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## A Mathematical Model of a Transformer-Connected with Combination of Static Converter and Synchronous Motor for Electromagnetic Transients.

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A MATHEMATICAL MODEL OF A TRANSFORMER-CONNECTED WITH COMBINATION OF  
STATIC CONVERTER AND SYNCHRONOUS MOTOR FOR ELECTROMAGNETIC TRANSIENTS

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نموذج رياضي للكهروديناميكه العابره لمحول موصل مع محرك تزامني  
ومحول استاتيكي

الخلاصه :  
يقدم هذا البحث نموذج رياضي لدراسة الكهروديناميكه العابره لمحول موصل على التوالي مع محرك تزامني وقد تم تمثيل  
النظام المتطير باستعمال تحليل العنصر الموزع في صورة تفاضله . مع اخذ نقطه التعادل في الاعتبار لتبسيط الحسابات . وقد تم في  
هذه الدراسة تمثيل الحمل على اساس دارة واستعمل عنده طرق لحل النموذج الرياضي بطريقه سهله معها حساب التيارات والجهود  
ومن نتائج هذا البحث ان النموذج الرياضي بسيط وعلى درجه عاليه من الدقه ويمكنه احتواء احوال اكثر للتحليل . هذا بالاضافه الى ان  
البرنامج المعدد للحسابات ذات مقدره لحساب الكهروديناميكه العابره للتنفيذيه مماثله للاحوال . كذلك فان اجزاه الحمل الالكترونيه  
ذات تأثير سالب على القيم العظمى للتيارات والجهود العابره كما في حاله المحرك المتزامن .

ABSTRACT

The purpose of this paper is to present a mathematical model for studying electromagnetic transients of a transformer connected in cascade with a synchronous motor. An exact equivalent circuit represents the system is developed using the distributed element analysis in the differentiating form. In addition, the model can be easily account for neutral point of the transformer which is connected to ground. In this study the load is represented in d,q,o reference frame.

Several methods for solving the mathematical model are used. The desired voltages and currents of this model can be easily computed. Generally, several cases are considered to explain the effects on system transients of different system parameters.

INTRODUCTION

The analysis of electromagnetic transient arising in power networks has a great interest and importance. This is mainly because such studies yield necessary information about the possible stresses on different network components, which well, in turn, determine their proper design, limits of operation as well as their pertinent protection strategies.

For a proper design and operation of power system as well as its constituent components, the transient voltage distribution in the system must be determined. To evaluate the transient stresses of a system components during its operation, these components should be modeled as a part of the integrated power system.

An attention has been paid to the transient voltage distribution along the winding turns of high voltage transformers [1-4]. Actually, the transformer transient input voltage is determined primarily by its location in this network and the overall transient response of the interconnected system. An illustrative example of this method is presented in Fig. (1). This example, however, is a transformer-connected with synchronous

motor.

In this present work, a new mathematical model for a transformer connected with a synchronous motor is proposed. This model can be used to analyze the transient voltage distribution along the transformer winding with several transformer loading conditions. In addition, the influence of the transformer neutral connection on the system's transient behaviour can easily be incorporated.

1- MATHEMATICAL MODEL OF STATICALLY TRANSFORMATION

The substitution circuit equations on converters by current contour have the following form

$$L_k \frac{d\bar{i}_k}{dt} + R_k \bar{i}_k = \bar{e}_k \tag{1}$$

where inductances and resistances contours matrices are as follows:-

$$L_k = K_t * L * K ; R_k = K_t * R * K ; e_k = K_t * e \tag{2}$$

Equation (1) must be rearranged, in order to calculate the contour currents, as follows:

$$d\bar{i}_k/dt = L_k^{-1} * (\bar{e}_k - R_k * \bar{i}_k) \tag{3}$$

Fig. (2) represents the case of line directions based on the substituting circuit. The solid and dotted lines denote the trees and span graphical respectively. From this figure with regard to (n=p-q+1), the number of equations which going on system composition, describing the electromagnetic conditions of Fig. (2) are (10-6+1=5). Therefore, the contour matrix of the system of Fig. (2) is given in table 1.

Using this table, the matrices of  $R_k$ ,  $L_k$  of contour parameters of the substituting circuit of Fig. 1 are written as the following:-

$R_k =$	<table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td><math>R_{11}</math></td><td><math>R_{12}</math></td><td><math>R_{13}</math></td><td><math>R_{14}</math></td><td><math>R_{15}</math></td></tr> <tr><td><math>R_{21}</math></td><td><math>R_{22}</math></td><td><math>R_{23}</math></td><td><math>R_{24}</math></td><td><math>R_{25}</math></td></tr> <tr><td><math>R_{31}</math></td><td><math>R_{32}</math></td><td><math>R_{33}</math></td><td><math>R_{34}</math></td><td><math>R_{35}</math></td></tr> <tr><td><math>R_{41}</math></td><td><math>R_{42}</math></td><td><math>R_{43}</math></td><td><math>R_{44}</math></td><td><math>R_{45}</math></td></tr> <tr><td><math>R_{51}</math></td><td><math>R_{52}</math></td><td><math>R_{53}</math></td><td><math>R_{54}</math></td><td><math>R_{55}</math></td></tr> </table>	$R_{11}$	$R_{12}$	$R_{13}$	$R_{14}$	$R_{15}$	$R_{21}$	$R_{22}$	$R_{23}$	$R_{24}$	$R_{25}$	$R_{31}$	$R_{32}$	$R_{33}$	$R_{34}$	$R_{35}$	$R_{41}$	$R_{42}$	$R_{43}$	$R_{44}$	$R_{45}$	$R_{51}$	$R_{52}$	$R_{53}$	$R_{54}$	$R_{55}$
$R_{11}$	$R_{12}$	$R_{13}$	$R_{14}$	$R_{15}$																						
$R_{21}$	$R_{22}$	$R_{23}$	$R_{24}$	$R_{25}$																						
$R_{31}$	$R_{32}$	$R_{33}$	$R_{34}$	$R_{35}$																						
$R_{41}$	$R_{42}$	$R_{43}$	$R_{44}$	$R_{45}$																						
$R_{51}$	$R_{52}$	$R_{53}$	$R_{54}$	$R_{55}$																						

$L_k =$	<table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td><math>L_{11}</math></td><td><math>L_{12}</math></td><td><math>L_{13}</math></td><td><math>L_{14}</math></td><td><math>L_{15}</math></td></tr> <tr><td><math>L_{21}</math></td><td><math>L_{22}</math></td><td><math>L_{23}</math></td><td><math>L_{24}</math></td><td><math>L_{25}</math></td></tr> <tr><td><math>L_{31}</math></td><td><math>L_{32}</math></td><td><math>L_{33}</math></td><td><math>L_{34}</math></td><td><math>L_{35}</math></td></tr> <tr><td><math>L_{41}</math></td><td><math>L_{42}</math></td><td><math>L_{43}</math></td><td><math>L_{44}</math></td><td><math>L_{45}</math></td></tr> <tr><td><math>L_{51}</math></td><td><math>L_{52}</math></td><td><math>L_{53}</math></td><td><math>L_{54}</math></td><td><math>L_{55}</math></td></tr> </table>	$L_{11}$	$L_{12}$	$L_{13}$	$L_{14}$	$L_{15}$	$L_{21}$	$L_{22}$	$L_{23}$	$L_{24}$	$L_{25}$	$L_{31}$	$L_{32}$	$L_{33}$	$L_{34}$	$L_{35}$	$L_{41}$	$L_{42}$	$L_{43}$	$L_{44}$	$L_{45}$	$L_{51}$	$L_{52}$	$L_{53}$	$L_{54}$	$L_{55}$
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$L_{51}$	$L_{52}$	$L_{53}$	$L_{54}$	$L_{55}$																						

The vectors representing the currents and voltages are:

$$\bar{i}_k = [i_1 | i_2 | i_3 | i_4 | i_5]$$

$$\bar{e}_k = [e_d | e_B - e_C | e_C - e_B + e_d | e_A - e_C | e_C - e_A + e_d]$$

The branches currents in the substituting circuit are limited by the following equation:

$$\bar{i} = K \bar{i}_k \tag{4}$$

Many methods in the literature is used to integrate differential equations but the fourth order Rung-Kutta method is preferred. This is

because it has sufficient high accuracy compared with other methods.

The values of integrating step follows select the calculation of minimum meaning of constant time of the modelling system is

$$n < C \cdot T_{\min}$$

Thus the currents in all branches of circuits are limited by increasing matrix contour currents (n). Moreover, in these moments the massive formation come out information, which lead out to stamp tables and graphics after finishing the integration process.

## 2- MATHEMATICAL MODEL OF A SYNCHRONOUS MOTOR

The mathematical process using electromechanical energy transformation in synchronous motor in back current sequences differs from certain complex sequences.

At unsymmetrical feeding voltage of stator windings of synchronous motor, it is useful to use the d, q, o reference frame. Therefore, the following equations are obtained:

$$U_d = \frac{d\phi_d}{dt} + \frac{R_s}{\sqrt{L_s}} \phi_d - \frac{MR_s}{\sqrt{L_r L_s}} \phi_{rd} \quad (5)$$

$$U_q = \frac{d\phi_q}{dt} + \frac{R_s}{\sqrt{L_s}} \phi_q - \frac{MR_s}{\sqrt{L_r L_s}} \phi_{rq} \quad (6)$$

$$0 = \frac{d\phi_{rd}}{dt} + \frac{R_r}{\sqrt{L_r}} \phi_{rd} - \frac{MR_r}{\sqrt{L_r L_s}} \phi_d + W_r \phi_{rq} \quad (7)$$

$$0 = \frac{d\phi_{rq}}{dt} + \frac{R_r}{\sqrt{L_r}} \phi_{rq} - \frac{MR_r}{\sqrt{L_r L_s}} \phi_q + W_r \phi_{rd} \quad (8)$$

$$\frac{dw_r}{dt} = -\frac{p}{J} (M_m - M_c(v_r, t)) \quad (9)$$

where,

$$M_m = (3/2) P \cdot (M/\sqrt{L_s L_r}) \cdot (\phi_q \phi_{rd} - \phi_{rq} \phi_d) \quad (10)$$

In equations (1-5), the current relations must be used instead of flux linkages to solve system equations as well as the synchronous motor. Therefore, the relations between currents and flux linkages are given in the following equations:

$$i_d = \frac{1}{\sqrt{L_s}} \phi_d - \frac{M}{\sqrt{L_s L_r}} \phi_{rd} \quad (11)$$

$$i_q = \frac{1}{\sqrt{L_s}} \phi_q - \frac{M}{\sqrt{L_s L_r}} \phi_{rq} \quad (12)$$

$$i_{dr} = \frac{1}{\sqrt{L_r}} \phi_{rd} - \frac{M}{\sqrt{L_s L_r}} \phi_d \quad (13)$$

$$i_{qr} = \frac{1}{\sqrt{L_r}} \phi_{rq} - \frac{M}{\sqrt{L_s L_r}} \phi_q \quad (14)$$

Since the equations in unsymmetrical systems can be written in phase coordinates, the equations of synchronous motor voltage in d, q coordinates are related to the for voltages,

$$\begin{aligned} U_d &= (1/3) * (2 * U_a - U_b - U_c) \\ &= (1/3) * (2 * U_{ab} - U_{bc}) \end{aligned} \quad (15)$$

$$\begin{aligned} U_q &= (1/\sqrt{3}) * (U_b - U_c) \\ &= (1/\sqrt{3}) U_{bc} \end{aligned} \quad (16)$$

and for currents

$$i_a = i_d \quad (17)$$

$$i_b = -0.5 * i_a + (\sqrt{3}/2) * i_q \quad (18)$$

$$i_c = -i_a - i_b \quad (19)$$

Substituting of the investigated equations (5 : 9) of synchronous motors with equations (10:15) and Eq. (16), the following equations are obtained ;

$$\frac{d\varphi_d}{dt} = \frac{1}{\sqrt{3}} (2U_{ab} - U_{bc}) - \frac{R_s}{\omega L_s} \varphi_d + \frac{M R_s}{\omega L_r L_s} \varphi_{rd} \quad (20)$$

$$\frac{d\varphi_q}{dt} = \frac{U_{bc}}{\sqrt{3}} - \frac{R_s}{\omega L_s} \varphi_q + \frac{M R_s}{\omega L_r L_s} \varphi_{rq} \quad (21)$$

$$\frac{d\varphi_{rd}}{dt} = \frac{M R_r}{\omega L_r L_s} \varphi_d - \frac{R_r}{\omega L_r} \varphi_{rd} - \omega_r \varphi_{rq} \quad (22)$$

$$\frac{d\varphi_{rq}}{dt} = \frac{M R_r}{\omega L_r L_s} \varphi_q - \frac{R_r}{\omega L_r} \varphi_{rq} + \omega_r \varphi_{rd} \quad (23)$$

$$\frac{d\omega_r}{dt} = -\frac{p}{j} \left[ -\frac{3}{2} p \frac{M}{\omega L_r L_s} (\varphi_q \varphi_{rd} - \varphi_{rq} \varphi_d) - M_c(\omega_r, t) \right] \quad (24)$$

Since electromagnetic and mechanical equations describing the synchronous motor which is connected in star with the neutral coil is isolated, the following equations are obtained [4] ;

$$U_d = d\varphi_d/dt + R_s i_d - \omega_r \varphi_{rd} \quad (25)$$

$$U_q = d\varphi_q/dt + R_s i_q - \omega_r \varphi_{rq} \quad (26)$$

$$U_f = d\varphi_f/dt + R_f i_f \quad (27)$$

$$0 = d\varphi_{sd}/dt + R_{sd} i_{sd} \quad (28)$$

$$0 = d\varphi_{sq}/dt + R_{sq} i_{sq} \quad (29)$$

$$-\frac{j}{p} \frac{d\omega_r}{dt} = -\frac{3}{2} p (\varphi_d i_q - \varphi_q i_d) - M_c(\omega_r, t) ; \quad (30)$$

$$\frac{d\theta}{dt} = \omega_r \quad (31)$$

where,  $\varphi_{sd}$ ,  $\varphi_{sq}$  are the magnetic flux linkage of stator and rotor coordinates of synchronous motor.

The first part of right hand side of Eq. (30) represents the electromagnetic moment of the motor.

The relations between magnetic-flux linkage and all contours currents of the synchronous motor is in the form ;

$$\phi_d = L_{dd} i_d + M_{df} i_f + M_{dD} i_D \quad (32)$$

$$\phi_f = L_{ff} i_f + (3/2) M_{df} i_d + M_{dD} i_{sd} \quad (33)$$

$$\phi_{sd} = L_{DD} i_{sd} + (3/2) M_{dD} i_d + M_{Df} i_f \quad (34)$$

$$\phi_q = L_{qq} i_q + M_{qQ} i_{sq} \quad (35)$$

$$\phi_{sq} = L_{QQ} i_{sq} + (3/2) M_{qQ} i_q \quad (36)$$

After obtaining the mathematical model which is described by the mentioned equations, these equations must be solved. Therefore, two methods have been used to solve these equations:

a) First Method

The first method depends on solving the equations (25-29) with relative to derivative of magnetic-flux linkage. Therefore, equation (32) to (36) have been used to substitute the currents into equations (25) to (29).

$$\bar{U} = -\frac{d\bar{\phi}}{dt} - R \bar{i} + \omega \bar{\phi} \quad (37)$$

where,  $\bar{U} = (U_d, U_q, U_f, 0, 0)$  is voltage vector at terminal coils of the motors ;

$\bar{i} = (i_d, i_q, i_f, i_{vd}, i_{vq})$  is current vectors of the motor,

$\bar{\phi} = (\phi_d, \phi_q, \phi_f, \phi_{rd}, \phi_{rq})$  is the magnetic flux-linkage vector of the motor.

$R$  = is the diagonal matrix of the motor coil active resistances.

$\omega$  is the square matrix of the rotating angular frequency of the rotor which is in the form

$$\bar{\omega} = \begin{bmatrix} 0 & -\omega_r & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Eqns. (32-36) of the vector matrix can be put in compact form as

$$\bar{\phi} = L \bar{i} \quad (38)$$

where  $L$  is a square matrix of self and mutual inductances of motor coils

$$L = \begin{bmatrix} L_{dd} & M_{df} & M_{dD} & 0 & 0 \\ (3/2)M_{df} & L_{ff} & M_{Df} & 0 & 0 \\ (3/2)M_{dD} & M_{Df} & L_{DD} & 0 & 0 \\ 0 & 0 & 0 & L_{qq} & L_{qQ} \\ 0 & 0 & 0 & (3/2)M_{qQ} & L_{QQ} \end{bmatrix}$$

The current can be obtained from Eq. (38) as

$$\bar{i} = L^{-1} \bar{\phi} \quad (39)$$

Substituting Eq. (39) in Eq. (37) and solving with relative to magnetic-flux linkage yields

$$-\frac{d\bar{\phi}}{dt} = \bar{U} - R L^{-1} \bar{\phi} - \bar{\omega} \bar{\phi} \quad (40)$$

while Eq. (39) is used for calculating the motor currents. Considering the values of rotating speed of rotor field and the rotating angle  $\theta$ , Eqns. (30), (31) of the mechanical condition of the rotor are solved.

b) Second Method

In this method the currents of the coils is obtained firstly by substituting Eq. (37) in Eq. (38) to yield the equation

$$-\frac{d\mathbf{i}}{dt} = L^{-1}(U - R \mathbf{i} - \omega L \mathbf{i}) \quad (41)$$

Then, the phase voltages of busbars, which connect with stator coil of synchronous motor can be written in d, q, 0 reference frame as

$$U_d = -\frac{1}{3} (2 U_{ab} + U_{bc}) \cos \theta + \frac{1}{\sqrt{3}} U_{bc} \sin \theta \quad (42)$$

$$U_q = \frac{1}{\sqrt{3}} U_{bc} \cos \theta - \frac{1}{3} (2 U_{ab} + U_{bc}) \sin \theta \quad (43)$$

The currents of the stator coils in phase quantity related to d, q, 0 reference frame is:

$$i_a = i_d \cos \theta - i_q \sin \theta \quad (44)$$

$$i_b = [(\sqrt{3}/2) i_q - 0.5 i_d] \cos \theta + (0.5 i_q + (\sqrt{3}/2) i_d) \sin \theta \quad (45)$$

$$i_c = -i_a - i_b \quad (46)$$

The final differential equations of electromagnetic loads results in the normal case are obtained from the above step. This is important and essential to reduce the machine time and necessary for calculating the transient process in unsymmetrical systems.

3- EQUIVALENT PARAMETERS DEFINITIONS OF SYNCHRONOUS MOTOR

The analysis mentioned earlier in the previous sections, show that its difficult to obtain an equivalent representation for a group of synchronous motors described by Eqns. (5 : 46) and if possible it did not give real pictures of motion process according to some neglecting. So that, the system calculations in which an equivalent synchronous motor is used can correctly be reproduced only at a certain peculiarity real transient process. The error in the calculations depend on the above reasons which need to satisfy the equivalent circuit. Therefore, choosing the equivalent criteria on the same significance [6].

In the present work, it may be taken to account, the given investigation of electromagnetic transient process. So that equivalent motor produced by the criteria of transient process coincide.

This criteria in the general form is ;

$$P(t) = \text{ideal} ; Q(t) = \text{ideal} \quad (47)$$

The practical parameters of the equivalent motor are calculated from the initial conditions of minimum interval time of investigation process

$$\int_0^t (P_n(U, w, t) - P_{eq}(U, v, t))^2 dt \rightarrow \min. \quad (48)$$

$$\int_0^t (Q_n(U, w, t) - Q_{eq}(U, w, t))^2 dt \rightarrow \min. \quad (49)$$

4- MATHEMATICAL MODEL OF TRANSFORMERS

The substitution circuit of transformer which is connected is shown in Fig. (3) while the direction graphic is shown in Fig. (4).

The differential equations of the composed system is based on the

contour currents method. In these equations, the coefficients  $L_{ga}$ ,  $L_{gb}$ ,  $L_{gc}$ , represent the non-linear dynamic inductances of phases on the short circuit principles of the magnetic flux, where  $L_{go}$ , is the inductance of magnetic flux for zero sequence. The coefficients  $L_{ga}$ ,  $L_{gb}$ ,  $L_{gc}$ , and  $L_{go}$  are determined by the approximation depending on the magnetic flux linkage and currents.

$$\varphi = f(i_{\mu}) \tag{50}$$

Since the differential equations of the circuit of Fig. (3) representing the system which contain three electrical no bounding circuits. So nine equations are obtained.

The compact form of these equations is given as follows:-

$$L_k \frac{d(i_k)}{dt} + R_k i_k = e_k \tag{51}$$

where,

$$L_k = \begin{bmatrix} L_{ia} + \Delta L_z & \Delta L_z & \Delta L_z & 0 & 0 & 1 & 0 & 0 & 0 \\ \Delta L_z & L_{ib} + \Delta L_z & \Delta L_z & 0 & 0 & 0 & 0 & 1 & 0 \\ \Delta L_z & \Delta L_z & L_{ic} + \Delta L_z & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & L_a^2 + L_b^2 + L_c^2 & L_a^2 & -L_{bL} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_a^2 & L_a^2 + L_{cL} & L_{cL} & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_{bL} & L_{cL} & L_c^2 + L_{cL} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & L_{ga} + L_{go} & L_{go} & L_{go} \\ 0 & 1 & 0 & 0 & 0 & 0 & L_{go} & L_{gb} + L_{go} & L_{go} \\ 0 & 0 & 1 & 0 & 0 & 0 & L_{go} & L_{go} & L_{gc} + L_{go} \end{bmatrix}$$

and the value of  $R_k$  is

$$R_k = \begin{bmatrix} R_{ia} + \Delta R_z & \Delta R_z & \Delta R_z & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta R_z & R_{ib} + \Delta R_z & \Delta R_z & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta R_z & \Delta R_z & R_{ic} + \Delta R_z & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_a^2 + R_b^2 + R_c^2 & R_a^2 & -R_a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_a^2 & R_a^2 + L_a^2 & R_{cL} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{cL} & R_{cL} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{bL} & R_{cL} & R_c^2 + R_{bL} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{cL} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



The mathematical model of the magnetic circuit of the transformer is indicated from their electrical equivalents. Dependence between the electrical and equivalent magnetic circuits in the substitution circuit is presented in mutual inductances. These mutual inductances appear in the relation between corresponding and mutual inductance coefficients,  $K_c = 1.0$  to the voltage of magnetic-flux linkages.

The non-linear phase of the dynamic inductances on the principle of magnetic flux in the model calculations are given by the values at the moment of the magnetic flux linkage of transformer phases. These, however, in the fitting zones of cutting approximation section of magnetic curves.

Calculations of magnetic transformer curves of the form  $\varphi = f(i_\mu)$

limits the following manners:

- i) By knowing the formula  $U_{t.n} = 2\pi f \varphi_{max}$ , the significance amplitude of nominal phase of magnetic-flux linkage is found from  
$$\varphi_{t.nph} = 2U_n / 2\sqrt{3} \pi f$$
- ii) The sign of  $I_{xx}$  % transformer and the coefficient form of magnetic current curve limit the amplitude of normal magnetic current of the working transformer.  
$$I_{n\mu} = I_{xx} \% I_n K_f / 100 = S_n K_f / \sqrt{3} \bar{U}_n$$
- iii) By the standard magnetic curve  $\varphi = f(i_\mu)$  electrotechnique of steel constructions c/c of magnetic transformer.
- iv) Obtaining the magnetic curve which limits dynamic inductances, the mathematical model of transformer allow to investigate as transients, as establishing processes.

#### 5- MATHEMATICAL MODEL FOR TRANSIENT PROCESS ANALYSIS INCLUDING NODAL POINTS OF LOADS

The substituting circuit of nodal points of loads is shown in Fig. (5) and that representing its graphic is shown in Fig. (6). As shown, the substituting circuit in composition appear in all consuming electrical energy. These depend on the principle of complex nodal points.

Regarding the equations which describe the electromagnetic condition of motor as well as, the differential equations which connect the voltages and condenser currents, the full differential equation become 27 equations [7, 8].

This model allows to limit currents in choosing contours, voltage in nodal points of parts and magnetic flux linkage in magnetic circuits.

The mathematical model allows also to investigate the electrical transmission systems at unsymmetrical e.m.f. of the feeding system. These e.m.fs appear in positive and negative sequences.

## 6- THE RESULTS

To examine the availability of the proposed model, a FORTRAN program has been constructed. The data required, for different types of loads, have been obtained. These, loads are static-converter, and complex load of a synchronous motor in conjunction with the static-converter. Also different system feeding conditions are studied. The results of these loads are discussed below.

### 6.1 Transformer Connected with Static Converter

Firstly, the static converter is considered alone as a load connected to the feeding transformer. The output of the transient conditions are given by curves of Fig. (7).

In Fig. (7a) group of curves illustrate the voltages, currents and fluxlinkages waveforms at load busbar are given. Fig. (7a) shows that when the voltage of the secondary coil of the feeding transformer is increased in one phase, the corresponding flux-linkage has a non-periodic components of a flux-linkage of the transformer. After 0.01 sec. (half-cycle), the flux of phase A reaches 1.95 of its maximum value.

Fig. (7b) indicates that the transient current, of the secondary coil of the transformer at the starting condition has a non-periodic form it is seen that the current reaches its maximum value at 0.02 sec.

Fig. (7c) illustrates the magnetizing current, while Figs. (7d), (7e) represent the d, q reference frame currents of the converter. In Fig. (7d) the current reaches its maximum transient value at 0.03 sec., while converter output voltages is not pure but it has some ripples in both the negative and positive regions as shown in Figs. (7g), (7f).

### 6.2 Transformer-Connected with a Combination of a Synchronous Motor and Static Converter

The second condition is to include a synchronous motor in the addition to the static converter.

Fig. (8) represents the output transients of such condition. Fig. (8a) illustrates the normal line voltages. In Fig. (8b), the transient voltage waveform of line AC is shown. The distortion in this wave is due to the electronic devices of the converter which always leads to this effect. From Fig. (8c), the converter transient current in the q axis reaches its maximum value at 0.04 sec. (two cycles). These, however, within a firing angle of 29-30. Fig. (8d) shows the secondary current of the transformer which reaches its steady-state at 0.03 sec. Fig. (8e) represents the waveform of the synchronous motor stator voltages. It is clear that in phase A its transient value is reached. Finally, in Fig. (8f), the transient flux reaches 2.0 of its maximum value.

### 6.3 Transformer Connected Loads With Unsymmetrical Feeding

In this case, the complex load section of 6.2 with the unsymmetrical feeding rate are considered as variables. The range of dissymmetry variations are 0.05, 0.1 and 0.2.

The effects of unymmetrical feeding are shown in Figs. (9, 10, 11). The output waveforms representing in Fig. (9) is obtained by using the rate of unymmetrical feeding 0.05, while, Figs. (10, 11) for 0.1 and 0.2.

#### 7- CONCLUSIONS

From this paper, the following conclusions may be considered:

- 1- A mathematical model for a transformer-connected synchronous motor and other loads is proposed and applied.
- 2- Equations describing the proposed model are deduced and presented in details.
- 3- The model has the ability to include more complex load nodes.
- 4- A realized program based on the model is constructed. This program has the ability to compute the electromagnetic transients due to unymmetrical feeding to the nodal loads.
- 5- The electromagnetic transients under static converter and synchronous motor are investigated in details.
- 6- The electronic load devices as well as synchronous motor has a negative effect on the maximum transient voltages and currents.
- 7- Increasing the unymmetrical feeding also has a negative effect on the transient conditions.
- 8- The model is a simple, accurate, and easy in uses. In the other hand, it can be extended to include more complex loads.

#### 8- NOMENCLATURE

$L_k$ & $R_k$	Square matrix of n-inductances and resistances respectively.
$i_k$	Contour current measured vectors.
$e_k$	Contour voltage measured vectors.
$K$	Contour matrix.
$K^t$	Transpose matrix of contours.
$R^t, L$	Square matrices of p-resistances and inductances order of substituting circuits branches.
$\bar{e}$	Measured vector of branches voltages.
$\bar{i}$	Measured vector of branches currents.
$T_{min}$	Minimum time of modeling system.
$C$	Constant of forth order Runge-Kutta method = 0.0271.
$U_d$ & $U_q$	d & q motor stator voltages.
$\psi_d, \psi_q, \psi_{rd}$ & $\psi_{rq}$	Magnetic flux linkage of stator and rotor coils in coordinates.
$\omega_r$	Electrical rotational speed motor rotor.
$L_s$ & $L_r$	Stator and rotor self inductance.
$M$	Mutual inductance between stator and rotor of synchronous motor.
$\nu$	Absolute spread coefficient, = $1 - M^2 / (L_s - L_r)$ .
$R_s$ & $R_r$	Synchronous motor stator and rotor resistance.

$P$ & $J$	Number of pole pairs and moment of inertia of synchronous motor.
$U_a, U_b, U_c$	Phases voltages combinations of stator.
$U_{ab}$ & $U_{bc}$	Line voltages combinations of stator.
$i_a, i_b$ & $i_c$	Phase currents of stator.
$U_f, i_f$ & $\varphi_f$	Voltage, current and magnetic-flux linkage of excitation coils.
$i_{sd}, i_{sq}, \varphi_{gd}, \varphi_{sq}$	Currents and magnetic-flux linkage of horizontal and vertical contours coils.
$\theta$	Angle between magnetic of phase axial of stator and horizontal axial
$R_s, R_f, R_{sd}$ & $R_{sq}$	Stator, excitation, horizontal and vertical coils resistances.
$L_{dd}, L_{qq}, L_{ff}, L_{DD}$ & $L_{QQ}$	Stator and rotor self inductances in horizontal and vertical axials.
$M_{df}, M_{dD}$ & $M_{fD}$	Mutual inductances between excitation, starting and stator coils in horizontal axial.
$M_{qQ}$	Mutual inductances between starting and stator coils in vertical axial.
$L_k$	Square matrix in order nine of contour coils for transformers, loads and phase dynamic inductance.
$R_k$	Square matrix in order nine of resistances for transformer coil and loads.
$\bar{i}_k$ & $\bar{\varphi}_k, \bar{e}_k$	Measured vectors current and magnetic-flux linkage in the high voltage side of the transformer. Measured vector e.m.f.
$L_{AL}, R_{AL}, L_{CL}$ & $R_{CL}$	Inductances and resistances of load circuits.
$\Delta L_2, \Delta R_2$	Supplementary self inductances and resistances of zero sequence for the transformer.
$U_n, U_{tn}$	Amplitude of nominal significance of line voltages on the side of feeding transformer.
$K_i$	Coefficient of magnetic current curve, =1.4-1.8.
$I_{n\mu}$	Normal magnetic current of the transformer.
$S$	Normal power of the transformer.
$M(\omega_c, t_r)$	Mechanism resistance moment.
$i_d$ & $i_c$	Stator currents of coils in coordinates.
$i_{dr}$ & $i_{qr}$	Rotor currents of coils in coordinates.
$L_{da}, L_{dh}, L_{qc}$	The non linear dynamic inductances of the magnetic flux.

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## TABLES

TABLE 1: THE CONTOUR MATRIX OF FIG. 2

NO. OF branches	No. of contours				
	1	2	3	4	5
1	1	0	0	0	0
2	0	1	0	0	0
3	0	0	1	0	0
4	0	0	0	1	0
5	0	0	0	0	1
6	1	-1	1	-1	1
7	0	0	0	1	-1
8	0	1	-1	0	0
9	0	-1	1	-1	1
10	1	0	1	0	1

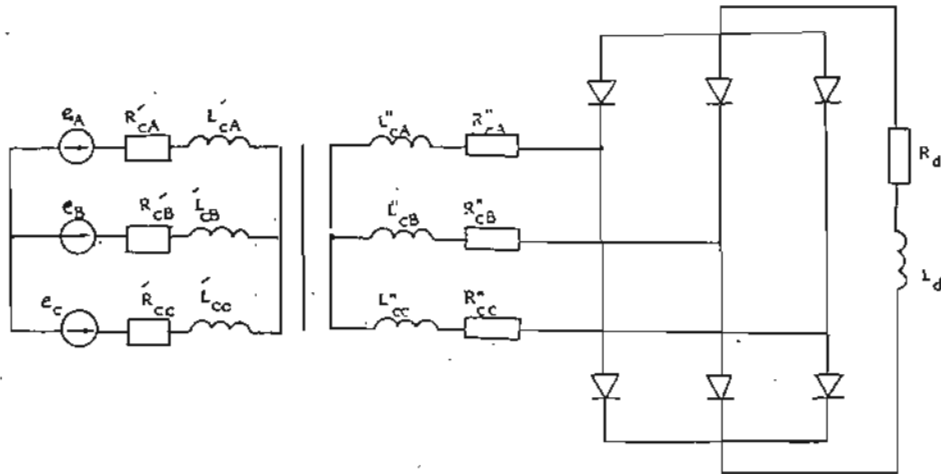


Fig. 1. Substitution circuit of converters connected with the secondary coils of the transformer.

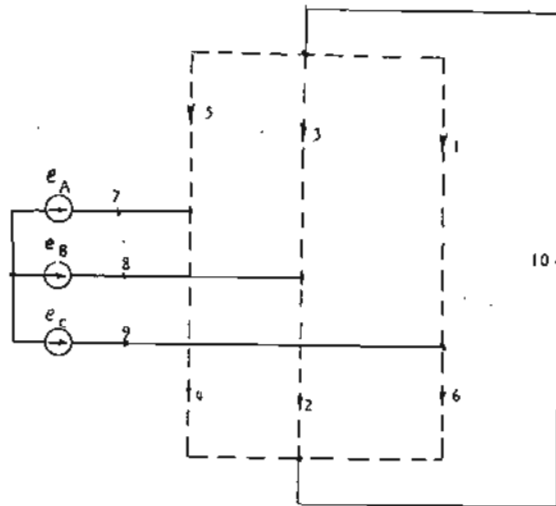


Fig.2. Substitution circuit of semiconductor converters in the case of linear direction graphic.

— Tree - graphic.  
 ..... Chord - graphic.

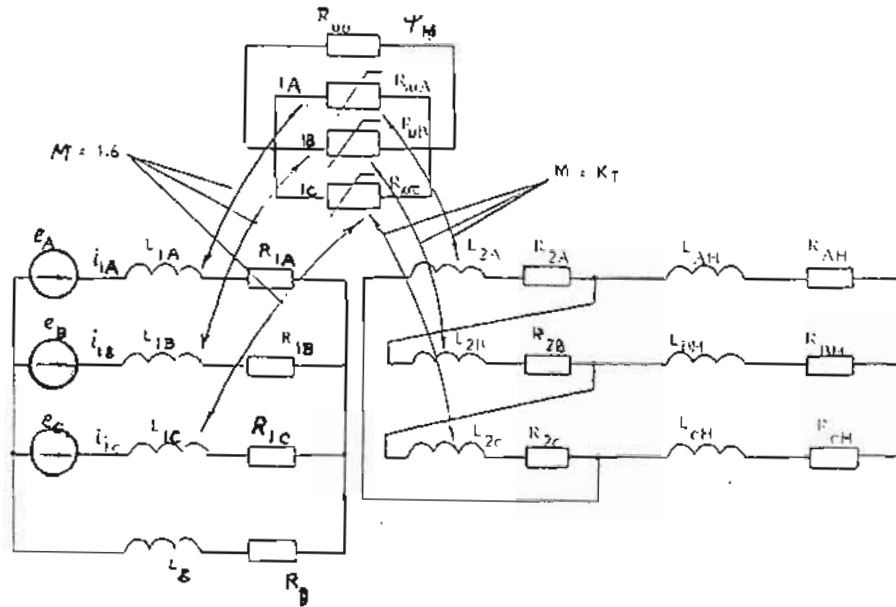


Fig. 3. Substitution circuit of magnetic circuits of the transformer.

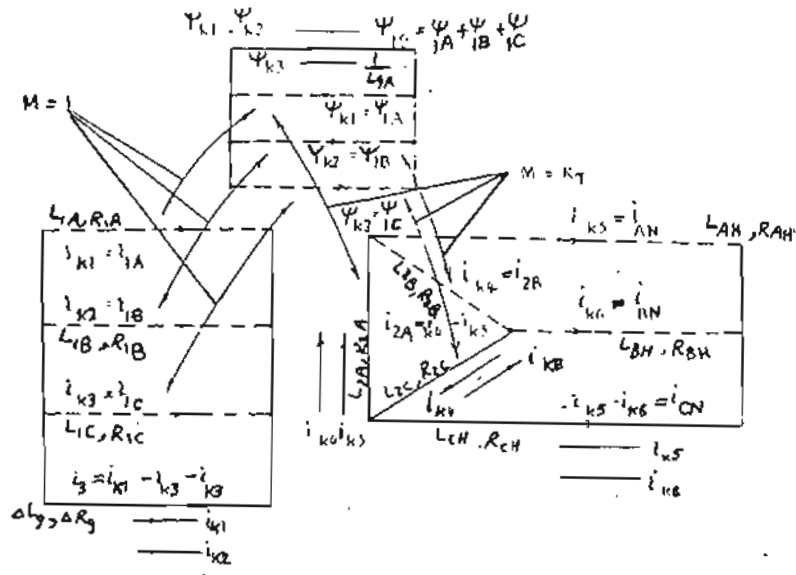


Fig. 4. Direction graphic of substitution circuit of electric and magnetic circuits of the transformer.

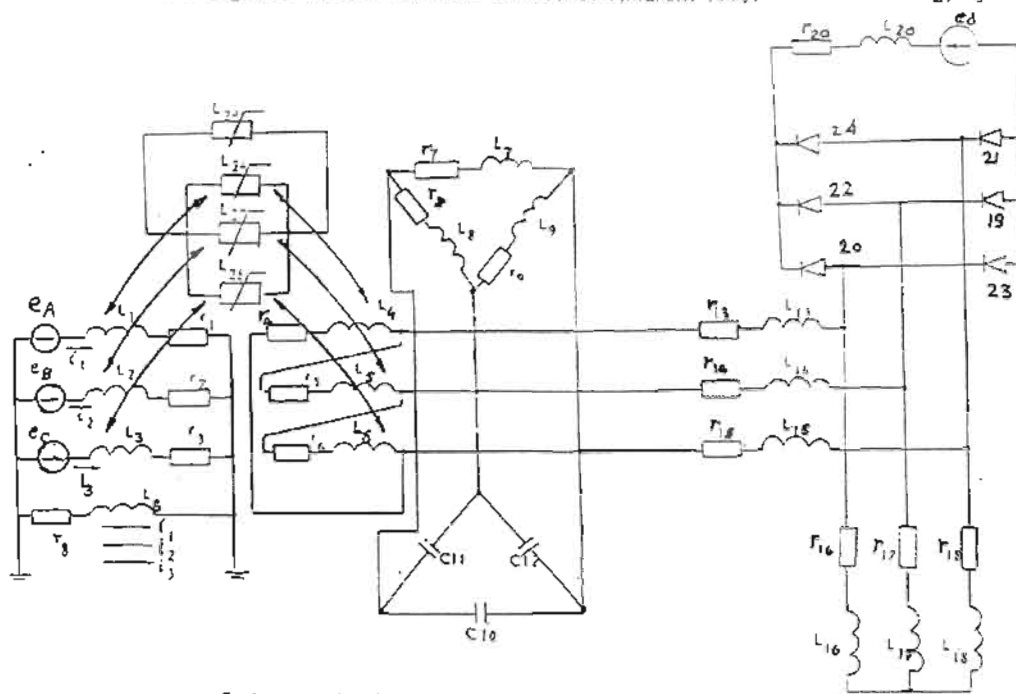


Fig. 5. Substituting circuit of modal loads for the mathematical model.

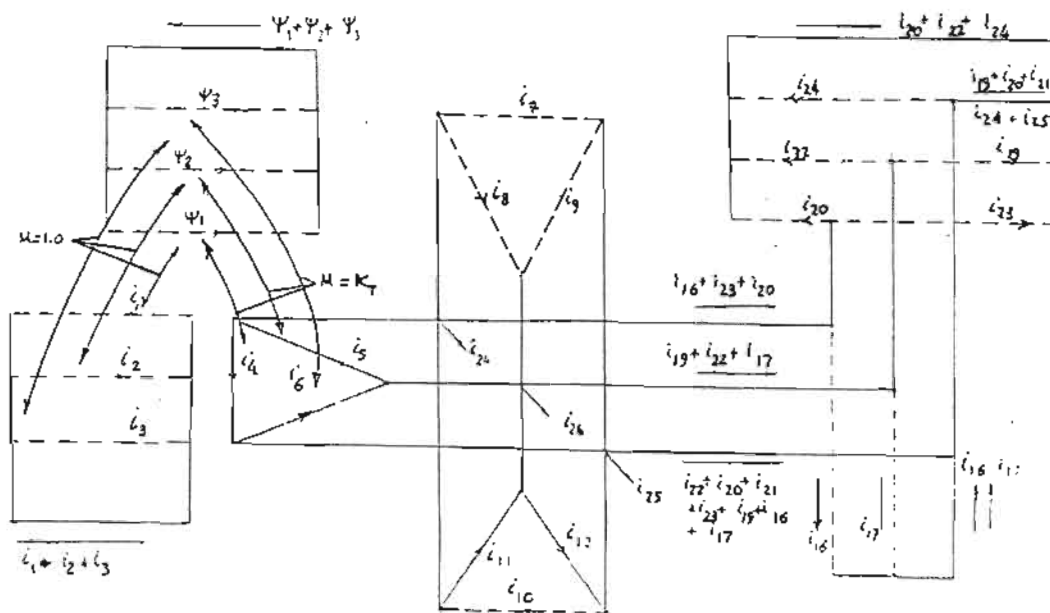


Fig. 6. Linear direction graphic substitution circuit of the complex modal loads.



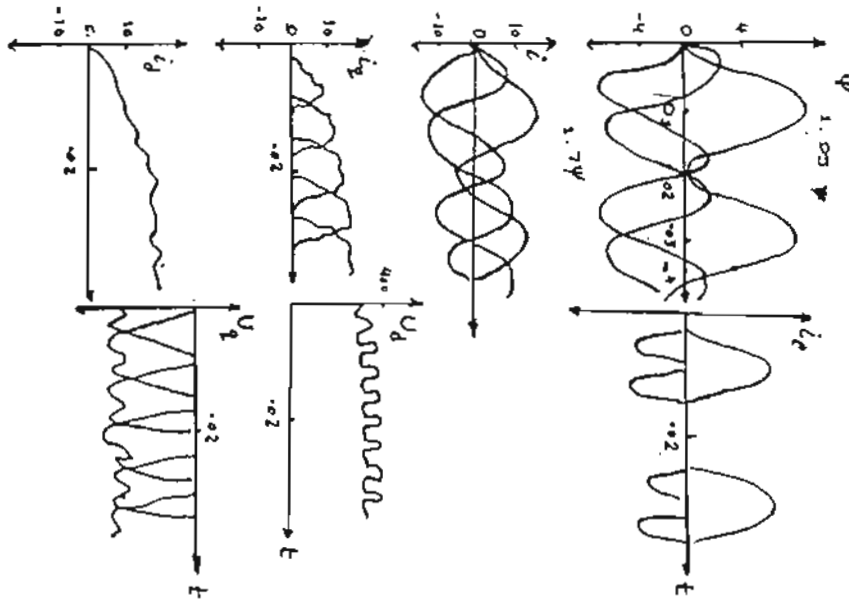


Fig. 7 Response of the System for a Static Converter Lead

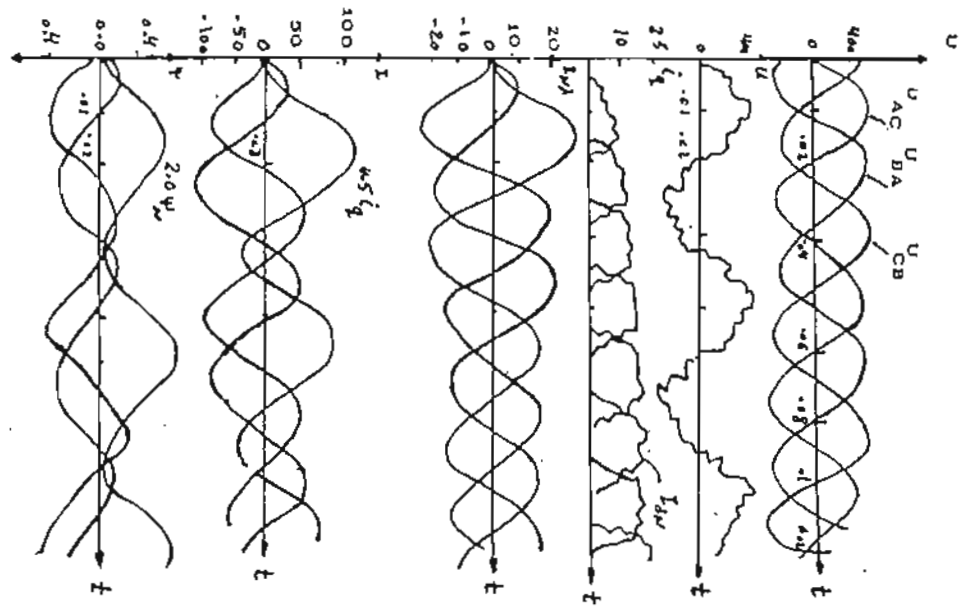


Fig. 8 Response of the System for a Static Converter and Synchronous Motor Load

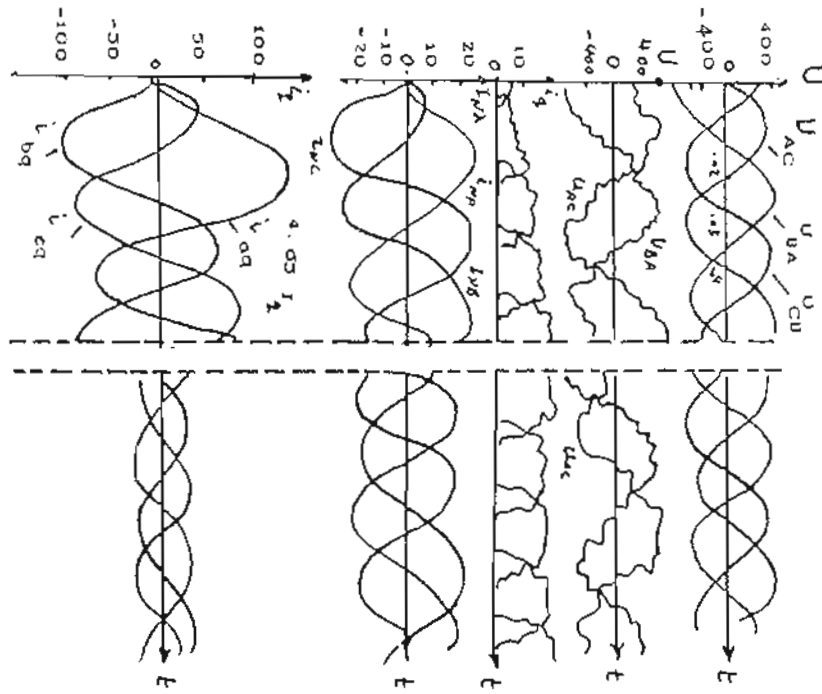


Fig. 9 Response of The system for Complex Load  
With 5% unsymmetrical feeding

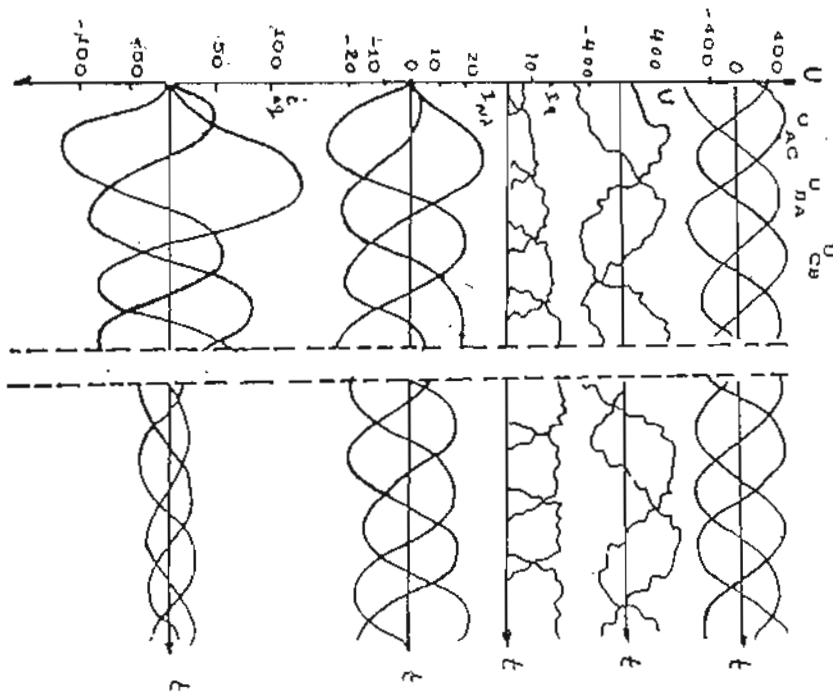


Fig. 10 Response of The system for complex Load  
With 10% unsymmetrical feeding

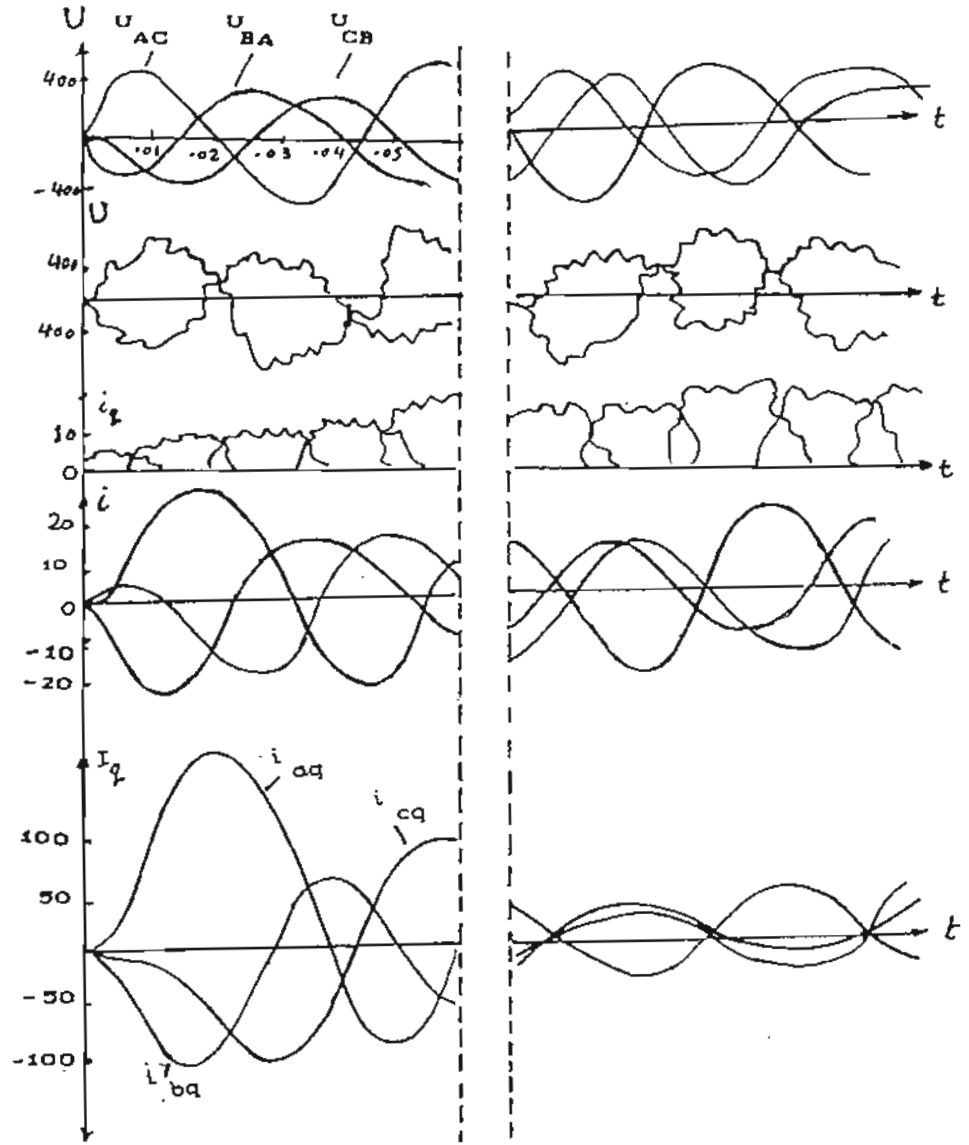


Fig 11 Response of The System for Complex Load with 20 %  
Unsymmetrical feeding