[Mansoura Engineering Journal](https://mej.researchcommons.org/home)

Volume 14 | Issue 2 Article 3

5-25-2021

# Plane Stress Analysis of Circular Plates by the Nodal Line Finite Difference Method.

Youssef Agag
*Associate Professor of Structural Engineering Department, Faculty of Engineering, Mansoura University, Mansoura, Egypt.*, yagag@mans.edu.eg



## Recommended Citation

Agag, Youssef (2021) "Plane Stress Analysis of Circular Plates by the Nodal Line Finite Difference Method.," *Mansoura Engineering Journal*: Vol. 14 : Iss. 2 , Article 3.
Available at: https://doi.org/10.21608/bfemu.2021.172223

# PLANE STRESS ANALYSIS OF CIRCULAR PLATES BY THE NODAL LINE FINITE DIFFERENCE METHOD

## تحليل الالواح الدائرية لحالة الاجهادات المستويه بطريقة الفروق المحددة لخطوط التقسيم

BY

DR. ENG. YOUSSEF AGAG

Assoc. Prof., Struct. Engrg. Dept.
Faculty of Engrg., Mansoura University, EGYPT

الخلاصة - يهتم هذا البحث بتطوير طريقه الباحث المسماه بطريقة الفروق المحددة لخطوط التقسيم Nodal Line Finite Difference Method بغرض استخدامها فى تحليل الالواح الدائريه المعرضة لقوى فى مستواها. وتتلخص الطريقة المتبعة فى هذا البحث بتقسيم اللوح الدائرى الى مجموعة من الدوائر المتوازية يتم التعبير عن مركبتى الازاحة على هذه الدوائر باستخدام الدوال المثلثية Trigonometric Series وبذلك امكن تحويل المعادلتين التفاضليتين الجزئيتين الاتيه اللتان تعبران عن سلوك هذه الالواح الى معادلتى الفروق الاتيه Nodal Line Difference Equations. ولقد تم تحليل بعض نماذج من الالواح الدائرية الحلقية والكاملة باستخدام الطريقة المذكورة. وبمقارنة النتائج التى تم الحصول عليها بتلك التى تم استنتاجها بالطرق التحليلية اظهرت المقارنة دقة وكفاءة هذه الطريقة.

ABSTRACT: *Plane stress analysis of elastic circular plates using the nodal line finite difference method, developed earlier by the Author, has been presented. The analysis deals with the in-plane displacements and requires the solution of two simultaneous second order partial differential equations. A trigonometric basic functions have been used to express the displacement components variation in the circumferential direction. Accordingly, the governing partial differential equations are reduced to two simultaneous nodal line difference equations by means of replacing the derivatives by difference expressions. Numerical examples are presented to demonstrate the efficiency and the accuracy of the method.*

## INTRODUCTION

The nodal line finite difference method NLFDM is a semi-analytical approach which reduces the partial differential equations into an ordinary differential equations by first of all adopting continuous basic functions which satisfy the boundary conditions in one direction. The solution of this ordinary differential equations is then obtained for chosen parallel lines on the actual structure referred to as nodal lines by means of replacing the differential operators by difference expressions at these nodal lines. The earliest formulation and the subsequent application of this method was developed by the Author [1,2,3,4,5] for the bending analysis of rectangular plates. The method has also been extended by the Author [6,7] to include the plane stress analysis of rectangular plates and the bending analysis of circular plates.

The present work aims to extend the application of the nodal line finite difference method NLFDM to the plane stress analysis of elastic circular plates. For the analysis of such type of plates, polar coordinates are preferred over the cartesian coordinates. In the present analysis, the solution is obtained for the radial and tangential displacement components for chosen circles on the plate referred to as circles of division or simply as the nodal circles. Two trigonometric basic functions are chosen to express the radial and tangential displacements along these nodal circles. Accordingly, the two partial differential equations describe the equilibrium and compatibility conditions are then reduced into two ordinary differential equations. These ordinary differential equations are cast into two simultaneous nodal line difference equations by means of using the central finite difference technique in the radial direction. Illustrative examples are presented to demonstrate the validity and the accuracy of the method, where the results have shown good agreement with those of analytical solutions.

## METHOD OF ANALYSIS

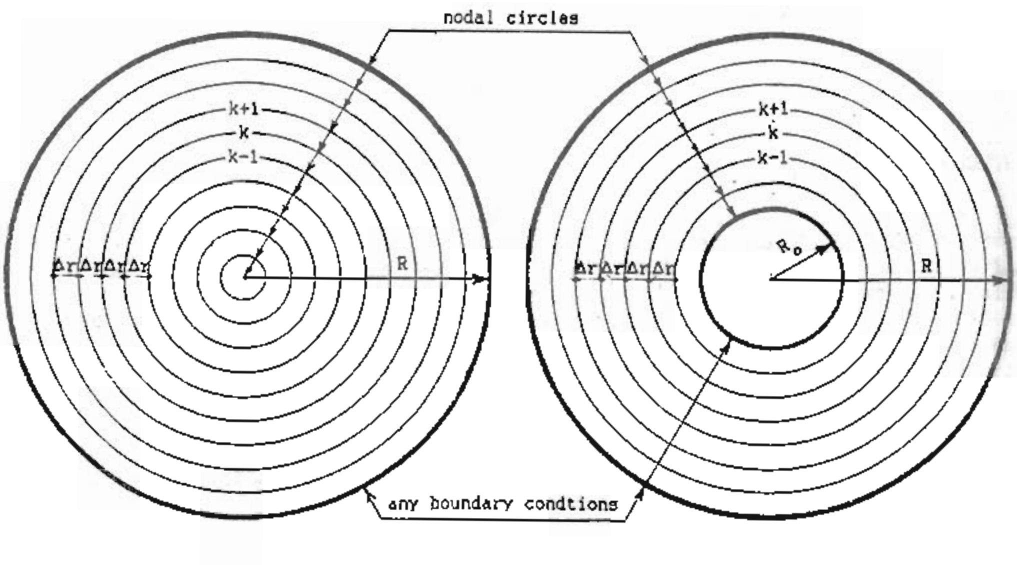
### a- Nodal line difference equations

In the plane stress analysis of circular plates, the in-plane displacement at any point within the plate can be resolved into two components u and v parallel to the radial and tangential directions respectively. The equilibrium and the compatibility conditions are cast into two simultaneous partial differential equations relating the displacement components $u(r,\phi)$ and $v(r,\phi)$ to the surface load intensity components $P_r(r,\phi)$ and $P_\phi(r,\phi)$. These partial differential equations can be written in the following form.

$$\left.\begin{aligned} 2\left[ u^{\bullet\bullet} + \varkappa u^{\bullet} - \varkappa^2 u \right] + (1-\nu)\varkappa^2 u^{\prime\prime} + (1+\nu)\varkappa v^{\bullet\prime} - (3-\nu)\varkappa^2 v^{\prime} &= -\frac{2}{D} P_r \\ (1-\nu)\left[ v^{\bullet\bullet} + \varkappa v^{\bullet} - \varkappa^2 v \right] + 2\varkappa^2 v^{\prime\prime} + (1+\nu)\varkappa u^{\bullet\prime} + (3-\nu)\varkappa^2 u^{\prime} &= -\frac{2}{D} P_\phi \end{aligned}\right\} \quad (1)$$

where $(\ )^{\bullet} = \frac{\partial}{\partial r}$ , $(\ )^{\prime} = \frac{\partial}{\partial \phi}$ , $\varkappa = \frac{1}{r}$ and

$D = \frac{Et}{(1-\nu^2)}$ is the in-plane stiffness




Full circular plate Fig. 1 Ring circular plate

The application of the nodal line finite difference method in the present analysis requires the division of the plate into a mesh of parallel fictitious nodal circles as shown in Fig. 1. The displacement components variation at any nodal circle, k, are assumed as a summation of trigonometric basic function terms multiplied by nodal circle parameters as follows

$$\left.\begin{aligned} u_k &= \sum_{m=0}^{L} U_{m,k} \cos m\phi \\ v_k &= \sum_{m=0}^{L} V_{m,k} \sin m\phi \end{aligned}\right\} \qquad (2)$$

The applied load intensity components at the nodal circle, k, are then expanded into a series similar to the displacement functions as

$$\left.\begin{aligned} P_{r,k} &= \sum_{m=0}^{L} p^{r}_{m,k} \cos m\phi \\ P_{\phi,k} &= \sum_{m=0}^{L} p^{\phi}_{m,k} \sin m\phi \end{aligned}\right\} \qquad (3)$$

Substitution of equations (2) and (3) into equations (1) reduces the partial differential equations to simultaneous ordinary differential equations. For each term of the used basic functions, these equations become

$$\left.\begin{aligned} 2\left[U''_{m,k} + \varkappa_k U'_{m,k}\right] - \beta^1_m \varkappa^2_k U_{m,k} + m\varkappa_k\left[(1+\nu)V'_{m,k} - (3-\nu)\varkappa_k V_{m,k}\right] &= -\frac{2}{D}\, p^{r}_{m,k} \\ (1-\nu)\left[V''_{m,k} + \varkappa_k V'_{m,k}\right] - \beta^2_m \varkappa^2_k V_{m,k} - m\varkappa_k\left[(1+\nu)U'_{m,k} + (3-\nu)\varkappa_k U_{m,k}\right] &= -\frac{2}{D}\, p^{\phi}_{m,k} \end{aligned}\right\} \qquad (4)$$

where $\beta^1_m = \left\{2+(1-\nu)m^2\right\}$ , $\beta^2_m = \left\{2m^2+(1-\nu)\right\}$

The above ordinary differential equations are then transformed into two nodal line difference equations by means of applying the central finite difference technique in the radial direction. Thus, we obtain

$$\left.\begin{aligned} \left[\, C^1_m \quad -C^2_m \quad C^3_m \quad C^4_m \quad C^5_m \quad C^2_m \,\right]\left\{\delta_m\right\} &= -\frac{p^{r}_{m,k}}{\lambda^2}\frac{R^2}{D} \\ \left[\, C^2_m \quad C^6_m \quad C^4_m \quad C^7_m \quad -C^2_m \quad C^8_m \,\right]\left\{\delta_m\right\} &= -\frac{p^{\phi}_{m,k}}{\lambda^2}\frac{R^2}{D} \end{aligned}\right\} \qquad (5)$$

where $C^1_m = (1-\frac{1}{2}\alpha_k)$ , $C^2_m = \frac{1+\nu}{4} m\alpha_k$ , $C^3_m = -\left[2 + \left\{1+\frac{1-\nu}{2}m^2\right\}\alpha^2_k\right]$

$C^4_m = -\frac{3-\nu}{2} m\alpha^2_k$ , $C^5_m = (1+\frac{1}{2}\alpha_k)$ , $C^7_m = -\left[(1-\nu)+\left\{m^2+\frac{1-\nu}{2}\right\}\alpha^2_k\right]$

$C^6_m = \frac{1-\nu}{2} C^1_m$ , $C^8_m = \frac{1-\nu}{2} C^5_m$ , $\alpha_k = \varkappa_k \Delta r$ , $\lambda = \frac{R}{\Delta r}$ and

$\left\{\delta_m\right\} = \left\{ U_{m,k-1} \quad V_{m,k-1} \quad U_{m,k} \quad V_{m,k} \quad U_{m,k+1} \quad V_{m,k+1} \right\}^T$

Equations (5) represent the nodal line difference equations required for the plane stress analysis of circular plates. The application of these difference equations at each nodal circle results in a system of simultaneous linear algebraic equations. Due to the uncoupling property of the adopted basic functions, this system of algebraic equations can be solved for each term, m, separately.

b - Internal forces

The internal forces per unit length at any point of thin elastic isotropic circular plate can be expressed in terms of the displacement components u and v as follows

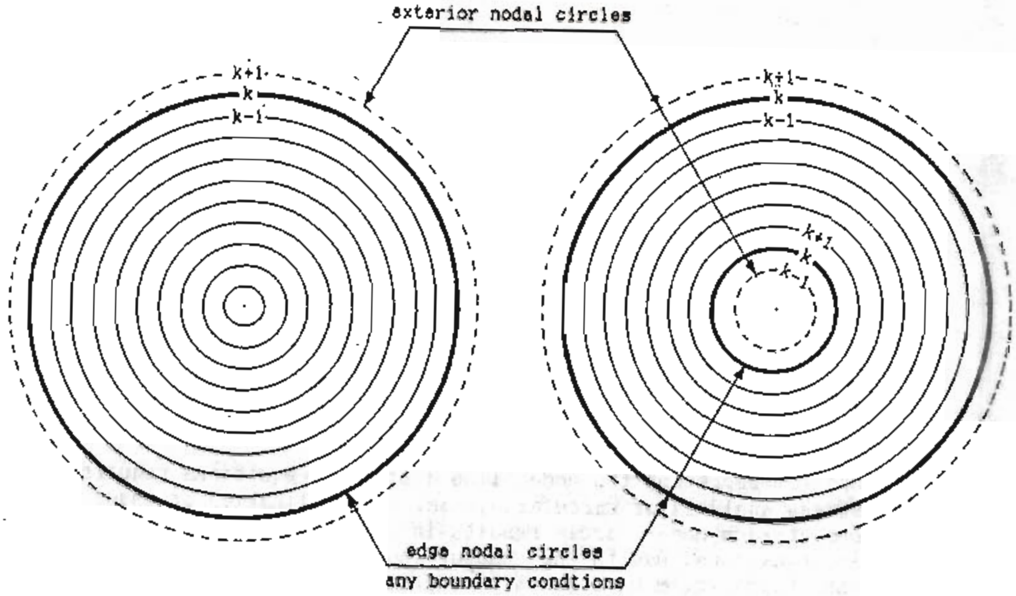
$$\left.\begin{aligned} N_r &= D\,[\,u^{\bullet} + \varkappa\nu\,(\,u + v^{\prime}\,)\,] \\ N_\phi &= D\,[\,\varkappa\,(\,u + v^{\prime}\,) + \nu u^{\bullet}\,] \\ N_{r\phi} &= \frac{1-\nu}{2}\,D\,[\,v^{\bullet} + \varkappa\,(\,u^{\prime} - v\,)\,] \end{aligned}\right\} \qquad (6)$$

Upon substitution of equations (2) into equations (6) and application of the finite difference technique in the radial direction, the internal forces at any nodal circle , k , can be written as

$$\left.\begin{aligned} N_{r,k} &= \frac{D\lambda}{2R}\sum_{m=0}^{L}\cos m\phi\,\begin{bmatrix}-1 & 0 & 2\nu\alpha_k & 2\nu m\alpha_k & 1 & 0\end{bmatrix}\{\delta_m\} \\ N_{\phi,k} &= \frac{D\lambda}{2R}\sum_{m=0}^{L}\cos m\phi\,\begin{bmatrix}-\nu & 0 & 2\alpha_k & 2m\alpha_k & \nu & 0\end{bmatrix}\{\delta_m\} \\ N_{r\phi,k} &= \frac{D\lambda}{2R}\sum_{m=0}^{L}\sin m\phi\,\begin{bmatrix}0 & -\frac{1-\nu}{2} & -(1-\nu)m\alpha_k & -(1-\nu)\alpha_k & 0 & \frac{1-\nu}{2}\end{bmatrix}\{\delta_m\} \end{aligned}\right\} \qquad (7)$$

c - Boundary conditions

The solution of the governing partial differential equations by the nodal line finite difference method requires proper finite difference representation of the boundary conditions at the edge nodal circles. When the nodal line difference equations (5) are applied to the edge nodal circle, the introduction of one fictitious nodal circle outside the plate as shown in Fig. 2 is required. According to the prescribed boundary conditions at the edge nodal circle, the exterior nodal circle parameters have to be expressed in terms of the edge and the adjacent interior nodal circle parameters.




Full circular plate Fig. 2 Ring circular plate

Ring circular plate

For ring circular plates, the boundary conditions at the inner and the outer edge nodal circles must be introduced in the analysis. According to the prescribed boundary conditions, the exterior nodal circle parameters for each term of the used basic functions can be expressed as

1 - Simply supported edge $\quad ( N_{r,k} = 0 \quad , \quad v_k = 0 )$

$$\left.\begin{aligned}
&\text{Inner edge: } (r_k = R_o) \quad U_{m,k-1} = 2\nu\alpha_k U_{m,k} + U_{m,k+1} \\
&V_{m,k-1} = \Big[ (C^3_m + 2\nu\alpha_k C^1_m) U_{m,k} + 2U_{m,k+1} + C^2_m V_{m,k+1} + P^r_{m,k} \frac{R^2}{\lambda^2 D} \Big] / C^2_m \\
&\text{Outer edge: } (r_k = R) \quad U_{m,k+1} = -2\nu\alpha_k U_{m,k} + U_{m,k-1} \\
&V_{m,k+1} = \Big[ -(C^3_m - 2\nu\alpha_k C^1_m) U_{m,k} - 2U_{m,k-1} + C^2_m V_{m,k-1} - P^r_{m,k} \frac{R^2}{\lambda^2 D} \Big] / C^2_m
\end{aligned}\right\} \qquad (8)$$

2 - Clamped edge $\quad ( u_k = 0 \quad , \quad v_k = 0 )$

$$\left.\begin{aligned}
&\text{Inner edge: } (r_k = R_o) \quad U_{m,k-1} = \Big[ \xi^1_m U_{m,k+1} + \xi^2_m V_{m,k+1} + \{ C^6_m P^r_{m,k} + C^2_m P^\phi_{m,k} \} \frac{R^2}{\lambda^2 D} \Big] / \xi^o_m \\
&V_{m,k-1} = \Big[ \xi^3_m U_{m,k+1} + \xi^4_m V_{m,k+1} + \{ C^2_m P^r_{m,k} + C^1_m P^\phi_{m,k} \} \frac{R^2}{\lambda^2 D} \Big] / \xi^o_m \\
&\text{Outer edge: } (r_k = R) \quad U_{m,k+1} = \Big[ \xi^4_m U_{m,k-1} - \xi^2_m V_{m,k-1} + \{ C^8_m P^r_{m,k} - C^2_m P^\phi_{m,k} \} \frac{R^2}{\lambda^2 D} \Big] / \xi_m \\
&V_{m,k+1} = \Big[ -\xi^3_m U_{m,k-1} + \xi^1_m V_{m,k-1} + \{ -C^2_m P^r_{m,k} + C^5_m P^\phi_{m,k} \} \frac{R^2}{\lambda^2 D} \Big] / \xi_m
\end{aligned}\right\} \qquad (9)$$

where

$$\xi^o_m = (C^2_m C^2_m - C^1_m C^6_m) \quad , \quad \xi_m = (C^2_m C^2_m - C^5_m C^8_m)$$

$$\xi^1_m = (C^2_m C^2_m + C^5_m C^6_m) \quad , \quad \xi^2_m = (1-\nu)\ C^2_m$$

$$\xi^3_m = 2\ C^2_m \quad , \quad \xi^4_m = (C^2_m C^2_m + C^1_m C^8_m)$$

3 - Loaded edge $\quad (N_{r,k} = N \quad , \quad N_{r\phi,k} = T )$

$$N = \sum_{m=o}^{l} N_m \cos m\phi \quad , \quad T = \sum_{m=o}^{l} T_m \sin m\phi$$

$$\left.\begin{aligned}
&\text{Inner edge: } (r_k = R_o) \quad U_{m,k-1} = 2\nu\alpha_k U_{m,k} + 2\nu m\alpha_k V_{m,k} + U_{m,k+1} - \frac{2R}{\lambda D} N_m \\
&V_{m,k-1} = -2m\alpha_k U_{m,k} - 2\,\alpha_k V_{m,k} + V_{m,k+1} - \frac{2}{1-\nu} \frac{2R}{\lambda D} T_m \\
&\text{Outer edge: } (r_k = R) \quad U_{m,k+1} = -2\nu\alpha_k U_{m,k} - 2\nu m\alpha_k V_{m,k} + U_{m,k-1} + \frac{2R}{\lambda D} N_m \\
&V_{m,k+1} = 2m\alpha_k U_{m,k} + 2\,\alpha_k V_{m,k} + V_{m,k-1} + \frac{2}{1-\nu} \frac{2R}{\lambda D} T_m
\end{aligned}\right\} \qquad (10)$$

| - Free edge $(N_{r,k} = 0 \;,\; N_{r\phi,k} = 0)$

$$\left.\begin{array}{ll} \text{Inner edge: } (r_k = R_o) & U_{m,k-1} = 2\nu\alpha_k U_{m,k} + 2\nu m\alpha_k V_{m,k} + U_{m,k+1} \\ & V_{m,k-1} = -2m\alpha_k U_{m,k} - 2\alpha_k V_{m,k} + V_{m,k+1} \\ \text{Outer edge: } (r_k = R) & U_{m,k+1} = -2\nu\alpha_k U_{m,k} - 2\nu m\alpha_k V_{m,k} + U_{m,k-1} \\ & V_{m,k+1} = 2m\alpha_k U_{m,k} + 2\alpha_k V_{m,k} + V_{m,k-1} \end{array}\right\} \quad (11)$$

'ull circular plates

In the plane stress analysis of circular plates under self equilibrium .oading conditions, it can be easily concluded that the displacement :omponents u and v at the central point of the plate are equal to zero. Therefore, the nodal circle parameters at the central point of the plate )ecome

$$U_{m,k} = V_{m,k} = 0 \quad (12)$$

These parameters are then introduced into the analysis as prescribed boundary :onditions through the application of the nodal line difference equations (5) ıt the first circle adjacent to the central point of the plate.

The internal forces of the full circular plate are then obtained by ıpplying of the nodal line difference equations (7) at any nodal circle except :hat of the central point of the plate where the radius is equal to zero. To )btain the internal forces at the central point of the plate, special polar :oordinates formulation of the internal forces has been derived from the :artesian coordinates representation of these forces in a difference form. Considering this formulation, it can be easily concluded that the contribution )f the odd terms of the adopted basic functions are equal to zero. For even :erms, the following relationships are obtained

$$\left.\begin{array}{l} N_{r,k} = \frac{D\lambda}{R}\left[\left\{1 + \nu(-1)^{\frac{m}{2}}\right\} U_{m,k+1}\right] \cos m\phi \\ N_{\phi,k} = \frac{D\lambda}{R}\left[\left\{\nu + (-1)^{\frac{m}{2}}\right\} U_{m,k+1}\right] \cos m\phi \\ N_{r\phi,k} = \frac{1-\nu}{2}\,\frac{D\lambda}{R}\left[\left\{1 + (-1)^{\frac{m}{2}}\right\} V_{m,k+1}\right] \sin m\phi \end{array}\right\} \quad (13)$$

## NUMERICAL EXAMPLES

Numerical examples are presented herein to demonstrate the applicability and the accuracy of the nodal line finite difference method in the plane stress analysis of circular plates. The present nodal line finite difference formulation is quite general and can be applied to the analysis of full and ring circular plates for axi-symmetry as well as for non axi-symmetry loading conditions either within the plate or at the edges. In the case of axi-symmetry problems, it can be easily concluded that the displacement component v in the circumferential direction and the shearing force $N_{r\phi}$ are equal to zero. Only the first term of the used basic functions (m=0) is required for the analysis of such problems

In the following numerical examples, Poisson's ratio, $\nu$, equal to 1/6 is considered.

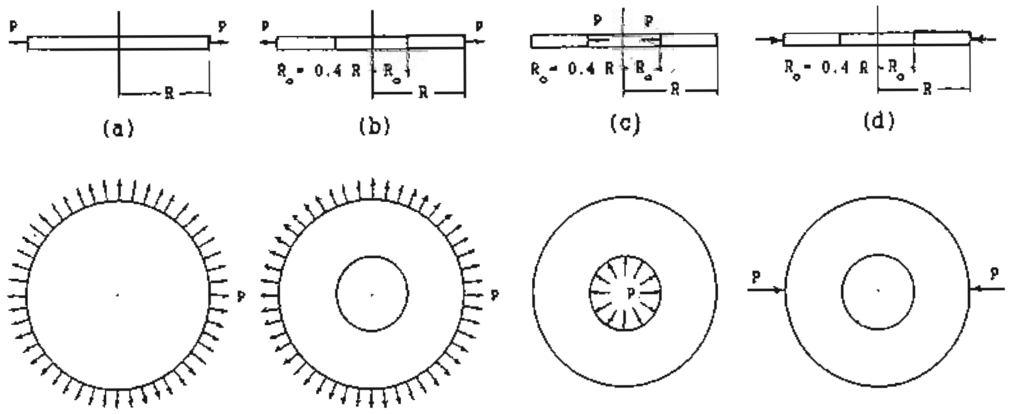



Fig. 3

Example 1: A full circular plate subjected to axi-symmetry edge distributed load as shown in Fig. 1-a, has been analyzed. The analysis was carried out by division of the plate into a mesh of twenty-one nodal circles ($\Delta r$ = R/20). The obtained results are presented in table 1 together with those obtained from the analytical expressions [9]. Due to the linear variation of the displacement component u, complete agreement is obtained.

Table 1 : displacement and internal forces

| $\frac{r}{R}$ | NLFDM | | | analytical [9] | | |
|---|---|---|---|---|---|---|
| | u | $N_r$ | $N_\phi$ | u | $N_r$ | $N_\phi$ |
| 0.0 | 0.0000 | 1.0 | 1.0 | 0.0000 | 1.0 | 1.0 |
| 0.1 | 0.0857 | 1.0 | 1.0 | 0.0857 | 1.0 | 1.0 |
| 0.2 | 0.1714 | 1.0 | 1.0 | 0.1714 | 1.0 | 1.0 |
| 0.3 | 0.2571 | 1.0 | 1.0 | 0.2571 | 1.0 | 1.0 |
| 0.4 | 0.3429 | 1.0 | 1.0 | 0;3429 | 1.0 | 1.0 |
| 0.5 | 0.4286 | 1.0 | 1.0 | 0.4286 | 1.0 | 1.0 |
| 0.6 | 0.5143 | 1.0 | 1.0 | 0.5143 | 1.0 | 1.0 |
| 0.7 | 0.6000 | 1.0 | 1.0 | 0.6000 | 1.0 | 1.0 |
| 0.8 | 0.6857 | 1.0 | 1.0 | 0.6857 | 1.0 | 1.0 |
| 0.9 | 0.7714 | 1.0 | 1.0 | 0.7714 | 1.0 | 1.0 |
| 1.0 | 0.8571 | 1.0 | 1.0 | 0.8571 | 1.0 | 1.0 |
| Multiplier | PR/D | P | P | PR/D | P | P |

xample 2: The ring circular plate with the geometrical relations shown in ig. 1-b has been analyzed. The plate is subjected to an axi-symmetry istributed load at the outer edge. In this analysis the plate was divided nto a mesh of twenty-five nodal circles ($\Delta r = R/40$). The obtained results are resented in table 2 together with those obtained from the analytical xpressions [9].

Table 2 : displacement and internal forces

| $\frac{r}{R}$ | NLFDM | | | analytical [9] | | |
|---|---|---|---|---|---|---|
| | u | $N_r$ | $N_\phi$ | u | $N_r$ | $N_\phi$ |
| 0.40 | 0.9771 | 0.0000 | 2.3749 | 0.9796 | 0.0000 | 2.3810 |
| 0.45 | 0.9649 | 0.2503 | 2.1264 | 0.9671 | 0.2499 | 2.1311 |
| 0.50 | 0.9654 | 0.4292 | 1.9486 | 0.9673 | 0.4286 | 1.9524 |
| 0.55 | 0.9750 | 0.5614 | 1.8170 | 0.9768 | 0.5608 | 1.8201 |
| 0.60 | 0.9915 | 0.6619 | 1.7169 | 0.9932 | 0.6614 | 1.7196 |
| 0.65 | 1.0133 | 0.7401 | 1.6390 | 1.0149 | 0.7396 | 1.6413 |
| 0.70 | 1.0393 | 0.8022 | 1.5771 | 1.0408 | 0.8017 | 1.5792 |
| 0.75 | 1.0686 | 0.8522 | 1.5272 | 1.0701 | 0.8519 | 1.5291 |
| 0.80 | 1.1006 | 0.8931 | 1.4864 | 1.1020 | 0.8929 | 1.4881 |
| 0.85 | 1.1349 | 0.9270 | 1.4526 | 1.1363 | 0.9268 | 1.4541 |
| 0.90 | 1.1710 | 0.9554 | 1.4242 | 1.1723 | 0.9553 | 1.4256 |
| 0.95 | 1.2087 | 0.9795 | 1.4002 | 1.2100 | 0.9794 | 1.4015 |
| 1.00 | 1.2477 | 1.0000 | 1.3797 | 1.2490 | 1.0000 | 1.3810 |
| Multiplier | PR/D | P | P | PR/D | P | P |

xample 3: The ring circular plate considered in the foregoing example is nalyzed for axi-symmetry distributed load at the inner edge as shown in Fig. -c. The analysis was carried out by dividing the plate into a mesh of wenty-five nodal circles ($\Delta r = R/40$). The obtained results are illustrated in able 3.

Table 3 : displacement and internal forces

| $\frac{r}{R}$ | NLFDM | | | analytical [9] | | |
|---|---|---|---|---|---|---|
| | u | $N_r$ | $N_\phi$ | u | $N_r$ | $N_\phi$ |
| 0.40 | -0.6343 | 1.0000 | -1.3749 | -0.6367 | 1.0000 | -1.3810 |
| 0.45 | -0.5792 | 0.7497 | -1.1264 | -0.5814 | 0.7501 | -1.1311 |
| 0.50 | -0.5368 | 0.5708 | -0.9486 | -0.5388 | 0.5714 | -0.9524 |
| 0.55 | -0.5035 | 0.4386 | -0.8170 | -0.5054 | 0.4392 | -0.8202 |
| 0.60 | -0.4772 | 0.3381 | -0.7169 | -0.4789 | 0.3386 | -0.7196 |
| 0.65 | -0.4562 | 0.2599 | -0.6390 | -0.4578 | 0.2604 | -0.6414 |
| 0.70 | -0.4393 | 0.1978 | -0.5771 | -0.4408 | 0.1983 | -0.5793 |
| 0.75 | -0.4257 | 0.1478 | -0.5272 | -0.4272 | 0.1481 | -0.5291 |
| 0.80 | -0.4149 | 0.1069 | -0.4864 | -0.4163 | 0.1071 | -0.4881 |
| 0.85 | -0.4063 | 0.0730 | -0.4526 | -0.4077 | 0.0732 | -0.4542 |
| 0.90 | -0.3996 | 0.0446 | -0.4242 | -0.4009 | 0.0447 | -0.4257 |
| 0.95 | -0.3944 | 0.0205 | -0.4002 | -0.3957 | 0.0206 | -0.4016 |
| 1.00 | -0.3905 | 0.0000 | -0.3797 | -0.3918 | 0.0000 | -0.3810 |
| Multiplier | PR/D | P | P | PR/D | P | P |

**Example 4**: A ring circular plate has been analyzed for the non axi-symmetry loading condition shown in Fig. 1-d. The plate is subjected to two opposite concentrated loads at the outer edge. In this analysis the plate was divided into a mesh of twenty-five nodal circles ($\Delta r = R/40$). The analysis was carried out by using seven even terms of the used basic function. For this case of non axi-symmetry loading condition the displacement component v in the circumferential direction and the shearing force $N_{r\phi}$ are not equal to zero. The obtained results are given in tables 4,5,6,7 and 8..

Table 4 : coefficients $\alpha_1$ for the displacement component u $u = \alpha_1 PR/D$

| $\frac{r}{R}$ | $\phi$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 15 | 30 | 45 | 60 | 75 | 90 |
| 0.4 | -2.479 | -2.131 | -1.260 | -0.204 | 0.750 | 1.402 | 1.633 |
| 0.5 | -2.510 | -2.149 | -1.259 | -0.193 | 0.766 | 1.421 | 1.653 |
| 0.6 | -2.559 | -2.159 | -1.242 | -0.186 | 0.756 | 1.402 | 1.630 |
| 0.7 | -2.673 | -2.180 | -1.216 | -0.175 | 0.744 | 1.379 | 1.600 |
| 0.8 | -2.869 | -2.195 | -1.189 | -0.161 | 0.730 | 1.363 | 1.569 |
| 0.9 | -3.150 | -2.179 | -1.175 | -0.140 | 0.706 | 1.360 | 1.531 |
| 1.0 | -3.492 | -2.127 | -1.165 | -0.110 | 0.671 | 1.371 | 1.486 |

Table 5 : coefficients $\alpha_2$ for the displacement component v $v = \alpha_2 PR/D$

| $\frac{r}{R}$ | $\phi$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 15 | 30 | 45 | 60 | 75 | 90 |
| 0.4 | 0.000 | 1.050 | 1.724 | 1.877 | 1.551 | 0.870 | 0.000 |
| 0.5 | 0.000 | 0.829 | 1.366 | 1.500 | 1.250 | 0.705 | 0.000 |
| 0.6 | 0.000 | 0.735 | 1.167 | 1.263 | 1.048 | 0.591 | 0.000 |
| 0.7 | 0.000 | 0.666 | 0.986 | 1.052 | 0.868 | 0.490 | 0.000 |
| 0.8 | 0.000 | 0.565 | 0.763 | 0.823 | 0.676 | 0.385 | 0.000 |
| 0.9 | 0.000 | 0.362 | 0.480 | 0.554 | 0.458 | 0.266 | 0.000 |
| 1.0 | 0.000 | -0.081 | 0.187 | 0.209 | 0.222 | 0.121 | 0.000 |

Table 6 : coefficients $\alpha_3$ for the internal force $N_r$ $N_r = \alpha_3 P$

| $\frac{r}{R}$ | $\phi$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 15 | 30 | 45 | 60 | 75 | 90 |
| 0.4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.5 | 0.032 | 0.141 | 0.127 | -0.092 | -0.313 | -0.450 | -0.498 |
| 0.6 | -0.641 | -0.127 | 0.156 | -0.031 | -0.277 | -0.423 | -0.485 |
| 0.7 | -1.453 | -0.298 | 0.161 | 0.023 | -0.210 | -0.291 | -0.382 |
| 0.8 | -2.402 | -0.230 | 0.102 | 0.086 | -0.193 | -0.114 | -0.310 |
| 0.9 | -3.451 | 0.090 | -0.096 | 0.206 | -0.258 | 0.116 | -0.315 |
| 1.0 | -4.138 | 0.318 | -0.318 | 0.318 | -0.318 | 0.318 | -0.318 |

Table 7 : coefficients $\alpha_4$ for the internal force $N_\phi$ $N_\phi = \alpha_4 P$

| $\frac{r}{R}$ | $\phi$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 15 | 30 | 45 | 60 | 75 | 90 |
| 0.4 | 4.412 | 3.245 | 0.819 | -1.444 | -3.087 | -4.074 | -4.404 |
| 0.5 | 1.726 | 1.169 | 0.098 | -0.896 | -1.661 | -2.143 | -2.305 |
| 0.6 | 0.755 | 0.218 | -0.370 | -0.722 | -0.987 | -1.178 | -1.235 |
| 0.7 | 0.163 | -0.504 | -0.659 | -0.593 | -0.527 | -0.553 | -0.516 |
| 0.8 | -0.508 | -1.207 | -0.757 | -0.460 | -0.126 | -0.071 | 0.074 |
| 0.9 | -1.875 | -1.685 | -0.700 | -0.287 | 0.216 | 0.389 | 0.569 |
| 1.0 | -5.502 | -0.860 | -0.991 | 0.260 | 0.181 | 1.186 | 0.681 |

Table 8 ; coefficients $\alpha_5$ for the internal force $N_{r\phi}$ $N_{r\phi} = \alpha_5 P$

| $\frac{r}{R}$ | $\phi$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 15 | 30 | 45 | 60 | 75 | 90 |
| 0.4 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.5 | 0.000 | 0.934 | 1.129 | 0.931 | 0.638 | 0.324 | 0.000 |
| 0.6 | 0.000 | 1.143 | 1.195 | 0.981 | 0.679 | 0.353 | 0.000 |
| 0.7 | 0.000 | 1.224 | 0.927 | 0.794 | 0.535 | 0.295 | 0.000 |
| 0.8 | 0.000 | 1.219 | 0.491 | 0.577 | 0.328 | 0.216 | 0.000 |
| 0.9 | 0.000 | 0.960 | 0.042 | 0.371 | 0.109 | 0.133 | 0.000 |
| 1.0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

## CONCLUSION

Polar coordinates formulation of the nodal line finite difference method NLFDM for the plane stress analysis of full and ring circular plates has been presented. In this formulation, the two governing partial differential equations are transformed into two ordinary differential equations by adopting simple trigonometric basic functions to express the displacement components variation along nodal circles on the plate. These ordinary differential equations are then cast into two simultaneous nodal line difference equations by means of replacing the derivatives by difference expressions. The application of these difference equations at each nodal circle results in system of linear algebraic equations. The adopted basic functions have uncoupling property; and therefore, the analysis can be carried out for each term of these functions separately. Moreover, the final matrix of the difference equations has the property of banded matrices with small half band width. This leads to a considerable reduction in the core storage and computation time. Numerical results obtained by using the proposed technique have been compared with those of the analytical solution. The comparison has shown a very close agreement which indicates the validity and the power of the method.

## APPENDIX I

## NOTATION

$u, v$ = radial and tangential displacement components.

$U_{m,k}, V_{m,k}$ = nodal circle parameters.

$R_o, R$ = inner and outer radius of circular ring plate.

$r_k$ = radius of any nodal circle k.

$\Delta r$ = constant distance in radial direction.

$E$ = modulus of elasticity.

$t$ = thickness of the plate.

$\nu$ = Poisson's ratio.

$D$ = in-plane stiffness of the plate.

$P_r, P_\phi$ = radial and tangential load intensity components.

REFERENCES

1- AGAG, Y. :Nodal Line Finite Difference Method for the Analysis of Elastic Plates with two Opposite Simply Supported Ends, The Bulletin of the Faculty of Engineering, El-Mansoura University, Vol.9, No.1, June 1984, P. C.136-C.147.

2- AGAG, Y. :The Nodal Line Finite Difference Method with Iteration Procedure in the Analysis of Elastic Plates in Bending, The Bulletin of the Faculty of Engineering, El-Mansoura University, Vol.9, No.2, December 1984, P. C.8-C.22.

3- AGAG, Y. :Nodal Line Finite Difference Method for the Analysis of Plates with Variable Flexural Rigidity, The bulletin of the Faculty of Engineering, El-Mansoura University, Vol.10, No.1, June 1985, P. C.13-C.30.

4- AGAG, Y. :Nodal Line Finite Difference Method for the Analysis of Rectangular Plates with Abrupt Change in Thickness, The Bulletin of the Faculty of Engineering, El-Mansoura University, Vol.13, No.2, December 1988, p. C.52-C.63

5- AGAG, Y. :Nodal Line Finite Difference Method in the Analysis of Rectangular Plates on Elastic Foundation, The Bulletin of the Faculty of Engineering, El-Mansoura University, Vol.14, No.1, June 1989, P. C.59-C.70.

6- AGAG, Y. :Plane Stress Analysis of Simply Supported Rectangular Plates by the Nodal Line Finite Difference Method,The Bulletin of the Faculty of Engineering, El-Mansoura University, Vol.14, No.1, June 1989, P. C.71-C.82.

7- AGAG, Y. :Bending Analysis of Circular Plates by the Nodal Line Finite Difference Method, Civil Engineering Research Magazine, El-Azhar University, Vol. 11, No.6, 1989, P. 44 - 54.

8- TIMOSHENKO, S. P and GOODIER, J. N. : Theory of Elasticity, International Student Ed., McGraw Hill, New York, 1982.

9- HAMPE, E :STATIK, Rotationssymmetrischer Flächentragwerke, Band 1, 3. Auflage, VEB VERLAG FÜR BAUWESEN, BERLIN, 1967.

10- RABICH, R :Ingenieur Tachenbuch Bauwesen, Band I, Grundlagen des Ingenieurbaus, B.G. TEUBNER VERLAGSGESELLSHAFT LEIPZIG 1963.