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# Effect of the Aspect Ratio on the Natural Convection in a Rectangular Porous Medium.

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# EFFECT OF THE ASPECT RATIO ON THE NATURAL CONVECTION IN A RECTANGULAR POROUS MEDIUM

# تاثير النسبه الباعيه للاطوال على الحمل الطبيعي في وسط مسامي ثنائي البعد في شكل متعامد

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خلاصـــــه :

يحتوى هذا البحث على دراسه عدديه لتاثير النسبه الباعيه على الحمل الطبيعي المستقر في حيز ثنائي البعد في شكل متعامد مملوء بوسط مسامي مشبع بالمائع. الجدران الراسيه للحيز غير موصله للحراره والافقيه ذات درجه حراره ثابته مع وجود انحدار حراري راسي فيه درجه حراره الجدار السفلي اعلى منها على الجدار العلوى . تمت الدراسه على نسب باعيه من ١ الى ٨ وبمدى كبير من عدد دارسي-رايلى يصل حتى ١٠٠٠ وتظهر نتائج الدراسه على هيئه مقارنه بين خطوط السريان وخطوط درجه الحراره الثابته وعدد نوسيلت الموضعي والكلي على الحوائط الافقيه وكذلك عدد خلايا السريان.

بزياده النسبه الباعيه للاطوال يظهر سريان متعدد الخلايا ثنائي البعد تزداد شدته بزياده عدد دارسي-رايلي. كما يزداد عدد خلايا السريان وذلك بزياده النسبه الباعيه للاطوال. اما سرعه السريان المتوسطه للمائع فهي لاتعتمد على نسبه الاطوال وتزداد خطيا بزياده عدد دارسي-رايلي. كما تقل داله السريان القصوى للمائع مع زياده نسبه الاطوال. اما نقط الانقلاب لمنحنى عدد نوسيلت الموضعي على الحوائط الافقيه فهي تحدث على نقط حدود خلايا السريان ويزداد عددها بزياده النسبه الباعيه للاطوال. كذلك لايعتمد عدد نوسيلت الكلي للحوائط الافقيه ذات درجه الحراره الثابته على النسبه الباعيه للاطوال ويزداد بزياده عدد دارسي-رايلي حسب العلاقه الاتيه

$$Nu = 1.59344 \ln Ra - 5.186$$

**ABSTRACT**

The effect of the aspect ratio on natural convection in a two dimensional rectangular cavity filled with fluid saturated porous media is examined numerically.. The vertical walls of the cavity are adiabatic and the horizontal walls are isothermal with lower hotter one and upper colder one. The test is done for aspect ratios 1 to 8 and wide range of Darcy-Rayleigh number Ra up to 800. Results are presented in terms of the streamlines and isotherms, maximum velocity and stream function, the local and global Nusselt number as well as the number of flow cells.

By the increase of the aspect ratio a two dimensional cellular pattern appears with vigorous motion as the Darcy-Rayleigh number increases. The number of flow cells increases with the aspect ratio. The non-dimensional average flow velocity is independent on the aspect ratio and increases linearly with the increase of Ra. The extremum value of non-dimensional stream function decreases with the increase of aspect ratio. The turning points in the local Nu at the isothermal walls occurs at the boundaries of the cells. Its number equals to the cells number and increases as the aspect ratio increases. The global Nu at the isothermal walls is independent on the aspect ratio and increases with Ra due to the relation Nu = 1.59344 ln Ra - 5.186.

## 1. INTRODUCTION

Owing to its numerous and wide ranging applications natural convection in porous medium has been the subject of many recent studies. For example, this phenomenon is encountered in the underground spreading of chemical wastes, water movements in geothermal systems, porous insulations, packed beds catalytic reactors, heat storage beds, nuclear waste disposal systems and sensible heat storage beds. Most of the previous studies have been theoretical, including natural temperature gradients [1,3], natural convection in boundary layer [4], and convective flows in horizontal porous layers heated from below [5,6].

In the previous work of the author [6] the phenomenon of natural convection in two dimensional rectangular enclosures filled with porous medium saturated with fluid under natural vertical temperature gradient was studied. An insight into the physical nature of the flow of the natural convection in the porous media showed that such convective flows strongly depends on the heating conditions, fluid type, porous material type and the system geometry [6].

The Darcy-Rayleigh number comes out as the coefficient of the buoyancy driving force along the flow. It includes all the parameters which express the porous media and the fluid. It expresses the porous media through the permeability and conductivity. It expresses the fluid through its density and viscosity. It includes also the operating driving conditions such as the temperature difference, coefficient of thermal expansion and the gravity effect. So, in the present work the effect of system geometry i.e. the aspect ratio, for a wide range of Darcy-Rayleigh number on the nature of the convective heat flow inside the rectangular porous medium was studied and analyzed. The horizontal rectangular porous media considered in this study has a ratio of the width to the height equal and greater than unity

## 2. FORMULATION OF THE PROBLEM

Consider a horizontal two dimensional rectangular porous material with two opposite walls at constant but different temperatures and the other two vertical thermally insulated with H units height and W units wide as depicted in Fig. 1. Here $T_H$ and $T_C$ represent the temperature of the hot and cold walls, respectively. All the walls of the cavity are further assumed to be impermeable. The fluid and the porous medium are treated as a homogenuous, isotropic system with an equivalent heat conductivity. The fluid is assumed to be normal Boussinesq fluid and the density $\rho$ is taken to be a linear function of the temperature T. A non dimensional form of the mass, momentum and energy equations, using the Darcy's law and the Boussinesq approximation are used in a model solved by the finite difference technique. Non dimensional variables X, Y, $\psi$, $\theta$, U and V are used for the distances x and y, stream function, temperature T and horizontal and vertical velocities respectively. Their values are expressed in the nomenclature.

The relevent hydrodynamic and thermal boundary conditions are X=0 and 1 , $\partial\theta/\partial X=0$; Y=0, $\theta=1$; Y=1, $\theta=0$; $\psi=0$ around the boundary.

The governing parameters for the present problem are the aspect ratio A and the Darcy-Rayleigh number Ra

$$A = W/H \quad \text{and} \quad Ra = Kg\beta(T_H-T_C)H\rho/\alpha\mu$$

where K, $\beta$, g, $\alpha$ and $\mu$ are the permesbility, coefficient of thermal expansion, acceleration due to gravity, effective thermal diffusivity and dynamic viscosity, reepectively.

The local and overall heat transfer are best characterized by the local and average Nusselt number for each isothermal wall of the enclosure, defined as follows:

$$Nu(x) = -\partial\theta/\partial Y \quad \text{and} \quad Nu = -\int_0^1 (\partial\theta/\partial Y)\, dX$$

The flow is characterized by two variables $\psi_{max}$ and a non-dimensional average fluid velocity over the area A, $U_m$ defined as follws:

$$\psi_{max} = \pm \max |\psi(x,y)| \quad \text{and} \quad U_m = \int_A (U^2 + V^2)\, dA$$

For the validity of this model, a comparison of some results is done with an experimental work of Close et al [7]. The comparisons is preaented in a previous work of the author [6] and ahowed good agreement. More detailed information about the formulation, the model and the numerical method of solution can be found in [6].

## 3. RESULTS AND DISCUSSION

Since the primary object of this study is to examine the effects of the aspect ratio on the nature of convective heat flow, a wide range of aspect ratio, 1≤A≤8 has been considered for a range of Darcy-Rayleigh number up to 800. Numerical results for the rectangular cavity filled with a saturated porous media has been obtained. The vertical walls of the cavity are adiabatic and the horizontal walls are isothermal with lower hotter one and upper colder one.

### 3.1 Temperature and Flow Field

The isotherms and flow patterns are first presented for aspect ratios A = 1, 2, 3, 4, 5 and 7 and Ra = 100 and 400 in

Figs 2, 3 and 4, respectively.

By the case of Darcy-Rayleigh number Ra=100 shown in Fig. 2, Because of the buoyancy effects, a circulating motion is established, filling the entire cavity and rotating slowly in the counter clockwise direction with single extremum value of stream function $\psi_{max}$ for the aspect ratio A=1. By the increase of the aspect ratio the fluid is required to move all the way up to the top surface to reject the heat gained at the base. Than at the top surface the colder and denser fluid tends to topple over and returns to the lower surface, this causes the appearance of the two dimensional cellular pattern. Three cells appeare by A=2 and 3, five cells by A=4, and seven cells by A=5 and 7. Therefore, besides the conduction heat transfer exists also the natural convective heat motion with multiple cells circulating in alternate directions

With the increase of Darcy-Rayleigh number, for an example by Ra= 400, the flow which is shown in Fig. 3, is characterized with vigorius motion and higher stream functions. In this case the number of cells increases for example from 1 to 2 by A=1, from 3 to 5 by A=3 and from 7 to 9 by A=7. The streamlines become denser near the boundaries of the cells than near the middle

The temperature fields are shown in Figures 2 and 3 for the cases of Ra=100 and 400 and different Aspect ratios. The temperature field is significantly modified in the regions where the convective transport is larger. The isotherms deviate up towards the cold surface as the hotter convective flow moves up after picking up the heat from the lower boundary and it deviates down towards the hot surface as the colder flow returns to the lower surface after rejecting the heat. As the Darcy-Rayleigh number increases the isotherms deviates more towards the horizontal isothermal upper and lower walls and becomes denser than in the middle of the cells .

Table 1 represents the number of cells as a function of the aspect ratio for a wide range of Darcy-Rayleigh number. The number of cells is equal to 1 for all the aspect ratios and the Ra <40 where pure conduction heat flow occurs. With the increase of the Ra and the existing of the convective heat transfer mode the number of cells increase for all the aspect ratios. With the more increase of the Darcy-Rayleigh number Ra the number of cells decreases again in some cases .

The non-dimensional average flow velocity $U_m$ is shown in Fig. 5 for the different aspect ratios and Darcy-Raylaigh number. The $U_m$ is nearly independent on the aspect ratio, but increases with the increase of Darcy Rayleigh number due to the increase of the driving buoyancy force. $U_m$ increases linearly with the increase of Ra as shown in Fig. 6 with the following relationship

$$U_m = 4.6676 \, Ra - 430.39$$

The extremum values of the stream function $\psi_{max}$ is shown in Fig. 7 for different values of aspect ratios and wide range of Darcy-Rayleigh number. $\psi_{max}$ decreraes with the increase of the aspect ratio, although it is nearly constant and independent on the aspect ratio. This indicates that the maximum velocities always occur at the boundariee of the cells, where the stream lines are denser and closer to each other than near the middle of the the cells, where the maximum values of stream functions occur.

Table 1 Number of cells as a function of the aspect ratio A and the Darcy-Rayleigh number Ra

| Ra \ A | 1 | 2 | 3 | 4 | 5 | 7 |
|---|---|---|---|---|---|---|
| 20 | 1 | 1 | 1 | 1 | 1 | 1 |
| 50 | 1 | 1 | 1 | 5 | 5 | 7 |
| 100 | 1 | 3 | 3 | 5 | 7 | 7 |
| 200 | 2 | 3 | 5 | 5 | 7 | 7 |
| 300 | 2 | 3 | 5 | 5 | 7 | 9 |
| 400 | 2 | 3 | 5 | 5 | 7 | 9 |
| 600 | 2 | 3 | 5 | 5 | 7 | 8 |
| 800 | 2 | 3 | 5 | 5 | 7 | 8 |
| 1000 | 2 | 3 | 5 | 5 | 6 | 7 |

3.2 Heat Transfer

Interesting features of the buoyancy-driven flow are further exhibited by the distribution of local Nusselt number on the horizontal walls which participate more effectively in the heat transfer process. The local Nusselt number Nu(x) on the lower hotter surface is preeented in Figuras 8 and 9, and on the upper colder surface are presented in Figures 10 and 11 for the aspect ratios 1, 3 and 7 and both the Darcy-Rayleigh numbers Ra= 100 and 400, respectively. As mentioned in [6], most of the heat is transferred at several localized spots at the walls, these corresponds to the boundaries between the cells. At these boundaries the flow of heat is directly from the hotter to the colder wall. So, the local Nu is maximum at the hotter wall and is minimum at the colder one, this phenomenon can be also shown in Figuree 8-11. In the other side, where the flow is in the opposite direction, i.e. returning from the colder wall to the hotter wall, a minimum point in tha local Nu occurs at the lower hotter wall and a meximum point in the local Nu occurs at tha upper colder one. Thus, the points of maximum and minimum values i.e. the turning points, in the local Nu occur at the boundaries between the cells. The number of these turning points is aqual to the number of cells in the flow. Therefore, the number of these cells increases as the aspect ratio increases.

Fig. 12 represents the global Nusselt number as a function of the aspect ratio at both the lower hotter and the upper colder horizontal walls for Darcy-Rayleigh number up to 1000. By the change of the aspect ratio the global Nusselt number remains nearly constant, i.e. it is independent on the aspect ratio. At the same time, the global Nusselt number increases with the increase of Darcy-Rayleigh number. A logarithmic relation between the global Nu and the Darcy-Rayleigh number is shown in Fig. 13 and takes the following important form:

$$Nu = 1.59344 \ln Ra - 5.186$$

## 4. CONCLUSIONS

A numerical study has been conducted to analyse the effects of the aspect ratio on the natural convection in horizontal two dimensional rectangular porous media with adiabatic vertical walls and isothermal horizontal hotter lower wall and colder upper one. A wide range of aspect ratio up to 8 with a wide range of Darcy-Rayleigh number up to 1000 was considered.

In the region of Darcy-Rayleigh number, where conduction heat transfer is accompanied with the convective heat transfer mode, exists the following:

- By the increase of the aspect ratio a two dimensional cellular pattern exists with multiple cells circulating in alternate directions and the motion becomes vigorious with higher stream functions as the Darcy-Rayleigh number increases..
- The isotherms is significantly modified in the regions where the convective transport is larger.
- The number of cells in the flow increases as the aspect ratio increases.
- The non-dimensional average flow velocity $U_m$ is independent on the aspect ratio and increases with the increase of the Darcy-Raylelgh number linearly by the relation:

$$U_m = 4.6676 \ Ra - 430.39$$

- The extremum value of stream function $\psi_{max}$ decreases with the increase of the aspect ratio and the maximum flow velocities occur at the boundaries of the cells, where the streamlines are denser and closer to each other
- The turning points in the local Nu occur at the boundaries of the cells with number equals to the number of cells, i.e. this number increases as the aspect ratio increases.
- The global Nu at the isothermal horizontal walls is independent on the aspect ratio and increases with the increase of the Darcy Rayleigh number by the following logarithmic relation

$$Nu = 1.59344 \ln Rs - 5.186$$

## 5. NOMENCLATURE

| | |
|---|---|
| A | aspect ratio = W/H |
| g | acceleration due to the gravity, $m^2/s$ |
| H | height of the porous material, m |
| K | permeability of theporous medium, $m^2$ |
| Nu | global Nusselt number at the walls |
| Nu(x) | local Nusselt number |
| Ra | Darcy-Rayleigh number= $g\ \beta\ K(T_H-T_C)/\alpha\nu$ |
| T | Temperature, K |
| $T_H$,$T_C$ | temperatures of hot and cold isothermal walls, K |
| u | field velocity in the x direction, = $\partial\varphi/\partial y$, m/s |
| v | field velocity in the y direction, = $-\partial\varphi/\partial x$, m/s |
| x,y | spatial coordinates |
| X, | dimensionless distances on x axes = x/W |
| Y | dimensionless distances on y axes = y/H |
| W | width of the porous material, m |
| $\alpha$ | therma diffusivity of porous material, $m^2/s$ |
| $\beta$ | coefficient of volumetric expansion, $K^{-1}$ |
| $\mu$ | dynamic viscosity of the saturated fluid, |
| $\nu$ | kinematic viscosity of the saturated fluid, $m^2/s$ |
| $\rho$ | fluid density |
| $\varphi$ | stream function |
| $\psi$ | dimensionless stream function = $\varphi$ H/$\alpha$W |
| $\psi_{max}$ | maximum extremum value of the atream function |
| $\theta$ | nondimensionsl temperature = $(T-T_C)/(T_H-T_C)$ |

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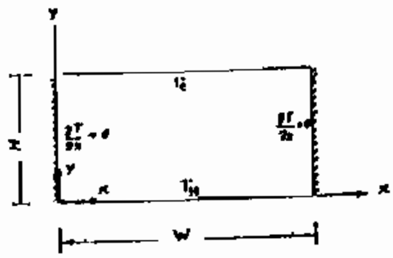


Fig. 1
Schematic diagram of the rectangular porous cavity, coordinate system and the thermal boundaries


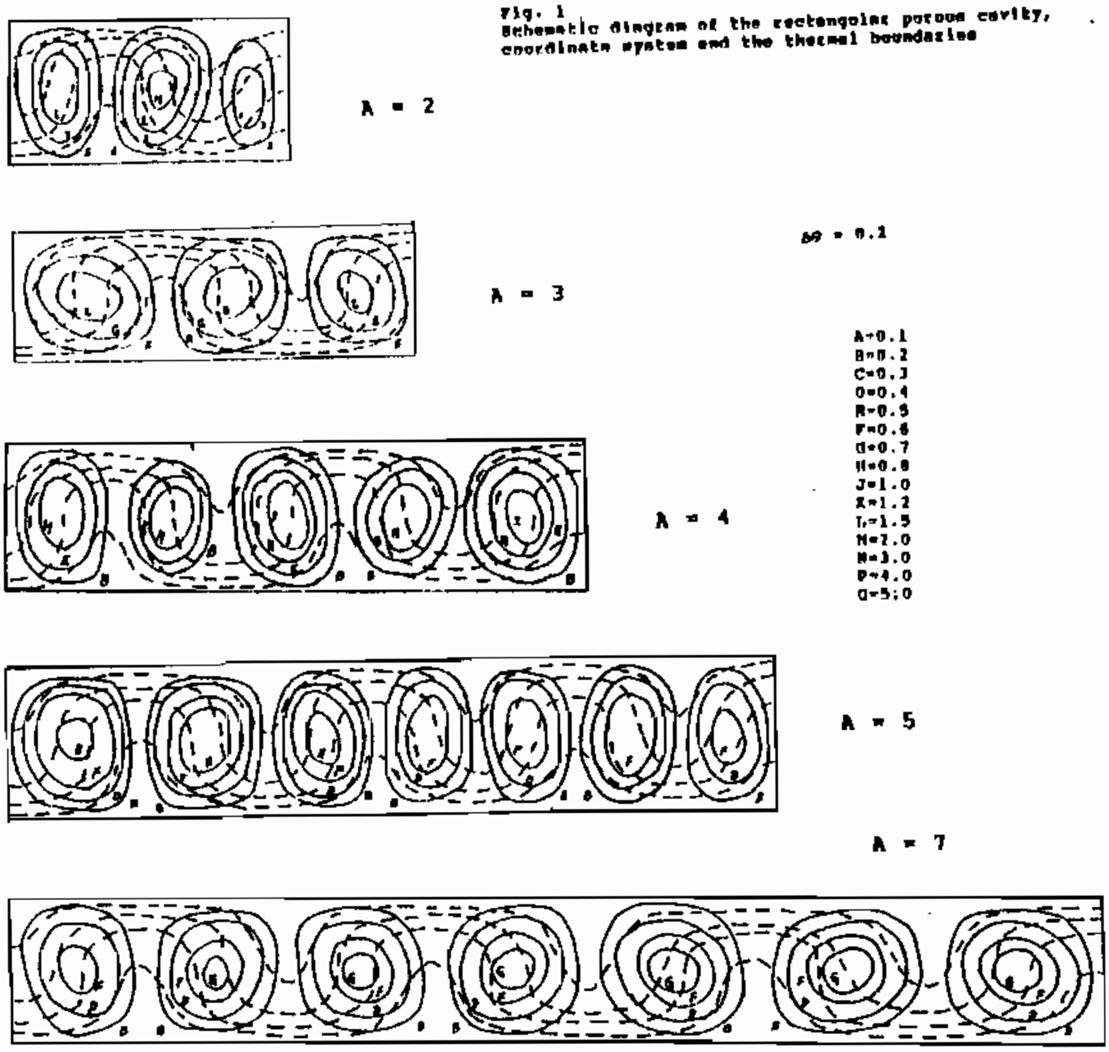


Fig. 2 Streamlines and isotherms for different aspect ratios and Darcy-Rayleigh number Ra=100

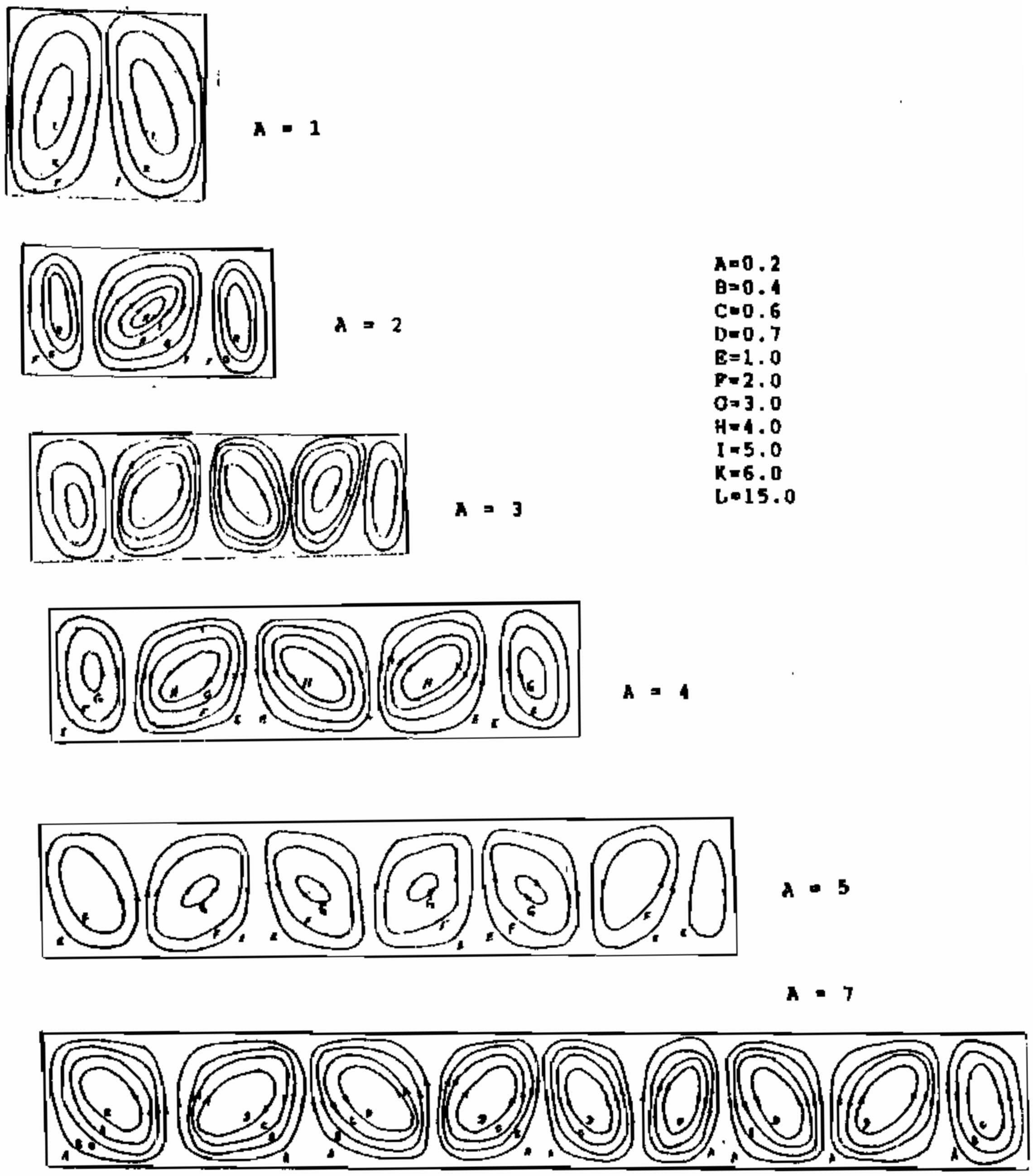


Fig. 3 Streamlines for different aspect ratios
end Darcy-Rayleigh number Ra = 400

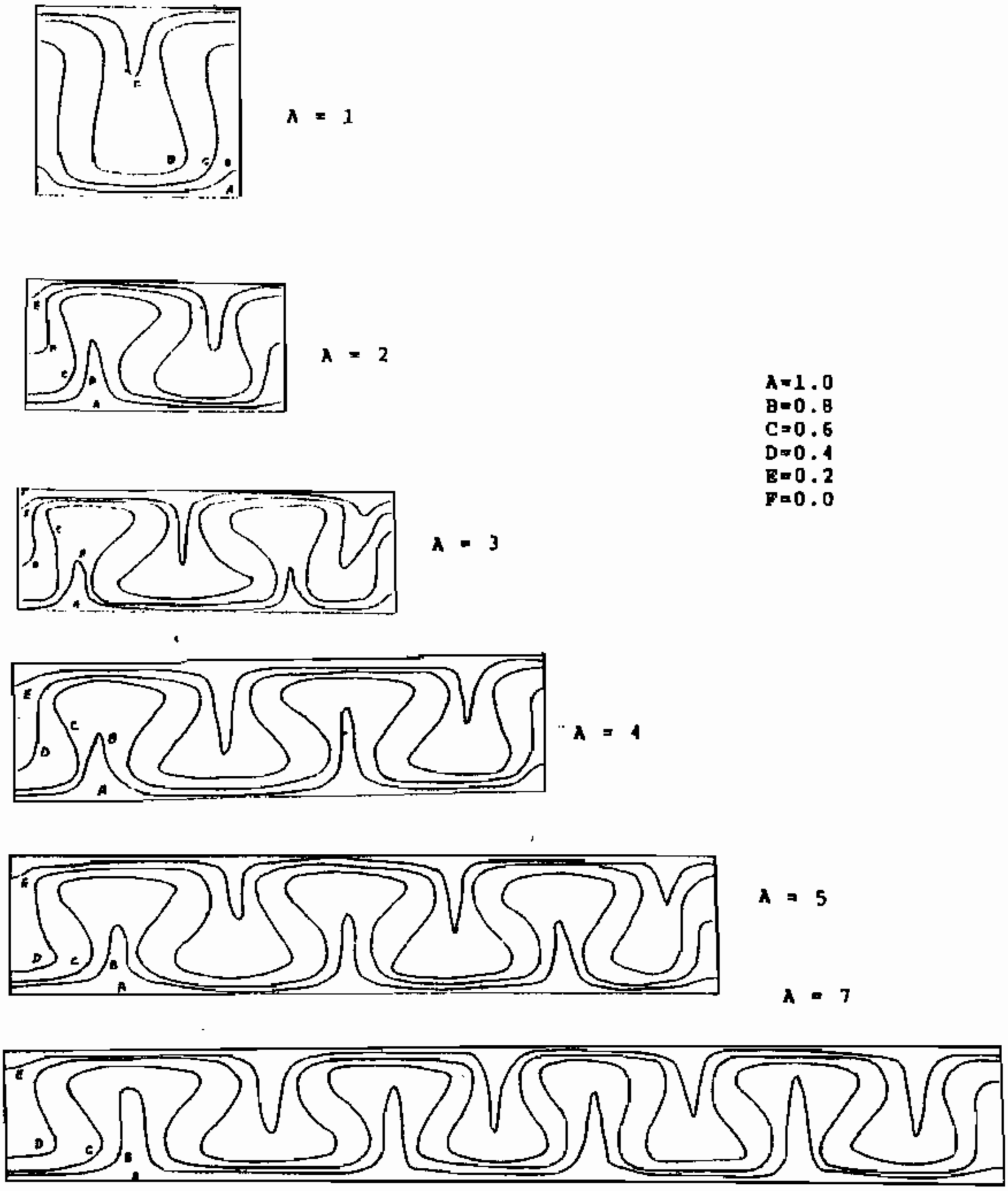


Fig. 4 Isotherms for different aspect ratios
and Darcy-Rayleigh number Ra = 400

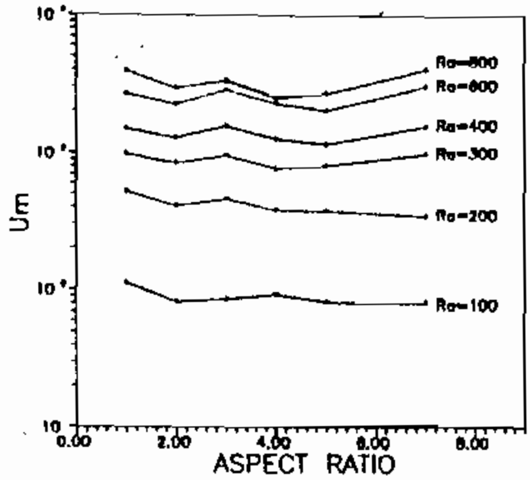


Fig. 5 Fluid non-dimensional average speed as function of aspect ratio for different Darcy-Rayleigh number

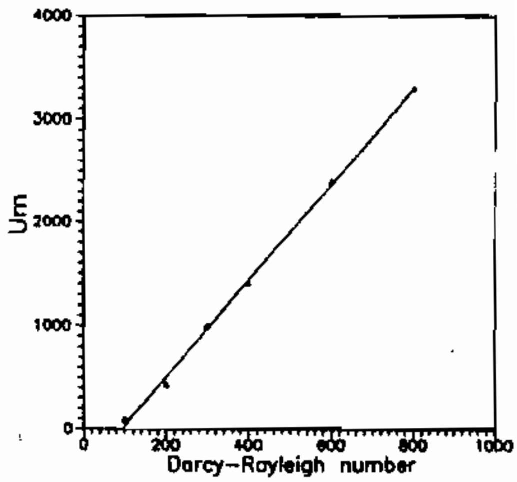


Fig. 6 Fluid non-dimensional average speed as a function of The Darcy-Rayleigh number

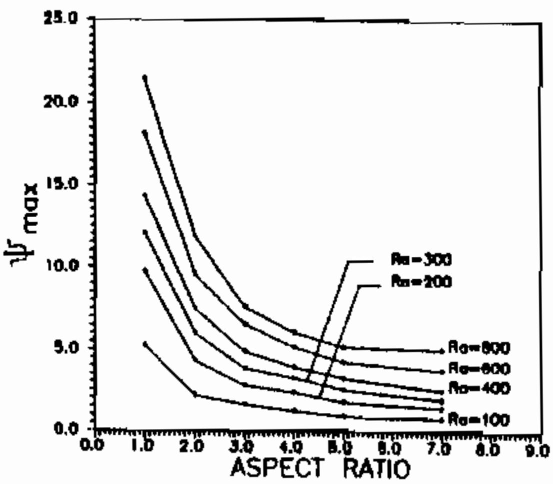


Fig. 7 Maximum non-dimensional stream function as function of aspect ratio for different Darcy-Rayleigh number

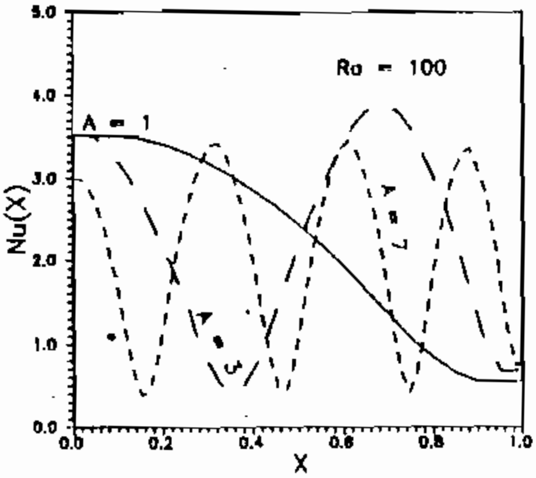


Fig. 8 Spatial variation of local Nusselt number at the lower hotter horizontal wall at Ra = 100

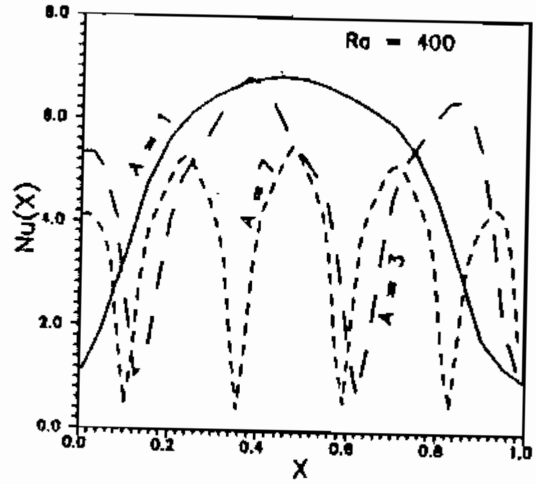


Fig. 9 Spatial variation of local Nusselt number at the lower hotter horizontal wall at Ra = 400

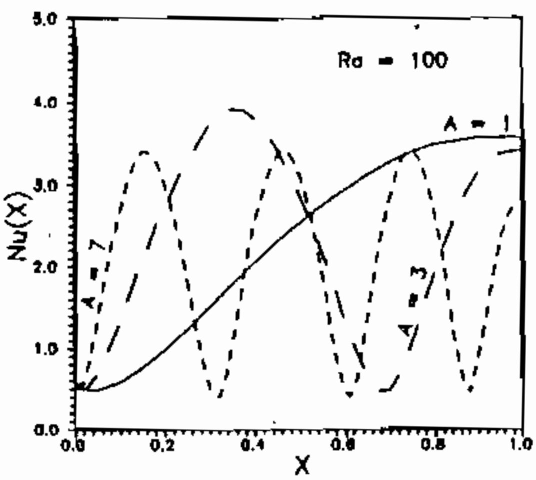


Fig. 10 Spatial variation of local Nusselt number at the upper colder horizontal wall at Ra = 100

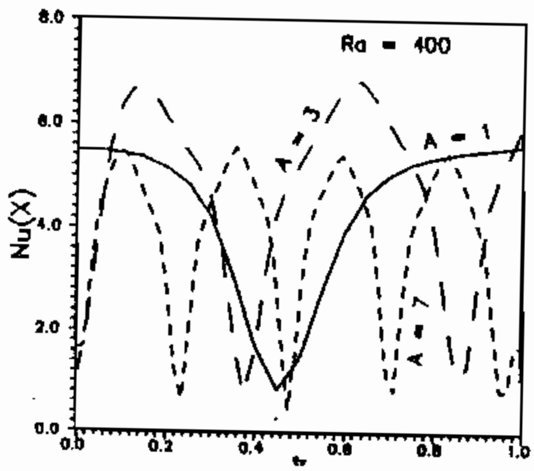


Fig. 11 Spatial variation of local Nusselt number at the upper colder horizontal wall at Ra = 400

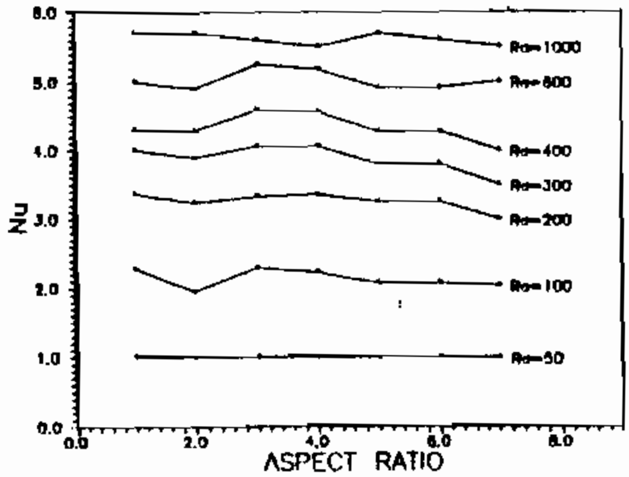


Fig. 12 Global Nusselt number as afunction of aspect ratio at the upper and lower isothermal walls

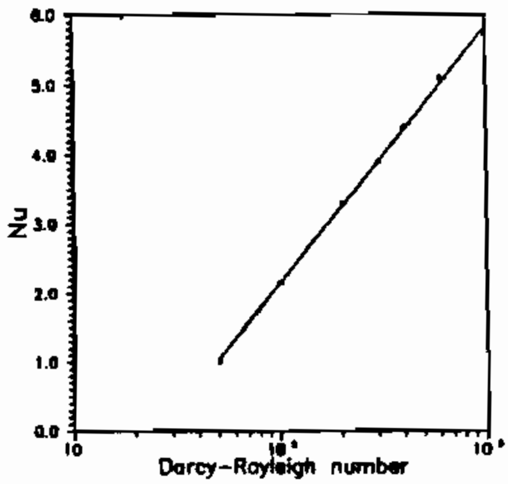


Fig. 13 The global Nusselt as a function of Darcy-Rayleigh number at the upper and lower isothermal walls.