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Orthogonal Transforms for Electrocadiographic Data Compression: A Comparative Study.

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ORTHOGONAL TRANSFORMS FOR ELECTROCADIOGRAPHIC DATA COMPRESSION: A COMPARATIVE STUDY

التحويلات المتعامدة لاختزال بيانات رسم القلب الكهربى : دراســـــة مقارنــــــة

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الخلاصـة — فى هذا البحث تم مقارنة أداء ست خوارزمـات للتحويلات المتعامدة فى اختزال بيانات رسم القلب الكهربى وهى : تحويل فورير ، تحويل الجيب ، تحويل الجيب تمام، تحويل هارتلى، تحويل والش وتحويل هار وقد استخدم لذلك عدد من التسجيلات الحية لرسم القلب لحالات عادية وأخرى غير عادية ، وقد أظهرت النتائج أن تحويل هارتلى يتميز بمقدرته على اختزال البيانات مع تواجد أقل نسبة مئوية للخطأ بين الاشارة الاصلية والاشـارة التى تم استعادتها حيث تم مقارنة معامل الاداء (النسبة المئوية لجذر متوسط مجموع مربعات الفروق RMS) لهذه التحويلات وتميز تحويل هارتلى بأقل نسبة تشوه فى الاشارة التى تم استعادتها وبذلك لاتتأثر الخصائـــــص الاكلينيكية للاشارة .

ABSTRACT- Discrete forms of the Fourier, Sine, Cosine, Hartley, Walsh, and Haar transforms are examined for their performance to compress the electrocardiographic (ECG) signals. To compare their effectiveness in accomplishing this goal, the percent RMS difference resulting for each transformation method at various compression ratios is computed for records of normal and abnormal cases. Based on this comaprison, it is found that the Hartley transform is most advantageous for compresssing the ECG signal as it provides higher compression ratios and lower percent RMS differences compared to the other five transforms.

I. INTRODUCTION

Orthogonal transforms are one of the powerful techniques for data compression. The compression procedure involves preprocessing the input signal by means of a linear orthogononal transform such as Fourier, Karhunen-Loeve, Walsh, Haar, ..., etc., and properly encoding the transformed output (expansion coefficients) and reducing the amount of data needed to adequately represent the original signal. Upon signal reconstruction, an inverse transformation is performed and the original signal is recovered with a certain degree of error.

Many discrete orthogonal transforms have been employed to electrocardiographic (ECG) data compression. The discrete Cosine transform (CT), the Haar transform (HT), and the Karhunen-Loeve transform (KLT) have been applied, resulting in a compression ratio (*CR*) of 3:1 [1]. Dual application of the KLT [2] to a vector lead ECG (X, Y, and Z leads of the vectorcardiogram in the Frank coordinate system) partitioned into a P wave and a QRST segment have resulted in a *CR* of 12 : 1. Use of Fourier transform (FT) in two-lead ECG data compression has also been reported [3] giving a *CR* greater than 7.4. Discussion on the employment of the fast Walsh transform in ECG signals is given in [4] and further study is reported in [5] resulting a *CR* of 8. Application of the discrete Hartley transform (DHT) has been reported in [6] and resulted in a *CR* of 12 :1. However, due to the diverse procedures that have been employed in each application, it would be improper to compare these methods in absolute terms of values given for CR and percentage error as they have been obtained under different conditions of sampling rate, presicion, and noise level, and their performance have been evaluated using different indexes of error. To arrive at an exact comparison, a large set of ECG's from a common database needs to be processed by all ECG compression techniques and their performance needs to be evaluated with a common measure of "goodness" [7].

In the present paper, a comparative study is presented to evaluate the performance of six orthogonal transforms. These are: Fourier, Cosine, Sine, Hartley, Walsh and Haar transforms. The study was conducted using records of human ECG's for normal and abnormal cases. In the following section of this paper, we present the theoretical basis behind data compression via orthogonal transforms and a general introduction to each transform is given. The third section includes a description to the database used for evaluation. The fourth section is devoted to the description of the compression scheme using orthogonal transforms and the implementation of each transform. Evaluation of the performance of the transforms techniques and some representative results are presented in the fifth section for a number of normal and abnormal cases by reconstructing real ECG signals.

II. ORTHOGONAL TRANSFORMS FOR DATA COMPRESSION

Consider a discrete data vector $X^T = [x_o, x_1, \ldots, x_{N-1}]$ and an orthogonal transform $P = [\psi_o, \psi_1, \ldots, \psi_{N-1}]^T$, where ψ_o, ψ_1, ..., ψ_{N-1} are orthogonal real-valued vectors, i.e. vectors having the property $\psi_j^T\psi_j = \delta_{ij}$ (Kronecker delta) or $P^TP = I$ (unit $N \times N$ matrix). Applying P upon X gives transformed vector components $y_i = \psi_i^T X$ or in matrix form

$$Y = PX, \quad Y^T = [y_o, y_1, \ldots, y_{N-1}] \tag{1}$$

Premultiplying (1) by P^T and using the orthogonality of P gives the inverse transform

$$X = P^T Y = \sum_{i=o}^{N-1} y_i\psi_i \qquad (P^T = P^{-1}) \tag{2}$$

Clearly, P and P^{-1} are linear inverse transforms. The basic problem is to select a family of vector functions ψ_o, ψ_1, ..., ψ_{N-1} such that the signal X can be reconstructed or estimated using a smaller number of terms $y_i\psi_i$, $i = o, 1, \ldots, M-1$. This implies that only $M < N$ data values y_i have to be used or transmitted for reconstructing X as

$$X = \sum_{i=o}^{M-1} y_i\psi_i + \sum_{i=M}^{N-1} c_i\psi_i \tag{3}$$

where c_i are preselected constants. Using fewer data points or transform coefficients y_i, we have what is known as "data compression" which allows to transmit the information signal X approximately with reduced information rate, a result which is mostly important in practice.

II.1 The Optimal Transform

Since we are interested in dropping some of the coefficients, we may also want the error (mean-square) in reconstruction of X, due to dropping of some coefficients be the least. Also we would like signal energy to be compacted into as few coefficients as possible. Fortunately, there is one transform that satisfies this criterion , and it is known as the Hotelling transform or Karhunen-Loeve transform (KLT) [8]. The optimum transform can be computed from the covariance matrix C_x

$$C_x = E\,\{(X - E(X)).(X - E(X))^T\} \tag{4}$$

where E is the statistical expectation and superscript T denotes transpose. Indeed, the mean square error V is given by

$$V = E\,\{(X - E(X))^T\,(X - E(X))\}$$

$$= \sum_{i=M}^{N-1} E\,\{(y_i - c_i)^2\} \tag{5}$$

and the optimality conditions $\partial V/\partial c_i = 0$, $\partial V/\partial \psi_i = 0$ subject to $\psi_i \psi_i^T = 1$ give, respectively:

$$c_i = E\,[\,y_i\,] = \psi_i^T E(X),$$

$$\Sigma_x\,\psi_i = \lambda_i\,\psi_i \tag{6}$$

where λ_i are the Lagrange coefficients for the constraints $\psi_i^T \psi_i = 1$. This shows that ψ_i is an eigenvector of Σ_x and λ_i is the corresponding eigenvalue. Now from Y = PX we obtain

$$\Sigma_y = P\,\Sigma_x\,P^{-1} = P\,\Sigma_x\,P^T = \text{diag}\,[\lambda_o, \lambda_1, \ldots, \lambda_{N-1}] \tag{7}$$

i.e. Σ_y is diagonal which implies that the components of y_i are uncorrelated.

The minimum value V_o is obtained by substituting (6) into (5), i.e.

$$V_o = \sum_{i=M}^{N-1} \psi_i^T\,\Sigma_x\,\psi_i = \sum_{i=M}^{N-1} \psi_i^T\,\lambda_i\,\psi_i = \sum_{i=M}^{N-1} \lambda_i \tag{8}$$

This shows that V_o is equal to the sum of the eigenvalues corresponding to the data components y_i which are discarded. Thus in practice, one must keep the M data components having the largest eigenvalues. The discarded data components are replaced by the constants $c_i = \psi^T E(x)$ which can be reduced to zero by preprocessing the given data such that $E(x) = 0$. This data compression criterion for selecting the set of y_i components with the largest variances is known as "variance data compression criterion". Note that for all other orthogonal transforms the covariance matrix Σ_y is not diagonal [9].

An alternative way of expressing the above is to use a set of basis vectors $\phi_1, \ldots, \phi_N$ which are orthonormal and then represent X as a linear sum of these basis vectors, i.e.,

$$X = \sum_{i=1}^{N} a_i \phi_i \qquad (9)$$

It is well known [9] that the optimum a_{iN} which minimize the error between X and its representation (i.e., $\sum_{i=1} a_i \phi_i$) are given by

$$a_i = X^T \phi_i \qquad (10)$$

Now instead of storing signal samples, we store $\{a_i\}$ only. In order to achieve the maximum degree of compression, we would like to store as few $\{a_i\}$ as possible, without a significant error in the reconstruction of X. The basis vectors which compact maximum energy in the fewest $\{a_i\}$ turn out to be the eigenvectors of the covariance matrix C_x. Thus the KLT provides us with the optimum basis functions for representing a signal.

Although the optimum transformation is explicitly known, its use in practice presents many problems [7]. First of all, the computational time needed to calculate the KLT basis vectors are based on determining the eigenvalues and corresponding eigenvectors of the covariance matrix of the original data, which can be a large symmetric matrix. Its implementation requires N^2 multiplications by constants which may be complicated (i.e., not simple powers of 2 for easy binary arithmetic). The second problem is in the computation of the eigenvectors of C_x. In many cases this matrix C_x turns out to be singular, and then some eigenvectors cannot be uniquely defined. The lengthy processing requirement of the KLT has led to the use of suboptimum transforms with fast algorithms [7].

II.2 Suboptimum Transforms

Many other transforms have been invented which produce less correlated coefficients than the signal itself and which are easier to implement. Some of the popular transforms are:

1) Discrete Fourier Transform (DFT)

$$A = \text{Matrix } \{a_{ij}\}$$

where $$a_{ij} = \frac{1}{\sqrt{N}} \exp\left[-2\pi \sqrt{-1}\ (ij)\right] \qquad (11)$$

2) Discrete Cosine Transform (DCT)

$$a_{ij} = \frac{2K(i)}{\sqrt{N}} \cos\left[(2j + 1)i\pi/2N\right] \qquad (12)$$

where $$K(i) = \begin{cases} 1/\sqrt{2}, & \text{for } i = 1 \\ 1, & \text{for } i = 2, \ldots, N \\ 0. & \text{otherwise} \end{cases}$$

3) Discrete Sine Transform (DST)

$$a_{ij} = \sqrt{\frac{2}{N + 1}} \sin\left[\pi(i+1)(j+1)/(N+1)\right] \qquad (13)$$

4) Discrete Hartley Transform (DHT)

$$a_{ij} = \cos(2\pi ij/N) + \sin(2\pi ij/N) \qquad (14)$$

5) Discrete Walsh Transform (DWT)

$$a_{ij} = \frac{1}{\sqrt{N}} (-1)^{b(i,j)} \qquad (15)$$

where $$b(i,j) = \sum_{l=0}^{k-1} i_l j_l$$

The terms i_l and j_l are the bit states of the binary representations of i and j, respectively.

6) Discrete Haar Transform (HT)

$$a_{ij} = 1 \qquad \text{for } 0 < n < 1$$

$$= 2^{i/2} \qquad \text{for } \frac{j-1}{2^i} \le n \le \frac{j - 1/2}{2^i}$$

$$= -2^{i/2} \qquad \text{for } \frac{j - 1/2}{2^i} \le n \le \frac{j}{2^i}$$

$$= 0 \qquad \text{elsewhere} \qquad (16)$$

The set of orthogonal functions may be any one of many different types. In the Fourier, Cosine, Sine and Hartley transforms, the orthogonal functions are discrete sampled sines and/or cosines as illustrated in Fig.1 for N = 16. In the Walsh and Haar transformations, the orthogonal functions are discrete sampled walsh functions [10] (Fig.1).

III. DATA ACQUISITION

35-sec Lead II ECG signals were recorded using a Philips ECG recorder type XV1503. The signals were separated into two classes, namely normal and abnormal. The abnormals showed old inferior infarction. The bandwidth of the signal was chosen to be 100 Hz. The output of the ECG recorder was then connected to a 12-bit analog-to-digital converter and sampled at a rate of 250 samples/sec. The digital data were transferred to an IBM PS2/80 computer by a program written in Basic.

IV. THE COMPRESSION SCHEME

IV.1 Preprocessing

The method of orthogonal transforms requires delineation of the QRS complexes. This is usually accomplished using a search procedure based on the first derivative of the signal. Once the QRS is detected, backward and forward searches are performed with a lower threshold to locate its beginning and end. Each ECG beat was represented by 256 samples. These samples were chosen such that the QRS complex and the T wave would always appear.

IV.2 Implementation

Fig.2 shows a block diagram of the compression scheme. The input to the scheme is the ECG signal X of length N samples. The forward transform yields the coefficients a_{ij}. The coefficients to be retained are lower order harmonics. The number of coefficients L to be retained to achieve a given compression ratio CR is given

$$L = N/CR \qquad (17)$$

The higher order harmonics for each transform are discarded. For reconstruction, N-L zeros are added to the L coefficients and the resulting sequence is inverse transformed. The reconstructed waveforms are compared to the original signals by visual examination of plotted waveforms and also by a measure of goodness as described in the next subsection.

The obvious difference between these transforms and the KLT is the ease of implementation. All the above transforms are unitatry, i.e., $A^{T*} = A^{-1}$ (where A^{T*} is the complex conjugate of A^T), and therefore inverse transformation which is done for reconstruction is as the transformation itself.

The discrete Fourier transform was implemented by using the fast Fourier transform technique [11]. Instead of the N^2 multiplications required for the computation of the DFT, $2N \log_2 N$ multiplications and additions are required. The discrete cosine transform was implemented by a technique developed by Makhoul [12] using $N \log_2 N$ real multiplications. The Sine transform was computed using a fast algorithm developed by Yip and Rao [13]. The discrete Hartley transform was

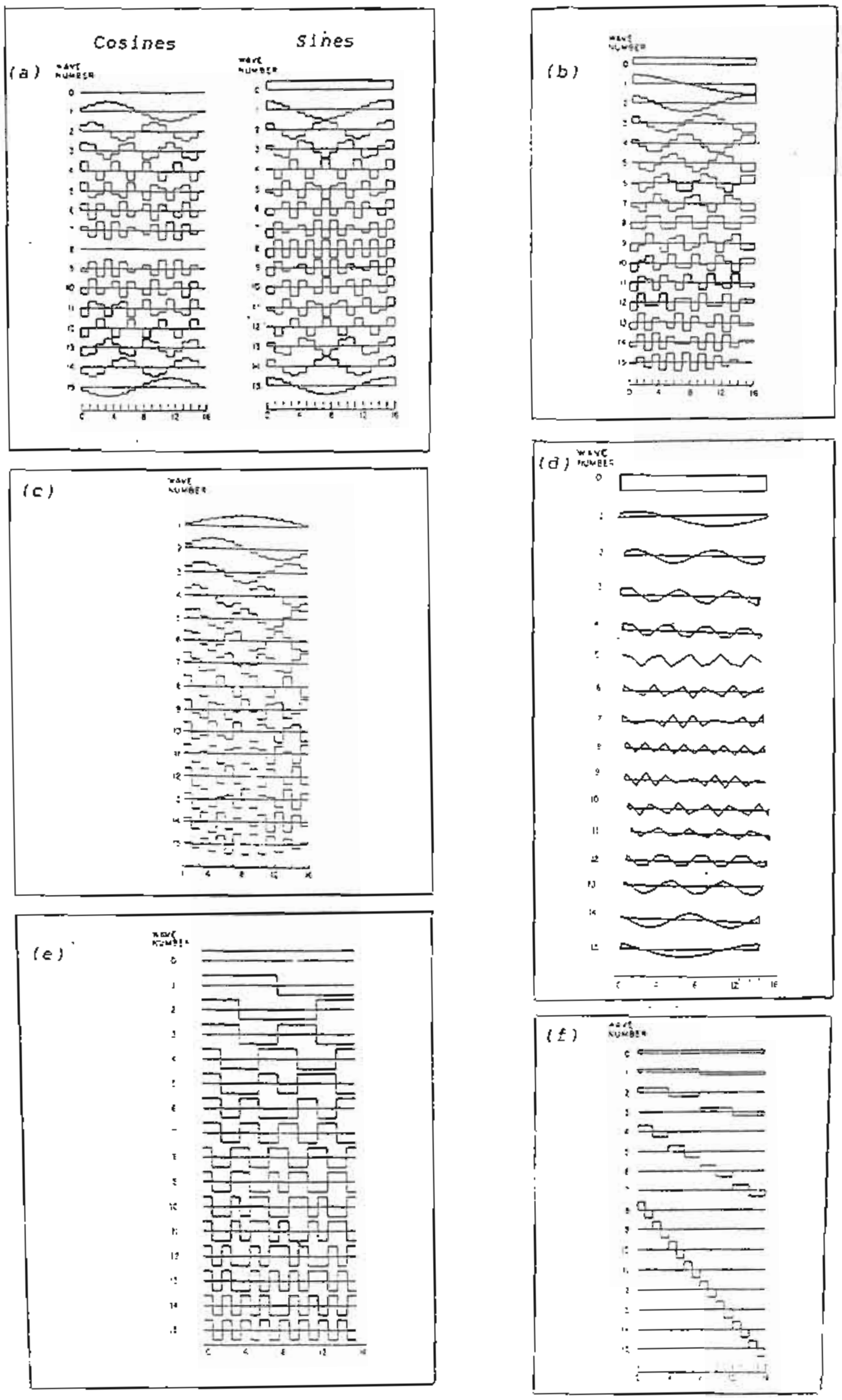

Fig.1 The basis orthogonal functions of the transforms used (N =16) (a) Fourier, (b) Cosine, (c) Sine, (d) Hartley, (e) Walsh, and (f) Haar.

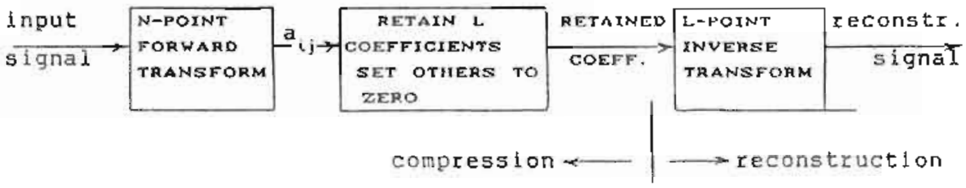

Fig.2 Block Diagram of data compression scheme using orthogonal transforms

implemented using a radix-2 decimation-in-time fast Hartley algorithm [14]. The walsh transform is the simplest to implement, since it consists of ± 1's and, therefore, only additions and no multiplications are required. However, fast algorithms have been developed that can compute an N-point Walsh transform in $N \log_2 N$ rather than N^2 operations [15]. The Haar transform was computed using a fast Haar transform algorithm [16]

IV.3 Performance Index

In order to assess the performance of the compression scheme, in addition to visual comparison, an index of performance was employed. It represents a measure of "goodness" of the reconstructed waveforms. This index is the percent RMS difference (PRD), which is computed as

$$PRD = \left[\frac{\sum_{i=1}^{N} \left[X(i) - \hat{X}(i) \right]^2}{\sum_{i=1}^{N} \left[X(i) \right]^2} \right]^{1/2} * 100 \qquad (18)$$

where X and $\hat{X}$ are samples of the original and reconstructed data sequence.

Using this method, we can either compute the error between the original and reconstructed waveforms for a specified compression ratio or obtain the permissible ratio for the specified error limit.

V. RESULTS

Fig.3 shows a typical normal ECG record. Fig.4 depicts its six spectra corresponding to the six orthogonal transforms.

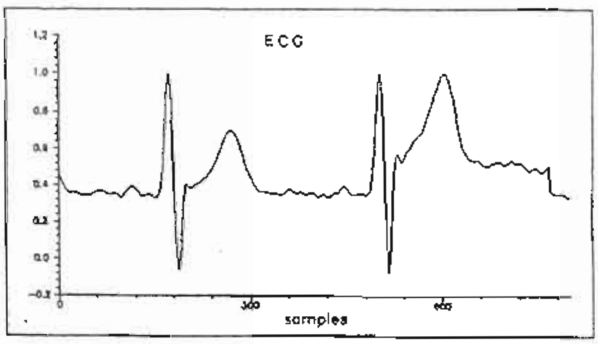

Fig.3 A typical ECG signal (normal record)

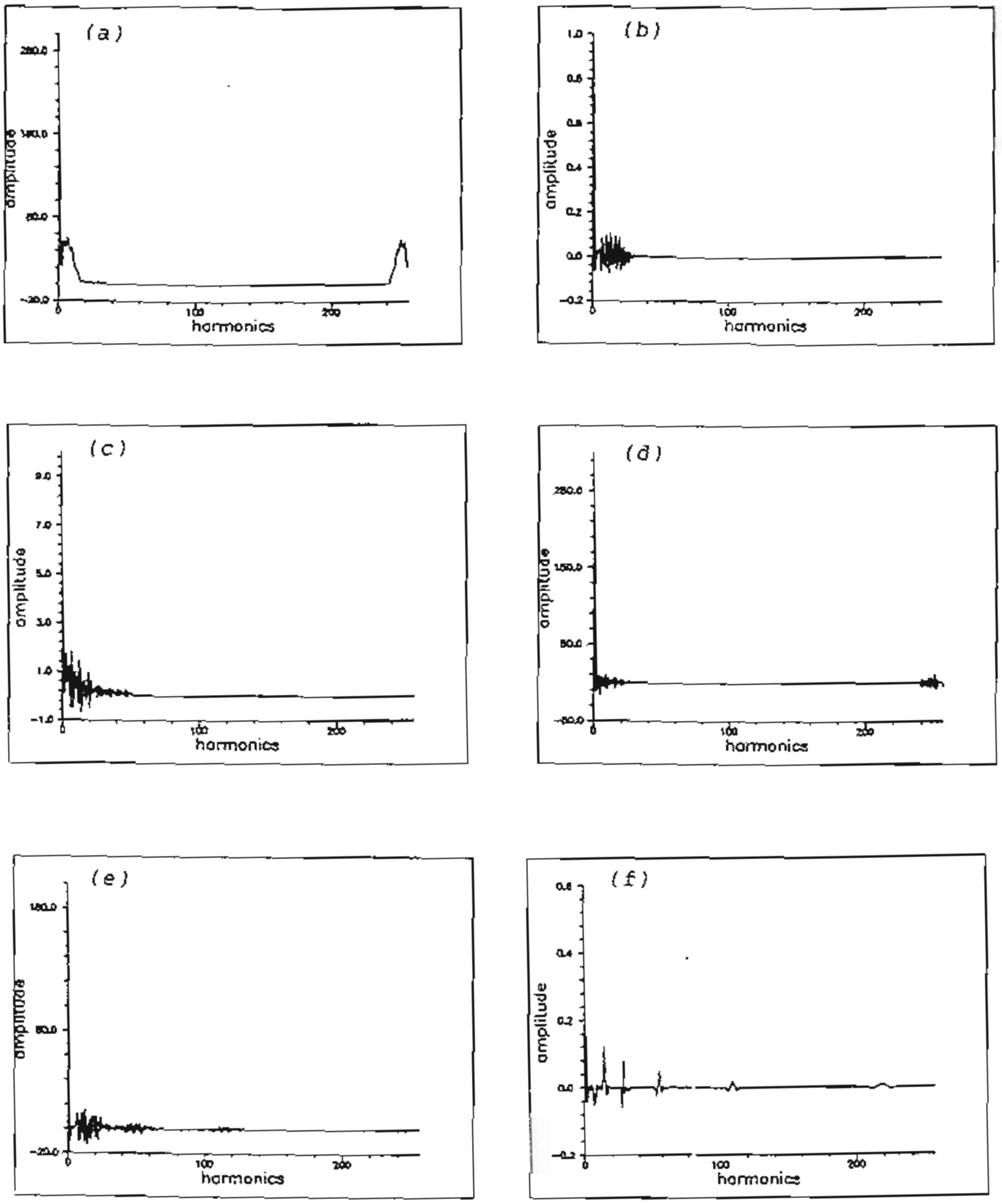

Fig.4 Spectra of the six orthogonal transforms of the signal of Fig.3. (a) Fourier, (b) Cosine, (c) Sine, (d) Hartley, (e) Walsh, and (f) Haar.

TABLE I

Percent *RMS* Differences for various orthogonal transforms (normal cases)

Compression ratio *CR*	Coeff. retain. L	APRD					
		DFT	DCT	DST	DHT	DWT	HT
5.1	20	36.60	26.16	6.37	1.97	9.07	9.49
6.4	30	36.92	36.07	8.67	2.52	11.27	11.43
8.6	40	37.97	36.15	11.19	3.65	13.38	15.06
12.8	50	46.70	49.36	20.30	13.97	23.30	28.24

Figs.(5,6) show reconstructed waveforms (dotted line) superimposed over the original signal (continuous line) for each transform using compression ratios *CR* = 6.4, 12.8 respectively. Fig.7 shows the results of compressing and reconstructing an abnormal record using *CR* = 8.6. As seen from the figures, as the *CR* value increases, the more distortion in the shape of the beat occurs. The figures also show that the two transforms resulting in the lowest distortion are the Sine and Hartley transforms. The other four transforms produced major variations even in the level of the isoelectric line of the ECG signal and, therefore, resulting changes in the magnitude of the R-wave with respect to the isoelectric line. Inspection of the figures of the Hartley transform shows that the information lost as a consequence of the data compression is not diagnostically significant: the Sine transform yielded more oscillatory variations.

The performance indexes described in Section IV.3 were computed for two hundred ECG beats of which half were normal and the other half were abnormal. Table I summarizes the average PRD's (APRD) for the different transforms using *CR* = 5.1, 6.4, 8.6, and 12.8 in case of normal records. The APRD values obtained for the abnormal signals are summarized in Table II. The results tabulated confirm the visual comparison. The Hartley transform possesses higher capability to restore the original signals with the lowest APRD's value.

TABLE II

Percent *RMS* Differences for various orthogonal transforms (abnormal cases)

Compression ratio *CR*	Coeff. retain. L	APRD					
		DFT	DCT	DST	DHT	DWT	HT
5.1	20	34.65	29.96	6.45	2.03	9.87	8.47
6.4	30	36.97	38.84	8.35	3.44	13.56	12.32
8.6	40	37.24	45.84	12.58	5.89	23.39	28.82
12.8	50	45.75	50.12	22.47	14.95	25.34	29.44

VI. CONCLUSION

In this study, six orthogonal transforms were implemented and evaluated for ECG data compression. The six transforms are: Fourier, Cosine, Sine, Hartley, Walsh, and Haar. Implementations of these transforms were carried out using fast techniques derived from their discrete versions.

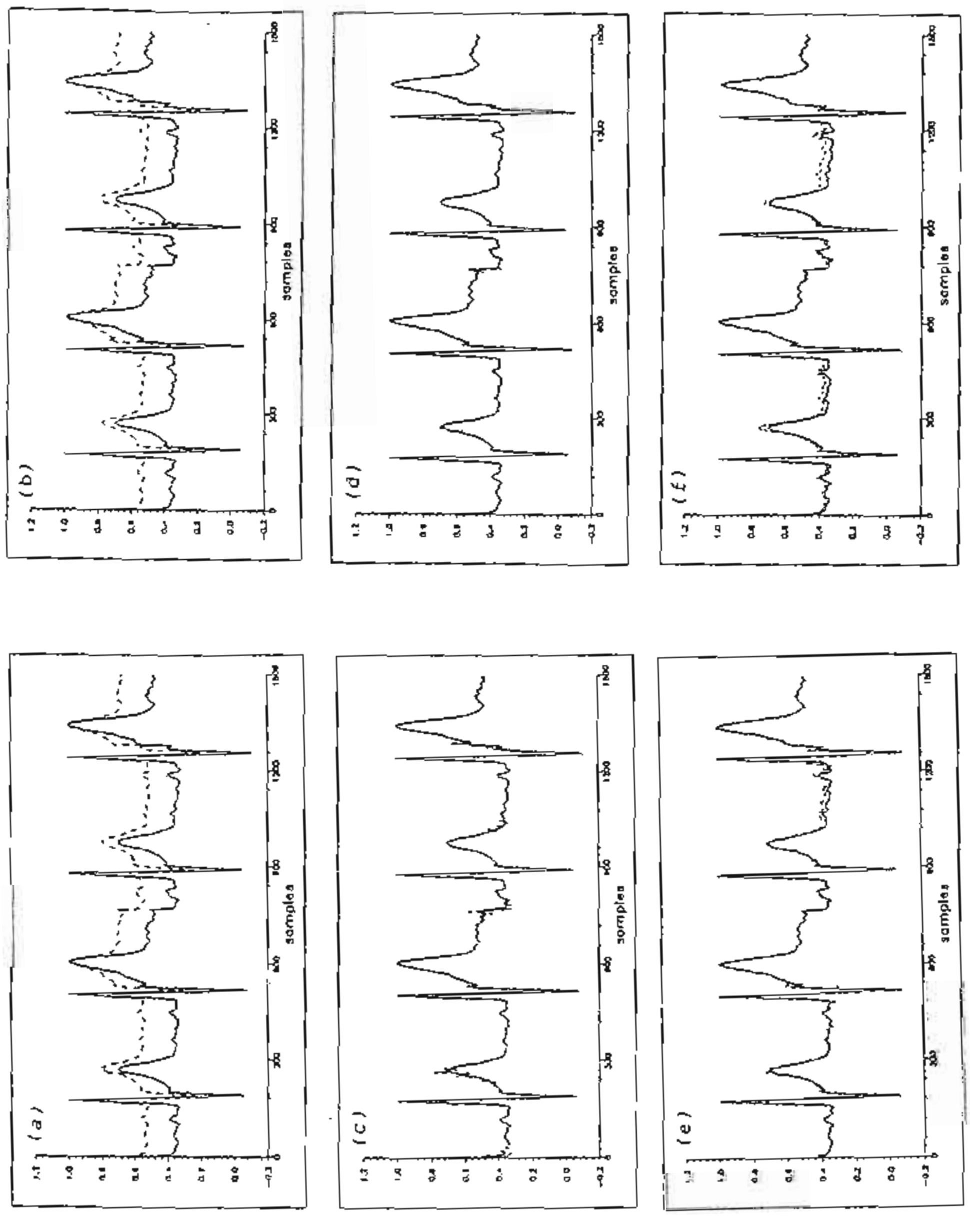

Fig.5 Reconstructed ECG data (- - - -) superimposed over the original signal (——) for a normal subject (CR = 6.4). (a) Fourier, (b) Cosine, (c) Sine, (d) Hartley, (e) Walsh, and (f) Haar.

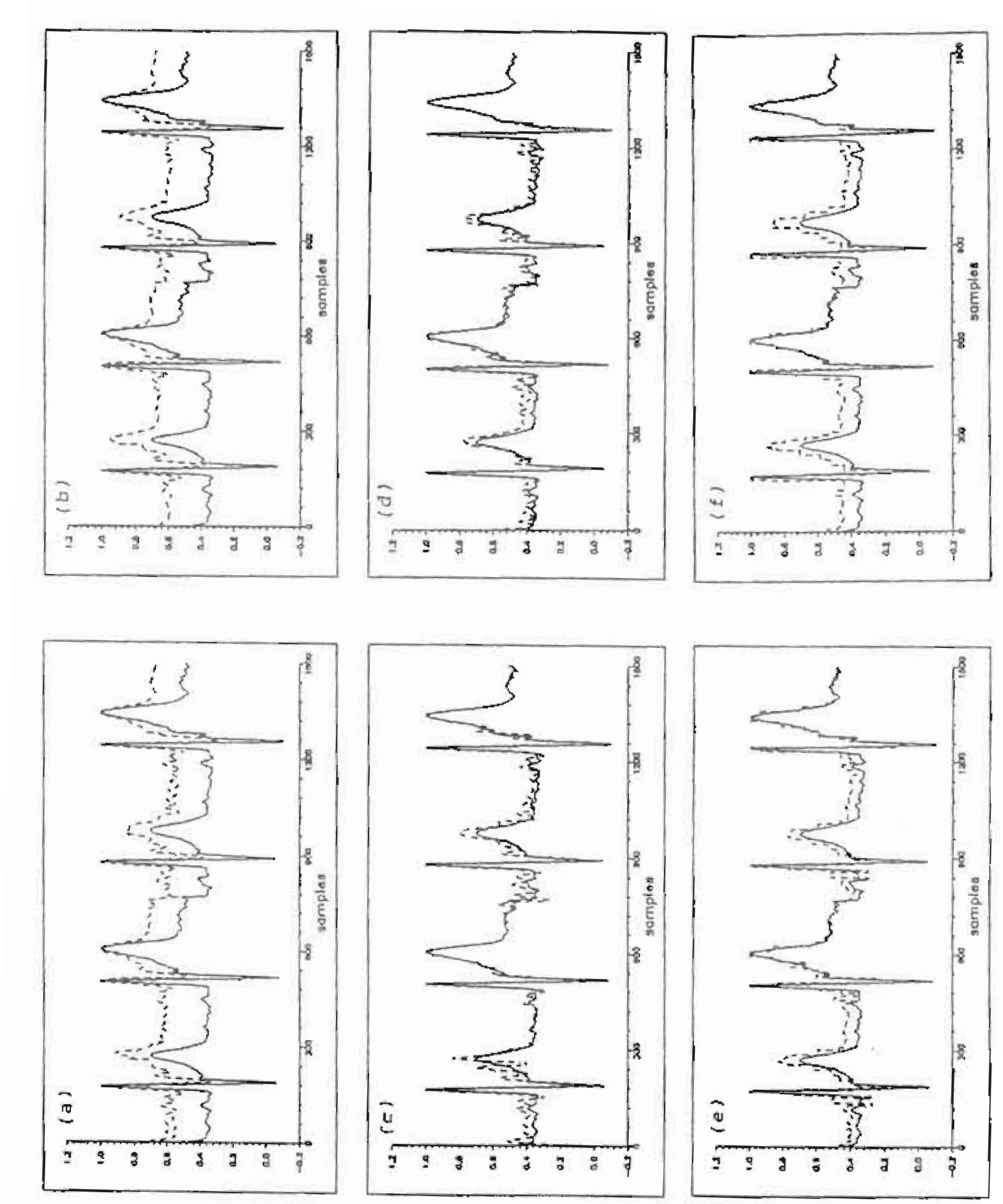

Fig.6 Reconstructed ECG data (- - -) superimposed over the original signal (——) for a normal subject (CR = 12.8). (a) Fourier, (b) Cosine, (c) Sine, (d) Hartley, (e) Walsh, and (f) Haar.

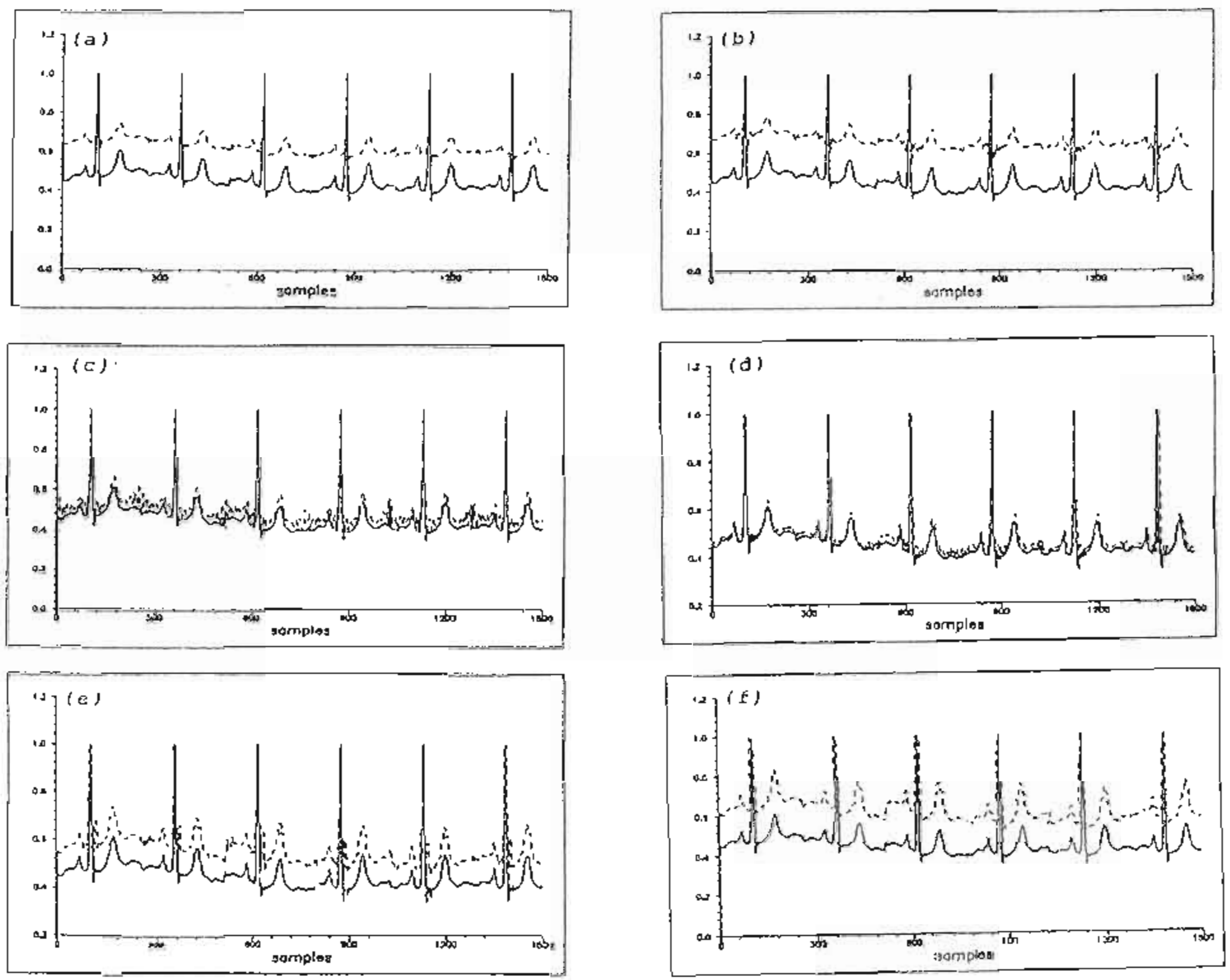

Fig.7 Reconstructed ECG data (- - -) superimposed over the original signal (——) for a subject having old inferior infarction (CR = 8.6).(a) Fourier, (b) Cosine, (c) Sine, (d) Hartley, (e) Walsh, and (f) Haar.

The performance of the six orthogonal transformations was investigated in terms of the percent RMS difference PRD between the reconstructed and the original waveforms of records of normal and abnormal cases. This has demonstrated that the Hartley transform is the most efficient and accurate transformation that can be used safely for ECG data compression. It allows the restoration of the compressed signal with the lowest PRD value for the same compression ratio.

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