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INVESTIGATION OF INCOMPRESSIBLE ROUGH-WALL BOUNDARY LAYER FLOWS

بحث فى انسياب الطبقة الجدارية اللاانضغاطية على الاسطح الخشنة

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خلاصة : اقترح "كولز" (١) ان يمثل توزيع السرعة فى الطبقة الجدارية المضطربة وفى وجود تدرج عكسى على الاسطح الملساء، كجمع خطى شامل لدالتين - احداهما قانون الحائط المعروف والاخر قانون الخصرية الذى يعرف الشكل عند نقطة الانفصال او الاتصال.

واهداف هذا البحث هو تطوير قانون "كولز" لتوزيع السرعة لحساب حسابات الطبقة الجدارية المضطربة على الاسطح الملساء والخشنة، ويدرس تحليليا معاملات الطبقة الجدارية المختلفة ومعامل السطح الخشن المشتق من توزيع السرعة المطور ونتائج بعض التجارب المعملية للانسياب على الاسطح الخشنة المعطاه فى مرجع (١٠).

ABSTRACT

COLES [1] suggested to represent the velocity distribution in turbulent boundary layers with adverse pressure gradient over smooth surfaces by a linear combination of two universal functions. One is the well known law of the wall, the other the law of the wake which is characterised by the profile at a point of separation or reattachement. The aims of this investigation are to modify Coles velocity distribution to perform the calculations for turbulent boundary layers over smooth and rough surfaces and also to study analytically the various boundary layer parameters and surface roughness function derived from the modified velocity distribution and experimental results with flows over rough surfaces given in [10].

NOMENCLATURE :

A	velocity profile parameter	
B	constant , eq. (2.2.1)	
c_∞	free stream velocity	(m/s)
$\bar{c}$	velocity at the outer edge of the boundary layer	(m/s)
c_τ	friction velocity $\sqrt{\tau_w/\rho}$	(m/s)
$c_\tau k/\nu$	dimensionless roughness height	
$c_\tau y/\nu$	dimensionless distance normal to the wall	
c_f	local skin friction coefficient , $\tau_w/0.5 \cdot \rho \bar{c}^2$	
c_x	velocity of the fluid inside the boundary layer in x-direction	(m/s)
c_y	velocity component inside the boundary layer in y-direction	(m/s)
$C(\frac{c_\tau k}{\nu})$	surface roughness function	
C_r	constant of surface roughness function	
H_{12}	boundary layer form parameter , δ^* / δ^{**}	

I	boundary layer shape parameter , $\int_0^{\infty} (\frac{\bar{c} - c_x}{c_\tau})^2 \cdot d(\frac{y\, c_\tau}{\delta^* \bar{c}})$	
k	roughness height	(m)
p	velocity profile parameter	
Re_k	roughness height Reynolds number , $\frac{\bar{c} \cdot k}{\nu}$	
$w(\frac{y}{\delta})$	wake function	
x	coordinate in the direction of the wall	(m)
y	coordinate normal to the direction of the wall	(m)
δ	boundary layer thickness	(m)
δ^*	boundary layer displacement thickness , $\int_0^{\infty}(1 - \frac{c_x}{\bar{c}})\, dy$	(m)
δ^{**}	boundary layer momentum thickness, $\int_0^{\infty} \frac{c_x}{\bar{c}}\,(1 - \frac{c_x}{\bar{c}})\, dy$	(m)
$\varkappa$	von Kármán's universal constant	
Λ	Euler number , $-\frac{1}{\bar{c}} \cdot \frac{d\bar{c}}{dx}\, \delta^{**}$	
ν	kinematic viscosity of fluid	(m^2/s)
π	pressure gradient parameter . $-\frac{1}{\bar{c}} \cdot \frac{d\bar{c}}{dx} \cdot \frac{\delta^{**}}{\tau_w / \rho\, \bar{c}^2}$	
ρ	density of fluid	(kg/m^3)
τ_w	wall shear stress	(N/m^2)

1- INTRODUCTION

Solution of boundary layer problems depends on the velocity distribution and whether the flow is laminar or turbulent. Of the two, the laminar boundary layer is more amenable to analytical solution and there is an extensive literature on laminar boundary layer alone. The complication increases rapidly when the boundary layer becomes turbulent. Its flow structure is so complex that it can not be solved by the exact methods. This complexity also increases when the flow is over rough surfaces. Calculation and analysis methods of the flow of turbulent boundary layers over rough surfaces make use of the smooth wall flow predication methods as a guide. These methods expressed the effect of the surface roughness on the velocity distribution by the roughness function C ($c_\tau \cdot k/\nu$). A summarised survey of these methods will be mentioned as a literature review.

Nikuradse [2] has shown in a pipe flow that roughness leads to a downward shift in c_x/c_τ . He adopted the velocity distribution :

$$\frac{c_x}{c_\tau} = A + B \cdot \ln \frac{y}{k} \qquad \ldots (1.1)$$

with B as a constant and A = 8.48 for sand - grain roughness

Rotta [3] and independently Ross- and Robertson [4] proposed a simple approximation for the velocity profile as :

$$\frac{c_x}{c_\tau} = \frac{1}{\varkappa} \cdot \ln \frac{c_\tau \cdot y}{\nu} + \frac{2A}{\varkappa} \cdot \frac{y}{\delta} + C\left(\frac{c_\tau \cdot k}{\nu}\right) \qquad 0 \leq y \leq \delta \qquad \ldots (1.2)$$

where A denotes a free parameter and $\varkappa$ von Kármán's constant. The roughness function $C(c_\tau \cdot k/\nu)$ takes the value 5.2 for smooth surface, and for completely rough surface it is given by :

$$C\left(\frac{c_\tau \cdot k}{\nu}\right) = C_r - \frac{1}{\varkappa} \ln \frac{c_\tau \cdot k}{\nu} \qquad \ldots (1.3)$$

with C_r constant of surface roughness function.

Rotta [5] plotted the relation $c_f = c_f\,(k/\delta^{**}, H_{12})$ for rough surface with sand grain roughness based on Nikuradse [2] (experimental results) with the assumption that the roughness function is not affected by the existense of pressure gradient.

In [6] an analytical study of equation (1.2) is introduced. The different boundary layer parameters are calculated on the basis of the experiments achieved in [10].

Clauser [7,8] and Hama [9] used the form of the velocity distribution for flow over rough walls with zero pressure gradient as :

$$\frac{c_x}{c_\tau} = \frac{1}{\varkappa} \cdot \ln \frac{c_\tau \cdot y}{\nu} + B - C\left(\frac{c_\tau \cdot k}{\nu}\right) \qquad \ldots (1.4)$$

with $\varkappa$ and B are 0.4 and 5.2 respectively.

The surface roughness function, $C(c_\tau \cdot k/\nu)$ in equation (1.4) is different than that in equation (1.2) because of the introduction of the constant, B. Its value is zero for flow over smooth walls.

The Clauser form of the roughness function for fully rough flow is :

$$C\left(\frac{c_\tau \cdot k}{\nu}\right) = \frac{1}{\varkappa} \cdot \ln \frac{c_\tau \cdot k}{\nu} + C_1 \qquad \ldots (1.5)$$

where C_1 is a constant.

Allan [11] suggested another velocity distribution. The various factors and functions are given for pipe flows, flat plate flows, and diffusing flows with uniform sand grain roughness are further examined in [12 , 13].

2. GOVERNING EQUATIONS

Coles [1] proposed a velocity profile expression that can be used to predict the mean velocity distributions. It is given by :

$$\frac{c_x}{c_\tau} = \underbrace{\frac{1}{\varkappa} \cdot \ln \frac{c_\tau \cdot y}{\nu} + B}_{\text{law of the wall}} + \underbrace{\frac{P}{\varkappa} \cdot w\left(\frac{y}{\delta}\right)}_{\text{law of the wake}} \qquad \ldots (2.1)$$

where P is the velocity profile parameter, and $w\left(\frac{y}{\delta}\right)$ is the law of the wake. $\varkappa$ and B are two empirical constants of 0.4 and 5.1 values.

2.1 Wake Function

The wake function $w\,(y\,/\,\delta)$ is normalized to be zero at the wall and have a value $w\,(y\,/\,\delta) = 2.0$ at $y\,/\,\delta = 1.0$ also $\int_0^2 (y\,/\,\delta)\, dw = 1$. For flows over smooth surfaces, Coles [1] gave it in a table form. Several forms of the wake function have been presented by different authors. It is given to a good approximation by the empirical fit:

$$w\left(\frac{y}{\delta}\right) = 2.\sin^2\left(\frac{\pi}{2}.\frac{y}{\delta}\right) = 1 - \cos\left(\pi.\frac{y}{\delta}\right) \qquad \ldots (2.1.1)$$

The disadvantage of this approximation is that $\partial c_x / \partial y$ evaluated from equation (2.1) is nonzero at $y = \delta$: it equals $c_\tau / \varkappa.\delta$.

Moses [14] gave the wake function in a polynomial approximation in the form:

$$w\left(\frac{y}{\delta}\right) = 6\left(\frac{y}{\delta}\right)^2 - 4\left(\frac{y}{\delta}\right)^3 \qquad \ldots (2.1.2)$$

A better choice for the wake, originally due to Finley et.al. [15], is to replace the term $P.w\,(y\,/\,\delta)$ by :

$$P.\,w\left(\frac{y}{\delta}\right) = (1 + 6P).\left(\frac{y}{\delta}\right)^2 - (1 + 4P).\left(\frac{y}{\delta}\right)^3 \qquad \ldots (2.1.3)$$

which for $\partial c_x / \partial y = 0$ at $y = \delta$.

Rotta [16] gave it as :

$$w\left(\frac{y}{\delta}\right) = 39\left(\frac{y}{\delta}\right)^3 - 125\left(\frac{y}{\delta}\right)^4 + 183\left(\frac{y}{\delta}\right)^5 - 133\left(\frac{y}{\delta}\right)^6 + 38\left(\frac{y}{\delta}\right)^7 \ldots (2.1.4)$$

Strehle [17] has put it in the following formula :

$$w\left(\frac{y}{\delta}\right) = 2.1\left(\frac{y}{\delta}\right)^2 + 14.3\left(\frac{y}{\delta}\right)^3 - 30\left(\frac{y}{\delta}\right)^4 + 20.5\left(\frac{y}{\delta}\right)^5 - 5\left(\frac{y}{\delta}\right)^6 \qquad \ldots (2.1.5)$$

which satisfies the first normalizing condition $w(o) = o$ but not the second condition and gives for $w(1) = 1.9$.

A mathematical correlation was done in [18] on Strehle's formula and leads to:

$$w.\left(\frac{y}{\delta}\right) = 2.124\left(\frac{y}{\delta}\right)^2 + 14.344\left(\frac{y}{\delta}\right)^3 - 30.027\left(\frac{y}{\delta}\right)^4 + 20.527\left(\frac{y}{\delta}\right)^5 - 4.968\left(\frac{y}{\delta}\right)^6 \ldots (2.16)$$

and satisfies both values for normalizing conditions.

2.2 Modified Velocity Distribution

The modified Coles velocity distribution which contains the contribution of the effect of surface roughness, $C(c_\tau.k/\nu)$, is expressed in the form:

$$\frac{c_x}{c_\tau} = \frac{1}{\varkappa}.\ln\frac{c_\tau.\,y}{\nu} + B + \frac{P}{\varkappa}.\,w\left(\frac{y}{\delta}\right) + C\left(\frac{c_\tau.\,k}{\nu}\right) \qquad \ldots (2.2.1)$$

The last term of equation (2.2.1), represents the surface roughness function which equals zero in the case of smooth surface, while the flow over completely rough surface it takes the form :

$$C\left(\frac{c_\tau.\,k}{\nu}\right) = C_r - \frac{1}{\varkappa}.\ln\frac{c_\tau.\,k}{\nu} \qquad \ldots (2.2.2)$$

where C is the constant of surface roughness function. At the outer edge of the boundary layer ($y \geq \delta$, $c_x = \bar{c}$) eq. (2.2.1) yields :

$$\frac{\bar{c}}{c_\tau} = \frac{1}{\varkappa} \cdot \ln \frac{c_\tau \cdot \delta}{\nu} + B + \frac{2P}{\varkappa} + C\left(\frac{c_\tau \cdot k}{\nu}\right) \qquad \ldots (2.2.3)$$

The velocity distribution inside the boundary layer can be deduced from equations (2.2.1) and (2.2.3), it yields :

$$\frac{c_x}{\bar{c}} = 1 + \sqrt{\frac{\tau_w}{\rho \cdot \bar{c}^2}} \cdot \left[\frac{1}{\varkappa} \cdot \ln \frac{y}{\delta} - \frac{2P}{\varkappa}\right] + \sqrt{\frac{\tau_w}{\rho \cdot \bar{c}^2}} \cdot \frac{P}{\varkappa} \cdot w\left(\frac{y}{\delta}\right) \qquad \ldots (2.2.4)$$

with $\frac{c_\tau}{\bar{c}} = \sqrt{\frac{\tau_w}{\rho \cdot \bar{c}^2}}$.

2.3 Boundary Layer Parameter

The two boundary layer parameters ; defined in the nomenclature ; the dimensionless displacement thickness δ^*/δ & the dimensionless momentum thickness δ^{**}/δ are obtained by substituting the value of $c_x/\bar{c}$ in their formula, so that :

$$\frac{\delta^*}{\delta} = \frac{1}{\varkappa} \sqrt{\frac{\tau_w}{\rho \cdot \bar{c}^2}} \cdot (1 + P) \qquad \ldots (2.3.1)$$

and

$$\frac{\delta^{**}}{\delta} = \frac{1}{\varkappa} \sqrt{\frac{\tau_w}{\rho \cdot \bar{c}^2}} \cdot (1 + P) - \frac{1}{\varkappa} \frac{\tau_w}{\rho \cdot \bar{c}^2} \cdot (2 + 3.2P + 1.522\, P^2) \qquad \ldots (2.3.2)$$

The remaining boundary layer parameters such as the form parameter, H_{12}, the shape parameter, I, and the pressure gradient parameter, π , may be represented by separate formulas. They are :

Form parameter $$H_{12} = \frac{\delta^*/\delta}{\delta^{**}/\delta} \qquad \ldots (2.3.3)$$

Shape parameter $$I = \left(1 - \frac{1}{H_{12}}\right) \cdot \frac{1}{\sqrt{\frac{\tau_w}{\rho \bar{c}^2}}} = \frac{2 + 3.2P + 1.522P^2}{\varkappa (1 + P)} \qquad \ldots (2.3.4)$$

Pressure gradient parameter $$\pi = \Lambda \cdot \frac{H_{12}}{\frac{\tau_w}{\rho \cdot \bar{c}^2}} \qquad \ldots (2.3.5)$$

where Λ is Euler number .

The profile parameter P can be estimated by solving equation (2.3.4) and given as:

$$P = \frac{1}{3.044}\left[\varkappa \cdot I - 3.2 + \sqrt{\varkappa^2 \cdot I^2 - 0.321\, \varkappa \cdot I - 1.936}\right] \qquad \ldots (2.3.6)$$

2.4 Roughness Function

The calculation of the constant of surface roughness function, C_r, can be achieve through the following procedure: substituting the roughness function $C\left(\frac{c_\tau \cdot k}{\nu}\right)$ from equatic (2.2.2) into equation (2.2.1) one gets :

$$\frac{c_x}{c_\tau} = -\frac{1}{\varkappa}\left[\ln\frac{k}{y} - P.\, w\left(\frac{y}{\delta}\right)\right] + B + C_r \qquad \ldots (2.4.$$

At the outer edge of the boundary layer ($y = \delta$, $c_x = \bar{c}$) eq. (2.4.1) gives :

$$\frac{\bar{c}}{c_\tau} = -\frac{1}{\varkappa}\left[\ln\frac{k}{\delta} - 2P\right] + B + C_r \qquad \ldots (2.4.$$

The ratio k / δ can be written as $(k / \delta^{**})(\delta^{**} / \delta^{*})(\delta^{*} / \delta)$ and using equations (2.3.1) ar (2.3.2) , the last equation becomes :

$$\frac{\bar{c}}{c_\tau} = -\frac{1}{\varkappa}\left[\ln\frac{k}{\delta^{**}} + \ln\frac{1}{H_{12}} + \ln\left(\sqrt{\frac{\tau_w}{\rho.\bar{c}^2}} \cdot \frac{1+P}{\varkappa}\right) - 2P\right] + B + C_r$$

and C_r can be given by :

$$C_r = \frac{1}{\sqrt{\frac{\tau_w}{\rho \cdot \bar{c}^2}}} - B + \frac{1}{\varkappa} \cdot \ln\frac{1+P}{\varkappa} - \frac{2P}{\varkappa} + \frac{1}{\varkappa}\ln\left(\frac{k}{\delta^{**}} \cdot \frac{1}{H_{12}} \cdot \sqrt{\frac{\tau_w}{\rho.\bar{c}^2}}\right) \qquad \ldots (2.4.$$

So, C_r can be evaluated when P, k/δ^{**}, H_{12} and $\sqrt{\tau_w/\rho.\bar{c}^2}$ are knowns .

The effect of the roughness height Reynolds number, Re_k, on the surface roughnes function $C\,(c_\tau \cdot k/\nu)$ can be achieved by defining it as :

$$Re_k = \frac{\bar{c} \cdot k}{\nu} = \frac{\bar{c}}{c_\tau} \cdot \frac{c_\tau \cdot \delta}{\nu} \cdot \frac{\delta^*}{\delta} \cdot \frac{\delta^{**}}{\delta^*} \cdot \frac{k}{\delta^{**}}$$

and from equation (2.2.3), (2.3.1) and (2.3.3),

$$Re_k = \frac{1+P}{\varkappa} \cdot \frac{1}{H_{12}} \cdot \frac{k}{\delta^{**}} \cdot \exp\left\{\varkappa\left[\sqrt{\frac{\rho \cdot \bar{c}^2}{\tau_w}} - B - \frac{2P}{\varkappa} - C\left(\frac{c_\tau \cdot k}{\nu}\right)\right]\right\} \qquad \ldots (2.4.4$$

The surface roughness function $C\,(c_\tau \cdot k/\nu)$ can be obtained from the previous equation it yields :

$$C\left(\frac{c_\tau \cdot k}{\nu}\right) = \sqrt{\frac{\rho \cdot \bar{c}^2}{\tau_w}} - B - \frac{1}{\varkappa} \cdot \left[\ln Re_k - \ln\left(\frac{1+P}{\varkappa} \cdot \frac{1}{H_{12}} \cdot \frac{k}{\delta^{**}}\right) + 2P\right] \qquad \ldots (2.4.5$$

Equation (2.4.5) was used to evaluate the surface roughness function, $C\left(\frac{c_\tau \cdot k}{\nu}\right)$, foi different values of the roughness height Reynolds number at constant ratios of k/δ^{**}.

3- Results And Discussion

The variation of the velocity profile parameter, P, with both of Euler number, Λ, the local skin friction coefficient, $\tau_w / \rho . \bar{c}^2$, and the pressure gradient parameter, π, for ratios of k/δ = 0.05, 0.08, 0.10 and 0.30 are given in figures (1), (2) and (3) respectively. For boundary layers at the same value of k/δ, the velocity profile parameter, P, increases as Euler number, Λ, increases; Fig. (1); this is in conjuction with a decrease in the value of the local skin friction coefficient, $\tau_w / \rho . \bar{c}^2$; Fig. (2). At separation, i.e., $\tau_w \rightarrow 0$, values of P tend to infinity [19]. For the same value of the of the velocity profile parameter, P=constant, both Euler number Λ and local skin friction coefficient, increases as the ratio of, k/δ, increases.

Fig. (3) represents the change in the velocity profile parameter, P, with the pressure gradient parameter, π. For flow over rough surfaces it is given empirically as [13]

$$\pi = (P - 0.55)(1.60456 + 0.420645\, P) \qquad \ldots (3.1)$$

with P = 0.55, i.e., flow over flat plate at zero incidence.

White [20] suggested a power-law expression :

$$P \approx 0.8\,(\pi + 0.5)^{0.75} \qquad \ldots (3.2)$$

which is arranged to fit the theoretical requirement [Mellor and Gibson [21]] that the wake vanish at π = - 0.5, corresponding to an assymptotically large favorable gradient.

Das and White [22] gave a new empirical correlation between, P, and , π, for flows with adverse gradient unseparated flow as :

$$\pi = 0.76\, P + 0.42\, P^2 \qquad \ldots (3.3)$$

Equations (3.1), (3.2) and (3.3) are represented in Fig. (3) for comparison with the data points. An approximate correlation between, P, and , π, for these data can be obtained in the form:

$$P = 1.4\sqrt{\pi} \quad \text{for} \quad \pi \geqslant 0.6 \qquad \ldots (3.4)$$

which shows a good agreement in the given range.

The range of the pressure gradient parameter, π, for self preserving boundary layers is $-0.5 \leq \pi \leq \infty$ given by Mellor and Gibson [21]. The decelerating flows and accelerating flows corresponds to $\pi > 0$, $\pi < 0$ respectively. A relation between the shape parameter, I, and the pressure gradient parameter exsists in the form $I = I(\pi)$ given in a table form [21]. Nash [23] give an approximate relation between I and π for self-preserving flows in the form:

$$I \simeq 6.1\,(\pi + 1.81)^{0.5} - 1.7 \qquad \ldots (3.5)$$

Figure (4) illustrates the relation between I and π for ratios of k/δ^{**} = 0.05, 0.08, 0.10 and 0.30. It is compared with the relations giver by Nash [23] and Mellor and Gibson [21]. Data points are adjarent to the curve of [21]. A correlation $I = I(\pi)$ can be derived from equations (2.3.4) and (3.4) and approximated as :

$$I \cong 4.2 - 5.33\sqrt{\pi} \quad \text{for} \quad \pi \geqslant 0.6 \qquad \ldots (3.6)$$

It is representec in the same figure and fits better with the experimental data .

Figure (5) represents the relation between the constant of surface roughness function,

C_r, and the ratio, k/δ^{**}, with Euler number as parameter.

The influence of Euler number, Λ, the dimensionless wall shear stress, $\tau_w/\rho \cdot \bar{c}^2$, and the velocity profile parameter, P, on the surface roughness function, C ($c_\tau \cdot k/\nu$), are illustrated in Figs. (6 to 8). These figures are given for values of the ratio k/δ^{**} = 0.05 and 0.30 with the roughness height Reynolds number, Re_k, as a parameter. At constant k/δ^{**} and Λ; Fig. (6) ; the function C ($c_\tau \cdot k/\nu$) increases as Re_k decreases. The same can be observed at constant k/δ^{**} and $\tau_w/\rho \cdot \bar{c}^2$; Fig. (7) ; or at constant k/δ^{**} and P ; Fig. (8). Also, at the same Re_k the function C ($c_\tau \cdot k/\nu$) decreases as the ratio k/δ^{**} increases.

In figure (9) the velocity profiles at ratios of k/δ^{**} = 0.05 and 0.30 for different Euler number ranging between 0.0 and $3 \cdot 10^{-3}$ are plotted. For boundary layer at the same ratio of k/δ^{**}, the velocity distribution, $c_x/\bar{c}$, decreases with the increase of Euler number, Λ. Also, for constant Euler number, Λ, the velocity distribution decreases slightly with the increase of the ratio k/δ^{**}.

With the increase of the value of, Λ, (i.e., $> 3 \cdot 10^{-3}$) the flow approaches to separation from the surface, the value of $c_\tau \rightarrow 0$, then the C ($c_\tau \cdot k/\nu$) and ($c_\tau \cdot k/\nu$) approaches zero. In this region the surface changes from fully rough to transitionally rough and finally smooth surface at a value of $c_\tau \simeq 0$.

4- CONCLUSIONS

On the basis of the modified Coles velocity distribution for turbulent boundary layer over rough surfaces and the application of the experimental results of [10], the following conclusions are obtained:

(1) Two new empirical correlations between the velocity profile parameter, P, and the pressure gradient parameter, π, and the shape parameter, I, and, π, have been derived from the experimental data.

(2) The velocity profile parameter, P, increases with the increase of Euler number, Λ, and the decrease of the ratio. k/δ^{*}.

(3) The surface roughness function, C($c_\tau \cdot k/\nu$); for the boundary layer with the same ratio of k/δ^{**} and the roughness height, Re_k; increases with the increase of both Euler number Λ, and the relocity profile parameter, P, and the decrease of the dimensionless wall shear stress, $\tau_w/\rho \cdot \bar{c}^2$.

REFERENCES

[1]. Coles, D. "The Law of the Wake in the Turbulent boundary Layer". Journal of Fluid Mech., Vol.1, pp-191-226, 1956.

[2] Nikuradse, J., "Strömungsgesetze in rauhen Rohren" V.D.I. Forschungsheft, no. 361, 1933.

[3] Rotta, J.C. "Beitrag Zur Berechnung der turbulenten Grenzschichten". Ing. Arch. 19, pp. 31-41, 1951.

[4] Ross, D., Robertson, J.M. "A Superposition Analysis of the Turbulent Boundary Layer in an Adverse Pressure Gradient". Journal of Applied Mech., Vol.18, pp. 95-100, 1951.

[5] Rotta, J.C "Turbulent Boundary Layer in Incompressible Flows". Progress in Aeronoutical Sciences, Vol.2, pp.1-220, Pergamon Press, 1962.

[6] Hanna, S.F., Djebedjian, B.O. "Geschwindigkeitsverteilung in turbulenter Grenzschicht, die gegen Druck längs rauhen wand strömt". Maschinenbautechnik, Berlin 37 (1988) H.11.

[7] Clauser, F.H. "Turbulent Boundary Layer in Adverse Pressure Gradients". Journal of Aeronautical Sciences, pp. 21-91, 1954.

[8] Clauser, F.H. "The Turbulent Boundary Layer" Advances in Applied Mechanics, Vol.4, pp. 1-51, Academic Press, 1956.

[9] Hama, F.R. "Boundary Layer Characteristics of Smooth and Rough Surfaces". Trans.society of Naval Architects and Marine Engineers, Vol.62, pp. 333 , 1954.

[10] Zancow, S. "Integration der Impulsgleichung mit Berücksichtigung der Wandrauhigkeit und des Druckgradienten der Aussenströmung." Dissertation TU-Dresden, 1974.

[11] Allan, W.K. "Velocity Distribution in Turbulent Flow". Journal Mech. Engg. Sci. Vol.12, No.6, 1970 , pp 391-399.

[12] Allan, W.K., Sharma, B. "An investigation of Two Turbulent Flows over Smooth and Rough Surfaces". Journal of Mechanical Engineering Sci., Vol.16, p.71, 1974.

[13] Aswatha Narayana, P.A. "Turbulent Boundary Layer Velocity Profiles over a Smooth Surface". Applied Scientific Research Vol.33, pp. 427-435, 1977.

[14] Moses, H.L. "Ph. D. Thesis, Massachusetts Institute of Technology, Cambridge, Mass. 1964. (Also, M.I.T. Gas turbine Lab. Rep.73).

[15] Finley, P.J., Khoe, K.C., Foh, C.J. "Velocity Measurements in a Thin Turbulent Water Layer". La Houille Blanche, Vol.21, No.6, pp. 713-721, 1966.

[16] Rotta, J.C., "Fluid Mechanics of Internal Flow". (G. Sovran ed.) Elsevier, Amsterdam, 1967.

[17] Strehle, E. "Zur Integration der Bewegungs Gleichungen Für ebene Wandgrenzschichten". Wissen. Zeitschrift der TU-Dresden 23 (1974) H.1.

[18] Saad, M.S. "An investigation of Some Turbulent Boundary Layer Parameters". M. Sc. Thesis, Mansoura University 1980.

[19] Hinze, J.O. "Turbulence" p.697. 2nd Edition, McGraw Hill Book company New-York, 1975.

[20] White, F.M. "Viscous Fluid Flow" McGraw Hill, New York, 1974, pp - 481-482.

[21] Mellor, G.L., Gibson, D.M. "Equilibrium Turbulent Boundary Layers" Journal of Fluid Mechanics, Vol.24, Part 2, pp. 225-253, 1966.

[22] Das, D.K., White, F.M. "Integral Skin Friction Prediction for Turbulent Separated Flows". ASME Journal of Fluids Engineering, Vol.108, Dec. 1986, pp. 476-4

[23] Nash, J.F. "Turbulent Boundary Layer Behaviour and the Auxiliary Equation". National Physics Laboratory Aeronautical Report 1137, England 1965. Also, AGARDOGRAPH 97, part 1, 1965, pp. 245-279.

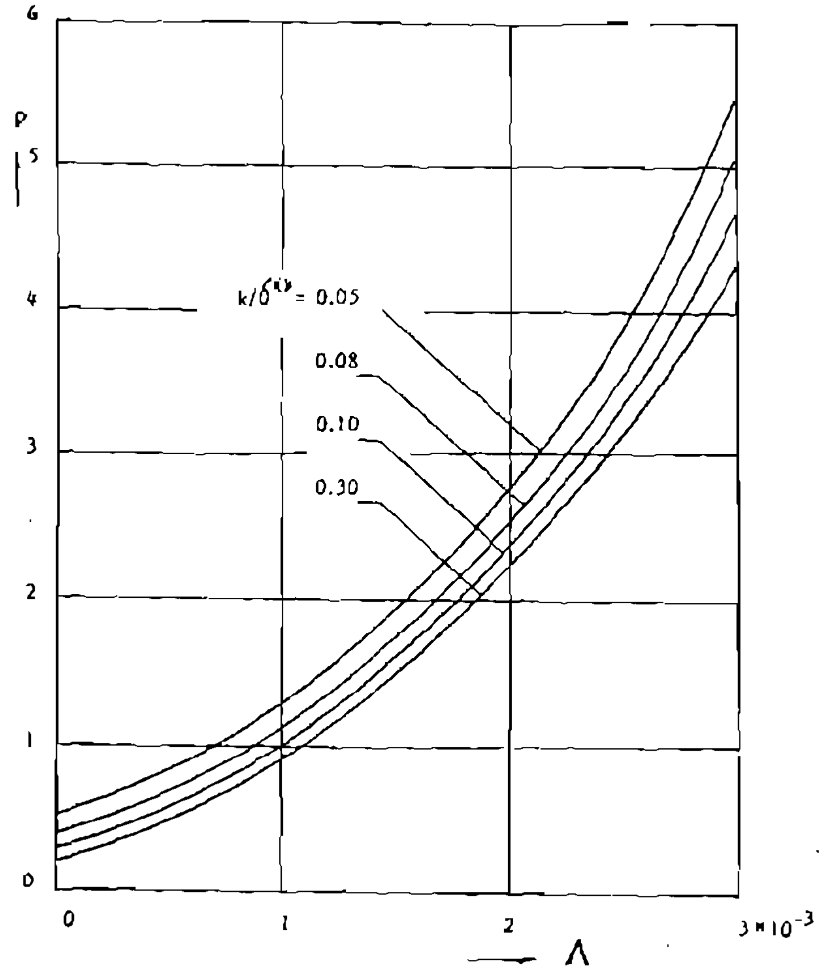

FIG. 1 VELOCITY PROFILE PARAMETER VERSUS EULER NUMBER WITH k/δ^{**} AS PARAMETER .

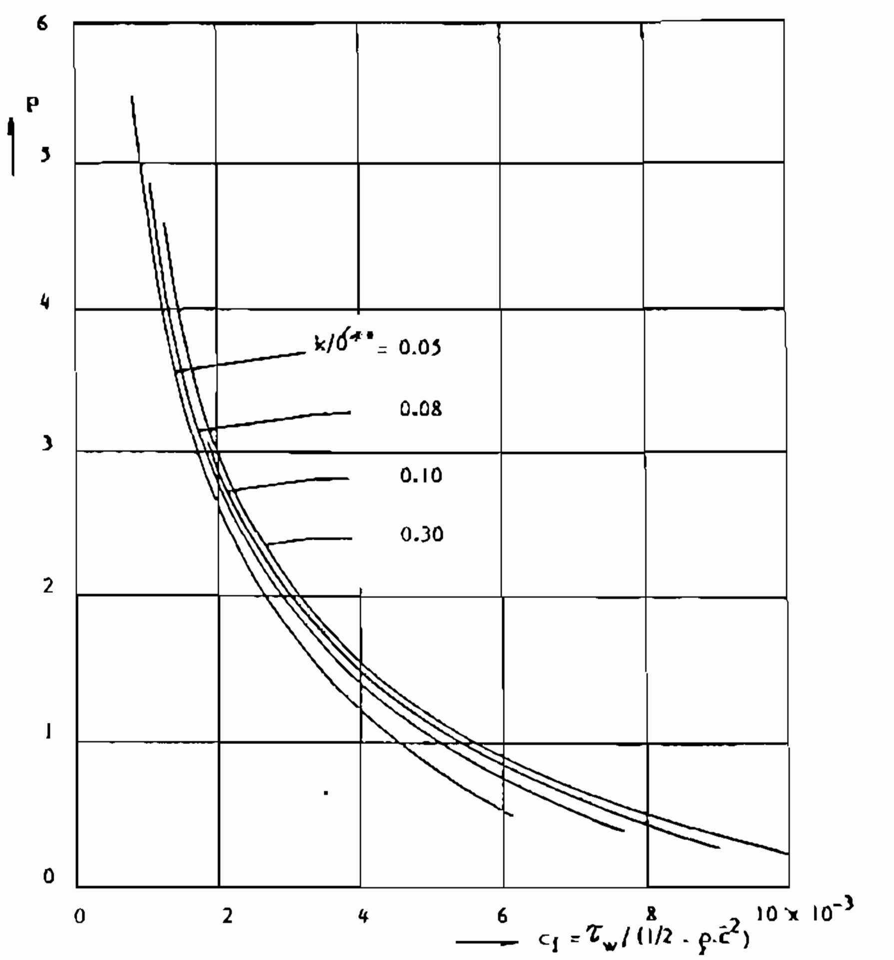

FIG. 2 VELOCITY PROFILES PARAMETER VERSUS LOCAL SHEAR STRESS COEFFICIENT WITH k/δ ** AS PARAMETER

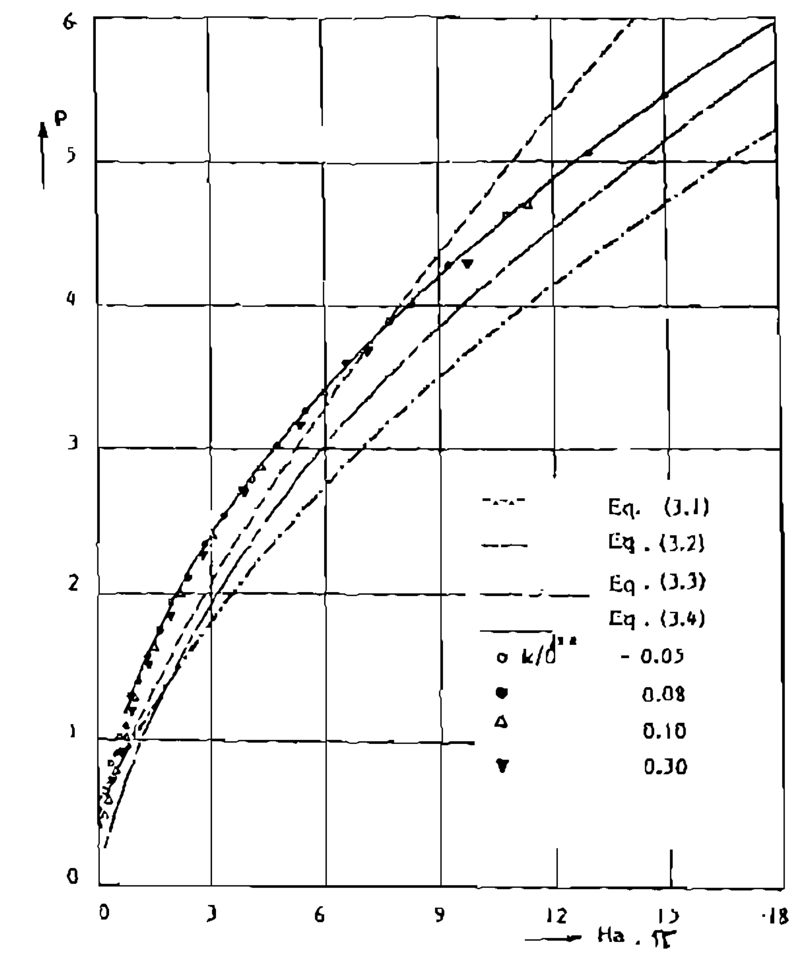

FIG. 3 VELOCITY PROFILE PARAMETER VERSUS PRESSURE GRADIENT PARAMETER

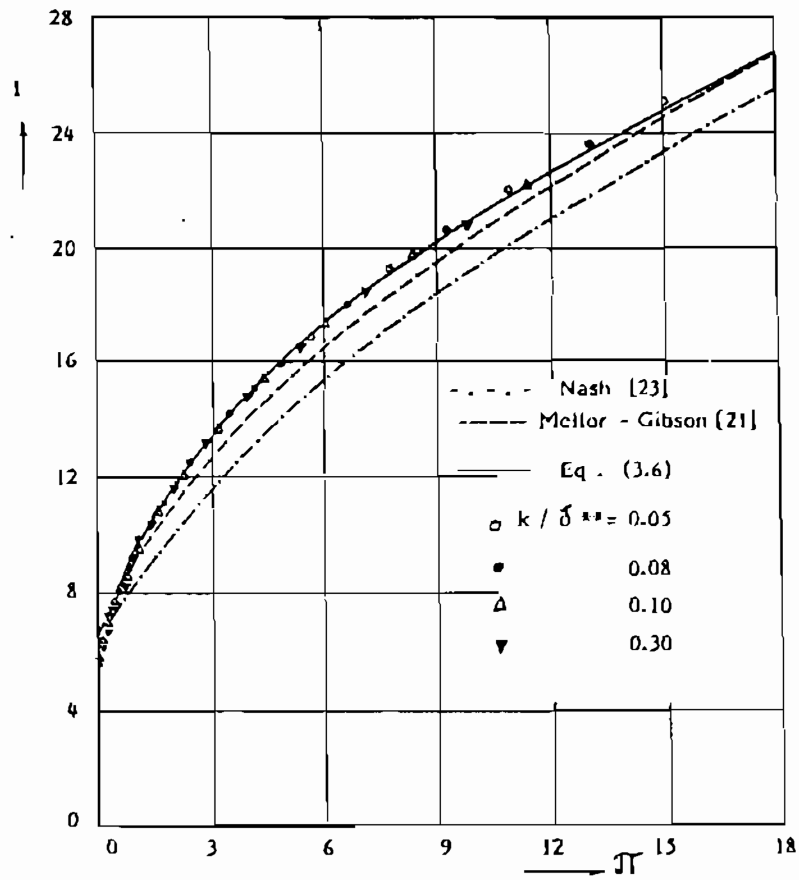

FIG. 4 SHAPE PARAMETER VERSUS PRESSURE GRADIENT PARAMETER

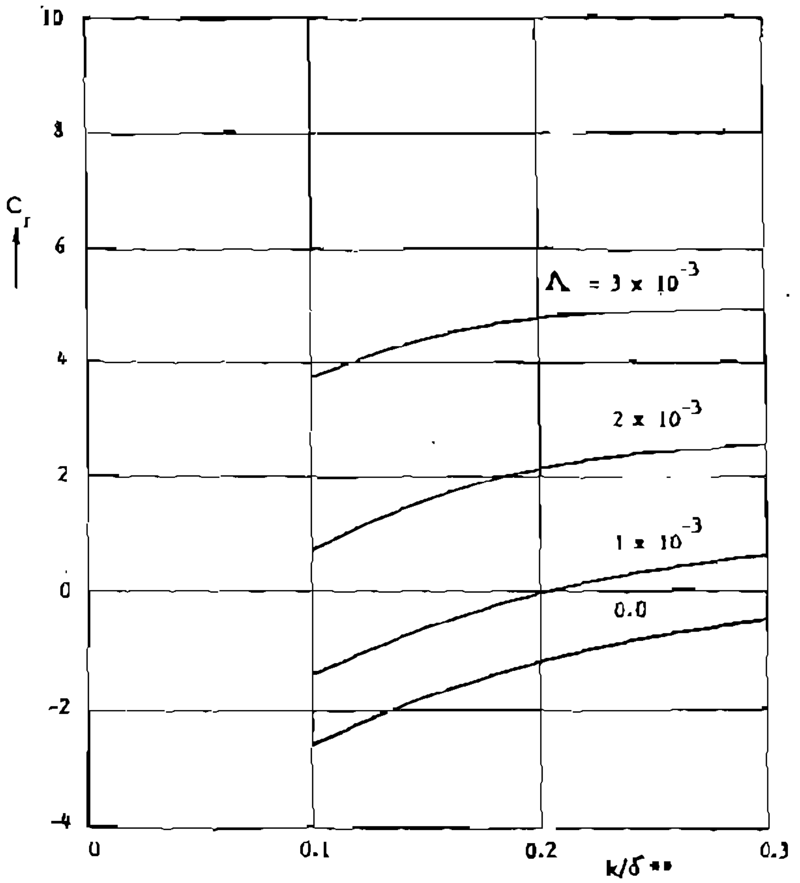

FIG. 5 CONSTANT OF SURFACE ROUGHNESS FUNCTION VERSUS k/δ^{**} WITH EULER NUMBER AS PARAMETER

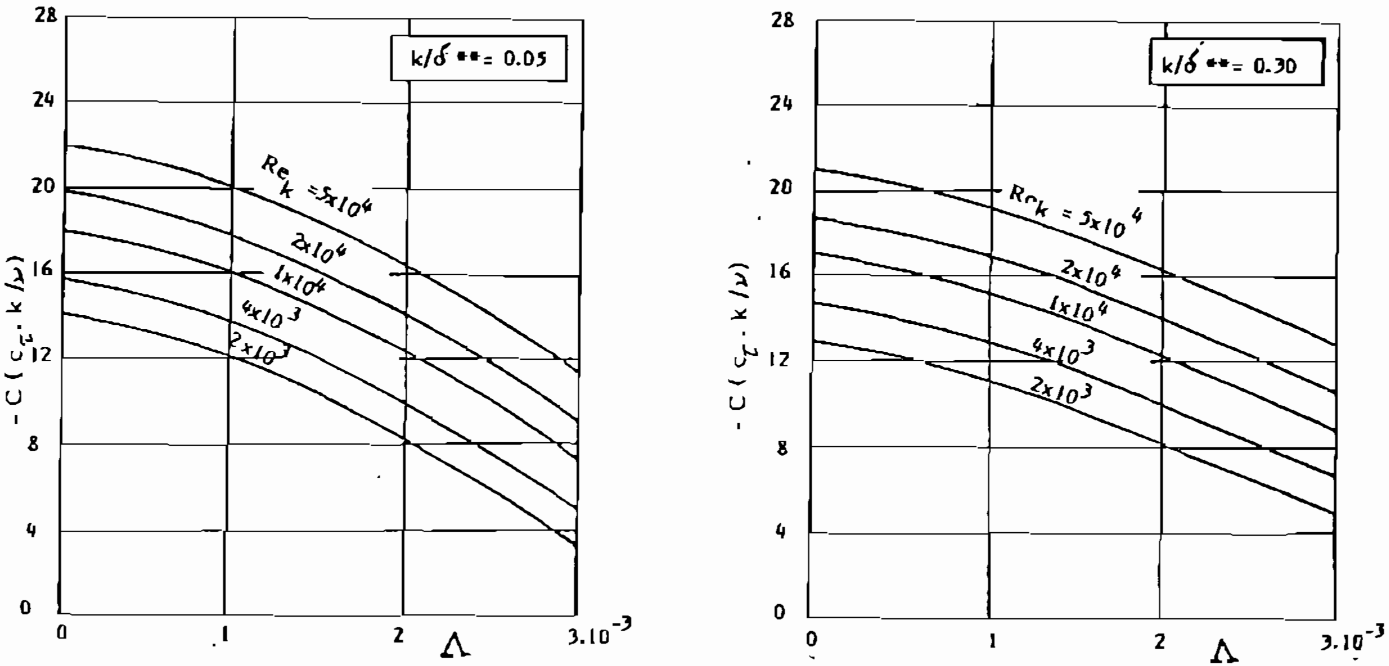

FIG. 6 SURFACE ROUGHNESS FUNCTION VERSUS EULER NUMBER WITH ROUGHNESS HEIGHT REYNOLDS NUMBER AS PARAMETER, AT CONSTANT RATIOS OF k/δ **

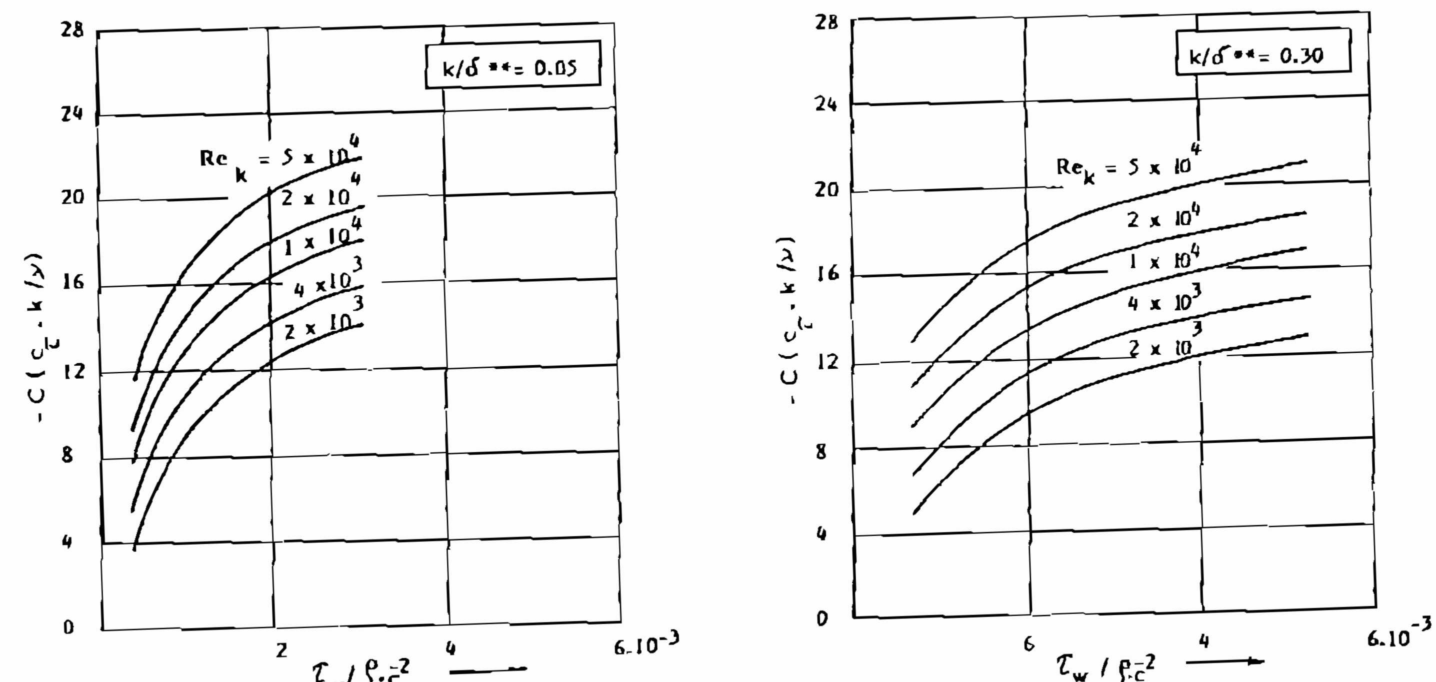

FIG. 7 SURFACE ROUGHNESS FUNCTION VERSUS DIMENSIONLESS WALL SHEAR STRESS WITH ROUGHNESS HEIGHT REYNOLDS NUMBER AS PARAMETER, AT CONSTANT RATIOS OF k/δ **

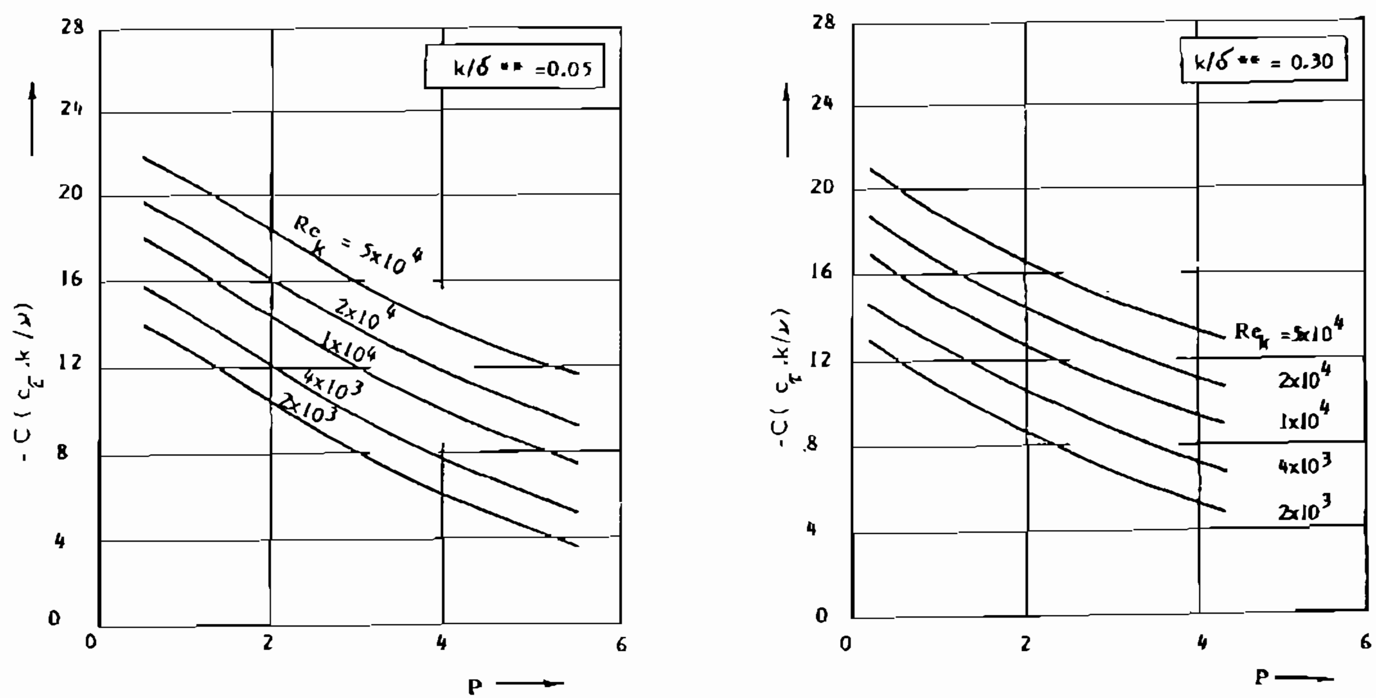

FIG. 8 SURFACE ROUGHNESS FUNCTION VERSUS VELOCITY PROFILE PARAMETER WITH ROUGHNESS HEIGHT REYNOLDS NUMBER AS PARAMETER, AT CONSTANT RATIOS OF k/δ**

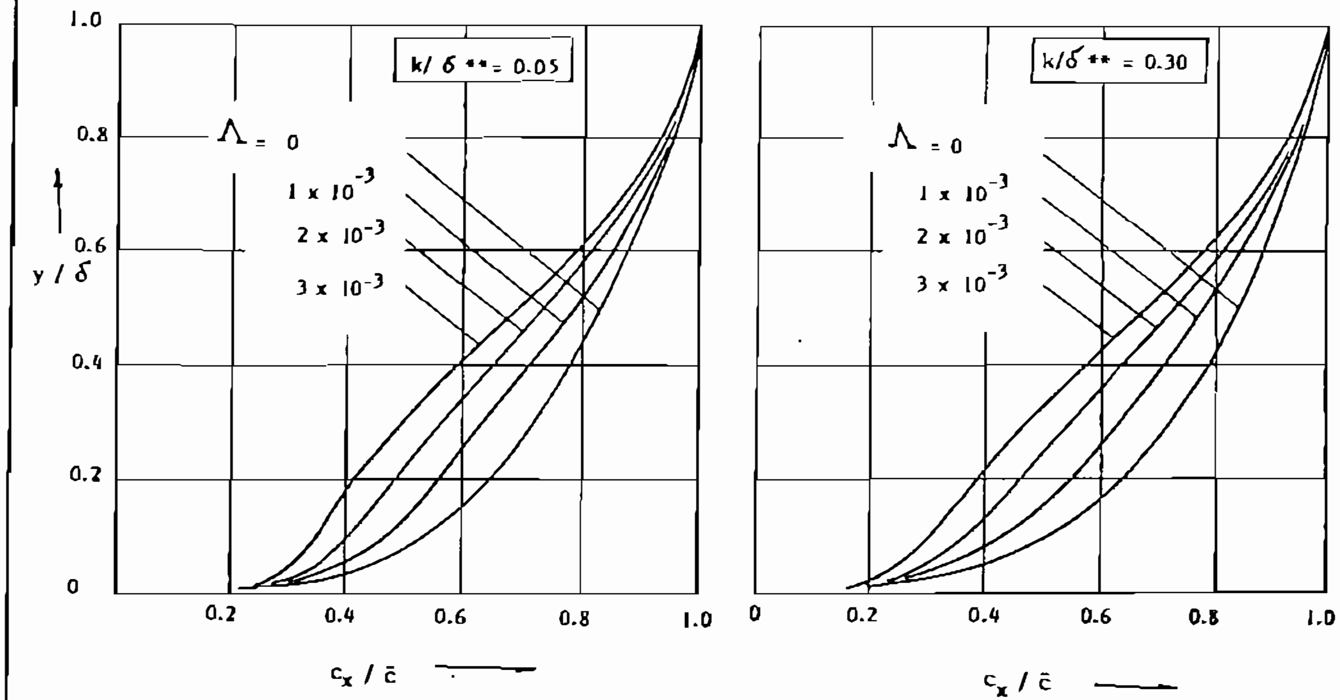

FIG. 9 VELOCITY PROFILES FOR DIFFERENT EULER NUMBERS AND CONSTANT VALUES OF k/δ^{**}