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OPTIMAL APPLICATION OF SHUNT COMPENSATION
FOR DISTRIBUTION SYSTEMS

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التطبيق الأمثل للتعويض على التوازي لسطم توزيع القوسى

الخلاصة:

نعمير تدنية المقاعد من اهم ما مدرس من شبكات الكهربية وقوسى
تناولت كثير من المراجع هذه الدراسة باستخدام مكثفات التوازي وذلك لتعويض
الطاقة الحثية .

تم فى هذا البحث دراسة تدنية المقاعد فى شبكات التوزيع باستخدام مكثفات
التوازي بتطبيق البرمجة الديناميكية لحساب القيم المثلى لكل من سعة ومكان وكذلك
زمن التوصيل لهذه المكثفات .

وتتلخص الاضافات فى هذا البحث فيما يلى :

- ١ - تطبيق البرمجة الديناميكية لحساب الاستخدام الأمثل لمكثفات التوازي الغير
دائمة الاتصال بالشبكة وذلك للمغذيات الشعاعية الوحيدة النهاية .
- ٢ - تطبيق البرمجة الديناميكية لحساب الاستخدام الأمثل لمكثفات التوازي وذلك
للمغذيات الشعاعية المتعددة النهايات سواء فى حالة المكثفات الدائمة
الاتصال أو الغير دائمة الاتصال بالشبكة .
- ٣ - مقارنة نتائج هذا البحث بنتائج الاساحات الاخرى التى تناولت نفس الموضوع .

Abstract

The minimization of power and energy loss in distribution
has a prominent role in power system design. A previous
developed procedures for optimizing the reduction of the
by using a specific number of shunt capacitive compensa
This paper presents a method to calculate the optimal
shunt capacitors in a distribution system with latera
The method is based on the dynamic programming
Numerical examples are tested and the results show
method provides more loss reduction than other appl

1-INTRODUCTION

The reduction of power and energy loss produced by the current flowing into a distribution system is an important objective . The shunt capacitors are used as a very effective tool for this concept. Particularly, with developing the switching and control schemes of these capacitors, their application should be more economical . Many papers have manipulated this aspect by using computational techniques to determine the optimal conditions. The optimality problem has been formulated to be subjected to minimizing the power and energy losses where the reactive power loss is neglected.

The optimal techniques have been used to solve this problem for radial feeders. These feeders have been taken as uniform size with uniform distributed loads connected with fixed and /or switched capacitors. The variables representing the optimal problem states are the size, location, time in service and the number of inserted capacitors [1-4].

The work done has extended to solve the problem for tapering feeders. An equivalent model has been derived. This model has a uniform resistance per unit length with the same losses of the original physical system. It has been analysed when applying shunt capacitors, fixed or switched, [5,6] . On the other hand, the optimal techniques used for this concept are either the dynamic programming (D.P.) [4], graphical methods [5], or decomposition approach [6]. In the later , the problem is decomposed into three subproblems :

- 1- Determination of optimum bank sizes using specified locations, and switching time .
- 2- Calculation of the optimum switching time using specified bank sizes and location.
- 3- Determination of optimum location, using specified bank size and switching time.

The optimal solution can be found by using iterative technique around the three subproblems.

The above stated works are dealing with the problems of capacitor location on such feeders using a sequential (straight away) feeder model which does not include lateral branches.

In [7], the author has presented an approach for capacitive compensation of distribution feeders involving lateral branches, which based upon the topological tree structure of the radial system. So, the radial distribution feeder may be considered to be composed of open paths. One of the open paths should be chosen as a main path. Additionally, each open path is considered as a set of continuous sections .For each section, the reactive current distribution function is constructed. Then, a function of a net saving due to an assumed number of fixed and /or switched capacitor can be developed. The optimal net saving can be found by using an iterative technique around the three subproblems namely, "optimal bank size, optimal location and optimal in service duration".

This paper introduces the applicability of DP to solve the optimal capacitive compensation problem. It is taken into consideration a sequential feeder and a feeder with lateral

branches connected with either fixed /or switched capacitors . It has been found that the developed technique in this paper leads to a saving in loss reduction greater than another applications, c.g., the decomposition approach used in (7). So, a complete comparison is introduced in this paper by applying the two techniques to four numerical examples, demonstrating the different forms of distribution feeders.

The next section illustrates the DP concept as a mathematical tool. The application of this concept to a sequential feeder with fixed and /or switched capacitors is explained in section 3. This application is developed in section 4 to apply the DP to a feeder with lateral branches. A brief presentation for the decomposition approach and the comparison are illustrated in sections 5 and 6, respectively. The conclusions are in section 7.

2- GENERAL MATHEMATICAL FORMULATION [8]

2-1 DYNAMIC PROGRAMMING APPROACH FOR SEQUENTIAL SYSTEMS

Consider a sequential system shown in Fig.1. Each stage m can be represented by the following input-output equation:

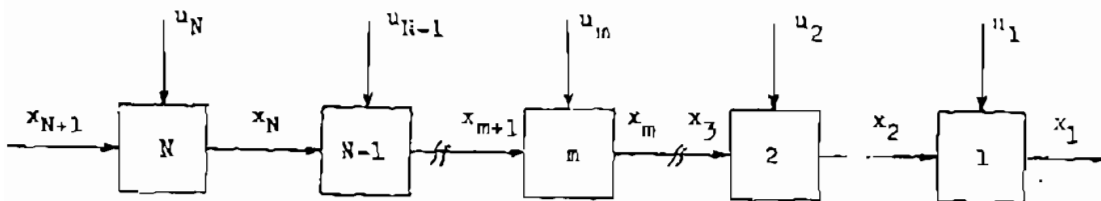


Fig. 1. A sequential system

$$x_m = T_m (x_{m+1}, u_m) \tag{1}$$

Where,

$$m = 1, 2, \dots, N$$

$$x_{m+1} = \text{input of stage } m,$$

$$x_m = \text{output of stage } m,$$

$$u_m = \text{the control variable of the stage } m,$$

$$T_m = \text{the function relating } x_{m+1} \text{ and } u_m$$

$$N = \text{no. of stages.}$$

Supposing that, the objective function Y is given in the form,

$$Y = \sum_{m=1}^N Y_m (x_m, x_{m+1}, u_m) \quad (2)$$

Where,

$Y_m (x_m, x_{m+1}, u_m)$ is the cost or profit function associated with stage m . Starting by stage no.1, the controller $u_1 \in U$ must be selected to optimize Y_1 at a specific value $x_2 \in X$.

The optimum value of the objective function Y_1 for stage no.1 is given by :

$$M_1(x_2) = (Y_1(x_1, x_2, u_2))^* \quad (3)$$

Where,

M_1 = the optimum value of the objective function Y_1 for stage no.1, and
 $*$ = the superscript denoting an optimal value.

Consequently, for the 2nd. stage :

$$M_2(x_3) = (Y_2(x_2, x_3, u_2) + M_1(x_2))^* \quad (4)$$

In general, for m th. stage, the cost is formulated by

$$M_m(x_{m+1}) = (Y_m(x_m, x_{m+1}, u_m) + M_{m-1}(x_m))^* \quad (5)$$

From equation (5), M_{m-1} & x_m are known from the calculation of stage no. $(m-1)$. Therefore, by selecting the controller $u_m \in U$ at a specified value of $x_{m+1} \in X$ which optimizes the m th function Y_m , the optimum cost M_m can be calculated. Varying m from 1 to N , the optimum controllers $u_m^* \in U$ and the corresponding optimal cost or profit Y_m are computed.

2.2 DYNAMIC PROGRAMMING APPROACH FOR SYSTEMS WITH LATERAL BRANCHES.

To apply the DP to a system with lateral branches, it is decomposed into subsystems to perform the sequential chains. Each sequential subsystem can be independently optimized. For example, consider the system shown in Fig.2. where at the link x_6

between stages no. 6 & 5, it is decomposed into two sequential subsystems I & II as shown in Fig.3. The input-output equations for the different stages of the subsystem I can be written as follows :

$$x_1 = T_1 (u_1, \bar{x}_2)$$

$$\bar{x}_2 = T_2 (u_2, x_3, x_2)$$

$$\bar{x}_3 = T_3 (u_3, \bar{x}_6)$$

$$x_5 = T_6 (u_6, x_7, x_6)$$

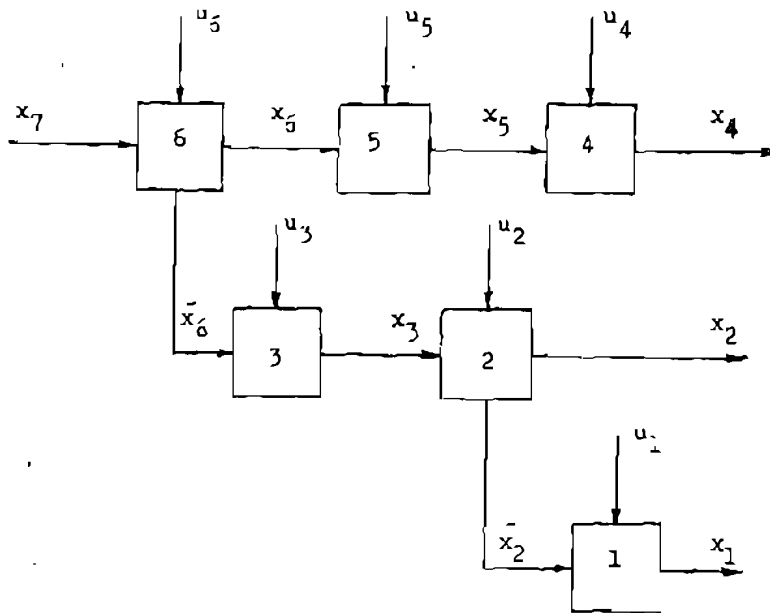


Fig. 2. Branching system

The optimum cost of subsystem I according to equation (5) is given by :

$$\begin{aligned}
 M_1(\bar{x}_2) &= (Y_1(x_2, x_1, u_1))^* \\
 M_2(x_3) &= (M_1(x_2) + Y_2(x_3, x_2, u_2))^* \\
 M_3(\bar{x}_6) &= (M_2(x_3) + Y_3(x_6, x_3, u_3))^* \\
 M_6(x_7) &= (M_3(x_6) + Y_6(x_7, x_6, u_6))^* \quad (6)
 \end{aligned}$$

For the sequential subsystem II the input-output equations are:

$$x_4 = T_4(u_4, x_5)$$

$$x_5 = T_5(u_5, x_6)$$

The optimal cost of stage no. 5 is defined by

$$\begin{aligned}
 M_4(x_5) &= (Y_4(x_5, x_4, u_4))^* \\
 M_5(x_6) &= (M_4(x_5) + Y_5(x_6, x_5, u_5))^* \quad (7)
 \end{aligned}$$

From equations (6) and (7), each subsystem can be optimized individually. The global optimum of the whole system can then be found by optimizing the link x_6 according to the following equation :

$$M(x_6) = (M_5 + M_6)^* \quad (8)$$

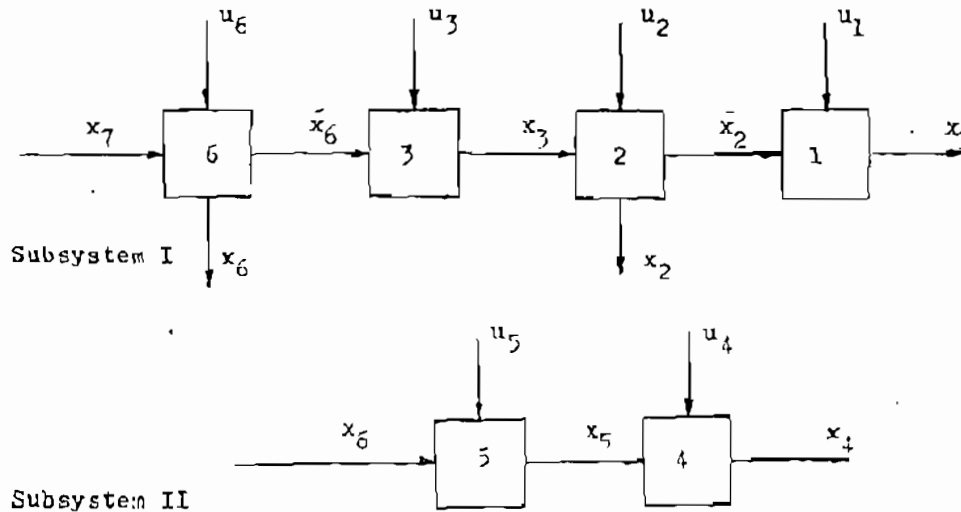


Fig. 3. The decomposed branching system

3- APPLICATION OF DYNAMIC PROGRAMMING TO RADIAL FEEDERS

3-1 THE SOLUTION ALGORITHM FOR RADIAL FEEDERS WITH FIXED CAPACITORS (SAF).

The following algorithm can be applied to the radial feeder (sequential) system shown in Fig.4. It is applied to find the optimal size, location and number of shunt capacitors required to minimize the power and energy losses by using the DP concept [4].

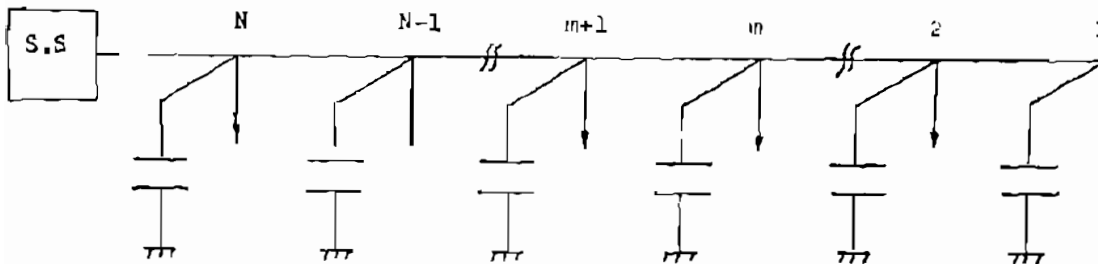


Fig. 4. Radial feeder.

The algorithm :

- The feeder nodes are labeled in an ascending order toward the source as shown in Fig.4.
- A uniform feeder of unity length equivalent to the physical feeder is defined[5]. This is formulated in two steps as follows :
 - a) L_u is the length of the equivalent feeder with a uniform resistance per unit length and can be calculated by

$$L_u = \sum_{m=1}^{N-1} \frac{r_m L_m}{r_k}$$

where,

L_m = length of branch m , and

r_k = resistance per unit length of branch k , which can be chosen as the resistance of the equivalent uniform feeder.

The physical length of section L_m must be modified to a length L_{um} where,

$$L_{um} = \frac{L_m r_m}{r_k}$$

- b) Divide each section length L_{uk} of the equivalent feeder by L_u to yield a normalized equivalent uniform feeder to unity length and uniform resistance r where,

$$r = \sum_{m=1}^{i=N-1} L_m r_m \text{ ohms per normalized unit length}$$

- A shunt capacitor bank may be installed at each node.
- Each node on the feeder can be considered as a stage of a sequential system. Fig.5, shows the node no. m , in which, i_{m+1} , i_m and i_{cm} represent the input, output and controller of the stage m , respectively.

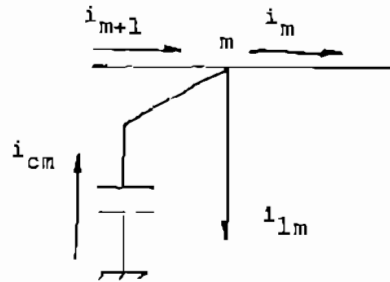


Fig. 5. The current at node m.

- Define the stage table as a table contains the elements :
 i_{m+1} , i_m , i_{cm} .

- For each node m, the elements of the stage table are formed as follows :

i) A discrete set I_m of output currents i_m at node m, which is

defined as $i_m \in I_m$ and $0 \leq i_m \leq i_{max}$, where i_{max} is

taken as the sum of the loads at nodes 1, 2, ..., m-1.

ii) A discrete set of capacitor size I_{cm} . It is chosen over an adequate range.

iii) A set of input currents, I_{m+1} , which is computed in terms of the elements of output current set and capacitor size set by using the following equation

$$i_{m+1} = i_m + i_{Lm} - i_{cm} \quad (9)$$

where,

$$i_{m+1} \in I_{m+1}$$

$$i_{cm} \in I_{cm} \quad \text{and}$$

$$i_{Lm} = \text{the load current at node m.}$$

iv) The saving S_m of section m which connects the node m+1 to node m and is defined as :

$$S_m = (k_p) (LP)_m + (k_e) (LG)_m - (i_{cm}) k_c \quad (10)$$

- where , $(LP)_m$ and $(IG)_m$ are the reduction of peak power and energy power losses at section m , respectively.
 k_p , k_e & k_c are the cost of peak power , the cost of energy and the cost of the installed capacitor at node m , respectively .
- v) The max.saving for section m can be depicted by using equations (9) & (10) for each pair of i_{m+1} and i_{cm} ;
- The output current at node no.1 equals zero.The optimal capacitor size and the input current at this node can be directly obtained. At the same time,the input current at node no.1 equals the output current at node no.2. Consequently,the optimal capacitor size and the input current at node no. 2 can be depicted. Repeating this process to compute the optimal variables at each node starting from the first node until the source node.

NUMERICAL EXAMPLE 1

The DP is applied to a radial feeder at 5.5 KV by using the algorithm SAF in sec. 3.1. Data of this feeder is tabulated in table 1 and Fig.6. shows the single-line diagram , which is taken from Fort-Found power network. The load curves at each node are as shown in Fig. 7. the base voltage and base volt-ampere are 5.5 KV & 3165 KVA, respectively. The cost constants are given as :

$$k_p = 120 \text{ LE /KW/year ,}$$

$$k_e = 0.015 \text{ LE/KWh/year}$$

$$k_c = \begin{cases} 6 & \text{LE/3-Ph. KVAR/year for switched capacitor} \\ 3.5 & \text{LE/3-Ph. KVAR/year for fixed capacitor.} \end{cases}$$

and the annual charge = 14.33 % per year .

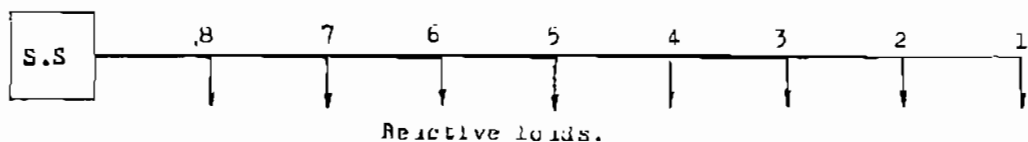


Fig. 6. One line diagram of example 1.

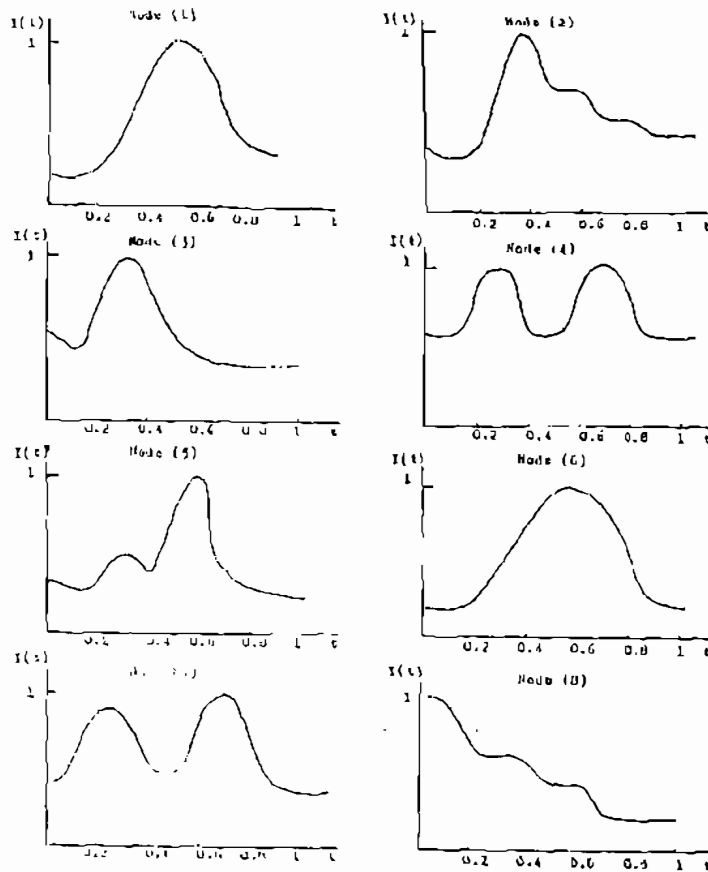


Fig. 7. Load curves

Table 1. Data of example (1).

| Section Number | Wire Size in mm | Overall Wiremeter mm | Resistanc CU /KN | Physical Branch Length in KM | KVAR Load at end sec. |
|----------------|-----------------------|----------------------------|------------------------|---------------------------------------|-----------------------------|
| 1 | 3x50 | 51 | 0.387 | 1.5 | 50 |
| 2 | 3x50 | 51 | 0.387 | 2.2 | 50 |
| 3 | 3x50 | 51 | 0.387 | 0.8 | 50 |
| 4 | 3x50 | 51 | 0.387 | 1.5 | 200 |
| 5 | 3x50 | 51 | 0.387 | 2.76 | 300 |
| 6 | 3x50 | 51 | 0.387 | 1.65 | 165 |
| 7 | 3x50 | 51 | 0.387 | 0.7 | 600 |
| 8 | 3x185 | 69 | 0.099 | 1.55 | 1750 |

Table 2. Stage table of node 8 for example 1.

| $I_{c8} \backslash I_B$ | 0.0 | 0.104 | 0.145 | 0.29 |
|-------------------------|---------------|--------------|---------------|---------------|
| 0.05 | 0.503 2.15 | 0.6 1.84 | 0.648 1.66 | 0.793 1.06 |
| 0.1 | 0.453 2.2 | 0.55 1.99 | 0.588 1.87 | 0.743 1.27 |
| 0.15 | 0.403 2.39 | 0.5 2.1 | 0.548 1.99 | 0.693 1.47 |
| 0.2 | 0.353 2.49 | 0.45 2.27 | 0.498 2.1 | 0.643 1.68 |
| 0.25 | 0.303 2.58 | 0.4 2.38 | 0.448 2.26 | 0.593 1.8 |
| 0.3 | 0.253 2.65 | 0.35 2.48 | 0.398 2.3 | 0.543 1.9 |
| 0.35 | 0.203 2.7 | 0.3 2.57 | 0.348 2.38 | 0.493 2.1 |
| 0.4 | 0.153 2.77 | 0.25 2.6 | 0.298 2.58 | 0.443 2.25 |
| I_g | | | | |
| $s_8 \backslash I_{c8}$ | 0.153 2.77 | 0.4 2.6 | 0.298 2.58 | 0.443 2.25 |

Considering node 8 as an example, the following results are obtained :

- The upper and lower elements of the output currents set are zero & 0.29 P.U., respectively.
- The chosen elements of capacitor sizes set are : 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35 & 0.4 P.U.

- The input current set I_g is computed by equation (9) to get the set elements 0.503, 0.453, 0.403, 0.353, 0.203 & 0.153 PU

Similarly, the saving of section 8 according to equation (10) is,

$$S_8 = 2.15, 2.2, 2.39, 2.49, 2.58, 2.65, 2.7 \text{ \& } 2.77 \text{ L.E/year}$$

Therefore the max. saving is 2.77 L.E/year at $I_{cg} = 0.04 \text{ KVAR}$ & $I_g = 0.153 \text{ KVAR}$. These results are depicted in table 2 (the stage table of node 8). The other stage tables for the rest of nodes can be constructed.

Table 3. Optimal policy for example 1.

| I_m m | 0 | 0.0157 | 0.051 | 0.07 | 0.08 | 0.104 | 0.106 | 0.145 |
|--------------|-------------|-------------|------------|--------------|----------------|---------------|----------------|--------------|
| 8 | | | | | | 0.25 2.5 | 0.4 | 0.239 2.5 |
| 7 | | | | | | | 0.145 0.077 | 0.1 |
| 6 | | | | | | 0.104 0.05 | 0.05 | |
| 5 | | | | | 0.104 0.019 | 0.05 | | |
| 4 | | | | 0.0 0.055 | 0.05 | | | |
| 3 | | | 0.047 0 | 0 | | | | |
| 2 | | 0.0157 0 | 0 | | | | | |
| 1 | 0.0157 0 | 0 | | | | | | |

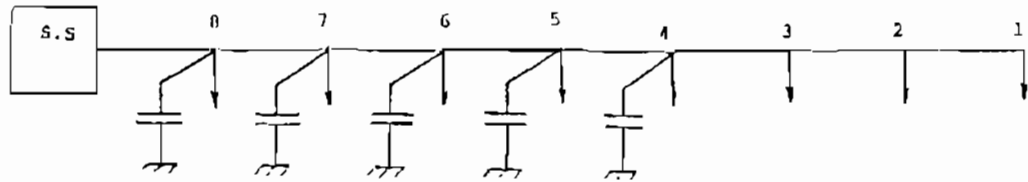


Fig. 8. Single line diagram of the feeder of example 1 showing the optimum compensation capacitor.

The optimal policy table, table 3, is derived by gathering the last row of stage tables to decide the optimal route. This optimal route gives a total saving of 15,221 LE/year. According to this policy the optimum capacitor sizes are tabulated in table 4. Fig. 8, shows the optimum compensation capacitors connected to the feeder of example 1.

Table 4. Optimum solution of example 1 according to the optimal policy shown in table 3.

| Capacitor location No. of node | Capacitor size KVAR | Annual Saving LE/year |
|-----------------------------------|------------------------|--------------------------|
| 4 | 158 | 269 |
| 5 | 158 | 1959 |
| 6 | 158 | 2051 |
| 7 | 475 | 2776 |
| 8 | 1266 | 8166 |
| Total | | 15221 |

3.2 THE SOLUTION ALGORITHM FOR SEQUENTIAL FEEDERS WITH SWITCHED CAPACITORS

The reactive power variation of a load necessitates to connect the feeder with switched capacitors to keep a desired matching between this variation and the compensators.

This section presents an algorithm, "Discretized Load Algorithm" (D.L.A), to solve the optimal capacitive compensation problem by using the DP. This solution is based upon discretizing the load curves at different nodes into incremental times. The reactive Table 5. Optimal policy for example 1 at the first time increment

Loads at each time increment are assumed to be constant. Then the SAF is applied for this increment to get the optimal parameters. By repeating this procedure for the succeeding increments, the optimal strategy can be obtained. The following example, example 2, illustrates the application of DLA to the feeder of example 1.

Example 2 : Find the optimal strategy of the feeder in example 1 when connecting it with switched capacitors. The load curves shown in Fig.7. should be considered.

Solution : The duration of load curves is divided into equal five increments of 0.2 P.H.

The basic principle of choosing the time increments is that the load variation during each increment is as small as possible. Considering the first time increment, the load currents at feeder nodes are shown in Fig. 9. according to the given load curves.

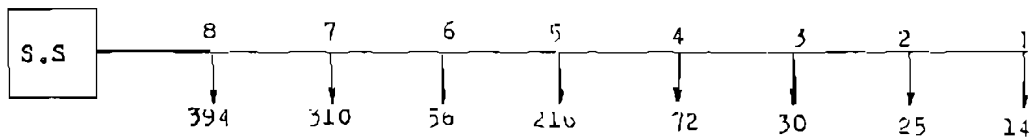


Fig 9. Reactive load at the first increment for example 1.

By applying SAF, the optimal policy table at this increment can be constructed as shown in table 5. Similar tables for the other increments are constructed to be able to deduce the optimal strategy. The rating of the multiple-tap capacitor, which realizes the optimal strategy for node 8, is specified in Fig. 10. The optimal strategy for the feeder by using multiple - tap and on / off switched capacitors is shown in fig. 11.

Table 5. Optimal policy for example 1 at the first time increment

| I_m | 0 | 0.012 | 0.034 | 0.035 | 0.049 | 0.092 | 0.119 |
|-------|------------|------------|----------------|---------------|---------------|----------------|--------------|
| 8 | | | | | | | 0.125 0.8 |
| 7 | | | | | | 0.119 0.576 | 0.25 |
| 6 | | | | | | 0.092 0.433 | 0.05 |
| 5 | | | | | 0.049 0.53 | 0.15 | |
| 4 | | | | 0.019 0.04 | 0.05 | | |
| 3 | | | 0.034 0.004 | 0.05 | | | |
| 2 | | 0.012 0 | 0 | | | | |
| 1 | 0.012 0 | | | | | | |

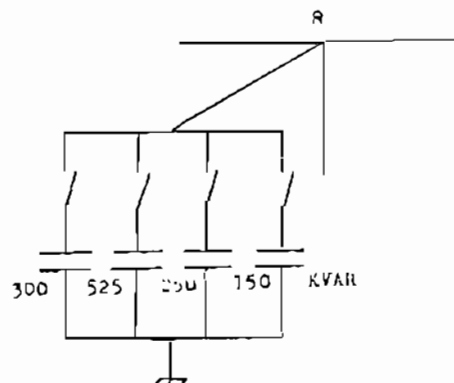


Fig. 10. The multiple- tap capacitor of node 8.

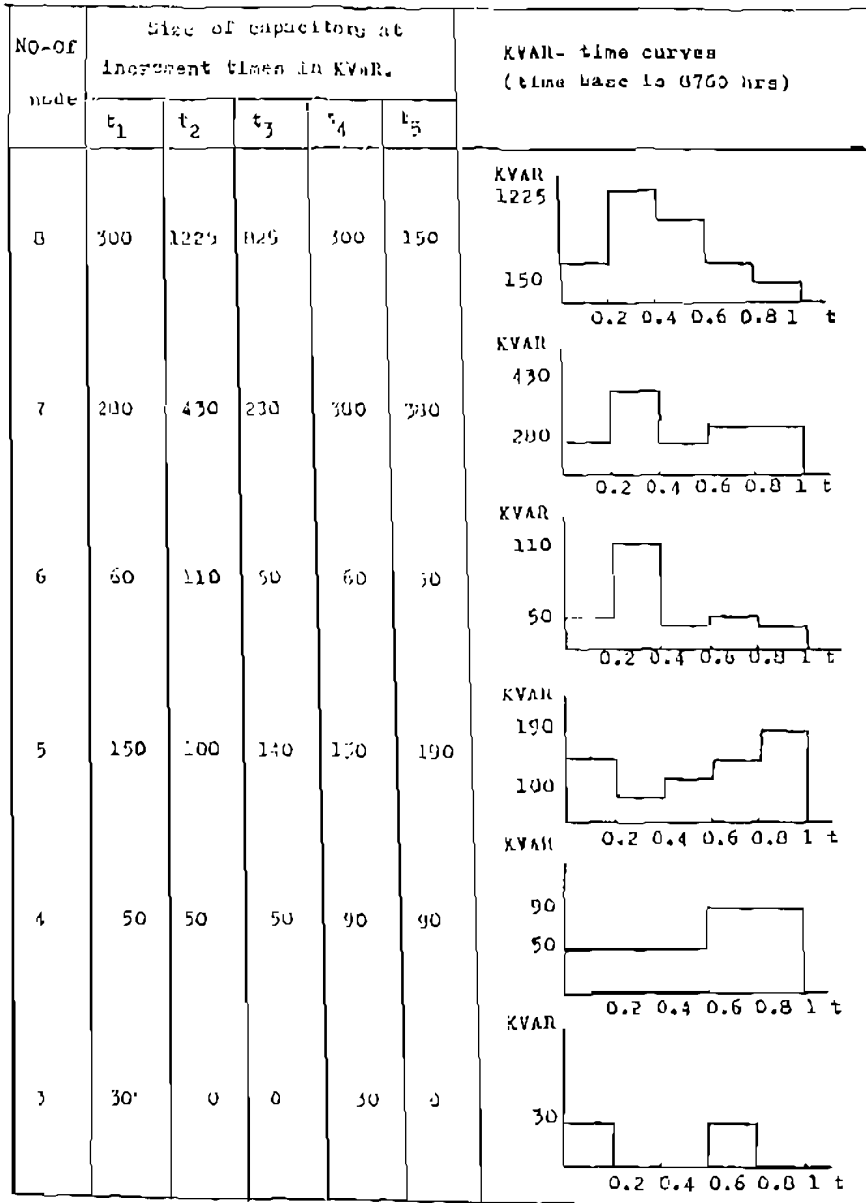


Fig. 11 . The optimal strategy of the compensation by using multi - tap and on / off switched capacitors.

4. APPLICATION OF DYNAMIC PROGRAMMING TO A FEEDER WITH LATERAL BRANCHES

4.1 APPLICATION TO FEEDER WITH FIXED CAPACITORS

As explained in section 2.2, the DP is applied by choosing a main path J starting at the substation (s.s) and terminated at an arbitrarily end point k , Fig. 12. The system laterals ($L = 1, 2, \dots, n$), separated from the main path, are manipulated individually by using SAF to calculate their own optimal policy. From which, the currents flowing into these laterals $i_1, \dots, i_L, \dots, i_n$ (input currents) are computed.

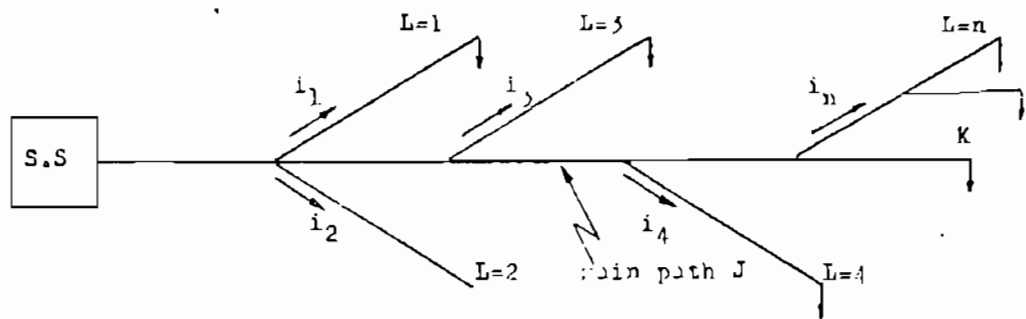


Fig. 12. Feeder with lateral branches.

Then, a global optimal strategy of the system is determined by applying the following summarized iterative concept :

- a desired lateral no. L , its input current i_L is considered as a control variable,
- the other input currents i_j , $j = 1, \dots, n$ & $j \neq L$, are considered by their updated values in time being,
- the DP is applied to the main path J to calculate the optimal current i_L^* ,
- replace i_L by i_L^* ,
- calculate the gain of saving S_g on the main path. It is the difference between the preceding saving and the saving due to the obtained optimal input current i_L^* ,
- calculate the loss of saving S_{loss} of the lateral L due to current change,

- If $S_g \geq S_{loss}$, the next steps are executed. Unless, the preceding input current is taken as an optimal value to implement the next steps,
 - repeat this procedure for all laterals L , $L = 1, \dots, n$.
 - the program is terminated when the change of system saving is within a specific tolerance,
- The detailed flow chart of this iterative concept is shown in Fig. 13.

NUMERICAL EXAMPLE 3

In order to illustrate the optimal specifications of fixed capacitors along a radial feeder involving lateral branches by using DP, a 5.5 Kv feeder is chosen as in Fig. 14. The data of this feeder is tabulated in table 6.

Table 6. Data of example 3.

| Section number. | Cross section area in mm ² . | Overall diameter in mm. | Resistance Ohm in Ω /km . | Physical branch length in Km. | KVAR at end of section. |
|-----------------|---|-------------------------|----------------------------------|-------------------------------|-------------------------|
| 0- 8 | 3 105 | 69 | 0.0991 | 1.65 | 0 |
| 8- 9 | 3 50 | 51 | 0.373 | 0.425 | 450 |
| 9-10 | 3 50 | 51 | 0.370 | 1.525 | 100 |
| 8-17 | 3 50 | 51 | 0.379 | 0.375 | 100 |
| 17-12 | 3 50 | 51 | 0.378 | 0.18 | 100 |
| 17-18 | 3 50 | 51 | 0.378 | 0.125 | 200 |
| 18-20 | 3 50 | 51 | 0.370 | 0.625 | 100 |
| 18-19 | 3 50 | 51 | 0.370 | 0.35 | 450 |
| 8- 7 | 3 50 | 51 | 0.370 | 1.3 | 250 |
| 7-11 | 3 50 | 51 | 0.370 | 0.7 | 400 |
| 7- 6 | 3 50 | 51 | 0.370 | 1.25 | 400 |
| 6-16 | 3 50 | 51 | 0.378 | 1.68 | 60 |
| 15-16 | 3 50 | 51 | 0.370 | 0.475 | 30 |
| 6- 5 | 3 50 | 51 | 0.378 | 0.55 | 75 |
| 5-12 | 3 50 | 51 | 0.370 | 2.76 | 0 |
| 12-14 | 3 50 | 51 | 0.370 | 1 | 0 |
| 12-13 | 3 50 | 51 | 0.373 | 1 | 100 |
| 5- 4 | 3 50 | 51 | 0.373 | 1 | 200 |
| 4- 3 | 3 50 | 51 | 0.370 | 1.5 | 200 |
| 3- 2 | 3 50 | 51 | 0.370 | 0.75 | 50 |
| 2-11 | 3 50 | 51 | 0.370 | 1.5 | 100 |

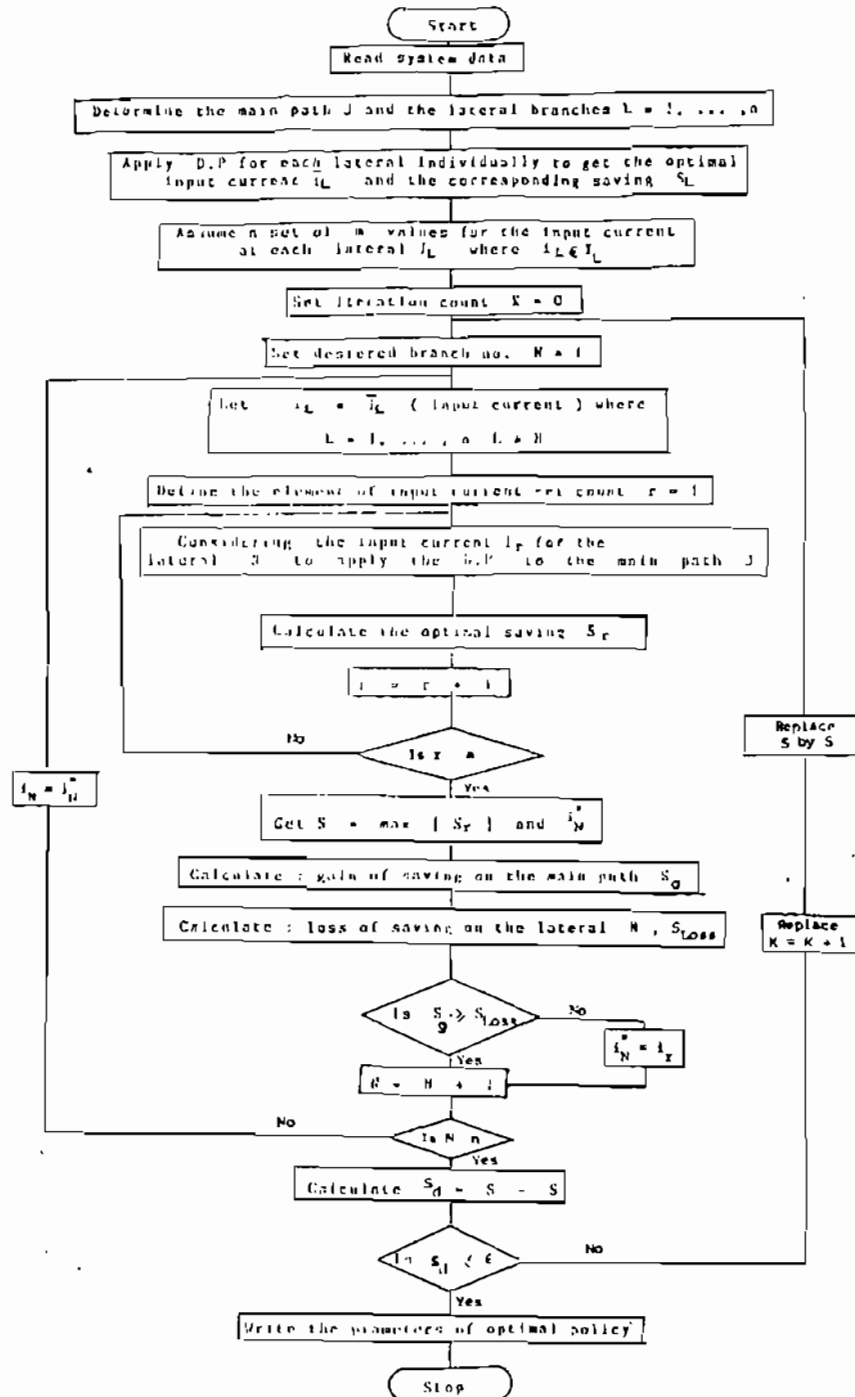


Fig. 13. Flow chart of the application of dynamic programming to a feeder with lateral.

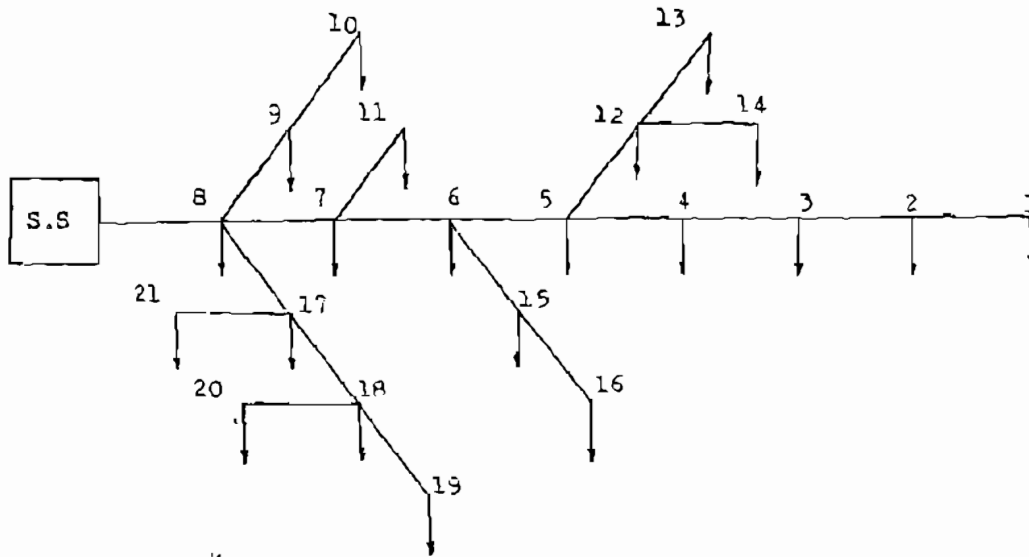


Fig. 14. Single line diagram of exaple 3.

The main path is the feeder starting from s.s. and terminated by node 1. The system contains eight lateral branches. By applying SAF algorithm to the lateral branch, $l = 1$, (nodes 8, 9 & 10) as an example, the stage tables for nodes 9 & 10 are computed and given in Table 7. & Table 8., respectively.

Table 7. The stage table for node 9.

| | | | | |
|--------------|----------------|----------------|----------------|----------------|
| I_{c9} | 0.0065 | 0.01 | 0.01 | 0.03 |
| 0.025 | 0.23 0.052 | 0.227 0.052 | 0.137 0.418 | 0.147 0.05 |
| 0.05 | 0.09 0.03 | 0.132 0.03 | 0.12 0.9 | 0.122 0.04 |
| 0.075 | 0.037 0.004 | 0.077 0.015 | 0.007 0.63 | 0.037 0.06 |
| 0.1 | 0.048 0.07 | 0.052 0.014 | 0.002 0.1 | 0.072 0.08 |
| 0.12 | 0.073 0.018 | 0.027 0.07 | 0.037 0.095 | 0.047 0.012 |
| I_{n9} | 0.022 | 0.074 | 0.115 | 0.047 |
| ϵ_9 | 0.12 0.037 | 0.1 0.044 | 0.1 0.037 | 0.12 0.037 |

Table 8. Stage table for node 10.

| | | | |
|----------------------------|-----------|--------|--------|
| I_{c10} \ \bar{I}_{10} | | 0 | |
| | | 0.0065 | 0.0095 |
| 0.025 | | 0.0065 | 0.025 |
| I_{10} \ S_{10} | I_{c10} | 0.0096 | |

\bar{I}_{10} is the output current of node 10
 I_{c10} is the capacitor current at node 10

Consequently , the optimal policy table of this lateral can be deduced . It is given in Table 9. Therefore, the input current following into this lateral at node 8 is 0.023 PU.

Table 9. Optimal policy table for lateral L = 1

| | | | | | |
|-----------------|--|--------|--------|--------|-------|
| m \ \bar{I} | | 0 | | 0.0065 | |
| | | | | 0.023 | 0.125 |
| 9 | | | 0.0996 | | |
| 10 | | 0.0065 | | | |
| | | 0.0096 | 0.025 | | |

\bar{I} is the output current of node under consideration

Similarly, the input currents for other branches are determined and shown in FIG. 19.

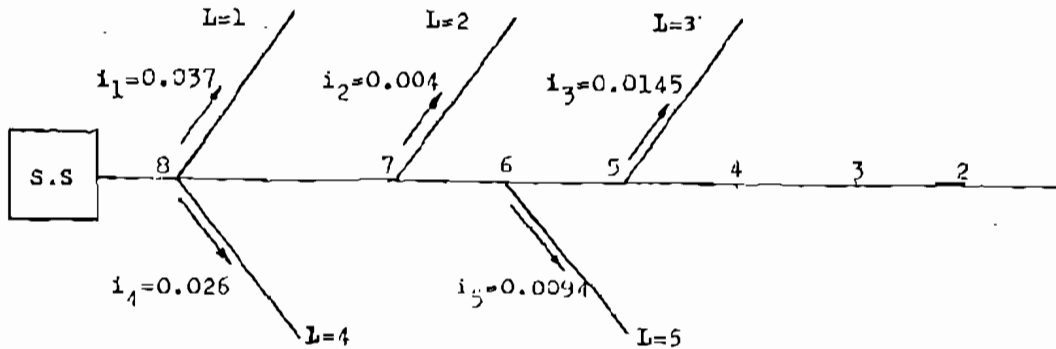


Fig. 15. Input currents for lateral branches (in PU)

Adding each input current to the load current (if any) to get the currents at the different main path nodes, Fig. 16. Then, the initial required capacitors are determined by using SAF, Fig. 17. The iterative concept is executed, and the results are given in Table 10 and illustrated by Fig. 18.

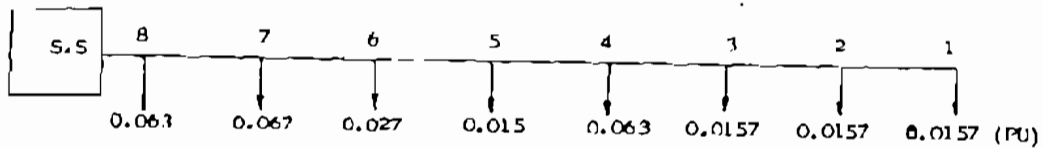


Fig. 16. The main path after adding the lateral currents.

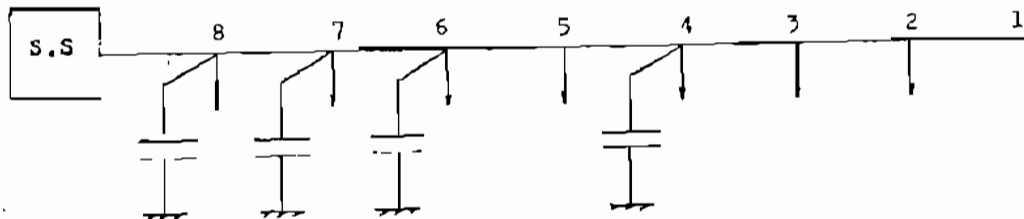


Fig. 17. The main path with initial required capacitors

4.2 APPLICATION TO FEEDERS WITH SWITCHED CAPACITORS

This problem is solved by applying the technique DLA in section 3.2 . It necessitates to discretize the load-duration curves into time increments with approximately constant loads. For each time increment, the supply is manipulated as a feeder with laterals using fixed capacitors . So, the technique in section 4.1 is implemented to get the optimal capacitor size and location at each time interval . Then, the optimal strategy for the total period can be obtained.

NUMERICAL EXAMPLE 4

Using the system in example 3 with load duration curves shown in Fig. 19 to get the optimal switched capacitors specifications (size, location and in service time duration).

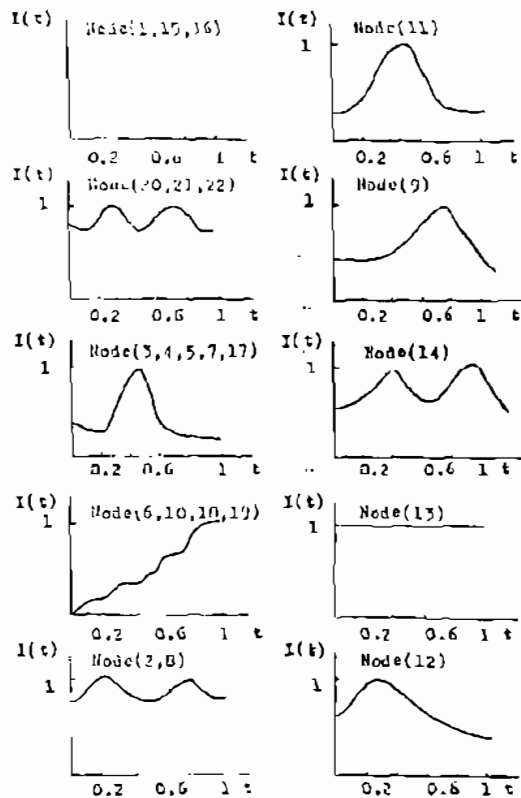


Fig. 19. Load current (PU) at each node of numerical example 4.

- The load duration curves are discretized into five increments, each of 0.2 PU. fig. 20 shows the loads at the different nodes during the first time increment as an example.
- Applying the technique of section 4.1 for each time increment. The optimal strategy can be obtained and the results are given in Fig. 21.

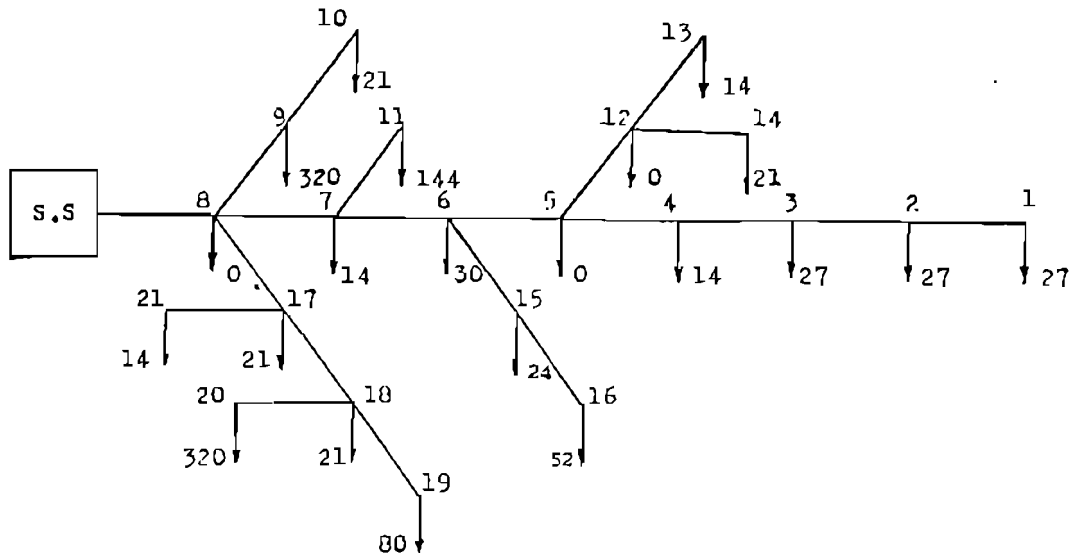


Fig. 20. Loads (in KVAR) at the first time increment

The effectiveness of the introduced techniques in this paper can be proved by comparing them with the technique " Decomposition approach " which has been done in [7]. The main concept of decomposition are illustrated in the next two sections.

5. DECOMPOSITION APPROACH

This approach is based upon a given number of capacitors. Its methodology can be summarised as follows :

- The problem is formulated as an optimisation problem.
- The system variables which must be calculated optimally are : capacitor size, location and its service time duration.
- An objective function is determined to represent the saving in terms of system variables.

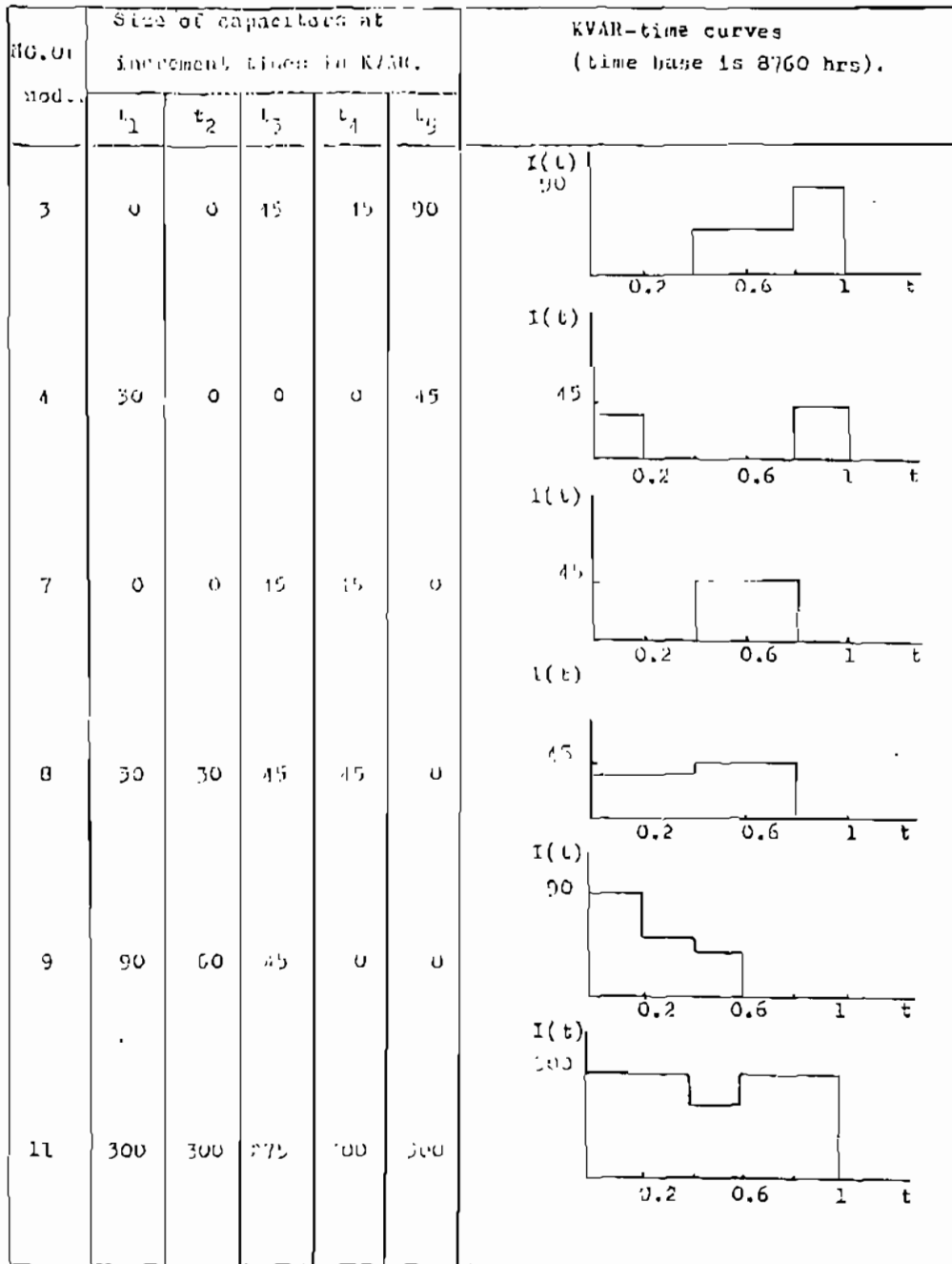


Fig. 21. Optimal strategy of the compensation by using multi-tap and on/off switched capacitors.

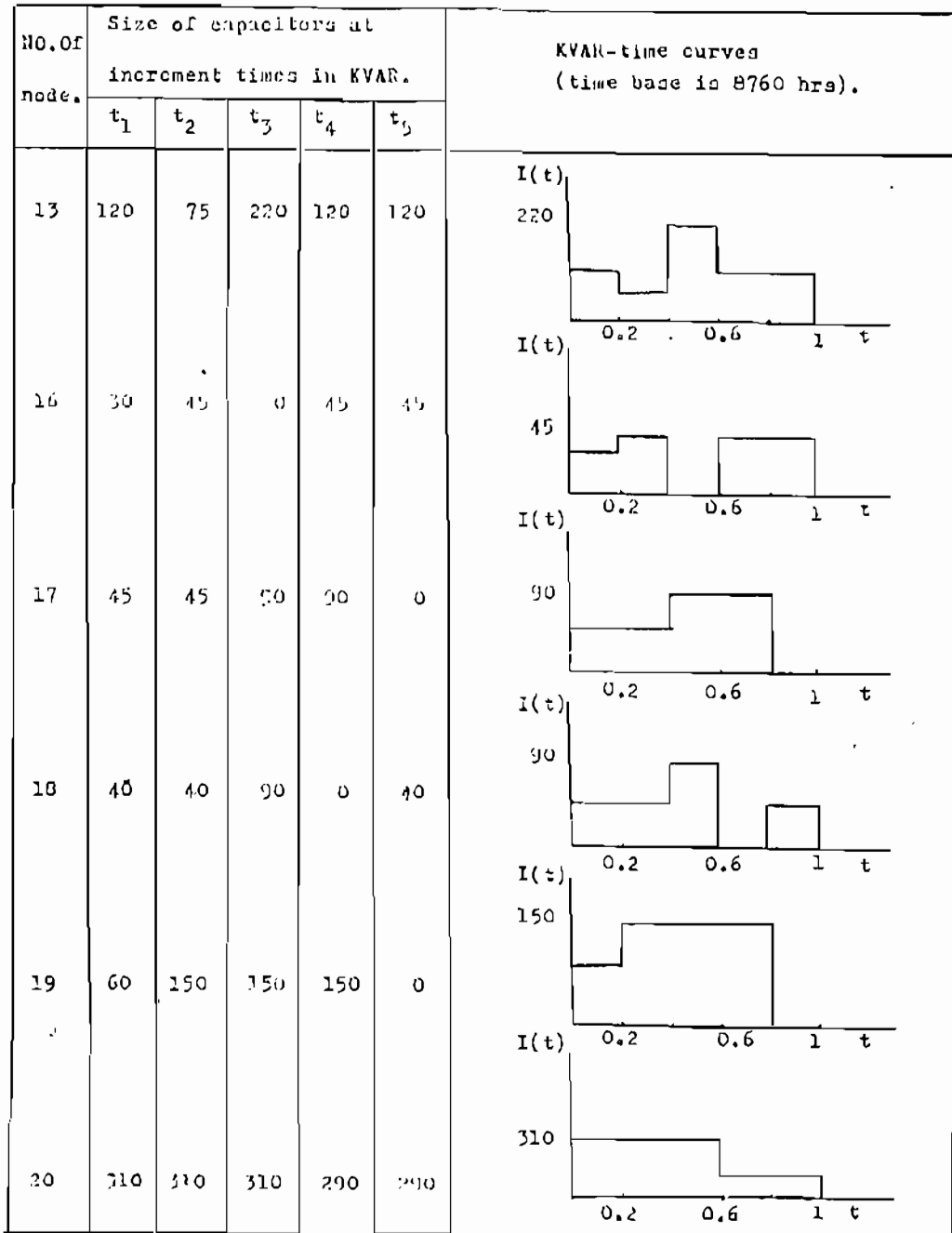


Fig. 22. Cont. optimal strategy of the compensation by using multi - tap and on / off switched capacitors.

- The problem is solved by decomposing it into three subproblems as shown in the flow chart, Fig 23.

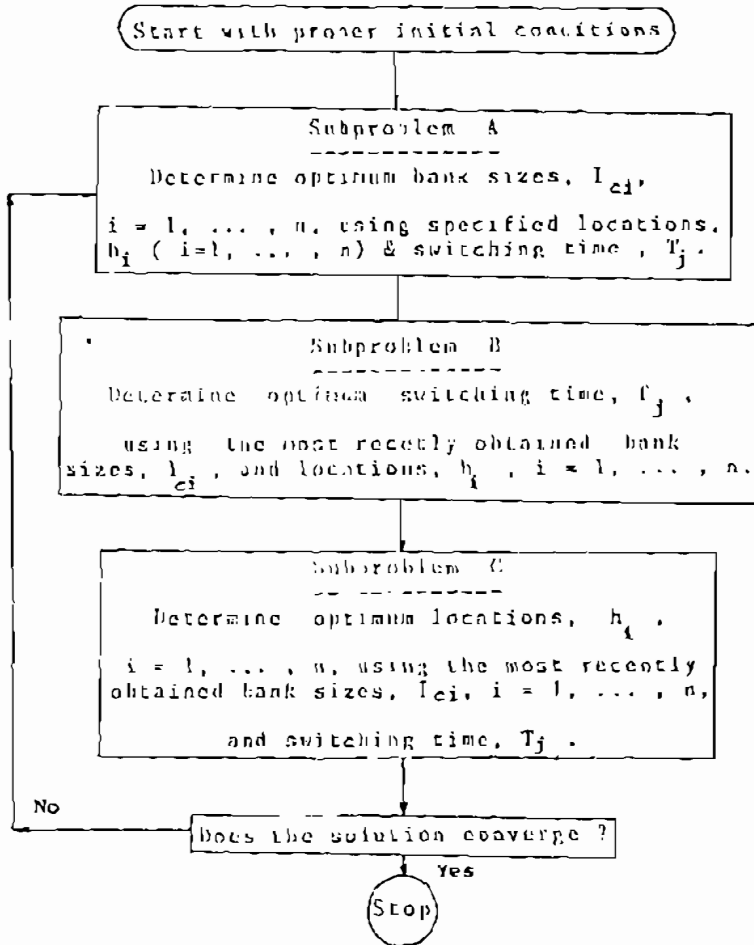


Fig. 23. Iterative solution using decomposition approach .

For more details, The reader is recommended to see reference [7].

6. Comparison

Applying the decomposition approach (D. A.) using three capacitors to examples 1, 2, 3 & 4, the results are tabulated in tables 11 and 12 to be compared with the result of DP application

From tables 11, 12 it is seen that :

| Application of DA | Application of DP |
|---|--|
| -It is restricted by a specific number of capacitors. | -It drives the optimal number of capacitors. |
| -The load duration curve is considered at the substation to calculate the switching time. | -Different load duration curves are considered at the feeder nodes which are more realistic. |
| -It gives a saving of 12042, 12585, 11925, 12960 LE/year | - It gives a corresponding saving of 15221, 16649, 12894, 16853 LE/year |

Table 11. Comparison table for fixed capacitors

| Dynamic Programming | | | Decomposition Approach | | |
|-----------------------------|---------------------|---------------------------|----------------------------|----------------|-----------------|
| capacitor locations (nodes) | optimal size (KVAR) | annual net saving LE/year | capacitor location (nodes) | opt. size KVAR | ann. net saving |
| 4 | 158 | | 5 | 443 | |
| 5 | 158 | | 7 | 369 | |
| 6 | 158 | 15222 | 8 | 1234 | 12042 |
| 7 | 475 | | | | |
| 8 | 1266 | | | | |
| 4 | 158 | | 11 | 373 | |
| 6 | 158 | | 20 | 512 | |
| 7 | 158 | 12894 | 9 | 474 | 11925 |
| 8 | 79 | | | | |
| 9 | 396 | | | | |
| 10 | 79 | | | | |
| 11 | 396 | | | | |
| 13 | 158 | | | | |
| 16 | 79 | | | | |
| 17 | 79 | | | | |
| 18 | 1158 | | | | |
| 19 | 237 | | | | |
| 20 | 477 | | | | |

Table 12. Comparison table for switched capacitors

| Dynamic Programming | | | | | | | | | Decomposition Approach | | | | |
|-----------------------------|--|-----|-----|-----|--|------|------|-------|---------------------------|----------------------------|-------------------|----------------------------|-------------------|
| capacitor locations (nodes) | optimal size for each capacitor tap KVAR | | | | in - service duration for each tap(FU) | | | | annual net saving LE/year | capacitor location (nodes) | optimal size KVAR | in - service duration (FU) | annual net saving |
| 3 | 30 | 00 | 00 | 00 | 0.4 | 0.00 | 0.00 | 0.00 | 0.00 | 5 | 614 | 0.6 | |
| 4 | 30 | 40 | | | 1 | 0.4 | | | | 7 | 718 | 1 | |
| 5 | 100 | 40 | 50 | | 1 | 0.4 | 0.4 | 16449 | | 8 | 1069 | 0.2 | 12355 |
| 6 | 50 | 10 | 100 | | 1 | 0.4 | 0.2 | | | | | | |
| 7 | 280 | 100 | 50 | | 1 | 0.6 | 0.2 | | | | | | |
| 8 | 150 | 150 | 525 | 700 | 1 | 0.8 | 0.4 | 0.2 | | | | | |
| | | | | | | | | | | | | | |
| 3 | 45 | 45 | 00 | 00 | 0.5 | 0.2 | 0.00 | | | 11 | 379 | 0.6 | |
| 4 | 30 | 15 | | | 0.4 | 0.2 | | | | 20 | 490 | 1 | |
| 7 | 45 | | | | 0.4 | | | 16853 | | 9 | 506 | 0.8 | 12960 |
| 8 | 30 | 15 | | | 0.4 | 0.4 | | | | | | | |
| 9 | 45 | 15 | 30 | | 0.8 | 0.4 | 0.2 | | | | | | |
| 11 | 275 | 25 | | | 1 | 0.8 | | | | | | | |
| 13 | 100 | 75 | 45 | | 0.2 | 1 | 0.8 | | | | | | |
| 16 | 30 | 15 | | | 0.8 | 0.8 | | | | | | | |
| 17 | 45 | 45 | | | 0.8 | 0.5 | | | | | | | |
| 18 | 40 | 50 | | | 0.2 | 0.2 | | | | | | | |
| 19 | 60 | 50 | | | 0.8 | 0.6 | | | | | | | |
| 20 | 290 | 20 | | | 1 | 0.6 | | | | | | | |

7. Conclusions

The application of DP to optimize the shunt capacitor influence on power system losses has been illustrated. This paper has manipulated two types of distribution feeders; sequential feeders and feeders with lateral branches. In addition, the use of shunt compensators is either by fixed or switched capacitors. A developed technique involving the DP concept has been introduced particularly for the feeder with laterals. This developed technique enables us to formulate the problem as an optimization problem in such way that the formulation is convenient for system conditions. Further more, it has been found that the developed technique in this paper leads to saving in loss reduction greater than that has been obtained by analysing the system by decomposing it into three subproblems (Decomposition Approach).

8. REFERENCES

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