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SLIDING MODE OSCILLATIONS AND STATIC CHARACTERISTICS OF ADAPTIVE SYSTEMS

العلاقة بين تردد الحركة الانزلاقية والخواص الاستاتيكية
لمنظومات التحكم التوالية .

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ملخص البحث :

يقدم البحث دراسة تحليليه عن الخواص الاستاتيكيه لمنظومات التحكم الكهروميكانيكيه والتى تحتوى مكوناتها متابعا (مرحلا) يعمل كجهاز للتحكم . وهذا المتابع يسبب وجود حركه انزلاقيه داخل المنظومة . ويتناول البحث العلاقة بين تردد هذه الحركة الانزلاقية والخواص الاستاتيكيه للمنظومة الكهروميكانيكية التى تتكون من محرك تيار مستمر محكوم من دائرة العضو الدوار - ويتم تغذية المحرك عن طريق موحد سيليكونى محكوم (ثيرستور) ويتضح من الدراسة التحليلية انه كلما قل تردد ذبذبات الحركة الانزلاقية كلما زاد تحسن الخواص الاستاتيكية لمنظومة التحكم قيد البحث والتى تعتبر منظومة تحكم توالى لوجود الحركة الانزلاقية التى تجعلها لا تتأثر بالاضطرابات الداخلية او الخارجية .

ABSTRACT :

The present paper introduces an analytical study about the static characteristics of electromechanical control systems, containing a relay element as a controller. Such relay element initiates a sliding mode inside the system . An analytical study about the correlation between the frequency of oscillations of the sliding regime and the static characteristics of the electromechanical system is introduced . The electromechanical system under consideration consists of an armature controlled d.c. motor, fed through a thyristor convertor.

Results of analytical study illustrated that : decreasing the oscillations frequency of the sliding regime insures a high quality static characteristic of the system .

INTRODUCTION :

It is well known [1,2,3] that, introducing a discontinuous control element (relay) into a control system , may initiate inside it a sliding motion . The system dynamic and static behaviours depend mainly, on the proper choice of the switching planes [4] . Choice of such switching planes may depend on a predetermind performance index [5]. The resulting sliding mode reduces the order of the system, under control by one degree [6] If a sliding mode is initiated, inside the control system , it becomes adaptive and invariant , i.e., the system becomes insensitive to both internal and external disturbances.

In the present work, the study concerns with searching a correlation between the frequency of oscillations of the sliding mode and the static behaviour of the electromechanical system shown in Fig.(1).

The system has the following dynamic equations :

$$R_a\ (d/dt) i_a = (1/T_e)\ \{k_m e_{th} - k_m e - R_a i_a\} \tag{1}$$

$$(d/dt) e = (1/T_m)\ \{ R_a i_a - R_a i_c \} \tag{2}$$

$$(d/dt) e_{th} = - (1/T_{th})\ e_{th} + (k_{th}/T_{th})\ u \tag{3}$$

where ;

R_a & L_a = armature resistance & armature inductance of the d.c. motor, respectively ;
i_a = armature current ;
i_o = nominal value of armature current ;
T_e = electrical time constant of the motor= L_a / R_a ;
T_m = mechanical time constant of the motor = J/f ;
j = moment of inertia ;
f = equivalant co-efficient of viscous friction ;
k_m = motor gain constant ,
e_{th} = thyristor output voltage
k_{th} = gain constant of the thyristor ;
u = discontinuous control signal

STATEMENT OF THE PROBLEM :

Assumming that the conditions of existance of the sliding mode , inside the system are verified [2], and the frequency of oscillations (f_o) of such sliding mode is detrmined, then, it is required to study effect of varying this frequency on the static behaviour of the relay electromechanical system, shown in Fig. (1). The dynamic behaviour of the system is based on the minimization of the quadratic performance index [1] :

$$J = \int_0^{\infty}((e - e_o)^2 + q_1(R_a i_a - R_a i_c)^2 + q_2 (e_{th} - e_{tho})^2)\, dt \quad (4)$$

where ;

q_1 & q_2 are positive weighting constants ;

e_{tho} = nominal value of the thyristor output .

MARHEMATICAL ANALYSIS :

The discontinuous control signal u^o can be written as :

$$u^o = \text{Sign}\ (b_o e_o - u_c)$$

$$= \text{Sign}\ (b_o e_o - e - b_1 R_a i_a - b_2 e_{th}) \quad (5)$$

when the sliding mode exists [2] , the system order will be reduced by one degree. Thus, the dynamic model of the system will be written as :

$$d/dt\, e = (1/T_m)\ (R_a i_a - R_a i_c) \quad (6)$$

$$R_a(di_a/dt) = (-1/T_e)k_m\ e-(1/T_e)R_a i_a+(k_m/b_2 T_e)\ (b_o e_o - e - b_1 R_a i_a) \quad (7)$$

It is clear from (6) & (7) that the variation of b_1 (and/or) b_2 will affect the system behaviour. Now consider the parameter (N) which will be defined as the ratio between the nominal (steady-state) speed e_o and the reduced speed e_r. Then from (6) & (7) we get :

$$\Delta r_o = ((b_1 + b_2)\ /\ (1 + b_2))\ R_a\ i_c \quad (8)$$

and since ,

$$\Delta e_o = R_a\ i_c\ , \qquad \text{then ,}$$

$$N = \Delta e_o\ /\Delta e_r = (1 + b_2)\ /\ (b_1 + b_2) \quad (9)$$

EFFECT OF OSCILLATIONS :-

To evaluate the characteristics of the adaptive electromechanical system under consideration, we shall introduce a new parameter (M) defined as the oscillation parameter and given by :

$$M = \max_{w \in (0,\infty)} \{\ A(w)\ /\ A(0)\ \} \quad (10)$$

where ;

A(w) = amplitude of the system transfer function G(s), in the frequency domain , and G(s) is given as :

$$G(s) = 1/(b_2 T_e T_m S^2 + (b_1 + b_2)T_m s + 1+b_2) = 1 /(a_o s^2+a_1 s+a_2) \quad (11)$$

From (11) , we have :

$$A(w) = 1 /((a_o w s^2 - a_2)^2 + a_1{}^2 w^2)^{0.5} , \quad A(o) = 1 /a^2 \quad (12)$$

$$M = a_2 / ((a_o w^2{}_{max} - a_2)^2 + a_1^2 \; w^2{}_{max})^{0.5} \quad (13)$$

$$w_{max} = ((2 a_o a_1 - a_1{}^2) / 2 a_o{}^2)^{0.5}$$

and is obtained from the relation :

$$dM / dw = 0$$

Thus for a chosen oscillation parameter (M) , we can tune the speed regulator to acheive the suitable static charateristics of the adaptive electromechanical system under study. This is illustrated in Fig. (2). The figure illustrates the relation between the parameter b_1 and the co - efficient (N), for different values of the oscillation parameter M , and taking T_e = 0.04 sec. & T_m = 0.1 sec. It is clear that , increasing M for the same value of b_1 will increase (N) . In this case , the equivalant time constant will take the form :

$$T_{equ.} = ((1 - Nb_1) /(N(1 - b_1)) \, T_e T_m)^{0.5} = \lambda(T_e T_m)^{0.5} \quad (14)$$

Hence; high quality static characteristics could be obtained over an allowed range of oscillations frequency for the sliding regime, i.e. , decreasing the factors b_1 & b_2 will increase the co-efficient (N) & hence; small time constants will be achieved - (region of insensitivity and time delay). [see Fig. (2)].

RESULTS AND DISCUSSION :

To illustrate the above obtained results, let us introduce in the nonlinear part of the system , an aperiodic group with a time constant T_o . The describing function of the nonlinear relay element is given by :

$$G_{N.L.} (a) = 4C / \pi a \quad (15)$$

where,

C = saturation voltage of the relay element ,

a = maximum amplitude of the input signal

The transfer function of the linear part is given by :

$$G_L(s) = \frac{b_2 T_o T_m S^2 + T_m(b_1 + b_2)s + b_2 + 1}{(1 + T_o s)(1 + T_{th} s)(1 + T_m s + T_o T_m s^2)} = \frac{U_{o.c.}(s)}{U_p(s)} \quad (16)$$

Solving the characteristic equation :

$$1 + G_L(jw)\, G_{N.L.}(a) = 0 \quad (17)$$

for different values of b_1 , b_2 & T_o , we can obtain the corresponding values for the frequency of oscillations of the sliding motion. Fig.(3) illustrates the lines determining the ranges of the co-efficient(N) , for a given minimum value of the frequency of oscillations f_c and a certain value of T_o .

Simialr characteristics could be obtained, when the relay element has not ideal characteristics . The time constant electromechanical systems, when the relay element is synthesized on the basis of intergrating operational amplifiers .

Synthesis of adaptive controllers , based on the sliding mode concept must take into account , the oscillations frequency of the sliding mition , to achieve high quality static characteristics.

CONCLUSTIONS :

In the present paper , the relation between the oscillations frequency of the initiated sliding regime and the static behaviour of electromechanical control systems, including a relay element with ideal characteristics , is introduced. It was concluded that the high quality static characteristics of such systems could be obtained, when the oscillations frequency of the initiated sliding regime, becomes as small as possible .

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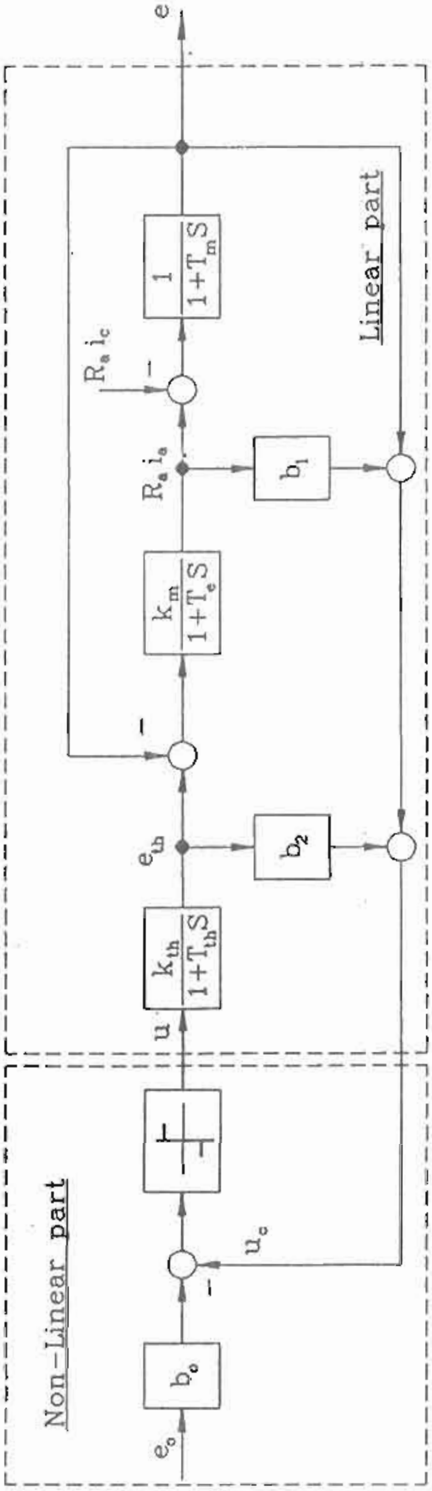

Figure (1)

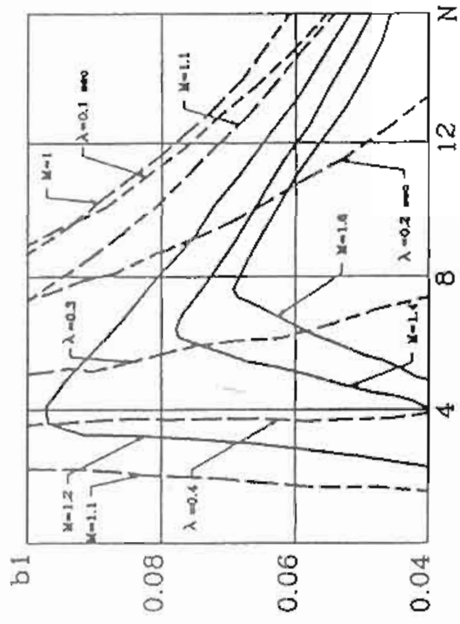

Figure (2)

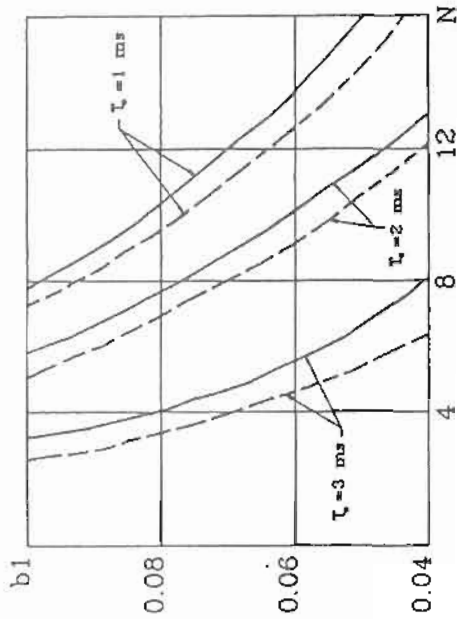

Figure (3)

- - - - f_c = 0.5 KHz
——— f_c = 1.0 KHz