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NUMERICAL SOLUTION OF LAMINAR FLOW AND HEAT TRANSFER OVER A FLAT PLATE IN A NONUNIFORM STREAM

الحل العددي للسريان الانسيابي وانتقال الحرارة من لوح مستو في مجال سريان غير منتظم

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خلاصة :

تمت دراسة السريان الانسيابي والانتقال الحراري على لوح مستو ذي درجة حرارة ثابتة في مجال سريان حر له توزيع سرعة غير منتظم . وقد تم حل المعادلات الحاكمة عدديا بواسطة نظرية الفروق المحدودة الضمنية مع استخدام طريقة الفروق المختلطة لتمثيل الحد السائل في الاتجاه المتعامد مع السريان . وقد اجريت دراسة عددية لتأثير عدم انتظام السريان الحر على خواص الطبقة الجدارية في مجالين تطبيقيين . الاول عندما يكون اللوح المستوي موجودا في مجال سريان منحسر خلف مروره على لوح مستو آخر . والثاني عندما يكون اللوح المستوي موجودا في طبقـــــة . قصية بين مجال ذو سرعة منتظمة ومجال ساكن . وقد تمت دراسة تحليلية للطبقة الجدارية في الاوضاع المختلفة للوح بالنسبة للسريان الحر .

ABSTRACT :

The laminar flow and heat transfer over an isothermal flat plate in a free stream with nonuniform velocity have been analysed. A fully implicit numerical method (with a hybrid representation for the convective term normal to the streamwise direction) is used for solving the governing equations. Two practical cases are considered. In the first case, a flat plate is located in the laminar wake of an upstream plate. In the second case, a flat plate is inserted in the laminar shear layer joining a uniform stream and a quiescent fluid. The dependence of the boundary-layer and heat transfer on the lateral and the downstream shifts of the plate have been investigated. It was found that the nonuniformity of the free stream velocity causes reductions in both the wall shear stress and heat transfer relative to the corresponding values for uniform free stream. These relative reductions decrease with increasing downstream distance along the plate in the upper half of the wake. In the lower half of the wake, these reductions increase with distance along the plate. It was also found that the minimum values for drag and overall heat transfer rate at a certain streamwise location correspond to a slight downward shift of the plate from the wake centreline. In the case of shear layer, the relative reductions in the wall shear and heat transfer are more pronounced in the low velocity region. These reductions decrease with increasing downstream distance along the plate. It was found that both the drag and overall heat transfer rate decrease continuously with downstream shift of the plate.

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Introduction

In the analysis of boundary-layer over bodies, it is usual to assume that both the velocity and temperature of the approach-flow are uniform. However, in many engineering applications, the approach-flow is nonuniform. For example, during airplanes take off, the approach-flow is nonuniform due to wind shear. The approach-flow to the wing section situated in the slipstream of the propeller is also nonuniform. Furthermore, the rotor blades of helicopters operating in the wakes of the other blades as well as the propellers of ships operating in the wake of the stern are examples for nonuniform approach-flow to propeller blades. Each of these flows are characterised by a transverse velocity and temperature gradients which may vary in magnitude depending on the particular configuration. Such transverse gradients affect the velocity and temperature boundary-layers development on the body, and consequently affect the body shear and heat transfer.

The effect of the approach-flow nonuniformity on laminar boundary-layer flow has been studied for many years. However, most of the previous investigators were concerned with a free stream velocity varying linearly in the direction normal to the plate [1-5]. Very little work has been reported on the case where the nonuniformities in the approach-flow are nonlinear. Sparrow et al. [6] analysed the laminar flow and heat transfer on a flat plate situated at the centreline of the laminar wake of an upstream plate. The finite-difference technique was used. They found that the effect of nonuniformities is to reduce the wall shear and heat transfer on the downstream plate relative to their values for uniform flow. El-Taher [7] investigated the momentum laminar boundary-layer over a flat plate in a wake or a jet using an integral method. It was found that the boundary-layer characteristics depend strongly on the lateral shift of the plate from the wake or jet centreline. El-Taher [8] extended his work to include the non-isothermal case and to investigate the effect of both transverse gradient and second derivative of free stream velocity and temperature at the plate leading edge on the boundary-layer flow and heat transfer. El-Sayed and El-Taher [9] investigated the laminar flow and heat transfer over a plate in a nonuniform flow using the local non-similar technique. Specific considerations are given to approach-flows of parabolic velocity and temperature distributions. The results showed that for the examined range of parameters both the wall shear stress and heat flux depend mainly on the magnitude of the approach-stream velocity and temperature at the plate leading edge. El-Taher [10] investigated the effect of uniform suction on the laminar flow over a flat plate in a nonuniform approaching-flow. A modified momentum integral method was used. Numerical results were obtained for both linear and parabolic shear flows. Sayed [11] used the integral technique to investigate the flow and heat transfer over a flat plate subjected to a propeller slipstream or a submerged-jet.

The present study deals with the steady, two-dimensional incompressible laminar flow and heat transfer over a flat plate in a free stream of a general nonuniform velocity distribution. The present work employs a fully implicit numerical method (with a hybrid representation for the convective term normal to the streamwise direction). The analysis is applied to two practical flow cases. In the first case, a flat plate is located in the laminar wake of an upstream plate. In the second case, a flat plate is inserted in the laminar shear layer joining a uniform stream and a quiescent fluid. The dependence of the boundary-layer parameters on the lateral and the downstream shifts of the plate have been investigated.

Analysis :

Consider an isothermal flat plate of temperature T_w subjected to a free stream with nonuniform velocity U and uniform temperature T_∞ ; refer to Fig.(1). The distribution of velocity in the free stream is given by

$$\frac{U}{U_\infty} = F\,(x,y) \tag{1}$$

Where F is a specified function and U_∞ is a reference velocity. It is assumed that the free stream pressure is constant.

In the cartesian coordinates system shown in Fig.(1), the usual boundary-layer equations for a steady, two-dimensional incompressible and laminar flow over the flat plate are as follows :

- Continuity equation :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

- Momentum equation :

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \nu\,\frac{\partial^2 u}{\partial y^2} \tag{3}$$

- Energy equation :

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \frac{K}{\rho\, C_p}\,\frac{\partial^2 T}{\partial y^2} \tag{4}$$

In Eq. (4) the thermal conductivity K is assumed constant and the viscous dissipation is neglected. The appropriate boundary conditions are :

At the wall, neither slip nor permeability is allowed and the surface temperature is specified; hence

$$u(x,y=0)=0,\ v(x,y=0)=0,\ \text{and}\ T(x,y=0)=T_w \tag{5}$$

Since the external inviscid flow has been effectively displaced by the displacement thickness δ^* (See Fig.(2)), the equivalent inviscid flow over the plate is

$$\frac{U}{U_\infty} = F\ (x,y - \delta^*) \tag{6}$$

Therefore, the boundary conditions at the outer edge of the boundary-layer are

$$u(x,y) \longrightarrow U(x,y-\delta^*),\quad T(x,y) \longrightarrow T_\infty\ \text{for}\quad y \longrightarrow \infty \tag{7}$$

The final condition is to specify the initial profiles

$$u(x=0,y) = u_i(y),\ T(x=0,y) = T_i(y) \tag{8}$$

Where $u_i(y)$ and $T_i(y)$ are the initial boundary-layer velocity and temperature profiles at the plate leading edge.

The boundary-layer equations are transformed into similarity form in order to obtain them in a form more appropriate for numerical solution. The new independent variables are

$$\xi = \frac{x}{L}\ ,\ \eta = y\ \left(\frac{U_\infty}{\nu\ x}\right)^{1/2} \tag{9}$$

When the new dependent variables

$$f'\ (\xi,\eta) = \frac{u}{U_\infty} \tag{10 a}$$

$$V\ (\xi,\eta) = \frac{v}{U_\infty}\ \left(\frac{U_\infty\ x}{\nu}\right)^{1/2} \tag{10 b}$$

and
$$\theta\ (\xi,\eta) = \frac{T^\infty - T_w}{T_\infty - T_w} \tag{10 c}$$

are introduced and the transformations are applied, the boundary-layer equations take the following forms :

- Continuity equation :

$$\xi\ \frac{\partial f'}{\partial \xi} - \frac{1}{2}\ \eta\ \frac{\partial f'}{\partial \eta} + \frac{\partial V}{\partial \eta} = 0 \tag{11}$$

- Momentum equation :

$$\xi\ f'\ \frac{\partial f'}{\partial \xi} + \overline{V}\ \frac{\partial f'}{\partial \eta} = \frac{\partial^2 f'}{\partial \eta^2} \tag{12}$$

- Energy equation :

$$\xi f' \frac{\partial\theta}{\partial\xi} + \bar{V} \frac{\partial\theta}{\partial\eta} = \frac{1}{Pr} \frac{\partial^2\theta}{\partial\eta^2} \qquad (13)$$

A new variable $\bar{V}$ has been introduced in Eqs. (12) and (13) for convenience. It is defined as

$$\bar{V} = V - \frac{1}{2} \eta f' \qquad (14)$$

The boundary conditions Eqs. (5),(7), and (8) are transformed as follows :

$$f'(\xi,0)=0, \ \bar{V}(\xi,0)=0, \ \theta(\xi,0)=0 \qquad (15)$$

$$f'(\xi,\eta \rightarrow \infty) = \frac{U}{U_\infty} = F(\xi,\eta-\Delta), \ \theta(\xi,\eta \rightarrow \infty) = 1 \qquad (16)$$

$$f'(0,\eta) = f_i'(\eta), \ \theta(0,\eta) = \theta_i(\eta) \qquad (17)$$

Where Δ is the dimensionless boundary-layer displacement thickness; $\Delta = \delta^* (\frac{U_\infty}{\nu x})^{1/2}$.

The initial profiles f_i' and θ_i in the above equation (17) are specified by solving third and second order equations obtained by introducing $\xi = 0$ into Eqs. (11) to (13) and assuming the flow to be self-similar

$$f_i''' + \frac{1}{2} f_i f_i'' = 0 \qquad (18)$$

$$\theta_i'' + \frac{1}{2} Pr f_i \theta_i' = 0 \qquad (19)$$

With boundary conditions

$$\eta = 0, \ f_i = f_i' = \theta_i = 0 \qquad (20)$$

$$\eta \rightarrow \infty, \ f_i' = \frac{U_h}{U_\infty}, \ \theta_i = 1 \qquad (21)$$

Where the prime means a derivative with respect to η and U_h is the free stream velocity at the leading edge of the plate.

The quantities of particular interest in boundary-layer calculations are the displacement thickness δ^*, shear stress at the wall τ, local heat flux q, drag force D and overall heat transfer rate Q. Equations for determining these quantities follow.

$$\int_0^{\delta^*} U \, dy = \int_0^{\delta} (U-u) \, dy \qquad (22)$$

Where U is given by equation (6)

$$\tau = \mu \frac{\partial u}{\partial y} \Big|_{y=0} \qquad (23)$$

$$q = -K \left.\frac{\partial T}{\partial y}\right|_{y=o} \quad (24)$$

$$D = \int_{o}^{L} \tau \, dx, \quad Q = \int_{o}^{L} q \, dx \quad (25)$$

for a unit width of plate.

The set of governing equations for laminar boundary-layer flow, Eqs.(11) through (13), are solved for the given boundary conditions, Eqs.(15) through (17), employing the hybrid finite difference Scheme [12]. The boundary-layer is divided with a grid of size $\Delta\xi$ and $\Delta\eta$ with $\xi_{I+1} = \xi_I + \Delta\xi$ and $\eta_{J+1} = \eta_J + \Delta\eta$. Where (I,J) is a typical mesh point. The finite-difference form of the momentum and energy equations (12) and (13) respectively are evaluated at the grid point (I+1,J) using a hybrid upwind/central difference formula for the convective term $\bar{V} \frac{\partial \phi}{\partial \eta}$. The finite-difference form of the continuity equation (11) is written at the point (I+1,J-1/2) employing central differencing. Details of the resulting equations and the solution procedure are given in Ref.[16].

Applications :

i- Flat plate in the laminar wake of an upstream plate

Fig.(3) shows a flow with uniform approach velocity U_∞ and temperature T_∞ past an upstream plate of length L. The temperature of the first plate is T_∞. So the temperature of the flow in the wake is constant and equal to T_∞. A second plate of the same length is situated in the laminar wake at a distance S downstream of the first plate and is shifted a distance h from the wake centreline.

The main concern is to determine the hydrodynamic and thermal response of the second plate (whose temperature is T_w) to the nonuniformity of the velocity distribution in the laminar wake of the first plate.

The velocity distribution in the laminar wake of the first plate is given by [14] :

$$\frac{U}{U_\infty} = 1 - \frac{0.664}{\sqrt{\pi}} \left(\frac{X}{L}\right)^{-1/2} \exp\left\{ - \frac{1}{4} \frac{U_\infty}{\nu X} Y^2 \right\} \quad (26)$$

Where

L : Length of upstream plate.

U_∞ : Velocity of flow approaching upstream plate.

X,Y: Coordinate axes with origin at trailing edge of first plate.

The pressure in the wake of the first plate is constant [14].

Now, considering the effective displacement of the external inviscid flow by the displacement thickness, the distribution of velocity in the equivalent inviscid flow over the second plate is given by :

$$\frac{U}{U_\infty} = 1 - \frac{0.664}{\sqrt{\pi}} \left(\frac{S+x}{L}\right)^{-1/2} \exp\left\{- \frac{1}{4} \left(\frac{L}{S+x}\right) \left[\frac{Y-\delta^*}{L} \sqrt{Re_L} + H\right]^2\right\} \quad (27)$$

Where : $Re_L = \dfrac{U_\infty L}{\nu}$, $H = \dfrac{h}{L} \sqrt{Re_L}$

It is worthy to mention that Eq.(26) was obtained from the solution of the conventional boundary-layer equations for the wake flow. This equation is not able to provide an accurate description of the flow in the immediate neighborhood of the trailing edge of the first plate. **Sparrow et al. [6]** estimated that Eq.(26) is entirely satisfactory provided that $S/L \geq 1$.

The numerical solutions provide values of wall shear stress τ and local heat flux q. The total drag D and overall heat transfer rate Q are also obtained. The results were compared with Blasius Similar Solution for a plate in a uniform free stream with velocity U_∞. The wall shear stress τ_∞ and the local heat flux q_∞ on a plate of constant temperature T_w subjected to a uniform flow of velocity U_∞ and temperature T_∞ are given from Blasius solution as follows [14].

$$\frac{\tau_\infty}{\rho U_\infty^2} = 0.33206 \left(\frac{\nu}{U_\infty x}\right)^{1/2} \quad (28)$$

$$\frac{q_\infty x}{K(T_w - T_\infty)} = 0.2927 \left(\frac{U_\infty x}{\nu}\right)^{1/2} \quad (29)$$

Using these equations, the ratios τ/τ_∞ and q/q_∞ may be evaluated, with both numerator and denominator corresponding to the same streamwise location x.

The development of momentum and thermal boundary-layer along the downstream plate are calculated for different values of relative plate spacing S/L and shifting distance H. The Prandtl number is fixed at 0.7. As expected, the solution is nonsimilar as a result of the nonuniformity of the upstream velocity. The variation of spacing and shifting give rise to significant changes in the velocity and temperature profiles across boundary layer. This is clearly observed from Figs. (4) and (5) which display the distribution of velocity and temperature across the boundary-layer.

The values of wall shear stress τ and local heat flux q over the plate are different from those of a plate located in the main uniform stream whose velocity is U_∞. This difference is due to the following two factors :

a- The magnitude of the local free stream velocity U_h in the wake at the plate leading edge is different from U_∞.

b- The shape of the free stream velocity profile in the wake is not uniform (i.e. $\frac{\partial U}{\partial y}$, $\frac{\partial^2 U}{\partial y^2}$, ... etc. $\neq$ 0).

The ratios τ/τ_h and q/q_h are plotted in Figs.(6) and (7), respectively, as functions of the dimensionless streamwise coordinate x/L for different values of the shifting distance H, and relative plate spacing S/L = 2. It is seen from the figures that τ/τ_h and q/q_h are greater than unity at the wake centreline. The figures also show that at H = 0 these ratios increase with increasing the value of x. At H=1, the figures show that τ/τ_h and q/q_h are greater than the corresponding values at H=0. However, at H=2 a decrease in τ/τ_h and q/q_h is observed compared to their values at H=1. A further decrease is observed at H = 3 and 4. These results can be explained in the light of the conclusions of El-Taher [8] who investigated the effect of nonuniformity of the free stream flow on the boundary-layer and heat transfer over a flat plate. One of the main conclusions of El-Taher [8] is that the positive approach- stream shear $\partial U/\partial y$ at the plate leading edge increases the wall shear stress and heat flux relative to their values for uniform flow. Negative approach-stream shear has an opposite effect. Likewise, positive shear derivative $\frac{\partial^2 U}{\partial y^2}$ of approach-flow increases wall shear stress and heat flux. The opposite is true for negative shear derivative. Now, if we examine the wake velocity profile at S/L=2, it will be noticed that at H = 0, the stream shear $\partial U/\partial y$ is zero and the shear derivative $\frac{\partial^2 U}{\partial y^2}$ is positive. This results in values of τ/τ_h and q/q_h greater than unity. At H = 1, both the stream shear and its derivative are positive and their combined effect is greater values for τ/τ_h and q/q_h compared to their values at H = 0. At H = 2, $\frac{\partial U}{\partial y}$ is positive and $\frac{\partial^2 U}{\partial y^2}$ is zero. So, the ratios τ/τ_h and q/q_h are less than the corresponding ratios at H = 1. At H = 3, $\frac{\partial U}{\partial y}$ is positive but $\frac{\partial^2 U}{\partial y^2}$ is negative and their combined effect is a decrease in τ/τ_h and q/q_h compared to their values at H = 1. With further a increase in the value of H, the magnitude of both the free stream shear and its derivative decrease and the resultant effect is a decrease in τ/τ_h and q/q_h approaching unity.

At H = -1, the free stream shear is negative while the shear derivative is positive. Therefore, their effects on τ/τ_h and q/q_h is opposite to each other. This results in the relatively small decrease in τ/τ and q/q compared to unity. At H= -2, $\frac{\partial U}{\partial y}$ is negative and $\frac{\partial^2 U}{\partial y^2}$ is zero. So, the resultant effect of negative stream shear is a more decrease in τ/τ_h and q/q_h compared to the corresponding values at H=-1. At H = -3, both $\frac{\partial U}{\partial y}$ and $\frac{\partial^2 U}{\partial y^2}$ are negative and their effects enhance each other resulting in the shown pronounced decrease in τ/τ_h and q/q_h. At H = -4, both the free stream shear and its derivative are negative but their magnitudes become small, therefore the curves for τ/τ_h and q/q_h starts to get back toward unity.

The distributions of the ratios τ/τ_∞ and q/q_∞ along the plate are shown in Figs. (8) and (9) respectively with shifting H as curve parameter for S/L = 2. The local free stream velocity U_h at any lateral positive or negative shift H is smaller than U_∞ and this reduces τ and q relative to τ_∞ and q_∞ respectively. The Figs. show that the drop in τ and q relative to τ_∞ and q_∞ in the lower half of the wake is greater than the corresponding drop in the upper half. It is seen from Figs. (8) and (9) that for $H \geq 0$, τ/τ_∞ and q/q_∞ increase with increasing values of x. Whereas at negative values of H, these ratios decrease with increasing distance x along the plate.

The effect of the nonuniformity of the approach-flow velocity on τ and q along a plate situated at the wake centreline (i.e. H=0) is shown in more details in Figs.(10) and (11) respectively. It is seen from these Figs. that for all values of S/L the ratios τ/τ_∞ and q/q_∞ are less than unity. These two Figs. also show that for a certain value of S/L, the effect of free stream nonuniformity on τ and q decreases with increasing distance x along the plate. However, this effect become less pronounced for large values of S/L. This is expected, because as S/L increases, the free stream become more uniform. Also, Fig.(10) compares the obtained τ with the numerical results of Sparrow et al. [6]. Agreement is remarkable for $S/L \geq 2$. However, deviations are observed for S/L = 1. These deviations are expected because Ref. [6] uses a numerical technique different from the one used in the present work. Also, the present work take into account the effect of the displacement of the free stream due to boundary-layer while Ref. [6] did not take this effect into account.

Figs. (12) and (13) show the variation of drag D and overall heat transfer rate Q with S/L and H. It is seen from these Figs. that the ratios D/D_∞ and Q/Q_∞ are less than unity. As expected, the most significant effects of the nonuniformity of velocity are evidence in the central part of the wake ($-4 < H < 4$). As the magnitude of H increases (i.e. U_h approaches U_∞), the curves tend to increase toward D/D_∞ and $Q/Q_\infty = 1$. These Figs. show that minimum values for D/D_∞ and Q/Q_∞ occur at $H \cong -1$. It is seen from Figs. (12) and (13) that at some values of H (e.g. -2, -1.0, and 1), D/D_∞ and Q/Q_∞ increase continuously with increasing S/L. At some other values of H(e.g. -3, 2 and 3) D/D_∞ and Q/Q_∞ first decrease with S/L until they reach a minima, then they increase with further increase in S/L. The behaviour of the curves at a certain value of H depends on the relative effects of the magnitude of the local free stream velocity U_h and the magnitude and sign of the local stream shear and its derivative.

2- Flat plate in the laminar shear layer joining a uniform stream and a quiescent fluid

Consider a flat plate inserted in a laminar shear layer joining two parallel uniform streams of the same fluid and the same temperature T_∞ but having different velocities, Fig.(14). The flow in the shear layer will be isothermal with temperature T_∞. The plate is at a distance S from the start of the shear layer and is shifted a distance h from the dividing streamline.

The velocity field in the shear layer is governed by boundary-layer equations with zero pressure gradient. No closed solution for the velocity distribution in the shear layer is known. However, solutions can be obtained numerically using a modified shooting method (15).

Let us consider the specific case where the velocity of the upper stream is U_∞ and that of the lower stream is zero. This corresponds to the shear layer at the lip of a jet issuing into a large room. Numerical results for the velocity distribution in this shear layer is given in table (1). It was found that the distribution of velocity in this shear layer can be reasonably represented by a relationship of the following form :

$$\frac{U}{U_\infty} = a + b \tanh\left\{C \left(\frac{U_\infty}{\nu X}\right)^{1/2} Y + d\right\} \qquad (30)$$

where

a,b,c, and d are constants.

X,Y coordinate axes with origin at the start of the shear layer.

The following values : a = 0.512, b =0.5, c =0.4 and d =0.18 were obtained by trial and error and the resulting relationship was found to fit the numerical results of table (1) to withen 1% accuracy.

The distribution of velocity in the shear layer referred to axes at the plate leading edge is given by

$$\frac{U}{U_\infty} = 0.512 + 0.5 \tanh\left\{0.4 \left[\frac{U_\infty}{\nu(S+x)}\right]^{1/2} (y+h) + 0.18\right\} \qquad (31)$$

To take account for the effective displacement of the external inviscid flow by the displacement thickness, the equivalent inviscid flow over the plate is calculated by using $(y-\delta^*)$ instead of y in Eq.(30).

From numerical calculations, the values of boundary layer parameters τ and q over the plate are different from those of a plate situated in the main uniform stream whose velocity is U_∞. This difference is due to the same two factors which have been mentioned in the first case (i.e. wake case); namely, the magnitude of the local free stream velocity and the nonuniformity of the free stream velocity profile.

The distribution of the ratios τ/τ_∞ and q/q_∞ along the plate are shown in Figs.(15) and (16) respectively with shifting distance H as curve parameter for S/L = 5. The local free stream velocity U_h decreases with decreasing the value of H and this results in a continuous decrease in τ and q relative to τ_∞ and q_∞ respectively. It is seen from these Figs. that for all values of H, ratios τ/τ_∞ and q/q_∞ are less than unity. The Figs. also show that for all values of H these ratios increase with increasing value of x. Figs.(17) and (18) show the effect of nonuniformity of the approach flow velocity on τ and q along a plate located at the dividing streamline of the laminar shear layer (i.e. H=0). These Figs. indicate that for all values of S/L, ratios τ/τ_∞ and q/q_∞ are less than unity. It is remarkable that for all values of S/L the ratios τ/τ_∞ and q/q_∞ start with constant values at the plate leading edge (x = 0). This is explained by the fact that along the line H = 0, U/U_∞ is constant and equal 0.586 for all values of S/L. The Figs. also show that for a certain value of S/L, τ/τ_∞ and q/q_∞ increase with increasing distance x along the plate. It is seen from Fig.(17) that as S/L decreases τ/τ_∞ increases. This is explained by the fact that although the local free stream velocity U_h is constant for all values of S/L, the free stream shear $\frac{\partial U}{\partial y}$ increases as S/L decreases.

Figs.(19) and (20) plot the variation of drag D and overall heat transfer rate Q with S/L and H. It is seen from these Figs. that the ratios D/D_∞ and Q/Q_∞ are less than unity. As expected, the most significant effects of the non-uniformity of velocity are in evidence at H = -2. As H increases (i.e. U_h approaches U_∞), D/D_∞ and Q/Q_∞ increase toward unity. It is also seen from the two figures that at (H > -1), D/D_∞ and Q/Q_∞ decrease continuously with increasing S/L. For all values of H, U_h increases with S/L causing an increase in D/D_∞ and Q/Q_∞ . On other hand, the magnitude of the stream shear and its derivative decrease with S/L, causing a decrease in D/D_∞ and Q/Q_∞ . The combined effect is a decrease in D/D_∞ and Q/Q_∞ with S/L.

Conclusions :

Solutions for the boundary-layer equations for a flat plate in a free stream with nonuniform velocity and uniform temperature were obtained using the hybrid upwind/central difference scheme. The results of the present analysis demonstrate the effects of the nonuniformity of the free stream velocity on the laminar flow and heat transfer on an isothermal flat plate in two cases. In the first case, the plate is located in the laminar wake of an upstream plate. In the second case, the plate is located in the laminar shear layer joining a uniform stream and a quiescent fluid. These effects are summarised as follows :

1- It was found that the boundary-layer parameters depend mainly on the lateral and the downstream shifts of the plate.
2- Upward or downward shift of the plate from the wake centreline causes a reduction in the local wall shear stress and local heat flux relative to their values for a uniform free stream. However, the drop in wall shear and heat flux in the lower half of the wake is greater than the corresponding drop in the upper half.
3- Reductions in wall shear and heat flux in the upper half of the wake relative to those for a uniform flow decrease with increasing downstream distance along the plate. In the lower half of the wake, these reductions increase with distance along the plate.
4- It was found that the minimum values for drag and overall heat transfer rate correspond to a slight downward shift of the plate from the wake centreline.
5- In the inner part of the wake, drag and overall heat transfer rate increase continuously with spacing between the two plates. In the outer part of the wake, drag and overall heat transfer rate first decrease until they reach minima, then they increase with further increase in spacing between the two plates.
6- In the shear layer case, At all lateral shifts of the plate from the dividing streamline the wall shear stress and local heat flux are smaller than the corresponding values for a uniform stream. Reductions in wall shear and heat flux relative to those for a uniform flow are more pronounced in the low velocity

region of the shear layer. Also, the extent of these reductions decrease with increasing downstream distance along the plate.

7- It was found that at all lateral shifts of the plate from the dividing streamline, drag and overall heat transfer rate decrease continuously with downstream shift of the plate.

References :

[1] **Li, T. Y.**, "Simple Shear Flow Past a Flat Plate in an Incompressible Fluid of Small Viscosity", Journal of the Aeronautical Sciences, Vol.22, No.9, Sep.1955.

[2] **Yen, K. T.**, "Approximate Solutions of the Incompressible Laminar Boundary-Layer Equations for a Flat Plate in Shear Flow", Journal of the Aeronautical Sciences, Vol.22, No.10, Oct. 1955.

[3] **Glauert, M. B.**, "The Boundary-Layer in Simple Shear Flow Past a Flat Plate", Journal of the Aeronautical Sciences, Vol.24, No.11, Nov.1957.

[4] **Murray. J. D.**, "The Boundary-Layer on a Flat Plate in a Stream with Uniform Shear", J. Fluid Mech., Vol.11, pp.309-316, 1961.

[5] **E. M. Sparrow and J. L. Gregg**, "The Effect of a Non-isothermal Free Stream on Boundary-Layer Heat Transfer", Journal of Applied Mechanics, pp. 161-165, June 1959.

[6] **Sparrow, E. M., Abdel-Wahed, R. M. and Patankar, S. V.**, "Effect of Approach-Flow Velocity and Temperature Nonuniformities on Boundary-Layer Flow and Heat Transfer", Int. J. Heat Mass Transfer. Vol.20, pp.975-979, 1977.

[7] **El-Taher, R. M.**, "Boundary-Layer over a Flat Plate in Parabolic Shear Flow", Bulletin of the Faculty of Engineering, Cairo University, Aeronautical Eng. Paper 3, 1977-1978.

[8] **El-Taher, R. M.**, "Laminar Flow and Heat Transfer on a Flat Plate with Approach-Flow of Parabolic Velocity and Temperature Distributions", J. Eng., Sci. University Riyadh, Vol.7, No.1, pp.23-32 (1981).

[9] Elsayed, M. M. and El-Taher, R. M., "Effect of Nonuniformities of Velocity and Temperature of an Approaching Flow on the Flow and Heat Transfer Over an Isothermal Flat Plate", Journal of Engineering and Applied Sciences, Vol.1, pp.77-87, 1981.

[10] El-Taher, R. M., "Effect of Uniform Suction on the Laminar Flow Over a Flat Plate in a Nonuniform Approaching Flow", J. eng. Sci., Univ. Riyadh, Vol.9, No.2, 1983.

[11] Sayed, S. A., "Effect of Approach-Flow Velocity and Temperature Non-Uniformilies on the Flow and Heat Transfer Over a Flat Plate" Bulletin of the Faculty of Engineering, El-Mansoura University, Vol., No.2, December 1986.

[12] Anderson, D. A., Tannehill, J. C. and Pletcher, R. H., "Computational Fluid Mechanics and Heat Transfer", McGraw Hill Inc., 1984.

[13] Roger Peyret and Thomas D. Taylor., "Computational Methods for Fluid Flow", Springer-Verlag, New York Inc., 1983.

[14] Schlicting, H., "Boundary Layer Theory", McGraw Hill, New York (1968).

[15] Cebeci, T. and Bradshaw, P., "Momentum Transfer in Boundary Layers", Hemisphere, Washington, Dec., 1977.

[16] Salem. S. A., "Numerical Solution of Laminar Boundary Layer Flow Over a Flat Plate in a Nonuniform Stream" M. Sc. Thesis. College of Engineering, Zagazig University, 1991.

Nomenclature

C_p	Specific heat at constant pressure
D	Drag force
F	Specified function for approach-flow velocity
f	Dimensionless stream function
f'	Streamwise velocity component, u/U_∞
h	Normal distance of the plate from wake centreline or from shear layer dividing streamline
K	thermal conductivity
L	Length of upstream plate, also plate characteristic length
O( )	Order of
Pr	Prandtl number, $\frac{C_p \mu}{K}$
q	Local heat flux
Re_L	Reynolds number, $\frac{U_\infty L}{\nu}$
S	Spacing between plates, also distance between plate and the start of the shear layer
T	Temperature
T_w	Wall temperature
T_∞	Approach-flow temperature (uniform), also temperature of upstream plate
U	Approach-flow velocity (nonuniform)
U_∞	Velocity of flow approaching upstream plate, also reference velocity
u,v	Velocity components parallel to the x and y axes respectively
V	Dimensionless normal velocity, $\frac{v}{U_\infty}\left(\frac{U_\infty x}{\nu}\right)^{1/2}$
X,Y	Coordinate axes with origin at trailing edge of the upstream plate also with origin at the start of the shear layer
x,y	Coordinate axes with origin at plate leading edge.

Greek Symbols

Δ	Dimensionless displacement thickness, $\delta^*\left(\frac{U_\infty}{\nu x}\right)^{1/2}$
$\Delta\xi, \Delta\eta$	Step sizes in ξ and η direction, respectively
δ	Boundary-layer thickness
δ^*	Displacement thickness of boundary-layer
η	Dimensionless independent variable, $y\left(\frac{U_\infty}{\nu x}\right)^{1/2}$
θ	Dimensionless temperature, $\frac{T - T_w}{T_\infty - T_w}$
μ	Dynamic viscosity

ν Kinematic viscosity
ξ Streamwise coordinate, x/L
ρ Density of fluid
τ Shear stress at the wall
ϕ Generalized dependent variable, f' for momentum and θ for energy.

Subscripts

i Initial profiles
w At the plate
∞ Outer edge of boundary-layer, also value for plate in uniform flow with velocity U_∞.

Table (1) Velocity Profile $\frac{U}{U_\infty}$ of the laminar Shear Layer Joining a Uniform Stream and a quiescent fluid

$Y\sqrt{\frac{U_\infty}{\upsilon S}}$	$\frac{U}{U_\infty}$	$Y\sqrt{\frac{U_\infty}{\upsilon S}}$	$\frac{U}{U_\infty}$	$Y\sqrt{\frac{U_\infty}{\upsilon S}}$	$\frac{U}{U_\infty}$
5.0	0.9993	1.6	0.8664	-1.8	0.2702
4.8	0.9990	1.4	0.8391	-2.0	0.2439
4.6	0.9986	1.2	0.8091	-2.2	0.2197
4.4	0.9980	1.0	0.7763	-2.4	0.1974
4.2	0.9970	0.8	0.7413	-2.6	0.1771
4.0	0.9960	0.6	0.7043	-2.8	0.1585
3.8	0.9936	0.4	0.6659	-3.0	0.1417
3.6	0.9910	0.2	0.6265	-3.2	0.1264
3.4	0.9873	0.0	0.5866	-3.4	0.1127
3.2	0.9824	-0.2	0.5467	-3.6	0.1003
3.0	0.9761	-0.4	0.5072	-3.8	0.0891
2.8	0.9680	-0.6	0.4687	-4.0	0.0791
2.6	0.9577	-0.8	0.4313	-4.2	0.0701
2.4	0.9450	-1.0	0.3953	-4.4	0.0621
2.2	0.9293	-1.2	0.3612	-4.6	0.0549
2.0	0.9117	-1.4	0.3288	-4.8	0.0485
1.8	0.8905	-1.6	0.2985	-5.0	0.0428

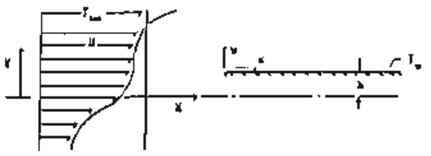

Figure (1) Flat plate in an approach flow with nonuniform velocity and uniform temperature.

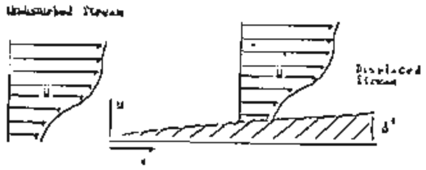

Figure (2) Displacement effect of boundary-layer on free stream flow.

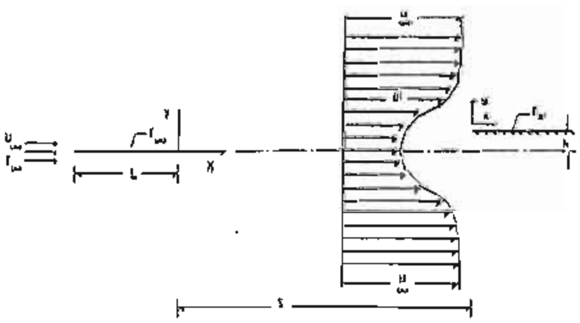

Figure (3) Flat plate in the laminar wake of an upstream plate.

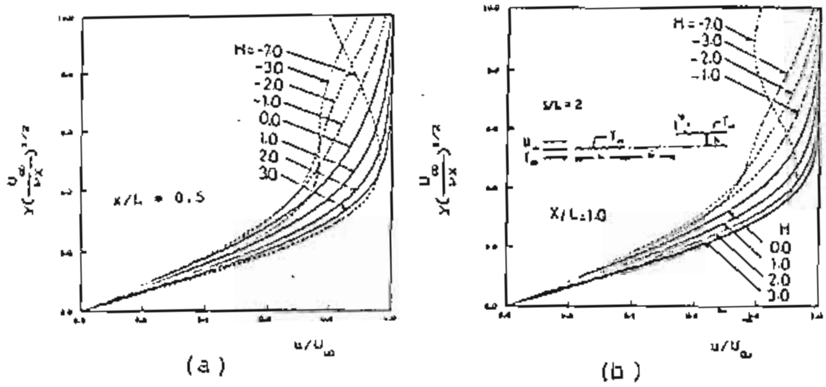

Figure (4) Velocity profiles across boundary-layer of a plate in the laminar wake of an upstream plate for S/L = 2.
(a) x/L = 0.5, (b) x/L = 1.0.

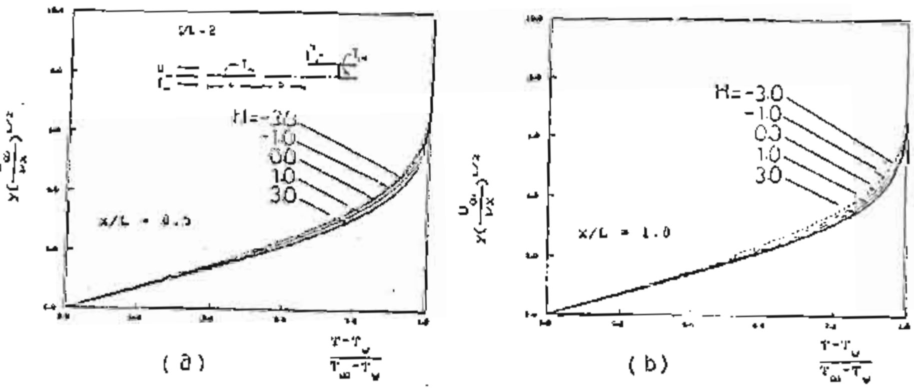

Figure (5) Temperature profiles across boundary-layer of a plate in the laminar wake of an upstream plate for S/L = 2.
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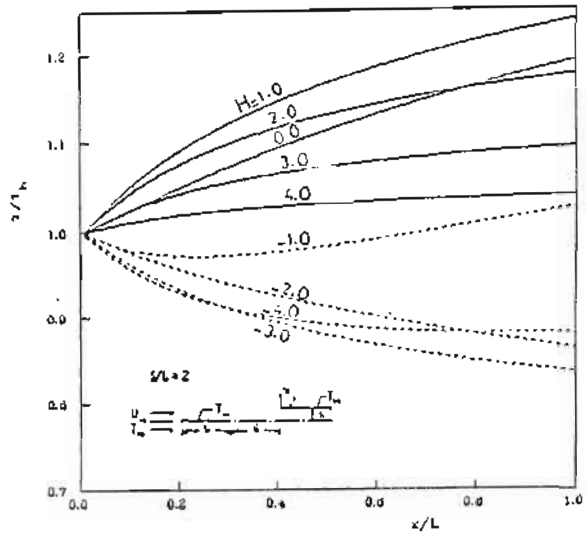

Figure (6) Variation of local wall shear stress τ/τ_h along a plate in the laminar wake of an upstream plate for S/L = 2.

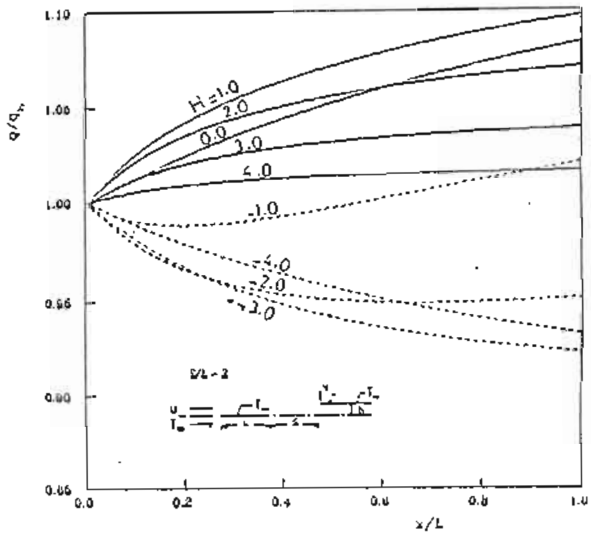

Figure (7) Variation of local heat flux q/q_h along a plate in the laminar wake of an upstream plate for S/L = 2.

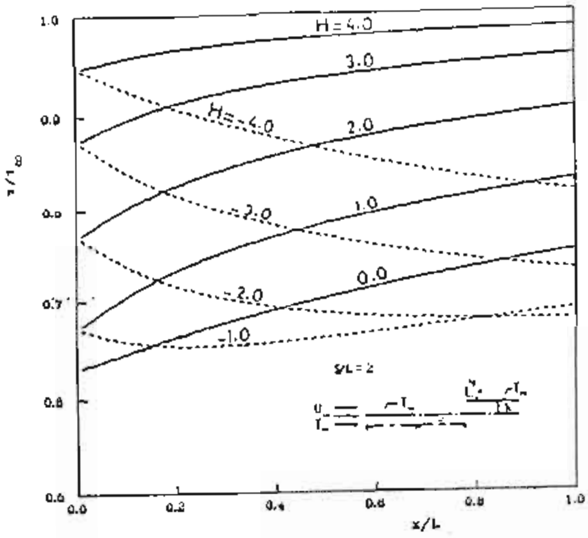

Figure (8) Effect of nonuniformity of the approach flow velocity on wall shear stress along a plate in the laminar wake of an upstream plate for S/L = 2.

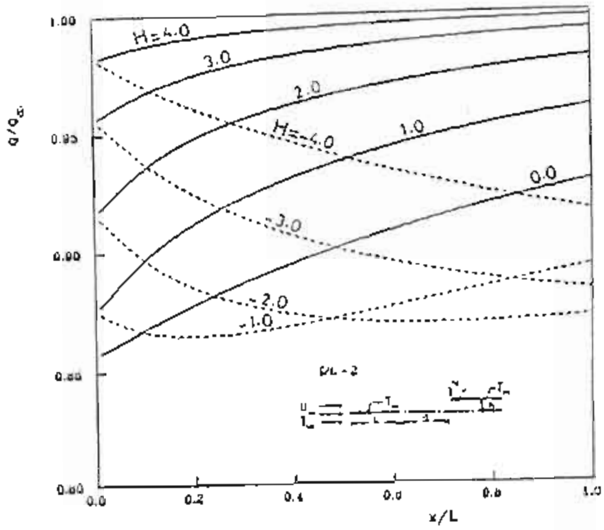

Figure (9) Effect of nonuniformity of the approach-flow velocity on local heat flux along a plate in the laminar wake of an upstream plate for S/L =2.

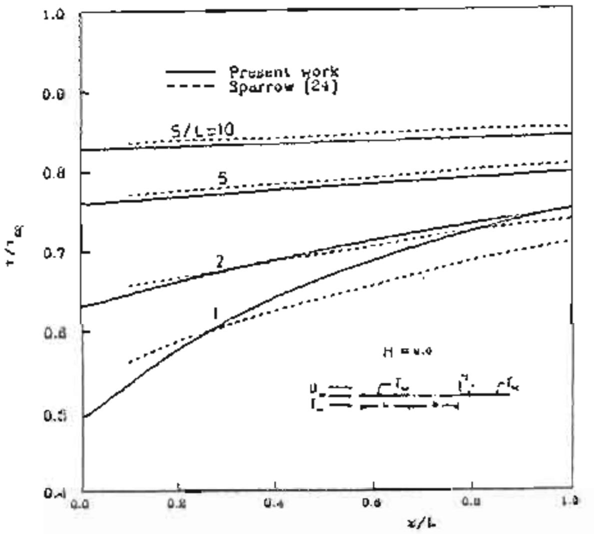

Figure (10) Effect of nonuniformity of the approach flow velocity on wall shear stress along a plate located at the wake centreline of an upstream plate.

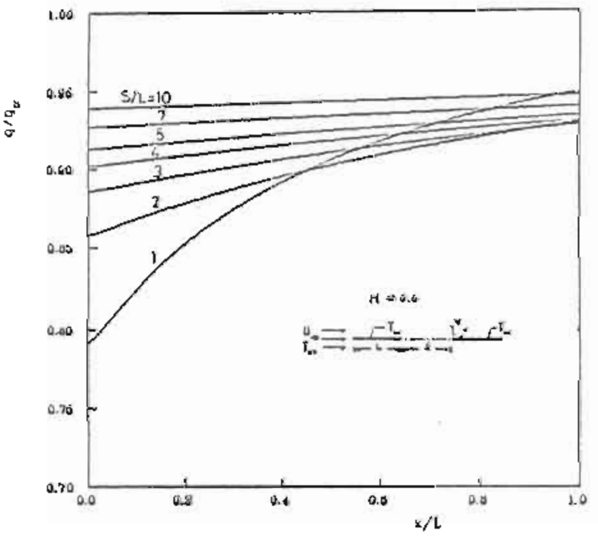

Figure (11) Effect of nonuniformity of the approach-flow velocity on local heat flux along a plate located at the wake centreline of an upstream plate.

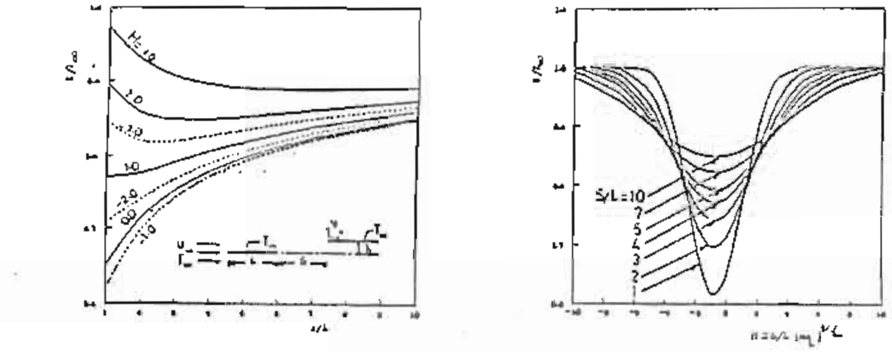

Figure (12) Effect of nonuniformity of the approach-flow velocity on drag force for a plate in the laminar wake of an ups*ream plate.

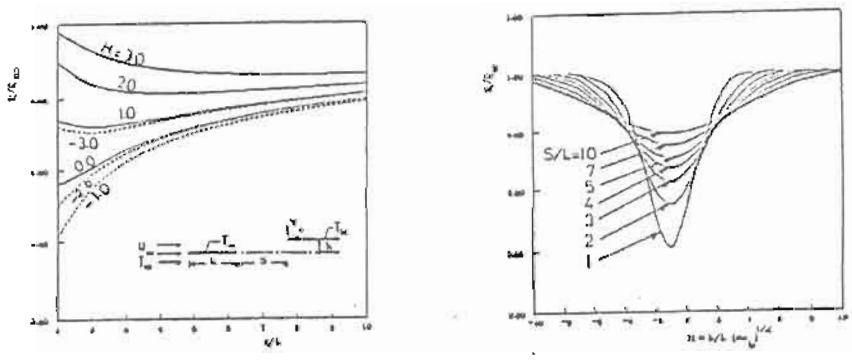

Figure (13) Effect of nonuniformity of the approach-flow velocity on overall heat transfer rate for a plate in the laminar wake of an upstream plate.

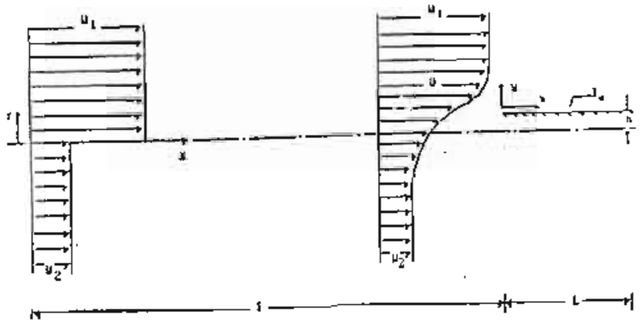

Figure (14):
Flat place in the laminar shear layer between two uniform streams.

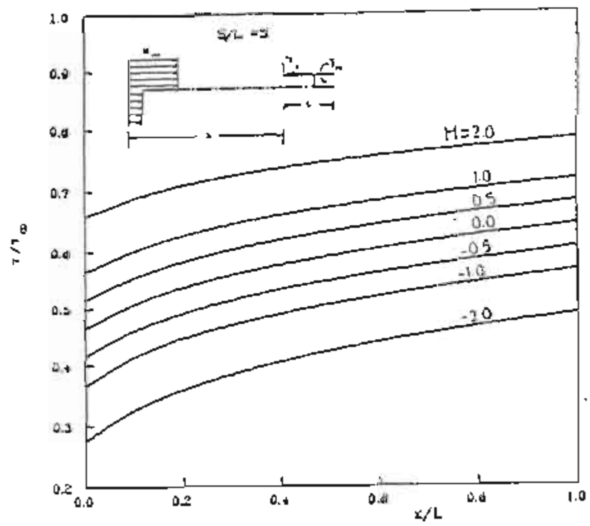

Figure (15) Effect of nonuniformity of the approach-flow velocity on wall shear stress along a plate in the laminar shear layer joining a uniform stream and a quiescent fluid for S/L = 5.

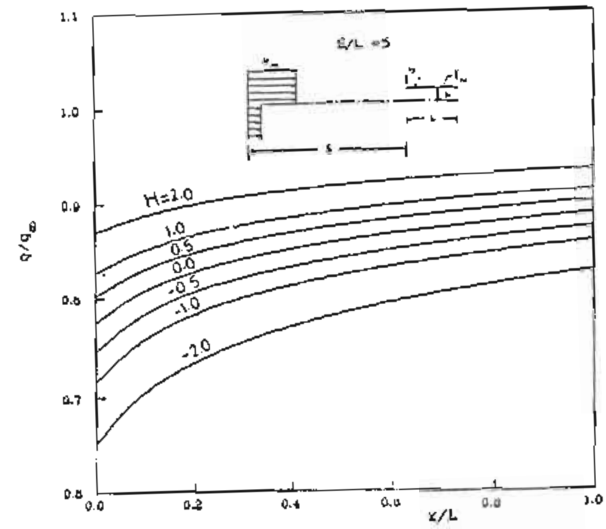

Figure (16) Effect on nonuniformity of the approach-flow velocity on local heat flux along a plate in the laminar shear layer joining a uniform stream and a quiescent fluid for S/L = 5.

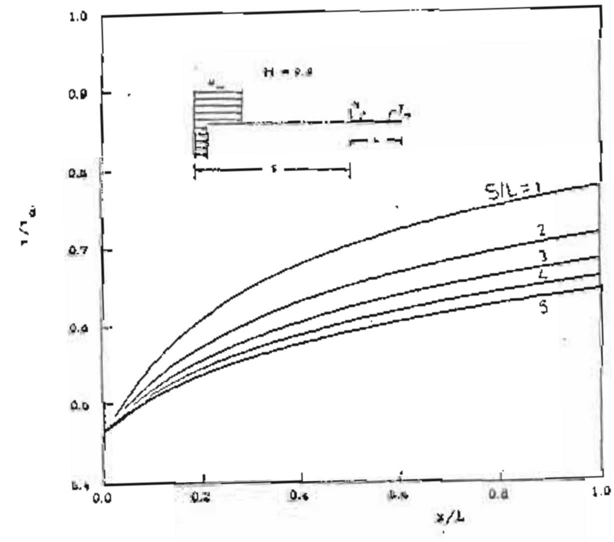

Figure (17) Effect of nonuniformity of the approach-flow velocity on wall shear stress along a plate located at the dividing streamline of the laminar shear layer.

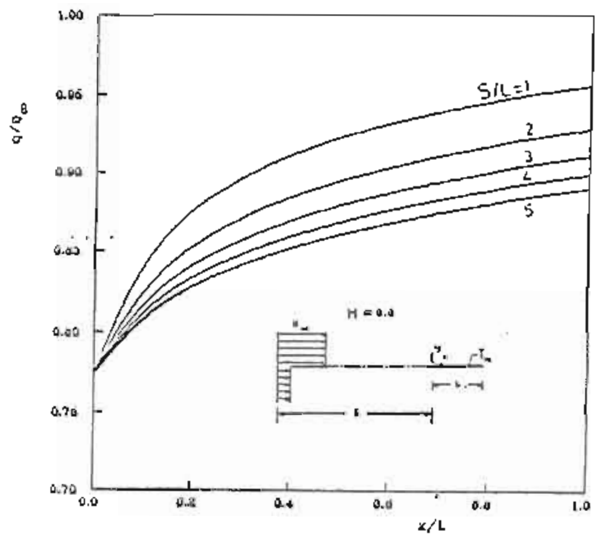

Figure (15) Effect of nonuniformity of the approach flow velocity on local heat flux along a plate located at the dividing streamline of the laminar shear layer.

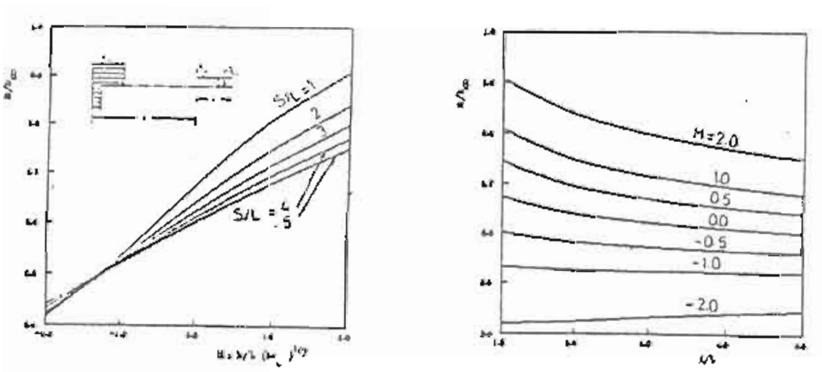

Figure (16) Effect of nonuniformity of the approach flow velocity on drag force for a plate in the laminar shear layer joining a uniform stream and a quiescent fluid.

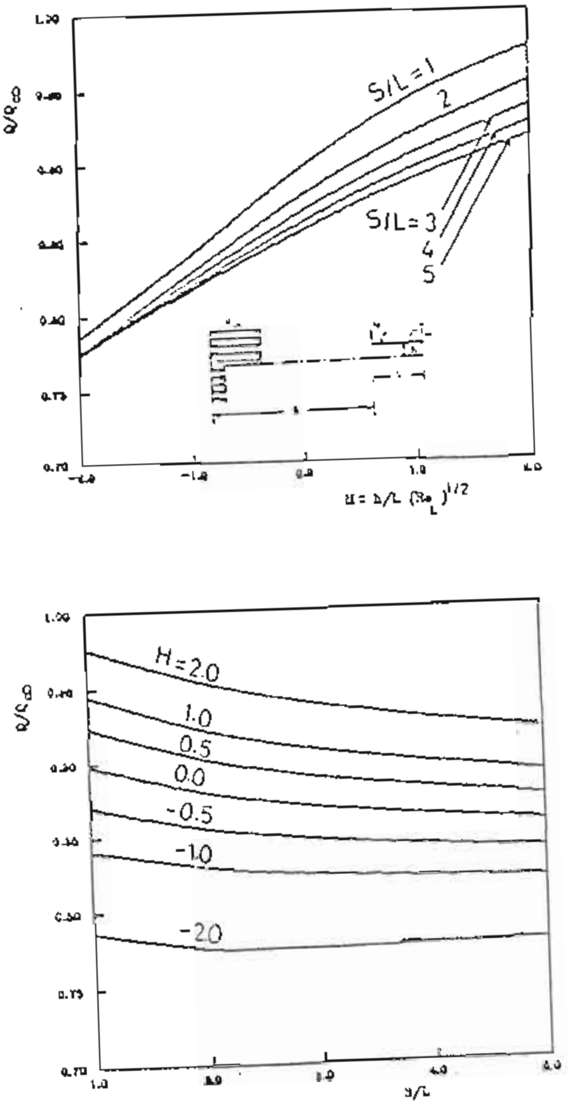

Figure (20) Effect of nonuniformity of the approach-flow velocity on overall heat transfer rate for a plate in the laminar shear layer joining a uniform stream and a quiescent fluid.