar at node 1, 100 Kvar at node 2 and 50 Kvar at node 3 with annual saving of 2380 L.E. and available released power capacity of 158 Kvar at substation node.

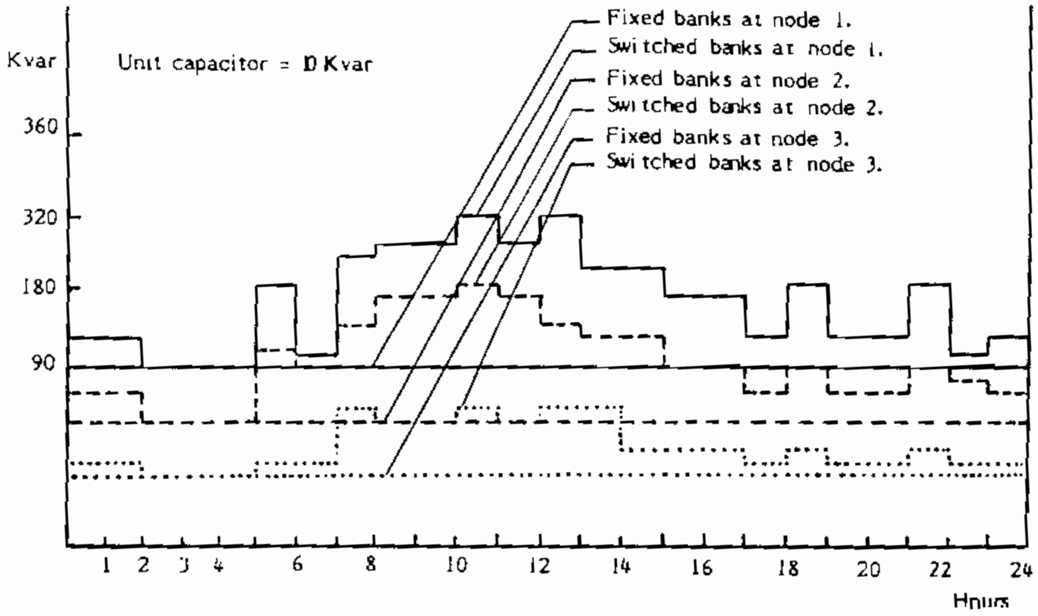

Fig.5 Time variation of fixed and switched capacitors at each node

8. CONCLUSIONS

The paper presents a proposed technique for optimal locating and sizing of fixed or fixed and switched capacitors in case of static loading or time-varying loads. The fixed banks are chosen according to the lower level of reactive power where the switched banks are optimally chosen according to the ammount of the power higher than the minimum one. A new analysis suitable for incorporating the time-varying loads in the optimal solution of capacitors is introduced. Also, special notation for defining the lateral and sublateral branches with the main feeder is presented. It was found that the growth in load, load factor and increase in energy cost must be taken into consideration when determining the optimal location and size of shunt capacitors. In corporation of these factors is very important specially for the planner engineers. The size of capacitor banks obtained by the proposed technique is a practical value on the contrary of the other methods which approximate the size of banks to the nearest one. The use of switched capacitors shown to be almost umeconomical, if the cost of released power capacity is not considered, due to its higher cost.

The proposed technique presented here is a simple generalized approach in solving location and size of shunt capacitors on distribution feeders and is applicable to any type of feeders with any pattern of loads. This technique is also very useful to carryout sensitivity analysis of the dependent cost components on capacitor compensation. The suggested algorithm has special advantage that the optimal capacitor allocation is limited by the off-peak period voltage-rise constraint and thus avoiding over voltage problems during that periods.

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