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J - INTEGRAL TEARING MODULUS ANALYSIS FOR CRACKED PLATES UNDER BIAXIAL BENDING

تحليل تكامل J ومعامل التمزق للألواح المشروخة تحت تأثير الإنحناء فى اتجاهين

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خلاصة : هذا البحث يستخدم تحليل تكامل J (تكامل المسار المستقل) ومعامل التمزق ، وذلك للتنبؤ باستقرار أو عدم أستقرار عملية نمو الشروخ فى الألواح المعدنية ذات اللدونة المحدودة فى منطقة الشرخ. هذه الألواح محتوية على شروخ منتظمة نافذة خلال السمك ومعرضة لإجهادات انحناء فى اتجاهين متعامدين. وقد تم إيجاد الحلول لحالات مختلفة من حيث نسبة طول الشرخ وكذلك حالة الإجهادات المؤثرة. كما تم تطبيق النتائج على مثال عملى لألواح مصنوعة من الصلب الكربونى 4130 وذلك بحساب القيمة الحرجة لعامل التمزق من المنحنيات المستنبطة من المعادلات النظرية وبمساعدة المنحنى العملى للمادة.

ABSTRACT

This paper presents a J - integral - tearing modulus analysis for thick plates containing uniform through cracks and subjected to biaxial bending stresses. The solutions are derived for various crack length ratios and applied stress conditions. They are useful for predicting stability of crack propagation in plates with limited plasticity in the vicinity of crack tip. The results are then applied to a carbon steel plate to predict the onset of crack growth instability. Similar applications may be made for other materials, plate geometry and stress conditions.

NOMENCLATURE

a	Crack Length
E	Modulus of Elasticity
f	Function
f_1	Function
h	Plate Thickness
J	Crack Path Integral
K	Stress Intensity Factor
M_1	Bending Moment Acting Normal to Crack
M_2	Bending Moment Acting Parallel to Crack
Q	Applied Stress Ratio
T	Tearing Modulus
T_σ	Tearing Modulus for Plane Stress Condition
T_ε	Tearing Modulus for Plane Strain Condition
U	Function
w	Plate Width
ν	Poisson's Ratio

σ_0 Reference Stress (Fracture Stress)
σ_1 Bending Stress Acting Normal to Crack
σ_2 Bending Stress Acting Parallel to Crack
ϕ Biaxial Stress Ratio

INTRODUCTION

The problem of predicting the onset of crack growth and failure of structures has been for a few decades, the subject of analytical and experimental investigations. Methods such as the stress intensity factor, the strain energy density, and the J-Integral have been used to provide solutions to this problem [1-3]. Recently, a method combining the J-Integral and its derivative with respect to the crack size, which is termed the tearing modulus T, has been developed by Paris et al [4]. The validity of the new method was proven for several cracked pipes applications [5-10]. In this method, the onset of crack growth instability may be predicted for a given load-material combination by combining the computed values of the J-integral and the tearing modulus T with the values of J and T obtained experimentally for the material. The method is yet to be verified for other load-crack-material combinations.

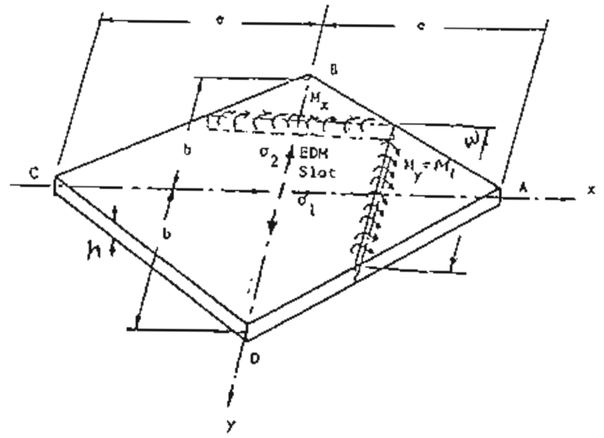
In this paper, the J-T analysis is applied to the problem of cracked plates subjected to biaxial bending moments. Such loading condition may be encountered in several engineering applications such as pressure vessels and turbine blades. The two bending moments are applied such that an arbitrary biaxial stress state is produced in the plate with the stress normal to the crack being always tensile. Depending on the stress parallel to the crack direction, the biaxial stress ratio may be positive, negative, or zero.

PLATE TEARING MODULUS ANALYSIS

The tearing modulus solutions for biaxial bending stresses applied to a thick plate containing through thickness crack are derived in this section, Figs. 1 & 2. The derivations are first carried out for the linear elasticity assumptions, which are then modified for crack tip plastic deformation for applications to small yielding situations.

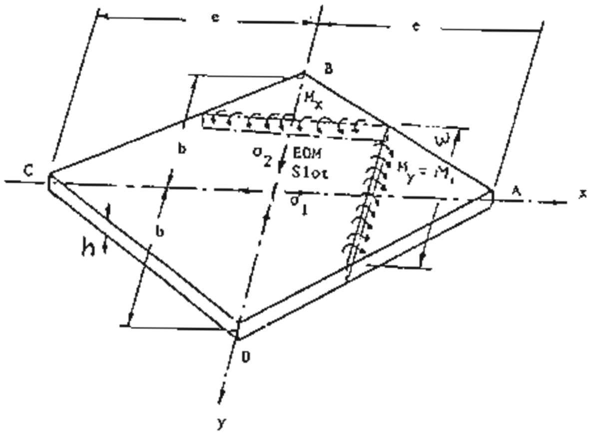
For convenience, the applied tearing modulus is defined by the following expression [4]:

$$T = \frac{E}{w\sigma_0^2}\frac{dJ}{da} \qquad (1)$$



Rhombic plate under biaxial bending (σ_2/σ_1 positive)

Figure 1



Rhombic plate under biaxial bending (σ_2/σ_1 negative)

Figure 2

where σ_0 is a reference stress or flow stress which is usually taken as the average of the yield and ultimate tensile strengths of the material. The elastic J-integral is obtained from the stress intensity factor and is given by:

$$J = \frac{K^2}{E'} \tag{2}$$

where $E' = E$ for plane stress condition

$= \frac{E}{1-\nu^2}$ for plane strain condition

The tearing modulus for the applied bending moment is derived next. The total derivative of equation (1) is evaluated for constant bending moment load (load control condition). The expression for the stress intensity factor for the crack geometry-loading condition combination, shown in Fig. (1, 2) by [11], using Muskheliskvili's Methods of complex potentials, as:

$$K = \sigma_1 \sqrt{\pi a}\; f(\tfrac{a}{w})\; U(Q, \phi) \tag{3}$$

$$\text{where } Q = \frac{\sigma_1}{\sigma_0}, \qquad \phi = \frac{\sigma_2}{\sigma_1} \tag{4}$$

The bending stresses normal and parallel to the crack are given by [12]:

$$\sigma_1 = \frac{6M_1}{h^2}, \qquad \sigma_2 = \frac{6M_2}{h^2} \tag{5}$$

$f(\frac{a}{w})$ = a function for correcting effects of plate width [13]

$$= 1 + 0.256\,(\tfrac{a}{w}) - 1.152\,(\tfrac{a}{w})^2 + 12.2\,(\tfrac{a}{w})^3 \tag{6}$$

$U(Q, \phi)$ = a function for correcting effects of yield strength and biaxial stress [11].

$$= 1 - 0.095\,(Q)(\phi - 1) - 0.19\sqrt{1 - 0.75\,(Q)^2(\phi - 1)^2} \tag{7}$$

The expression for the J - integral becomes:

$$J = \frac{36 w\pi M_1^2 U^2}{h^4 E'} \left[(\tfrac{a}{w})^{1/2} + 0.256\,(\tfrac{a}{w})^{3/2} - 1.152\,(\tfrac{a}{w})^{5/2} + 12.2\,(\tfrac{a}{w})^{7/2}\right]^2 \tag{8}$$

The tearing modulus T which is defined by Eqn. (1) becomes:

$$T = \frac{E}{w\,\sigma_0^2}\,\frac{dJ}{da}$$

$$= \frac{72\pi\, M_I^2}{\sigma_0^2\, h^4}\,\frac{E}{E'}\,U^2 f_1\left(\frac{a}{w}\right) \qquad (9)$$

where

$$f_1\left(\frac{a}{w}\right) = 0.5 + 0.512\left(\frac{a}{w}\right) - 2.88\left(\frac{a}{w}\right)^{3/2} - 0.478\left(\frac{a}{w}\right)^2 - 0.737\left(\frac{a}{w}\right)^{5/2}$$
$$+ 48.358\left(\frac{a}{w}\right)^3 + 3.318\left(\frac{a}{w}\right)^{7/2} + 15.616\left(\frac{a}{w}\right)^4$$
$$- 35.136\left(\frac{a}{w}\right)^{9/2} - 49.19\left(\frac{a}{w}\right)^5 + 520.94\left(\frac{a}{w}\right)^6 \qquad (10)$$

The applied tearing modulus for plane stress condition is:

$$T_\sigma = \frac{72\,\pi\, M_I^2}{\sigma_0^2\, h^4}\,U^2 f_1\left(\frac{a}{w}\right)$$

$$= 6.28\,(Q)^2\,U^2 f_1\left(\frac{a}{w}\right) \qquad (11)$$

Similarly, the applied tearing modulus for plane strain condition is:

$$T_\varepsilon = \frac{6.28}{1 - \nu^2}\,(Q)^2\,U^2 f_1\left(\frac{a}{w}\right) \qquad (12)$$

In this paper, the analysis is carried out for the following conditions.

(a) Applied Stress Ratio, Q ranging from 0.3 to 0.9

(b) Plate Thickness Ratio, $\frac{h}{w} = 0.35$

(c) Crack Length Ratio, $\frac{a}{w}$ from 0.025 to 0.3

(d) Biaxial Stress Ratio, ϕ ranging from - 1.000 to + 1.00

RESULTS AND DISCUSSION

The J-integral-tearing modulus solutions presented in this paper are valid for plates containing a uniform through-thickness flaw. The plate is subjected to two out-of-plane bending moments. The solutions are presented for various crack lengths and stress conditions. The solution are useful for predicting stability of crack propagation in plates with small-scale yielding, whereas it is assumed that plastic deformation is small and limited to the vicinity of the crack tip. The crack propagation is considered stable as long as the calculated tearing modulus is lower than the tearing modulus for the material when the applied J-integral equals J-integral of the material.

Care must be taken when using these solutions to assure that the combination of crack size and applied load is such that small scale yielding is valid. Analysis are presented for various maximum stress to failure stress ratios. A screening criterion that defines the limit of the small scale yielding may be based on the convergence of the plastic zone adjusted crack length. The adjusted crack length may be calculated as outlined in reference [8]. This adjusting process involves several iteractions on the crack length. The iterative calculation is terminated when the value of the crack length resulting from two successive iterations is within 5 percent of each other. The resulting values of J and T are expected to be accurate to within 10% of those calculated from the elastic plastic analysis. The predicted failure stress is then expected to accurate to within 3 to 6 percent of the elastic-plastic stress. Such an accuracy is considered acceptable for most engineering applications.

Figure 3 through 6 show the variation of the applied J-integral with crack length ratio for a cracked plate subjected to two out-of plane bending moments which produce a state of biaxial stress at the crack tips. From these figures, crack instability can be predicted by comparing the applied J-integral values against the material J-resistance curve. Figures 7 through 10 show the same graphs except that the J-integral has been replaced by a normalized J-integral which is given by the equation [10]:

$$J_n = \frac{E\,J}{w\sigma_0^2} \qquad (13)$$

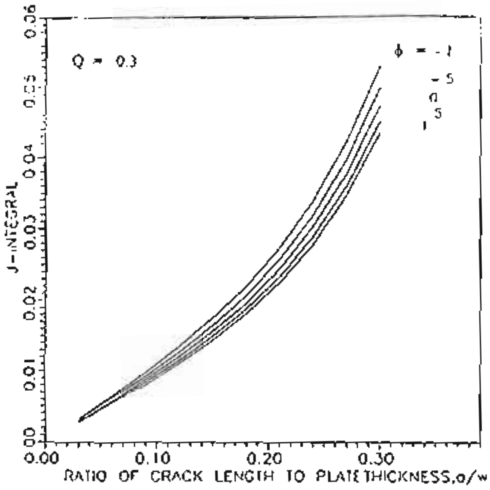
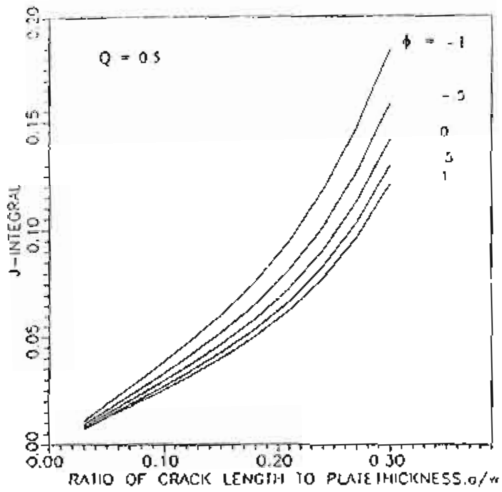


Figure 3



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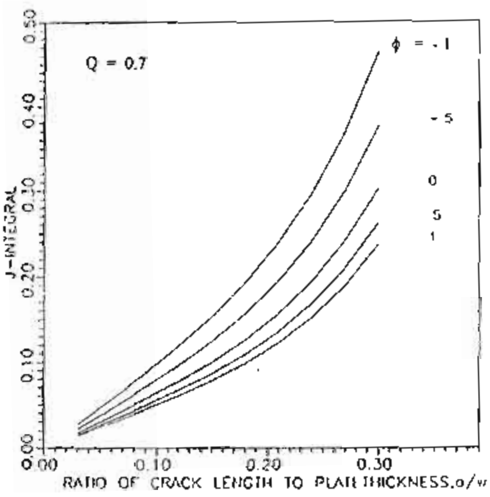

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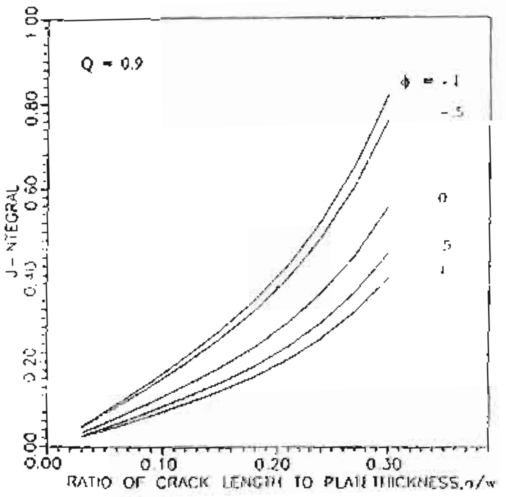

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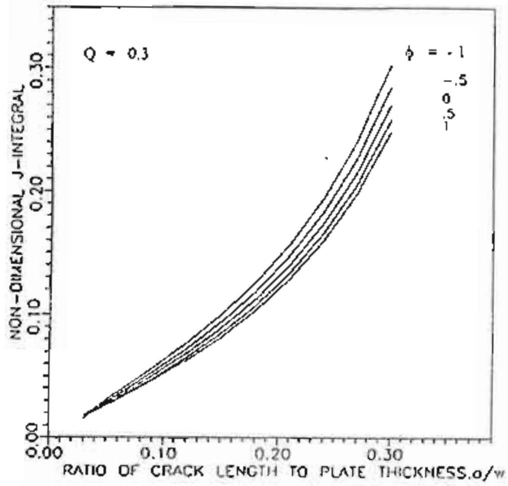

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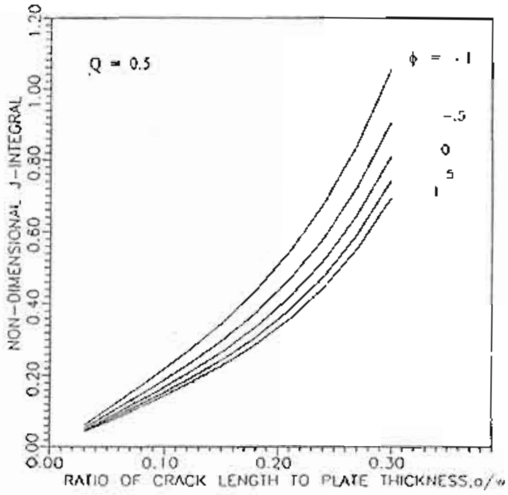

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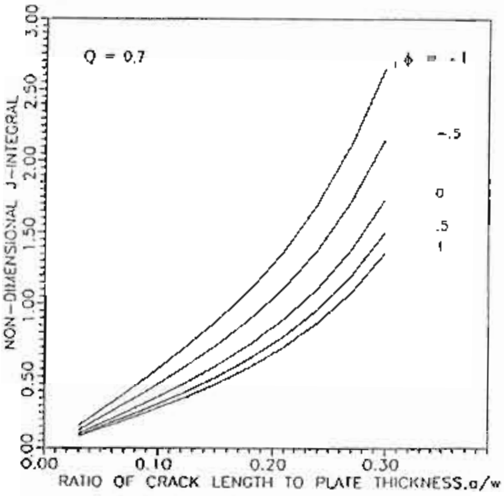


Figure 9

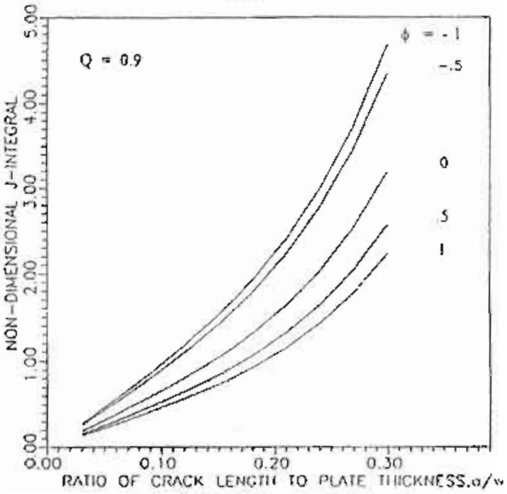


Figure 10

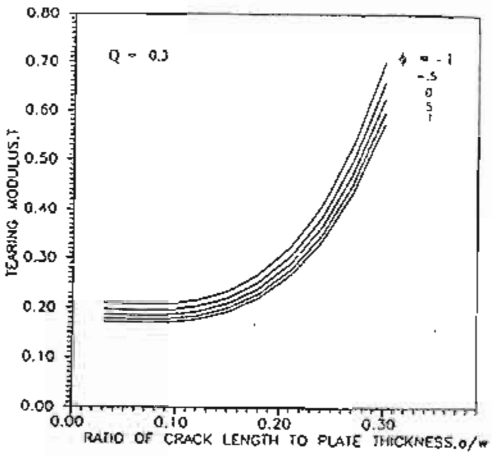
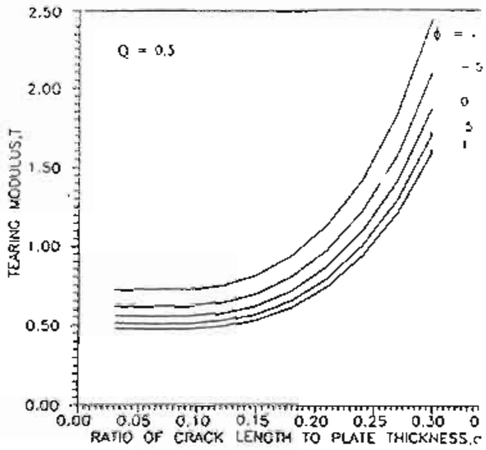


Figure 11



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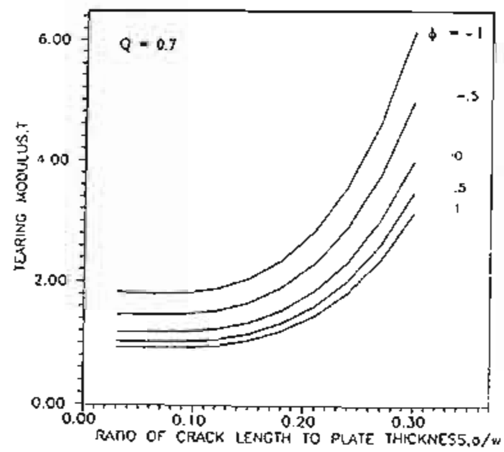


Figure 13

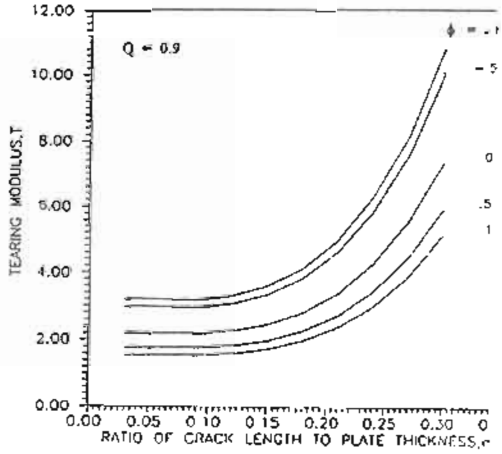


Figure 14

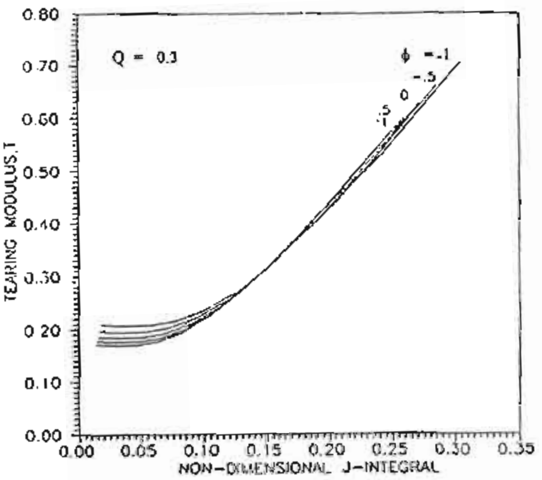


Figure 15

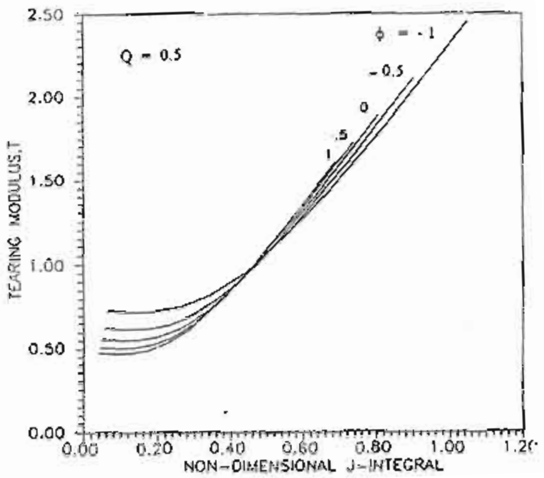


Figure 16

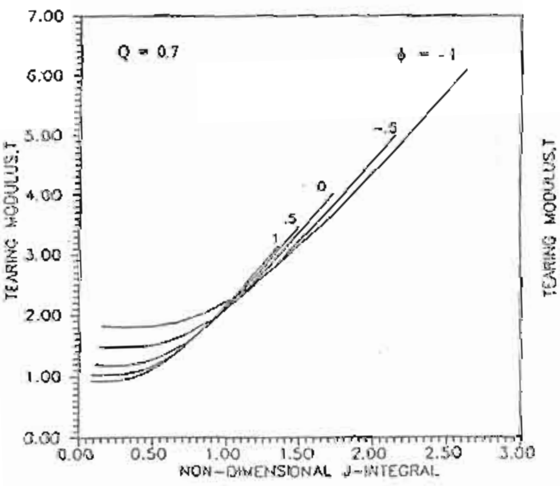
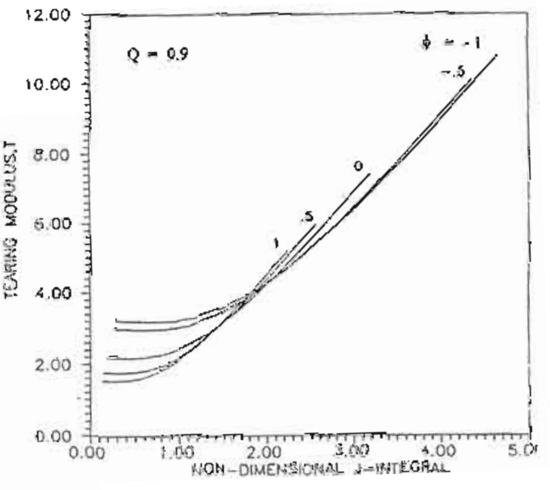


Figure 17



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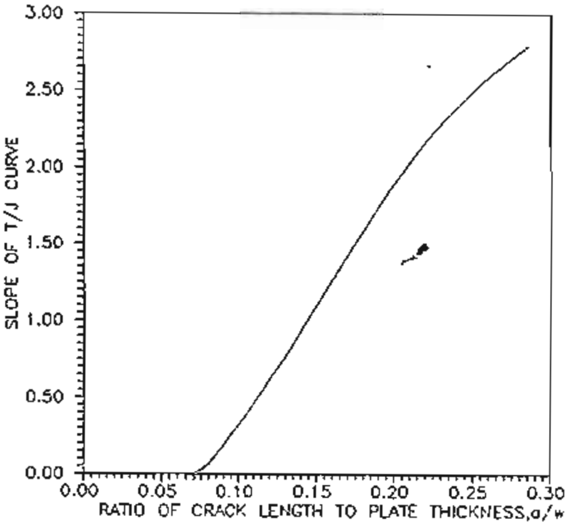

Figure 19

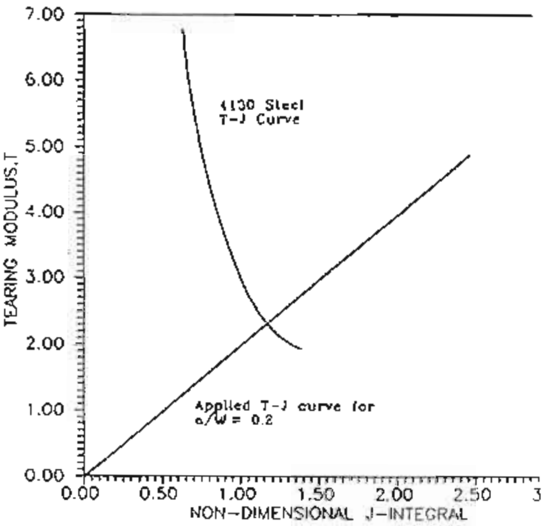

Figure 20

Figures 11 through 14 show the variation of the applied tearing modulus with crack length ratio. Such curves may be used, too, for predicting crack instability. In Figures, 15 through 18, the applied tearing modulus, given by Equation (9), versus the normalized J-integral, given by Equation (13) are shown for various crack length ratios, biaxial stress ratios, and applied stress ratio. In Fig. 19, the slope of the tearing modulus versus normalized J-integral is shown for the previously presented crack size, biaxial stress, and applied stress ratios. It can be seen that for all values of biaxial stress and applied stress ratios, the slope of T-Jn curves versus a/w coincide, i.e. the slope of T-J versus a/w curve is independent of ϕ and Q. It is pointed out that, because of the plate thickness, a plane strain condition is assumed in computing the quantities presented in all figures. The result of Figures 7 through 10 or Fig. 19 are used along with the material J- resistance curves or the material Tearing modulus versus normalized J-integral curves such as that shown in Fig. 20 in order to assess crack growth stability. For a specified crack length, a/w. Fig. 19 gives the slope of the applied T-Jn curve. This slope is then drawn on Fig. 20 to determine whether an intersection with the material T-Jn curve can be found.

As an example, for a/w of 0.2, Fig. 19 gives an applied T-Jn slope of 1.9 at all values of ϕ and Q. At this slope, Fig. 20 gives a critical tearing modulus T of 2.2.

According to the present method, stable crack growth is expected as long as the applied loadings are such that the applied tearing modulus T, calculated from Equation (9), is below the critical value (2.2 in this example).

This procedure may be conveniently followed; provided the material's T-Jn curve, such as that shown in Fig. 20 for 4130 steel, is known. Such a curve can be obtained experimentally in the same manner the material's stress-strain curve is obtained.

REFERENCES:

(1) Paris, P.C., and Sih, G.C.,"Stress Analysis of Cracks," Fracutre Toughness Testing and its Applications, ASTM-STP 381, American Society for Testing and Materials, 1965.

(2) Rice, J.R., "A Path-Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks," ASME Journal of Applied Mechanics, vol.35,1968.

(3) Begley, J.A., and Landes, J.D., "The J-Integral as a fracture Criterion." Fracture Toughness, Part II, ASTM-STP 515, American Society for Testing and Materials, 1972.

(4) Paris, P.C., Tada, H., Zahoor, A., and Ernst, H., "The Theory of Instability of the Tearing Mode of Elastic-Plastic Crack Growth", Elastic-Plastic Fracture, ASTM-STP 668, American Society for Testing and Materials, 1979.

(5) Zahoor, A. and Kanninen, M.F., "A Plastic Fracture Mechanics Prediction of Fracture Instability in a Circumferentially Cracked Pipe in Bending- Part I: J-integral Analysis," ASME Journal of Pressure Vessel Technology, Vol. 103, Nov. 1981, pp. 352-358.

(6) Wilkowski, G.M., Zahoor, A., and Kanninen, M.F., "A Plastic Fracture Mechanics Prediction of Fracture Instability in a Circumferentially cracked Pipe in Bending - Part II: Experimental Verification on a Type 304 Stainless Steel Pipe," ASME Journal of Pressure Vessel Technology, Vol. 103, Nov 1981, pp. 359-365.

(7) Zahoor, A., "J-Integral and Tearing Modulus Estimates for Circumferentially Cracked Pipes in Torsion," ASME Journal of Pressure Vessel Technology, Vol. 106, Oct. 1984, pp. 495-500

(8) Rajab, M.D. and Zahoor, A., "Crack Instability in Pipes Containing Circumference Internal Surface Flaw," AME Journal of Engineering Materials and Technology, Vol. 111, No. 3, July 1989, pp. 243-247.

(9) Rajab, M.D. and Zahoor, A., "Fracture Analysis of Pipes Containing Full Circumferences Internal Part Throughwall Flaw," The International Journal of Pressure Vessels and Piping, Vol. 41, No. 1, Apr. 1990.

(10) Rajab, M.D. and Zahoor, A., "Tearing Modulus Analysis for Pipes Containing Constant Depth Internal Flaw, "Transactions of the ASME, Vol. 113, Jan. 1991, pp. 156 - 160.

(11) Shabara, M.A.N., "A Method for the Prediction of Crack Growth Direction in a Biaxial Stress Field," Proceedings of the International AMSE Conference on Modelling & Simulation, Vol. 17, Solid Mechanics & Robotics, France 1982.

(12) Timoshenko, S.P., andGoodier, J.N., "Theory of Elasticity", McGraw-Hill, Inc., New York, 1970.

(13) Sih, G.C., "Handbook of Stress Intensity Factors," Bethlehem, Pennsylvania, Lehigh University, 1973.