Mansoura Engineering Journal

Volume 15 | Issue 2 Article 9

5-22-2021

Effects of Components Nonidealities upon the Performance of Adaptive Antenna Array Processors.

Fayez Zaki
Department of Electrical Engineering Faculty of Engineering, Mutah University, Jordan., fwzaki@mans.edu.eg

Recommended Citation

Zaki, Fayez (2021) "Effects of Components Nonidealities upon the Performance of Adaptive Antenna Array Processors.," *Mansoura Engineering Journal*: Vol. 15 : Iss. 2 , Article 9.
Available at: https://doi.org/10.21608/bfemu.2021.171233

EFFECTS OF COMPONENTS NONIDEALITIES UPON THE PERFORMANCE OF ADAPTIVE ANTENNA ARRAY PROCESSORS

FAYEZ W. ZAKI

Department of Electrical Engineering

Faculty of Engineering

MU'TAH UNIVERSITY, JORDAN

تأثير عدم المثالية في مكونات الدوائر على اداء صفوف الهوائيات ذات المعالجة المتهايئة

ملخص :

يصف هذا البحث التأثيرات الناتجة عن عدم مثالية مكونات الدوائر على اداء صفوف الهوائيات المتهايئة القياسية منها والرقمية • ويبين ان البناء القياسي يختلف بشكل كبير عن ذلك الموجود في الحالات المثالية حيث ان تأثيرات الانحراف في الجهد وعدم الخطية في الخواص تؤدي الى زيادة في متوسط مربع الخطأ بطريقة تتناسب عكسيا مع معامل التحكم في الاستقرار ومعدل التقارب للخواريزم المستخدم (ملحوظة : تتعارض هذه النتيجة مع الدراسات السابقة) •

ويخلص البحث الى تقديم طريقة جديدة لبناء المضاعفات تهدف الى التقليل من التأثيرات المشار اليها وذلك بواسطة كسب دائرة المضاعف حول مسار الاشارة •

ABSTRACT

This paper describes the effects of component nonidealities upon the performance of analog and sampled data adaptive antenna arrays. It is shown that the effects of analog implementation differ significantly from those encountered in ideal cases. It is also shown that the effects of nonzero mean errors, such as offset voltages and nonlinearities of the input multipliers contribute an excess mean square error which is inversely proportional to the parameter which controls the stability and the rate of convergence of the algorithm. A configuration is presented which minimises these effects by distributing the loop gain of the first multiplier around the signal path.

I-INTRODUCTION

An adaptive antenna is one whose parameters are caused to vary as a function of the interference field so that it always stays in an optimal or fairly optimal condition. In other words, adaptive antenna can be considered as a filter which has variable parameters. If the desired antenna output can be defined, then the antenna parameters can be varied until the mean square error between the desired output and the actual output is minimised. The problem with fixed parameter optimal solutions is that interference is seldom constant in space or time and an antenna which is initially optimum in some location will rapidly become very suboptimal as the interference changes either because of the appearance of new interference transmission, changes in the local scattering environment through movement of vehicles, ships, aircraft etc., variation in the orientation of the antenna if scanning or if it is mounted on a moving vehicle or aircraft, or changes in operating frequency. As an example, the null steering antenna arrays are extremely sensitive to some of these changes and it is hardly attempting to use them as fixed parameter arrays.

The interest in implementation of analog and sampled data adaptive antenna has recently increased because of advances in charge coupled devices and discrete design for UHF adaptive antenna array processors. The first known work to consider component nonidealities was performed in 1963 by P. Low [1]. He proved that systems constructed with imperfect components could be expected to converge to a solution if such a solution existed. Other previous work in adaptive filtering by Kaunitz [2] assumed an added random noise. Widrow [3] had shown that the effect of an additive zero-mean noise in the weight vector is an excess mean square error which is proportional to the step size μ. This noise was added to the weights to model the errors caused by the estimation of the actual gradient of the mean square error performance surface. Rosenberger [4] assumed that a zero-mean band-limited random process with a finite variance was added into the output of an adaptive noise canceller before it was fed back to the weights. His results showed that the maximum achievable echo suppression, for this case, was inversely proportional to μ, i.e., the smaller the μ the better the system worked. Thomas [5] make the same assumptions as Rosenberger, and showed that the choice of μ which allowed the most convergence is the smallest value of μ. Other papers considering nonideal multipliers [6,7,8] showed that the qualitative behavior of adaptive filters, using nonideal multipliers is similar to that of ideal adaptive filters. These results agreed with the general belief that adaptive filters can adapt around their own internal errors. It will be shown in this paper that this is not the case when one considers the effects of internal nonzero mean errors.

To the author's knowledge, no body considered the effects of many of the common errors found in adaptive antenna array processors. It was felt that by providing information on the sources of errors and limitations resulting from these errors as well as solutions for these errors, adaptive antenna arrays might gain greater acceptance among researchers.

II-ANALYSIS OF ADAPTIVE ANTENNA ARRAY PROCESSORS

For a narrow-band applications, an adaptive array controlled by the discrete-time least-mean-square (LMS) algorithm takes the form shown in Fig. 1, which illustrates the processing of the output of a single array element. In this diagram, the first local oscillator following the antenna element selects the R.F. frequency of interest w, and mixes it to a constant frequency w_0, where the desired signal bandwidth is passed. The output of the bandpass filter is mixed to intermediate frequency (I.F) w_I. The mixer outputs are image rejected and the weights are applied. The in-phase and quadrature (I & Q) weights are controlled by separate LMS feedback loops. The control attempts to minimise the mean square error (MSE) between the array output y(t) and a reference signal d(t); the latter waveform could, for example, be obtained by demodulating the array output itself [9], or from a separate reference antenna element [10].

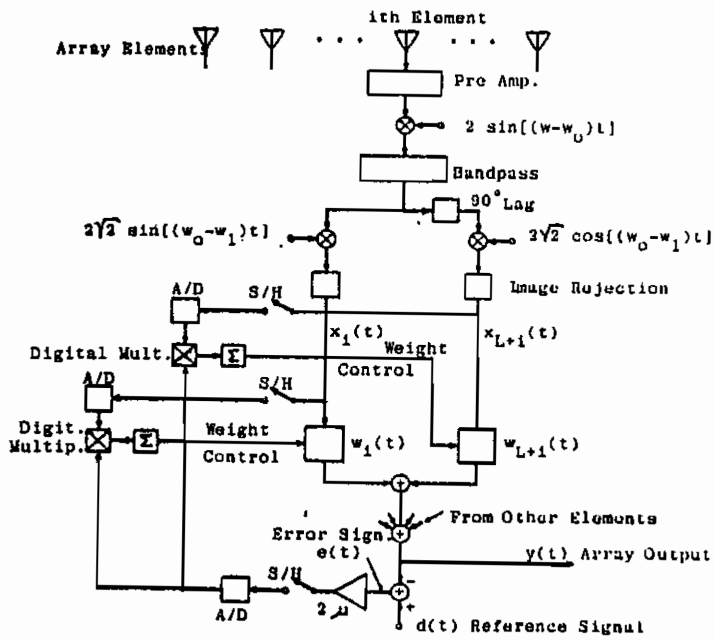

Fig. 1, Adaptive Array Processor Using LMS Algorithm.

For an L-element array, the discrete-time in-phase and quadrature element output are defined to be

$$\underline{X}(n) = [x_1(n) \quad x_2(n) \; \ldots\ldots\ldots \; x_{2L}(n)]^T \tag{1}$$

The 2L variable weights vector is defined as

$$\underline{W}(n) = [w_1(n) \quad w_2(n) \; \ldots\ldots\ldots \; w_{2L}(n)]^T \tag{2}$$

The array output is given by

$$y(n) = \underline{W}^T(n)\underline{X}(n)$$
$$= \underline{X}^T(n)\underline{W}(n) \tag{3}$$

Therefore, the error signal is given by

$$e(n) = d(n) - y(n)$$
$$= d(n) - \underline{W}^T(n)\underline{X}(n) \tag{4-a}$$

or

$$e(n) = d(n) - \underline{X}^T(n)\underline{W}(n) \tag{4-b}$$

The squared value of e(n) is therefore,

$$e(n)^2 = d(n)^2 - 2d(n)\underline{W}^T(n)\underline{X}(n) + \underline{W}^T(n)\underline{X}(n)\underline{X}^T(n)\underline{W}(n) \tag{5}$$

The MSE or cost function ξ is defined as

$$\xi(n) = E[e(n)^2]$$
$$= E[d(n)^2] - 2E[d(n)\underline{X}^T(n)]\underline{W}(n) + \underline{W}^T(n)E[\underline{X}(n)\underline{X}^T(n)]\underline{W}(n) \qquad (6)$$

where E[.] denotes expectation operation.

Define the cross-correlation between the reference signal d(n) and the vector $\underline{X}(n)$ as

$$\underline{S}(n) = E[d(n)\underline{X}(n)] \qquad (7)$$

and the Input correlation matrix as

$$\underline{R}(n) = E[\underline{X}(n)\underline{X}^T(n)] \qquad (8)$$

Therefore, the MSE $\xi(n)$ at time n can be expressed in terms of $\underline{S}(n)$ and $\underline{R}(n)$ as

$$\xi(n) = E[d(n)^2] - 2\underline{S}^T(n)\underline{W}(n) + \underline{W}^T(n)\underline{R}(n)\underline{W}(n) \qquad (9)$$

The MSE in Eq.(9) is a quadratic function of the weights $\underline{W}(n)$. Such a function has only one minimum. The object of the adaptive algorithm is to adjust the weights $\{w_i, i=1,2,3,....2L\}$ so that the minimum mean square point is reached. Gradient methods are commonly used for this purpose [3].

The gradient vector $\underline{G}(n)$ of the MSE can be obtained by differentiating Eq.(9) with respect to $\underline{W}(n)$ as

$$G(n) = \begin{bmatrix} \partial\xi(n)/\partial w_1(n) \\ \partial\xi(n)/\partial w_2(n) \\ .... \\ .... \\ \partial\xi(n)/\partial w_{2L}(n) \end{bmatrix} = -2\underline{S}(n) + 2\underline{R}(n)\underline{W}(n) \qquad (10)$$

For stationary input processes, the optimum weighting vector $\underline{W}^*$ can be obtained by setting $\underline{G}(n)$ in Eq.(10) equal to zero, i.e.,

$$2\underline{S} - 2\underline{R}\underline{W}^* = 0 \qquad (11)$$

and therefore,

$$\underline{W}^* = \underline{R}^{-1}\underline{S} \qquad (12)$$

Note that R(n) and S(n) are replaced by $\underline{R}$ and $\underline{S}$ for stationary input process. Eq.(12) is referred to as Weiner-Hopf equation [11]

A)Method of Steepest Descent

The major task of the adaptive algorithm is to find a recursive solution to Eq.(12) avoiding matrix inversion. One way of doing this would be to use the steepest descent method [3]. In this method, the adaptation starts with an arbitrary set of initial values, $\underline{W}(0)$, for the weights. An iterated change in the weighting coefficients in the direction of the negative gradient of the MSE is performed until the minimum point is reached.

Therefore, the weighting coefficients are updated by the steepest descent method as follows

$$\underline{W}(n+1) = \underline{W}(n) + \mu(- \underline{G}(n)) \quad (13)$$

where μ is a positive constant which controls stability and the rate of convergence.

B)Least Mean Square Method

The method of steepest descent described above requires the determination of the gradient vector at successive points on the MSE performance surface. In practice, the true values of the gradient are not available (the calculation of expectation is not feasible in practice). To overcome this difficulty, the least mean square (LMS) method provides a practical solution for implementing the method of steepest descent. In this case, an estimate of the gradient of the MSE is used instead of the true gradient [3]. This gradient estimate is determined by considering the square value of the instantaneous error signal as an estimate of the MSE. Therefore, by differentiating Eq.(5) with respect to $\underline{W}(n)$, yields

$$\hat{\underline{G}}(n) = \begin{bmatrix} \partial e(n)^2/\partial w_1 \\ \partial e(n)^2/\partial w_2 \\ \cdots\cdots \\ \cdots\cdots \\ \partial e(n)^2/\partial w_{2L} \end{bmatrix} = -2e(n)\underline{X}(n) \quad (14)$$

Substituting Eq.(14) into Eq.(13) results in the LMS algorithm as

$$\underline{W}(n+1) = \underline{W}(n) + 2\mu e(n)\underline{X}(n) \quad (15)$$

Sampled data adaptive antenna array using LMS algorithm has been given already in Fig. 1. Although this was mainly intended to illustrate mathematical procedures and basically a block diagram representation of the equations, it is probably the most efficient implementation in terms of adaptation rate and quality of solution. It is also very expensive to implement demanding real time digital data throughput and for practical purposes it is better to adopt cheaper methods either analog or hybrid analog/digital schemes which have a convergence rate penalty but give just as good as solution eventually. In many practical interference cases these slower implementation are entirely adequate.

Analog implementation of the LMS algorithm Eq.(15), is done simply by setting [12]

$$d\underline{W}(t)/dt = -\mu\hat{G}(t) = 2\mu e(t)\underline{X}(t) \quad (16)$$

and solving with a set of integrators

$$\underline{W}(t) = \underline{W}(0) + \mu \int_0^t \hat{G}(t)\, dt$$

$$= W(0) + 2\mu \int_0^t e(t)\underline{X}(t)\, dt \quad (17)$$

Continuous time implementation of adaptive antenna with LMS algorithm in Eq.(17) is shown in Fig. 2.

III-EFFECTS OF COMPONENT NONIDEALITIES

For a practical multiplier, the output will differ from the theoretical product of its inputs by an amount ϵ , as defined by

$$V_o = K_1 V_x V_y + \epsilon (V_x, V_y) \qquad (18)$$

where V_o is the multiplier output voltage, V_x and V_y are the multiplier inputs, and $K_1 V_x V_y$ is the true multiplier product. The error term can be expanded into terms directly related to the error sources in the multiplier circuit [13]. In Fig. 2, each of the input multipliers has two inputs defined as $x_i(t)$ and $2\mu e(t)$, where i is the tap number. Therefore, the output voltage of the ith first multiplier is given by

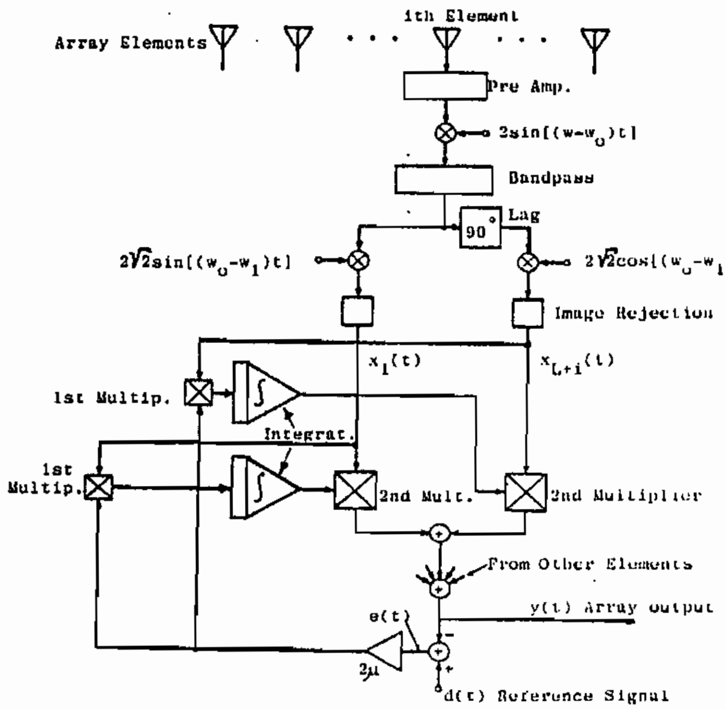

Fig. 2, Adaptive Array Processor Using Continuous-Time LMS Algorithm.

$$V^1_{oi} = K_1 [(x_i(t) + x_{os})(2\mu e(t) + y_{os}) + z_{os} + f(x_i(t), 2\mu e(t))] \qquad (19)$$

where the superscript 1 refers to first multiplier. The three sources of d.c. errors in an analog multiplier are: input offsets x_{os}, y_{os}, output offset z_{os}, and nonlinearity $f[x_i(t), 2\mu e(t)]$. Therefore, Eq.(19) can be rewritten as

$$V^1_{oi} = 2\mu K_1 e(t)x_i(t) + K_1[x_i(t)y_{os} + 2\mu e(t)x_{os} + z_{os} + k_2 x_i(t)^2 + k_3(2\mu e(t))^2] \quad (20)$$

If the multiplier is followed by a gain amplifier such that the true product is $2\mu x_i(t)e(t)$, then one can assume that $K_1 = 1$. This yields

$$V^1_{oi} = 2\mu e(t)x_i(t) + [(x_i(t)y_{os} + 2\mu e(t)x_{os}) + z_{os} + k_2 x_i(t)^2 + k_3(2\mu e(t))^2] \quad (21)$$

where $x_i(t)y_{os} + 2\mu e(t)x_{os}$ are feedthrough terms due to input offset voltages, z_{os} is an output offset voltage independent of $x_i(t)$ and $e(t)$, and $k_2x_i(t)^2$ and $k_3(2\mu e(t))^2$ are nonlinear terms due to transistor mismatch. The feedthrough terms can be neglected in high frequency applications (since the output of the first multiplier goes into an integrator), but must be considered in low frequency applications. The terms $k_2x_i(t)^2$ and $k_3(2\mu e(t))^2$ result in a nonzero d.c. component being added to the weights. The output offset voltage z_{os} is added to the true product. The nonlinear terms are important because the value of their d.c. component is dependent on input signal power. Therefore, although one could adjust the d.c. balance of the multiplier so that $z_{os} + k_2x_i(t)^2 + k_3(2\mu e(t))^2$ is zero for particular values of $x_i(t)$ and $e(t)$, the balance will be destroyed when one of the signal power levels is changed.

The effects of component nonidealities will be presented in four parts. First, the effects of input multiplier output offset voltages will be explained; second, the combined effects of input multiplier output offset voltages and nonlinearities will be presented; third, the effects of integrator offset voltages and bias currents will be shown; and fourth, the other nonlinearities will be considered.

a)The Effects of Input Multiplier Output Offset Voltages

Assuming that all of the circuit components used in the construction of a discrete-time adaptive antenna array are ideal, except that an output offset voltage error occurs in the first multipliers, the output of the first multiplier in the ith tap can be expressed as

$$V^1_{oi}(n) = 2\mu e_i(n)x_i(n) + z_{osi} \quad (22)$$

Define the vector

$$\underline{Z}_{os} = [z_{os1} \quad z_{os2} \quad \cdots\cdots \quad z_{os2L}]^T \quad (23)$$

whose elements z_{osi} are the output offset voltages for the ith input multiplier, where z_{osi} is a random variable which can take on a value between zero and z_{os} (max.). These values can be obtained from manufacturer's data sheets. The expected value of z_{os} is considered to be a constant d.c. voltage for all time after the power switch is turned on. Therefore,

$$\underline{Z} = E[\underline{Z}_{os}] \quad (24)$$

Applying Eq.(23) into the LMS algorithm in Eq.(15) yields

$$\underline{W}(n+1) = \underline{W}(n) + 2\mu e(n)\underline{X}(n) + \underline{Z}_{os} \quad (25)$$

Substituting for e(n) from Eq.(4-b), then

$$\underline{W}(n+1) = \underline{W}(n) + 2\mu\underline{X}(n)[d(n) - \underline{X}^T(n)\underline{W}(n)] + \underline{Z}_{os} \tag{26}$$

Taking the expected value of both sides of Eq.(26) and assuming $\underline{W}(n)$ to be fixed, then

$$\underline{W}(n+1) = \underline{W}(n) - 2\mu E[\underline{X}(n)\underline{X}^T(n)]\underline{W}(n) + 2\mu E[d(n)\underline{X}(n)] + E[\underline{Z}_{os}]$$

$$= [I - 2\mu\underline{R}]\underline{W}(n) + 2\mu\underline{S} + \underline{Z} \tag{27}$$

where I is a 2Lx2L unit matrix and $\underline{R}$ and $\underline{S}$ are as defined before. As the autocorrelation matrix $\underline{R}$ is positive definite, it can be expressed in normalised form as

$$\underline{R} = \underline{Q}^{-1}\underline{\Lambda}\,\underline{Q} \tag{28}$$

where $\underline{\Lambda}$ is a diagonal matrix of the eigenvalues of $\underline{R}$. The matrix $\underline{Q}$ is a square matrix called the modal matrix of $\underline{R}$. Its columns are assumed to be orthonormal eigenvectors of $\underline{R}$. Consequently [3],

$$\underline{Q}\underline{Q}^T = I \qquad \text{and} \qquad \underline{Q}^{-1} = \underline{Q}^T \tag{29}$$

Now, let us study the transient response of Eq.(27). First we make a rotation of coordinates [3] into the "primed" coordinates system such that

$$\underline{W}(n) = \underline{Q}\underline{W}'(n) \tag{30}$$

This will cause a rotation of coordinates into the principal axis of R. Substituting Eqs.(28), (29), and (30) into Eq.(27) and premultiplying both sides by $\underline{Q}^T$, Eq.(27) becomes

$$\underline{W}'(n+1) = [I - 2\mu\underline{\Lambda}]\underline{W}'(n) + 2\mu\underline{Q}^T\underline{S} + \underline{Q}^T\underline{Z} \tag{31}$$

Define

$$\underline{S}' = \underline{Q}^T\underline{S} = \begin{bmatrix} s_1 \\ s_2 \\ \cdots \\ \cdots \\ s_{2L} \end{bmatrix} \tag{32}$$

and

$$\underline{Z}' = \underline{Q}^T\underline{Z} = \begin{bmatrix} z_1 \\ z_2 \\ \cdots \\ \cdots \\ z_{2L} \end{bmatrix} \tag{33}$$

Therefore, Eq.(31) may be expressed as

$$\underline{W}'(n+1) = [I - 2\mu \underline{\Lambda}]\underline{W}'(n) + 2\mu\underline{S}' + \underline{Z}' \tag{34}$$

The general solution of Eq.(34) depends on the eigenvalues of the $\underline{R}$ matrix. A scalar expression for each of the primed weights can be deduced from Eq.(34) as

$$w_i'(n+1) = (1 - 2\mu\lambda_i)w_i'(n) + 2\mu s_i' + z_i' \tag{35}$$

where λ_i is the ith eigenvalue of $\underline{R}$ and s_i' and z_i' are the ith primed cross correlation and offset voltage error respectively. With initial weight vector $\underline{W}'(0)$, n+1 iteration of Eq.(35) yield

$$w_i'(n+1) = (1 - 2\mu\lambda_i)^{n+1} w_i'(0) + 2\mu s_i' \sum_{m=0}^{n} (1 - 2\mu\lambda_i)^m + z_i' \sum_{m=0}^{n} (1 - 2\mu\lambda_i)^m \tag{36}$$

If μ is made small enough so that the element $(1 - 2\mu\lambda_i)$ has magnitude less than one, then as the number of iterations increases, the limit [14]

$$\lim_{n\to\infty} (1 - 2\mu\lambda_i)^{n+1} \longrightarrow 0 \tag{37}$$

This requires

$$|1 - 2\mu\lambda_i| < 1$$

or

$$0 < \mu < 1/\lambda_i \tag{38}$$

Considering the second and third terms in the right hand side of Eq.(36) when μ satisfies condition (38) so that the element $(1 - 2\mu\lambda_i)$ is less than unity, it can be shown by summing the geometric series that

$$\lim_{n\to\infty} \sum_{m=1}^{n} (1 - 2\mu\lambda_i)^m \longrightarrow 1/2\mu\lambda_i \tag{39}$$

Substituting Eqs.(37) and (39) into Eq.(36) yields

$$\lim_{n\to\infty} w_i'(n+1) = 0 + s_i'/\lambda_i + z_i'/2\mu\lambda_i$$

or

$$w_i'(n+1) = w_i^{*\prime}(n) + z_i'/2\mu\lambda_i \tag{40}$$

Hence for positive eigenvalue λ_i and μ satisfies Eq.(38), the effect of the input multiplier output offset voltage is to alter the steady-state solution of the weight by a value equal to $z_i'/2\mu\lambda_i$. On the other hand, if λ_i is zero, the limit in Eq.(39) tends to ∞ and

$$w_i'(n+1) = w_i^{*\prime}(n) + z_i' \sum_{m=0}^{\infty} 1^m = \infty \tag{41}$$

Thus the input multiplier output offset voltages cause the weights corresponding to zero (negative) eigenvalues to increase indefinitely. Therefore, these weights never reach a steady state solution, they saturate instead. Even those weights corresponding to positive eigenvalues differ greatly from the ideal. From Eq.(40) it can be seen that as μ approaches zero, w_i^{ξ} approaches infinity. This is a very interesting result and contradicts all previous results assuming ideal case [3]. Let us examine this result more closely. Applying Eq.(12) into Eq.(9), the minimum MSE in the ideal case can be expressed as

$$\xi_{min} = E[d(n)^2] + \underline{W}^{*T}\underline{R}\underline{W}^* - 2\underline{S}^T\underline{W}^*$$
$$= E[d(n)^2] - \underline{S}^T\underline{W}^* \qquad (42)$$

Now, return to the problem of calculating the MSE when the input multiplier has a nonzero output offset voltage and recall from Eq.(40) that the weight vector was suboptimal. We therefore, want a means of expressing its deviation from optimum and the resulting increase in MSE. Define an error vector as

$$\underline{V}(n) = \underline{W}(n) - \underline{W}^* \qquad (43)$$

Substituting this value for $\underline{W}(n)$ in Eq.(9) yields

$$\xi = \xi_{min} + (\underline{W}(n) - \underline{W}^*)^T\underline{R}(\underline{W}(n) - \underline{W}^*) \qquad (44)$$

When $\underline{R}$ is nonsingular, the weights approach a steady-state solution which is found by substituting $(\underline{S} + \underline{Z}/2\mu)$ in Eq.(27) for $\underline{S}$ in Eq.(12), therefore,

$$\underline{W} = \underline{W}^* + (1/2\mu)\underline{R}^{-1}\underline{Z} \qquad (45)$$

Hence the effect of the input multiplier output offset voltage is to shift the weights from their optimum point by an amount $(1/2\mu)\underline{R}^{-1}\underline{Z}$. From Eq.(45), we have

$$\underline{W} - \underline{W}^* = (1/2\mu)\underline{R}^{-1}\underline{Z} \qquad (46)$$

Substituting Eq.(46) into Eq.(44) shows that the steady-state MSE will be

$$\xi_{ss} = \xi_{min} + (1/2\mu)^2\underline{Z}^T\underline{R}^{-1}\underline{Z} \qquad (47)$$

This is the result we were looking for. Eq.(47) reveals that the input multiplier output offset voltages increase the mean square error by an amount $(1/2\mu)^2\underline{Z}^T\underline{R}^{-1}\underline{Z}$ over its optimum value. Therefore, even when $\underline{R}$ is nonsingular there is an extremely large increase in MSE, which is inversely proportional to the square of μ. This result contradicts Widrow's theory [3], which states that the excess MSE is directly proportional to μ.

b)The Combined Effects of Input Multiplier Nonlinearities and Offset Voltages

Consider the input multiplier nonlinearities and assume all other circuit components are ideal, therefore, Eq.(21) can be reduced to

$$V_{oi}^1 = 2\mu e(n)x_i(n) + k_2 x_i(n)^2 + k_3(2\mu e(n))^2 \qquad (48)$$

Define

$$f_i(n) = k_2 x_i(n)^2 + k_3(2\mu e(n))^2 \qquad (49)$$

as the nonlinear terms lumped together, then combining Eq.(22) and Eq.(49) gives the output of the ith input multiplier as

$$v^{1}_{oi} = 2\mu e(n)x_{i}(n) + \delta_{i}(n) \tag{50}$$

where $\delta_i(n)$ represents the noise in the multiplier output, i.e.,

$$\delta_i(n) = z_{osi} + f_i(n) \; .$$

Define the vector $\underline{\Delta}_s$ as

$$\underline{\Delta}_s = [\, E[\delta_1(n)] \quad E[\delta_2(n)] \;\ldots\ldots\; E[\delta_{2L}(n)] \,]^T \tag{51}$$

Following the same procedure as in the previous section, it is easy to obtain the steady-state weighting vector as

$$\underline{W}_{ss} = \underline{W}^* + (1/2\mu)\underline{R}^{-1}\underline{\Delta}_s \tag{52}$$

and

$$\xi_{ss} = \xi_{min} + (1/2\mu)^2\, \underline{\Delta}_s^T \underline{R}^{-1} \underline{\Delta}_s \tag{53}$$

c)Effects of Integrator Offset Voltages and Bias Currents

It is easily shown [8] that the integrator errors due to offset voltages and bias currents can be grouped together such that the steady-state weight vector is given as

$$\underline{W}_{ss} = \underline{W}^* + \underline{V}_{os} \tag{54}$$

where $\underline{V}_{os} = [\, E[v_{os1}] \quad E[v_{os2}] \;\ldots\ldots\; E[v_{os2L}] \,]^T$ and v_{osi} is the integrator error due to offset voltage and bias current at ith tap.

Substituting Eq.(54) into Eq.(43) yields

$$\xi = \xi_{min} + \underline{V}_{os}^{\,T}\, \underline{R}\, \underline{V}_{os} \tag{55}$$

That is, there is also excess MSE due to the integrator offset voltages and bias currents.

d)Summer Nonidealities

The summer used to form the output of the adaptive filter can possess an offset voltage. This offset voltage will affect the steady-state solution of the weight vector and MSE. In this case, the adaptive filter output will be given by

$$y(n) = y^*(n) + \gamma \tag{56}$$

where $y^*(n)$ is the output of an ideal summer, and γ is a random variable representing the offset error. Therefore, the weight vector is expressed as

$$\underline{W}(n+1) = \underline{W}(n) + 2\mu\underline{X}(n)[d(n) - \underline{X}^T(n)\underline{W}(n) - \gamma\,] \tag{57}$$

Taking the expectations and rearranging terms yields

$$\underline{W}(n+1) = (I - 2\mu\underline{R})\underline{W}(n) + 2\mu(\underline{S} + \underline{\Gamma}) \qquad (58)$$

where

$$\underline{\Gamma} = E[X(n)\gamma]$$

Following the same procedure given in section III part (a), it is easy to show that

$$\underline{W}_{ss} = \underline{W}^* + \underline{R}^{-1}\underline{\Gamma} \qquad (59)$$

and

$$\xi_{ss} = \xi_{min} + \underline{\Gamma}\underline{R}^{-1}\underline{\Gamma} \qquad (60)$$

IV-SOLUTIONS

This section presents two techniques which offer promise in solving some of the problems resulting from adaptive system internal circuit component nonidealities.

a)Differential Integrator

A leaky integrator with a resistor added in the feedback loop in parallel with the integration capacitor provides a feedback path which may reduce the drift errors associated with the standard integrators. But this circuit still has two drawbacks; 1)the drift errors are not completly eliminated, and 2)the added resistor causes the integrator to have a finite memory. A better solution, known as differential integrator, is shown in Fig. 3. This circuit can be used to minimise the effects of bias currents and offset voltages for an analog integrator.

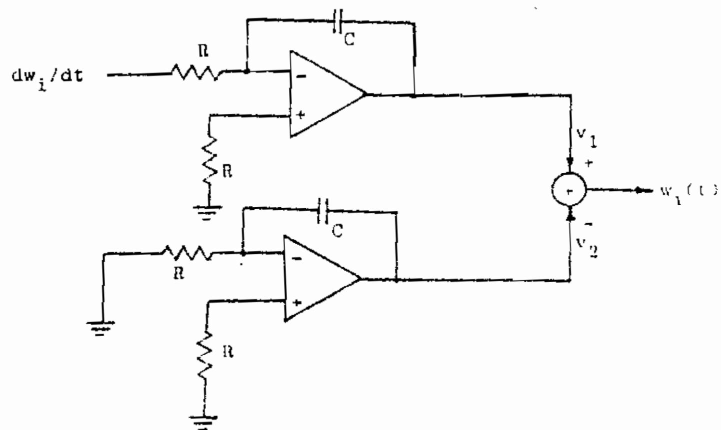

Fig. 3, Differential Integrator

The operation of the circuit shown in Fig. 3 is as follows: Assume the output of the top integrator is v_1, and the output of the bottom integrator is v_2. Since the input to the bottom integrator is zero, its output will only be the error terms due to bias currents and offset voltages. the output of top integrator will be $w_i(t)$ plus these same error terms. For identical OpAmps (a matched pair on a single integrated circuit), and identical resistor and capacitor values, the error terms from top and bottom integrators should be identical. After subtraction, the desired quantity $w_i(t)$ is left free of error terms and with infinite memory.

b)Distributed Loop Gain Multiplier

The effects of the most dangerous errors, those caused by the input multipliers, are configuration dependent. Therefore, a modification in the standard configuration can reduce these errors. The new configuration "called distributed loop gain multiplier" is shown in Fig. 4. In this figure the noise is reduced by an amount K while the required output signal remains the same.

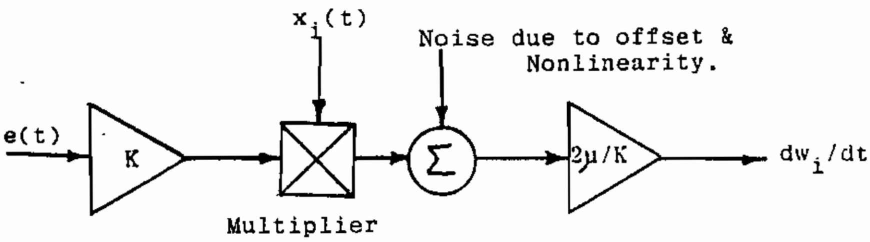

Fig. 4, Distributed Loop Gain Multiplier.

The output from Fig. 4 is given as

$$dw_i/dt = (2\mu/K)[Ke(t)x_i(t)] + (2\mu/K)[z_{osi} + k_2x_i^2(t) + k_3(Ke(t))^2] \quad (61)$$

The first term on the right hand side of Eq.(61) represents the required output signal and the second term represents the noise due to output offset and nonlinearities. This term will be referred to as

$$\delta_{iD} = (2\mu/K)[z_{osi} + k_2x_i^2(t) + k_3(Ke(t))^2]$$
$$= (2\mu/K)[z_{osi} + k_2x_i^2(t)] + 2\mu k_3Ke^2(t) \quad (62)$$

As $K \rightarrow 0$, $\delta_{iD} \rightarrow \infty$, because of the first two terms, and as $K \rightarrow \infty$, $\delta_{iD} \rightarrow \infty$, because of the last term. Therefore, there must be some value of K which is optimum for this configuration, i.e., optimum in the sense that it reduces the effects of the errors. To find this value of K which minimises the noise power, first square and take the expectations of both sides of Eq.(62), therefore,

$$E[\delta_{iD}^2(t)] = (2\mu/K)^2E[z_{osi} + k_2x_i^2(t)]^2 + E[2\mu k_3Ke^2(t)]^2$$
$$+ 8\mu^2k_3E[e^2(t)(z_{osi} + k_2x_i^2(t))]$$

Taking the derivative of the above equation with respect to K and setting the result equal to zero yields