Mansoura Engineering Journal

Volume 16 | Issue 1 Article 9

5-1-2021

Using Multiple Objective Techniques to Model Hierarchical Production Planning (HPP) Problems (Part 1: Theoritical Study).

M. Rasmy
Operations Research Program ISSR., Cairo University.

S. Ismail
Central Bank., Cairo.

Recommended Citation

Rasmy, M. and Ismail, S. (2021) "Using Multiple Objective Techniques to Model Hierarchical Production Planning (HPP) Problems (Part 1: Theoritical Study).," *Mansoura Engineering Journal*: Vol. 16 : Iss. 1 , Article 9.
Available at: https://doi.org/10.21608/bfemu.1991.169624

Using Multiple Objective Techniques to Model Hierarchical Production Planning (HPP) Problems (Part I: Theoritical Study)

مدخل البرمجه الهدفيه لبناء نموذج لتخطيط الانتاج الهرمى
(الجزء الأول : دراسه نظريه)

M.H. Rasmy
Operations Research Program
ISSR, Cairo Univ.

S. Ismail
Central Bank, Cairo.

خلاصه : فى هذا البحث تم تطوير نموذج رياضى متعدد الأهداف لحل مشكلة تخطيط الانتاج فى أطار ذات مستويات متعددة لاتخاذ القرار. طبقا لهذا النموذج يمكن تقليل تكاليف التخزين وتداول المواد والتحكم فى مستويات المخزون . ومن مميزات هذا النموذج هو امكانية حله باستخدام طرق حل البرمجة الخطية العاديه . ينتج عن حل هذا النموذج تحديد لكميات الانتاج المخصصه لكل نوع فى كل فترة زمنيه تحتويها الفترة التخطيطية لنموذج تخطيط الانتاج الاجمالى للأنواع. فى هذا البحث تم بناء النموذج فى ظل هدفين لهما نفس المستوي من الأولوية .

Abstract- This paper proposes a new approach for solving the production planning and scheduling problem called "Goal Programming approach to Hierarchical Production Planning". This approach combines the attractive feature of both goal programming as a powerful tool for multi-objective analysis and the hierachical system as an effective framework for decision making in a single-stage batch processing environment.

The proposed procedure assumes that there are two levels of the product aggregation in the production structure from the Hax and Meal framework [8]. Production items may be aggregated into families and families aggregated into types. Type is a collection of items that have the same demand pattern, the same unit costs, direct costs (excluding labour costs), holding costs per unit per period, and the production time required per unit. A family is a set of items within a type such that the items share a common setup. This form of aggregation may result in partitioning the production planning and scheduling problem into two subproblems in a hierarchy. The two subproblems are the aggregation production planning subproblem and the family disaggregation subproblem. The aggregation production planning subproblem, the highest level of planning in the hierarchical system, is concerned with the effective allocation of production resoures amongst product types to satisfy demand over a specified planning horizon. Typical decisions to be made at this level are the determination of production and inventory levels for each product type and regular and overtime workforce levels in each time period. The family diaggregation subproblem, the second level of planning, is concerned with the disaggregation of aggregate production plan for each type into production schedules for families belonging to that type over a short scheduling horizon. Typical decisions to be made at this level are the determination of production and inventory levels for each family within a type in each time period in the scheduling horizon.

INTRODUCTION

Various methods for solving the single stage production planning and scheduling problem, have been reported in the literature. These methods range from simulation to search procedures, heuristic rules, and explicit mathematical solution. The mathematical programming methods can be classified into two distinct approaches:-

The first approach termed a Monolithic approach, formulates the problem as a large mixed-integer linear programming problem (e.g. Manne [14], Dzielinski and Gomory [4], Lasdon and Terjung [11]). Typically the mathematical programme is solved approximately each planning period with only the immediate periods decision being implemented. The most common solution procedure (e.g. [14], [4], [1]) has been shown to be equivalent to using a lagrangean relaxation to solve the dual to the mixed-integer programme. The dual solution is then rounded to obtain a good feasible solution.

The second approach is a Hierarchical approach which partitions a production planning and scheduling problem into a hierarchy of subproblems (e.g. Hax and Meal [9]). In any planning period, the subproblems are solved sequentially, with the solutions of subproblems from the upper hierarchy imposing constraints on the lower hierarchy subproblems. Again the system only implements the decisions for the immediate period.

The hierarchical approach has potentially three advantages over the monolithic approach. The first advantage is that it is computationally simpler than a monolithic approach which must solve a large mixed-integer programming problem. The second advantage is that the hierarchical approach may require less detailed demand data, in that it needs only aggregate product demand data over the planning horizon (for the aggregate planning subproblem), with detailed product demand over a much shorter scheduling horizon (for the scheduling subproblem). The monolithic approach usually requires detailed demand data for all the planning horizon. The third advantage of the hierarchical approach is that it increases the interaction between the planning system and the decision-markers each hierarchical level, and improves the co-ordination of objectives throughout the organisation.

Several goal programming models have been reported in the literature for solving the production planning and scheduling problem. Lee [13], Lawrence and Burbridge [12], addressed the use of GP to solve planning problem at the aggregate level in terms of defining the production levels of different products, that required processing in a specified planning horizon. This previous research work in GP planning models did not attack the scheduling function of the problem in terms of determining how the available production resources should be allocated to each item in order to provide the best attainment of management goals for a given scheduling horizon. Gonzalez and Reeves [6] suggested a GP for developing a master production schedule for a batch production system. They formulate the production planning and scheduling problem as a large, linear, zero-one goal programme. Their model based upon the idea of considering the different potential production sequences as a parameter which must be defined outside the model and designed so that production plus avaiable inventory in each period will be sufficient to satisfy known demand for product in each period. They use the proof of Manne [14] about considering only such schedules that at given time periods produce either zero or the sum of consecutive demands for some number of periods in the future and that the number of production schedules limited to a maximum of 2^{T-1}, for every product, where T is the specified planning horizon to be convered by the schedule (in their illustrative examples they consider a planning horizon consisting of six periods). Once a particular schedule is defined, its characteristics in terms of costs, inventory levels, resource requirements, etc. are computed and used as input parameter to the GP model. Their model is a large-linear zero-one goal programme, which is quite difficult to solve for realistically-sized problems. This is because they fail to recognise the levels of product aggregation and hence cannot exploit implementation and computational opportunities suggested by these product aggregates. They consider three goals in their GP, minimizing total economic cost, minimising the comulative level of inventory and minimizing the over and/or under-unilization of the desired level of limited resources. They include the setup time in the goal constraint of minimising the total economic cost. They solve their model by relaxing the integrality restriction and using continuous decision variables (insted of the zero-one variables) then be rounded to give a feasible solution.

Generally, the Linear Goal Programming Problem is in the following specific form

Minimize : $\underline{Z} = (Z_1, Z_2, \ldots, Z_k, \ldots, Z_k)$ (1)

Such that:

$$\sum_{j=1}^{n} g_{ij} x_j + d_i^- - d_i^+ = g_i^*, \quad i=1,\ldots\ldots,q \qquad (2)$$

(Goal constraints)

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i=1,\ldots,m \qquad (3)$$

(Regular constraints)

and

$$x_j,\ d_i,\ d_i^+ \geq 0, j=1,\ldots,n; \quad i=1,\ldots q \qquad (4)$$

(non-negativity constraints)

with

$$Z_k = f_k(\underline{d}^-, \underline{d}^+)$$

where:

x_j: the $j^{\underline{th}}$ decision variable

$\underline{Z}$: a vector valued function and denoted as the achievement function; a row vector measure of the attainment of the objectives or constraints at each priority level.

$f_k(\underline{d}^-, \underline{d}^+)$ is a scalar valued function (linear function) of the deviation variables associated with objectives or goal constraints at priority level k.

k : is the total number of priority levels in the model.

g_i^*: is the desired level for the i^{th} objectives or goal constraints.

$\sum_{j=1}^{n} g_{ij} x_j$ is a linear function that measures the actual attainment of the i^{th} objective or goal constraint.

d_i^-, d_i^-: are respectively the positive and negative deviation variables associated with goal constraint i or in other definition, the under-achievement and the over-achievement of of the i^{th} objective or goal constraint, $\sum_{j=1}^{n} g_{ij} x_j$ from its desired level g_j^*

THE RELEVANT CHARACTERISTICS OF THE HPP PROBLEM

It is a suitable time to focus on the production planning and scheduling problem in a single stage batch processing environment in the Hierarchicl framework. The relevant characteristics of such problems are:-

a) The production-inventory system is concerned with multi-product.
b). There is only one facility to process all the products.
c) Only one product can be made at any given time.
d) For each product, the production rate is deterministic.
e) The demand data for each product is not rigid, because demand forecasts for periods farther into the future are likely to be of poorer quality. Hence, the demand data is continually revised as the demand forecasts become less uncertain.
f) The quantity produced in each production run for a given product is variable.
g) Backorders are not allowed, in other words, it is assumed that the sales lost in one period (i.e. the unfilled demand) cannot be recovered in the next period.
h) For our study we assume that there are two levels recognised in the product structure (as identifiled by Hax and Meal [9]):

   Product Type: is a collection of items that have the similar seasonal demand pattern and the same unit cost, direct costs (excluding labour), holding costs per unit period, and production rate as defined by the number of units that can be produced per unit time.

   Family: is a set of items within a type, such that the items share a common setup.

i) The setup costs to be considered are those major setup costs associated with each family, assuming that the minor setup costs associated with items within each family is zero. This means that the setup cost depends only on the family being made. That is whenever a machine is prepated to produce an item of a family, all other items in the same family also be produced with minor change in setup. Seeking for simplicity, these minor changes in setups are neglected in this work.
j) It is assumed that the setup times for all families are zero. This means that the capacity feasibility constraint will not include the setup times.
k) It is assumed that there is no lead time. It should be noted that the problem can be easily generalized to the simulations

where lead times are non zero, by solving the problem with zero lead time and transforming the resulting solution into a feasible plan by adjusting the time of production by the amount of lead time.

e) For coping with uncertainty of demand forecasts made at different points in time and reducing the complexity of the solution process, the production planning and scheduling problem is broken into a hierarchy of subproblems. Two subproblems are considered and based on the levels of product aggregation. In any planning period, the two sub-problems are solved sequentially, with the solutions of subproblem from the upper hierarchy imposing constraints on the lower hierarchy subproblem and the system only, implements decisions for the immdediate period. The first subproblem is the aggregated planning subproblem which allocates production capacity among product types over the planning horizon of types (normally one year divided into twelve months). The second subproblem is the family disaggregation subproblem which allocates the production for each product type among the families belonging to the type by disaggregating the results of the aggregate planning model for some periods defined by the length of the scheduling horizon of families. Each planning subproblem in the hierarchical system has its own characteristics, including length of planning horizon, level of details of the required information, and management's goals.

Given the above twelve characteristics, the problem of the Hierarchical production planning in a single-stage production system is to determine the production quantity for each product (i.e. item) in each time period so that the management's goals associated with each subproblem in the hierarchical planning system are met.

THE AGGREGATE PLANNING MODEL FOR PRODUCT TYPES

1- Management Goals

There are several objectives under which the management of the firm wishes to consider the aggregate production plan for product types over the multiperiod planning horizon. These include be following:-

1) Avoiding any underutilization of normal production capacity in each time period (no layoffs of production workers).
2) Limitation of the overtime operation to the desired level at each time period.
3) Satisfaction of demand requirements for all product types by production in each time period.
4) Realization of the desired ending inventory levels at the last period in the planning horizon for all product types.

The first goal reflects the desire of the firm to ensure that regular time is fully used before overtime is employed. Moreover, the elimination of any underutilization of normal production capacity provides job security to the firm employees by considering fixed work force level.

The second goal illustrates that although the stable employment level with accasional overtime is a better practice than the unstable employment with no overtime, the management feels that overtime more than certain limits in each time period should be avoided. Overtime worked more than its upper limit is not desired because of the declining productivity caused by fatigue

The management considers that goal 1 and goal 2 must be exactly achieved. Therefore, the two goals are formulated as regular rigid constraints. The normal production capacity constraints are expressed in the form of strict equations to justify that it is better to use all the available resource than to waste part of it. The overtime operation constraints are expressed in the form of inequilities of stipulate that the maximum capacity levels on the availability of production facility during overtime working hours should not be exceeded.

The third goal, to satisfy demand requirements for all product types in each time period by production in the same period reflects the desire of the firm to schedule production such that the demands are met, and the inventory holding costs, the handling costs and the transportation costs are to be minimized. This goal is practically implemented by minimizing the difference between the beginning and ending inventory levels for each product type in each time period.

The fourth goal, the realization of the desired ending inventory level for all product types at the last period of the planning horizon reflects the management's view of controlling the inventory levels of all product types while providing a reasonable level of safety stocks for each product in the each period. This goal is implemented in practice by setting a level of inventory for each product type, then scheduling production such that these levels are met. Normally this level is calculated on the basis of maintaining enough inventory to satisfy average demand for certain number of periods. Consequently, the magnitudes of the desired ending inventory level for each product, to be controlled, will depend upon what management considered reasonable stockout protection, the quality of demand forecasts and the level of customer service to be provided.

2- Model Structure

The above discussion indicates that the goal programming model for aggregate planning of product types includes only two goals. These goals are as follows:-

1) Minimization of the overachievement and the underachievement of the amount of production not directly used to satisfy demand during the period of production for each product type, at each time period.

2) Minimization of the overachievement and the underachievement of the desired ending inventory level for each product type at the last period of the planning horizon.

These goals for the goal programming model are similar to the management's goals 3 and 4, and to be formulated as the goal constraints of the model. The management's goals 1 and 2 related to the elimination of the underutilization of normal production capacity, and the limitation of overtime operation, respectively, are included in the set of the physical limitations of the production system and take the form of regualr constraints.

The goal programming model includes five basic sets of elements:-

a) Decision Variables

Let P_{it} be the number of units to be produced of product type i during time period t (where i=1,...,N; t=1,...,T).

$$P_{it} = P^{r}_{it} + P^{o}_{it}$$

Then the decision variables are:-

P^{r}_{it} = number of units to be produced of type i in regular time hours in period t (where i=1,...,N; t=1,...,T)

P^{o}_{it} = number of units to be produced of type i in overtime hours in period t (where i=1,...,N; t=1,...T)

I_{it} = Closing inventory of product type i at the end of period t (where i=1,...,N; t=1,...,T).

b) Parameters

D_{it} = Forecast demand for product type i in period t

m_i = Production time (hours) required per unit of product type i

$(rm)_t$ = total regular production time available in period t (i.e. normal production capacity in hours).

$(om)_t$ = total overtime hours available in period t

I_{io} = initial inventory level for product type i

T = time horizon, in periods

N = Total number of product types

I^{*}_{iT} = The desired closing inventory level for product type i in the last period T.

c) Goal Constraints

The goal constraints of the model are as follows:-

(i) The first goal seeks to minimize the overachievement and the underachievement of the amount of production not directly used to satisfy demand during the period of production for each product type, at each time period.

To achieve this goal exactly it is needed to schedule production for all types, in each time period, just to satisfy demand in the period of production. This means that the amount of production not direcly used to satisfy demand during the period of production needs to be zero. Given that this amount for type i in period t equal $(P_{it}-D_{it})$, then it is needed to let $P_{it}-D_{it} = 0$. But from the inventory balance relations we can show that $P_{it}-D_{it}=0$ implies that $I_{it}-I_{i,t-1}=0$.

Therefore, the set of constraints associated with the first goal takes the following form:-

$$I_{i,t} - I_{i,t-1} + d_{it}^{-} - d_{it}^{+} = 0 \qquad (5)$$

(for i=1,...,N; t=1,...,T)

where

d_{it}^{-} = the underachievement of the amount of production not directly used to satisfy demand (or the excess of demand over production level) for product type i in period t.

d_{it}^{+} = the overachievement of the amount of production not directly used to satisfy demand (or the excess of production level over demand) for product type i in period t.

Deviation variables to be minimised are d_{it}^{-}, d_{it}^{+}

(ii) The second goal seeks to minimise the underachievement and the overachievement of the desired ending inventory level for all product types at the last period of the planning horizon. These constraints take the following form:-

$$I_{iT} + d_{iT}^{*-} - d_{iT}^{*+} = I_{iT}^{*} \qquad (6)$$

where

d_{iT}^{*-} = the underachievement of the desired inventory (or inventory below desired level) for product type i in the last period T.

d_{iT}^{*+} = the overachievement of the desired inventory (or inventory above desired level) for product type i in the last period T.

Deviation variables to be minimised are d_{iT}^{*-}, d_{iT}^{*+}

d) Regular Constraints

(i) Production-Inventory Balance Constraints

$$P_{it}^{r} + P_{it}^{o} + I_{i,t-1} - I_{it} = D_{it} \qquad (7)$$

(ii) Regular Production Capacity Constraints

To assure the regular production time in each period is fully used

$$\sum_{i=1}^{N} m_i P_{it}^{r} = (rm)_t \qquad \text{(for } t=1,\dots,T) \qquad (8)$$

(iii) Limitation of Overtime Operation

To set upper limit for the total overtime available in each period

$$\sum_{i=1}^{N} m_i P^o_{it} \leq (om)_t \qquad (9)$$

(iv) Non-Negativity Constraints

$$P^r_{it},\ P^o_{it},\ I_{it},\ d^-_{it},\ d^+_{it},\ d^{*-}_{iT},\ d^{*+}_{iT} \geq 0 \qquad (10)$$

(for i=1,...,N; t=1,...,T)

It can be noticed that constraints (5-7) and (5-10) imply that no backordering is allowed.

e) Objective Function

The two goals of the model are of the same priority level, that is the proposed model is a weighted linear goal programming. The objective function is formulated as a scalar valued function of the deviational variables associated with the two goals. This function of this goal programming is as follows:-

$$\text{Min } Z = \sum_{i=1}^{N} \sum_{t=1}^{T} (u_{it} d^-_{it} + v_{it}\ d^+_{it} + u^*_{iT}\ d^{*-}_{iT} + v^*_{iT}\ d^{*+}_{iT})$$

with u_{it}, v_{it}, u^*_{iT}, v^*_{iT} of similar order (11)

where,

u_{it} = Scalar weighting factors assigned to the negative deviational variables d^-_{it}, i=1,...,N; t=1,...,T associated with the set of constraints of goal 1.

v_{it} = scalar weighting factors assigned to the positive deviational variables d^+_{it}, i=1,...,T associated with the set of constraints of goal 1.

u^*_{iT} = scalar weighting factors assigned to the negative deviational variables d^{*-}_{iT}, i=1,...,N associated with the set of constraints of goal 2.

v^*_{iT} = scalar weighting factors assigned to the positive deviational variables d^{*+}_{iT}, i=1,...,N associated with the set of constraints of goal 2.

The constraint sets of (5) - (10) are linear, and the objective function (11) is a linear function; the formulation can be solved by using linear programming methods.

The model is considered a simple planning of aggregate production. It considers only one constrained production resource and it incorporates only a single option for varying the resource level.

The given goal programming model structure has been developed as a decision making aid for the management of a production firm in allocating the available resource to product types in an effective manner. It allows the management of the firm to view the effects of changes in magnitudes of demands of product types according to different cases assumed to the forecast errors, production resource levels or desired inventory levels. More just producing a single aggregate plan, it produces a set of significantly different aggregate plants by changing the weighing factors associated with each goal involved in the objective function, which reflects the uncertainty typically related to the establishing of the management's goals and their weights.

THE FAMILY DISSAGGREGATION MODEL

In the following we propose a simple linear goal programming formal for modelling the families belonging to each corresponding type can be scheduled over the next three periods. We think that the horizon of three periods lenght is desirable for providing a disaggregation scheme that takes into consideration the advantage of minimizing the total number of setups over a suitable number of periods in future, moveover, the demand estimates for families for three later periods are easy to implement. The proposed multifamily, triple-period, scheduling problem would be implemented on a periodic basis. That is our disaggregation procedure requires forming a rolling horizon by solving a finite horizon triple-period problem and implementing only the first period decisions. One period later than triple-period problem is updated as better demand forecasts for families, and the corresponding aggregate planning decisions for types become available, and the procedure is repeated.

1- Scheduler's Goals

In scheduling families production in the next three periods, three goals are considered as follows:-

1) Co-ordinating production schedules of families belonging to a product type with production schedule that type.
2) Using production facility efficiently.
3) Controlling families inventory levels to ensure that no overstocks will occur.

These will be called the scheduler's goals.

The first goal reflects the desire to schedule families production such that the consistency between the aggregate production plan for types and the family disaggregation process would be assured. This goal is implemented by setting the amount detrmined by the aggregate plan for a type as a level for the sum of the production of the families in this type for each period in the scheduling horizon of three periods, then scheduling families production such that these levels are met.

The second goal reflects the desire to schedule production of famililties in a product type while minimizing the setup costs. This goal is implemented by setting as a goal the minimization of the total setup time of the families an a type in the next three periods.

The third goal reflects the desire to produce families in the correct quantitites such that storage requirements for each family in each time period not to be violated. This goal is implemented by setting an overstock limit for each family in each time period, then scheduling families production such that inventory above overstock limit for each family in each period is minimized.

2- Procedure Development

The family production scheduling procedure to be presented assumes the existence of demand schedules for each family over a scheduling horizon of three periods. In formulation the production schedules of the families belonging to a product type, the scheduler consideres the set of possible production sequences for each family such that the net demand requirements in each time period are met. Manne [14] suggested that it is enough to consider the dominant production sequences (or schedules) for each family, that is the production at any given period is either zero or the sum of consecutive demand for some number of periods into the future. When dealing with a time horizon of T periods, the total number of dominant production schedules to be considered are 2^{T-1}.

Thus, for a scheduling horizon of 3 periods the total number of dominant production schedules for any family j is $2^{3-1} = 4$ and these schedules can be constructed as follows:-

$$Q_{jt} = 0 \text{ or } Q_{jt} = \sum_{\ell=t}^{r} \hat{D}_{j\ell} \quad , t=1,2,3; \quad \ell \le 1, \; r \le 3 \qquad (12)$$

where,

$\hat{D}_{jt}$ = the net demand requirements of family j in period t

Q_{jt} = production quantity of family j in period t

We mean by net demand in a given period, that demand which cannot be satisfied from initial inventory in this period. If the initial inventory of a family is not zero, subtract it from the demand requirements in the first period to obtain the net demand for that period. If the initial inventory exceeds the first-period demand, continue with the adjustment process until all the inventory is used up.

In general, if D_{jt} is the forecast demand for family j in period t, I_{j0} is its corresponding initial inventory, the net demand $\hat{D}_{jt}$ of family j for period to is given by:-

$$\hat{D}_{jt} = \begin{cases} \max(0, \sum_{\tau=1}^{t} D_{j\tau} - I_{j0}), & i=1,2,3 \text{ if } D_{j,t-1} = 0 \\ D_{jt} & \text{otherwise} \end{cases} \qquad (14)$$

It is clear that, in using the policies defined in (13) for formulating the production schedules of families, may positively influence the attainment of the production scheduler's goal (2) but can violate goal (1) as previously defined. Goal (1) is attained only if

$\sum_{j \in J(i)} Q_{jt} = P^{*}_{it}$ for all t, t=1,2,3;

where J(i) is the set of families belonging to type i, P^*_{it} is the production of type i in period to determined by the aggregate plan, and Q_{jt} is the production of family j in period t. This means that for each time period it is required to produce families such that the type production equals the sum of the production quantities for its families.

The cases where there exists unit deviations of $_{j\ J(i)}\ Q_{jt}$ from P^*_{it} for all t, that is when either the case of $_{j\ J(i)}\ Q_{jt}\ \ P^*_{it}$ or the case of $_{j\ J(i)}\ Q_{jt}\ \ P^*_{jt}$ are met, Goal (1) is not attained and the resulting GP output needs to be adjusted. Since the disaggregation procedure would be implemented on a periodic basis, the adjustment is made only for the first period decisions (if it is required) to realise that Goal (1) is attained in the immediate period. The adjustment is based upon the application of a set of decision rules.

3- Model Structure

The goal programming model for the family disaggregation supproblem includes only two goals. These goals are as follows:-

1) Minimisation of the over-achievement and the under-achievement of the goal level for the sum of the productions of the families in a given type, at each time period.

2) Minimisation of the over-achievement of the goal level for the overstock of each family in a given type, at each time period.

These goals for the goal programming model corresponds to the production scheduler's goals (1) and (3). Production scheduler's goal (2) relates to the minimisation of the number of setups to be made for a given schedule, and is not explicitly considered.

The goal programming model includes five basic sets of elements:-

a) Input Parameters

Q_{jrt} = quantity to be produced of family j by means of production schedule r in period t; j J(i); r = 1,2,3,4; t=1,2,3.

The four production schedules for each family j are calculated outside the model as shown overleaf:-

Schedulte r for family j	Time periods t		
	t=1	t=2	t=3
r=1	$Q_{j11}=\hat{D}_{j1}+\hat{D}_{j2}+\hat{D}_{j3}$	$Q_{j12} = 0$	$Q_{j13} = 0$
r=2	$Q_{j21}=\hat{D}_{j1}+\hat{D}_{j2}$	$Q_{j22} = 0$	$Q_{j23} = \hat{D}_{j3}$
r=3	$Q_{j31}=\hat{D}_{j1}$	$Q_{j32}=\hat{D}_{12}+\hat{D}_{j3}$	$Q_{j33} = 0$
r=4	$Q_{j41}=\hat{D}_{j1}$	$Q_{j42}=\hat{D}_{j2}$	$Q_{j43} = \hat{D}_{j3}$

where $\hat{D}$. is the net demand requirement for family j in period t, j $J(i)$; t=1,2,3; $\hat{D}_{jt}$ is given by:

$$\hat{D}_{jt} \begin{cases} \max(0, \sum_{\tau=1}^{t} D_j = I_{j0}), \ t=1,2,3 \text{ if } D_{j,\tau-1} = 0 & (13) \\ D_{jt} \text{ otherwise} \end{cases}$$

D_{jt} = represents the demand forecast for family j in period t, and

I_{jo} is its corresponding initial inventory.

P^*_{it} = the goal level for the sum of the productions of all the families belonging to type i, t=1,2,3; P^*_{it} have been determined by the aggregate planning model for product types.

I_{jrt} = closing inventory in period t for family j produced according to production schedule r,r=1,2,3,4 & $\forall_{j,t}$ I_{jrt}'s are calculated outside the model as follows:-

$$I_{jrt} = I_{j0} + \sum_{\tau=1}^{t} (0_{jr\tau} - D_{j\tau}) \qquad (14)$$

OS_{jt} = goal level of overstock for family j in period t, $j \in J(i)$,t=1,2,3

J(i) = set of indices of all families belonging to type i.

b) <u>Decision Variables</u>

The decision variables are the θ_{jr}'s, j J(i); r=1,2,3,4 where

$$\theta_{jr} = \begin{cases} 1 & \text{if family j is produced according to production schedule r} \\ 0 & \text{otherwise} \end{cases}$$

c) Goal Constraints

(i) The first goal seeks to minimise the over-achievement and the under-achievement of the goal level for the sum of production quantities of all families belonging to a type, at each time period.

These constraints take the following form:-

$$\sum_{j \in J(i)} \sum_{r=1}^{4} Q_{jrt}\,\theta_{jr} + d^-_{P^*_{jt}} - d^+_{P^*_{jt}} = P^*_{it} \qquad (15)$$

(for t=1,2,3)

where,

$d^-_{P^*_{it}}$ = under-achievement of the goal level of the sum of the production quantities of all the families belonging to product type i in period t; t=1,2,3.

$d^+_{P^*_{it}}$ = over-achievement of the goal level of the sum of the production quantities of all the families belonging to product type i in period t; t=1,2,3.

Deviational variables to be minimised are $d^-_{P^*_{it}}$, $d^+_{P^*_{it}}$, t=1,2,3.

(ii) The second goal seeks to minimise the over-achievement of the goal level for the overstock of each family belonging to a type, at each time period. These constraints take the following form:-

$$\sum_{r=1}^{4} I_{jrt}\,\theta_{jr} + d^-_{os_{jt}} - d^+_{os_{jt}} = OS_{jt} \qquad (16)$$

(for j J(i); t=1,2,3)

where,

OS_{jt} = goal level for the overstock of family j in period t j ε J(i), t=1,2,3

$d^-_{os_{jt}}$ = under-achievement of the goal level of the overstock of family j in period t

$d^+_{os_{jt}}$ = over-achievement of the goal level of the overstock of family j in period t

Deviational variables to be minimised are $d^+_{os_{jt}}$.

d) Regular Constraints

The regular constraints of the model take the following form:-

(i) Assignment Restrictions:

Total fractional production of each family j must be added to unity, i.e.

$$\sum_{r=1}^{4} \theta_{jr} = 1 \qquad (17)$$

(for every j, $j \in J(i)$)

(ii) The integrality restrictions for the θ_{jr}'s

$$\theta_{rj} = 1 \quad \text{or} \quad 0 \qquad (18)$$

(for $j \in J(i)$; r=1,2,3,4)

(iii) Non-negativity Constraints

$$\theta_{jr},\ d^-_{P^*_{it}},\ d^+_{P^*_{it}},\ d^-_{os_{jt}},\ d^+_{os_{jt}} \geq 0 \qquad (19)$$

(for $j \in J(i)$; r=1,2,3,4; t=1,2,3)

It can be noticed that constraints (17) and (18) imply that each family be produced according to a single schedule

e) Objective Function

The two goals of the disaggregation model are of the same priority level, that is the proposed model is a weighted linear, zero-one goal programming. The objective function is formulated as a scalar valued function of the deviational variables to be minimised that associated with each goal. This function for this goal programming is as follows:-

$$\text{Min} \quad Z = \sum_{t=1}^{3} W_{P^*_{it}} \left(d^-_{P^*_{it}} + d^+_{P^*_{it}}\right) + \sum_{j \in J(i)} \sum_{t=1}^{3} W_{os_{jt}} \left(d^+_{os_{jt}}\right)$$

with $(W_{P^*_{it}}, W_{OS_{jt}})$ of similar order (20)

(for $j \in J(i)$, t=1,2,3).

This linear, zero-one goal programming can be solved using linear, zero-one programming methods, or by relaxing the integrality restrictions of the zero-one variables and using continuous variable, if the problem is of the large size problem or by generating the possible effective combinations for the solutions of the zero-one variables, if the problem is of the small size problem.

DECISION RULES

As we discussed above, the family diaggregation model may in result a families production schedule that does not achieve goal 1, which relates to the co-ordination of the production schedule of the families in a type with the production schedule of that type. The attainment of this goal is necessary for assuring consistency between the aggregate production planning model and the family disaggregation model. The disagreement between the two models happends model. The disagreement between the two models happends whenever the solution of the disaggregation model includes positive values either for $d^-_{p^*_{it}}$ or for $d^+_{p^*_{it}}$ at any time period t, t=1,2,3; that is when the total amount to be allocated among all the famillies belonging to type i and any time period will either be below or have the desired goal level which is the amount to be produced of type i in the corresponding period, and has been determined by the aggregate planning model.

Since the hierarchical planning system would implement the results of the family disaggregation model only for the first period of the scheduling horizon, it is necessary to adjust the first period decisions so as to make the sum of the production quantities of all the families in a type equal to the produc ion quantity of this type in the immediate period. The required adjustment is made according to the following decision rules:

Decision Rule 1:

No more than one family is to be chosen (whenever it is possible) for making the necessary adjustment for the sum of the production of all the families in a type in the first period.

Decision Rule 2:

$D^-_{p^*_{it}}$ is added to production of the family with the greated production figure in the first period such that no overstock will occur for this family as a result of the adjustment made.

Decision Rule 3:

$d^+_{p^*_{it}}$ is subtracted from production of the family when its overstock is violated in the first period, or from production of the family with the greatest production figure for statisfying demands for some periods in future. In the case of all families being produced to only satisfy first period demands, $d^+_{p^*_{i1}}$ is subtracted from production of the family with the greatest production figure for this period.

SUMMARY

The proposed approach for hierarchical production planning assumes that for planning purposes production items may be aggregated into families, and families aggregated into types: Aggregate plan for types is generated for each time period in the planning horizon, by means of a convenient linear goal programming model. The planning horizon of this model is six periods. For each type, the corresponding production quantitutes in the first three periods are disaggregated among all the families in that type to obtain families production schedule over the next three periods. The families schedule is generated by means of a goal programming formulation for the family diaggregation subproblem. The results of the family disaggregation model only for the first period of the scheduling horizon are implemented after adjusting the total production quantitative of all the families in a given type. This

adjustment is made by means of three decision rules, to assure the equality between the type production and the sum of the production quantity for its family in the immediate period. At the end of every time period, new information becomes available, that is used to update the aggregate planning model for types, and consequently the family disggregation model, and the proposed hierarchical planning process is repeated. This means that, for obtaining a production schedule for one year (i.e. 12 months) the proposed planning process is repeated twelve times

Although we have adopted two goal programming models for carrying out the hierarchical planning process, it is apparent that the four goals of these two models together, can be generally considered as the hierarchical planning system's goals. Since goal 1 of the family disaggregation model is permanently attained in the first period that would be implemented (as a result of the application of the decision rules, to the results of this model, for the immediate period), one can exclude this goal from the set of the effective hierarchical planning system's goals. Thus set includes the following goals:

I- To produce all types in quantities equal to their corresponding demand in each time period.

II- To control inventory levels for all types in the last period in the year to realise the corresponding planned levels.

III- To control inventory levels for all families in each time period so as to realize that no overstock will occur for any family at any time period.

The effectiveness of the proposed hierarchical planning approach described above is evaluated according to the the sume of the deviational variables associated with the three goals I, II and III and also according to the number of setups required by all the families in all types during one year.

Finally, this work will be applied on a practical problem in the future. All computational results will be published in coming paper.

REFERENCES

[1] Bitran, G.R., Haas, E.A. and Hax, A.C. "Hierarchical Production Planning: A Single Stage System", Operations Research, Vol. 29, No.4, pp. 717-743 (July-August 1981).

[2] Bitran, G.R., and Hax, A.C., "On the Design of Hierarchical Production Planning Systems", Decision Science 8, pp. 28-55 (1975).

[3] Dempster, M.A.H., Jansent, L., Lagewege, B.J., Lenstra, J.K. and Rinnooy Kan, A.H.S., "Analytical Evaluation of Hierarchical Planning Systems". Operations Research, Vol.29, No.4, pp 707-716 (July-August, 1981).

[4] Dzielindki, B.P., and Gomory, R.E., "Optimal Programming of Lot Sizes, Inventory and Labour Allocations", Management Science, Vol. 11, No.9, PP 874-890 (july 1956).

[5] Elsayed, E.A. et al. "Analysis and Control of Production Systems", Prentice-Hall Inc., 1985).

[6] Gonzalez, J.J., and Reeves, G.R., "Master Production Sche duling: a Multiple-Objective Linear Porgramming Approach", International Journal of Production Research, Vol.21, No.4, PP 553-562, (1963).

[7] Graves, S.C., "Using Lagrangean Techniques to Solve Hierarchical Production Planning Problems", Management Science, Vol. 28, No.3, pp. 260-275 (March 1982).

[8] Hax, A.C., "Production and Inventory Management", Prentice-Hall, Inc. Englweood Cliffs, New Jersy, 1984.

[9] Hax, A.C., and Meal, H.C., "Hierarchical Integration of Production Planning and Scheduling" in Studies in Management Science, Vol. 1, Logistics, M.A. Geisler (ed). North Holland-American Elsevier, New York, (1985).

[10] Khorshneir et al. "Aggregate Planning Models incorporating Productivity-An Overview" International Journal of Prod. Res. Vol.2, No.5, (1983).

[11] Lasdon, L.S., and Terjung, R.C., "An Efficient Algorithm for Multi-item Scheduling", Operations Research, Vol.19, No.4 pp. 946=989 (July-August 1971).

[12] Lawrence, K.D., AND Burbridge, J.J., "A Multiple Goal Linear Programming Model for Co-ordinated Production and Logistics Planning", International Journal of Production Research, Vol.14, PP. 215, (1976).

[13] Lee, S.M., "Goal Programming for Decision Analysis", Auerbach Publishing Co., Philadelphia. (1972).

[14] Mann, A.S., "Programming of Economic Lot Sizes", Management Science Vol.4, No.2, PP 115-135 (January 1958).