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STABILITY AND CONTROL FOR INTERCONNECTED MULTIVARIABLE SYSTEMS

دراسة الإتزان والتحكم في الأنظمة المركبة متعددة المتغيرات

By

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يقدم البحث دراسة وتحليل وتصميم نظام تحكم لعملية كيميائية صناعية تمثل وحدة من وحدات عملية تخليق الأمونيا بمصنع سماد اليوريا بطلخا · حيث يعتبر هذا النظام مركبا ومتعدد المتغيرات بالإضافة إلى وجود عوامل ربط داخلية بين مكوناته . وحيث أن النموذج الرياضى للنظام عبارة عن علاقة زمنية مباشرة بين المدخلات والمخرجات · فقد تم تحويل هذه الصيغة إلى صيغة ال ARMA MODEL التى تتناسب مع التحليل والتصميم باستخدام الحاسبات الرقمية . كما يقدم البحث دراسة دقيقة حول عوامل الربط بين مفردات النموذج وتأثيرها على الإتزان الكلى للنظام فى حالة دائرة التحكم المقترحة وبالتالى تصميم وحدات إضافية ثابتة تضاف إلى عوامل الربط لضمان عدم تأثيرها على الإتزان الكلى للنظام المركب تأثيرا سلبيا .

وتتم هذه العملية باستخدام الطريقة المعروفة باسم "مصفوفة الكسب النسبى". كما تم تصميم عملية التحكم بدائرة مغلقة وتغذية خلفية باستخدام طريقة "تخصيص الأقطاب" ، والتى أدت إلى نتائج جيدة تعزز فعالية النظام المقترح.

ABSTRACT:

The paper presents the analysis and control for an interconnected multivariable system, which simulates the behaviour of a subsystem of real chemical ammonia conversion process in SEMADCO. An ARMA model is constructed - using a discrete dynamic system - in a difference equation form able for testing stability, and in a matrix form for applying control techniques. Since relative gains provide a useful measure of interaction indices, so relative gain matrix procedure is followed for modifying the interconnected static gains through introducing computing relays with adjustable gains in the open loop control system for ensuring composite system stability.

Pole assignment technique is used for designing the suitable closed-loop strategy for composite system. Numerical results agree with the effectiveness of both of the suggested interconnected computing relays and the designed control strategy.

INTRODUCTION:

In a previous work [9], modelling and simulation for multivariable chemical system in Semadco Urea-plant were presented. The simulated system is the High Pressure Boiler Feed Water Heater (HP-BFW-E127), which governs the inlet temperature of ammonia converter R-107.

Based Upon practical knowledge, experimental investigations and mathematical simulation techniques, multivariable interconnected dynamic model is constructed as shown in figure (1) (without the double circled elements). Now, to accomplish the job of designing appropriate control technique for governing and modifying the system performance, reformulation of the deduced model in a suitable form for applying control methods on multivariable interconnected system is necessary. Afterwords, deduction of proper interaction between input / output channels represents a next step. Thus, this work is consisting from the following main three parts:

1- Discretization and ARMA Model construction:
Discrete model is a suitable form for computer applications. Several methods can be used for representing the corresponding diference equations or discrete transfer functions [3, 6, 8, 10, 12]. The ARMA model [4, 5] representation is a suitable and efficient one for testing linearized multivariable systems. So, We follow this representation for constructing our physical model in a suitable form for applying control strategies.

2- Interconnected system stability:
Interaction between control loops is a significant factor affecting the goodness and stability of complete system. Interaction conditions make different loops to help; or to fight each other. In other words, regulating action for some control loops may de-regulate and disturb the outputs of the other loops. So, system is said to be fully coupled if interaction improves stability. The relative gain matrix procedure [1,2] provides a useful technique for testing and then ensuring the interconnected stability conditions for the open loop system, that is by designing suitable interconnection modification factors.

3- Pole Assignment control strategy:
The pole placement [10,11] is one of the simplest direct design procedures. For which, the key idea is to find a feedback law such that the closed loop poles have the desired locations. The classical pole-assignment method [11] does not require that the interaction matrix to be minimum phase, and makes no constraints upon the channel time delays. So, it is a proper choice for obtaining control law for the practical chemical system under investigation. In order to apply this technique, the mathematical model has to be presented in a form suitable for computer control application as follows:

I- DISCRETIZATION AND ARMA MODEL CONSTRUCTION:

The pulse transfer function is H * Gp(z) (the hold element and the process), and can be related to the continuous transfer functions of hold H(s) and Gp(s) by a proper choice of T [4,5]. For the physical system of figure (1) with $T = 0.5$ sec. and $\Delta = 1.05$, we have [7]:

$$H\,G_{v1}(Z^{-1}) = \frac{0.004\ Z^{-1}}{1-0.87\ Z^{-1}} \qquad (1\text{-a})$$

$$H\,G_{v2}(Z^{-1}) = \frac{0.0021\ Z^{-1}}{1-0.87\ Z^{-1}} \qquad (1\text{-b})$$

$$H\,G_{p31}(Z^{-1}) = \frac{Z^{-2}\ (-0.000242\ Z^{-1})}{(1-0.999\ Z^{-1})} \qquad (1\text{-c})$$

$$H\,G_{p32}(Z^{-1}) = \frac{Z^{-2}\ (-0.000181\ Z^{-1})}{(1-0.999\ Z^{-1})} \qquad (1\text{-d})$$

$$H\,G_{p33}(Z^{-1}) = \frac{Z^{-2}\ (-0.008\ Z^{-1})}{(1-0.999\ Z^{-1})} \qquad (1\text{-e})$$

For general interconnected multivariable system, a process model is given by:

$$Y(t) = \frac{B(s)}{A(s)}\ U(t-\Delta) + d(t) \qquad (2)$$

where : Δ is the time delay

$d(t)$ is a disturbance, and it may be:

a- control value
b- slowly varying value
c- fed through a feed forward element
d- stocastic signal

The discrete time equivalent of equation (2) is given as:

$$Y(t) = \frac{B(Z^{-1})}{A(Z^{-1})}\,U(t-k) + D(z) \qquad (3)$$

or written in an ARMA Model form as [3, 8, 10]:

$$A(Z^{-1})\ Y(k) = Z^{-k}\ B(Z^{-1})\ U(k) + D(k) \qquad (4)$$

where $A(Z^{-1})$ and $B(Z^{-1})$ are polynomials of orders n_a and n_b respectively, and $D(k)$ is the disturbance model.
Substitution of system equations (1-a) to (1-e) into equation (4), produces the following ARMA model representation of our plant, with $Y_3(k)$ is the disturbanc/ channel:

$$Y_1(k) = -a_1 Y_1(k-1) + b_1\ U_1(k-1) + b_2\ U_2(k-1) \qquad (5)$$

$$Y_2(k) = -a_2 Y_2(k-1) + b_3\ U_1(k-1) + b_4\ U_2(k-1) \qquad (6)$$

$$Y_3(k) = -a_3 Y_3(k-1) - a_4 Y_3(k-2) + b_5\ U_1(k-4) + b_6\ U_2(k-4) + b_7\ U_3(k-1) + b_8\ U_3\ (k-4) \quad (7)$$

where : $a_1 = -0.87$, $b_1 = 0.004$, $b_5 = -0.968 * 10^{-6}$

$a_2 = -0.87$, $b_2 = 0.0021$, $b_6 = -0.38 * 10^{-6}$

$a_3 = -1.869$, $b_3 = 0.00036$, $b_7 = 0.008$

$a_4 = 0.869$, $b_4 = 0.0021$, $b_8 = -0.00696$

Since $Y_3(k)$ represents the disturbance, the following assumptions are considered acceptable [5, 7]:

$Y_3\ (k-2) = Y_3\ (k-1)$, $U_1(k-4) = U_1(k-1)$

$U_2\ (k-4) = U_2\ (k-2)$, $U_3(k-4) = U_3(k-3) = U_3(k-1)$

and substituting:

$a_5 = a_3 + a_4 = -1$, $b_9 = b_7 + b_8 = 0.00104$

The ARMA model will be:

$$\begin{bmatrix} Y_1(k) \\ Y_2(k) \\ Y_3(k) \end{bmatrix} = - \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \begin{bmatrix} Y_1(k-1) \\ Y_2(k-1) \\ Y_3(k-1) \end{bmatrix} + \begin{bmatrix} b_1 & b_2 & 0 \\ b_3 & b_4 & 0 \\ b_5 & b_6 & b_9 \end{bmatrix} \begin{bmatrix} U_1(k-1) \\ U_2(k-1) \\ U_3(k-1) \end{bmatrix} \quad (8)$$

where the interaction matrix is:

$$B(Z^{-1}) = \begin{bmatrix} b_1 & b_2 & 0 \\ b_3 & b_4 & 0 \\ b_5 & b_6 & b_9 \end{bmatrix} \quad (9)$$

Through which, the overall interconnected open loop stability can be decided when all roots of the characteristic equation;

$\left| Z I - B \right| = 0$ lie inside the unit circle disk.

For our system, the calculated roots are:

$$\begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} 0.00104 & 0.00434 & 0.00176 \end{bmatrix}$$

Although these roots tell us that, the overall interconnected system have to be highly stable, their approximate zero values represent dead-beat operations, which may produce contracted and unrelaxed performance pushing the system to instability, that is due to weak and unsuitable interconnections.

Truly, when this system is used for applying control techniques, results show diverging unstable response. so, the following section presents detailed study about the interconnected system stability conditions, with designing proper interconnection factors to convert the original open loop system to more reliable one suitable for applying differential control techniques.

II-INTERCONNECTED SYSTEM STABILITY:

The relative gain matrix procedure [1,2] used for testing, and then accomplishing the interconnected stability conditions is carried out in the following steps:

1- Constructing the interconnected multivariable system input- output relationship in the form:

$$Y_i(s) = \sum_{j=1}^{n} G_{ij}(s) \; U_j(s) \; , \; i=1,2,\ldots\ldots n \tag{10}$$

where G_{ij} is the T.F between input j and output i,

n is the system order

2- Determining the steady state gain matrix (S.S.G.M.):

$$K = \left[K_{ij} \right] \quad , K_{ij} = \left. \frac{\delta Y_i}{\delta U_j} \right|_{u_i = \text{constant} \; , \; i \neq j} \tag{11}$$

Where : K_{ij} is the open loop static gain when all other manipulated variables are kept constant.

Applying for our practical system (Figure (1) but without the double circled gains), the S.S.G.M. will be:

$$K = \begin{bmatrix} 0.031 & 0.0164 & 0 \\ 0.0093 & 0.0164 & 0 \\ -0.0075 & -0.00297 & 8 \end{bmatrix}$$

\- Converting equation (10) to relate the deviations between variables:

$$\Delta Y_i = \sum_{j=1}^{n} K_{ij} \ \Delta U_j \qquad , \ i = 1.2, \ldots, n \qquad (12)$$

4- Calculating gain for each loop when all other loops are closed, and all other controlled variables are returned to their set points:

$$K'_{ij} = \left. \frac{\Delta Y_i}{\Delta u_j} \right|_{\Delta Y_i = 0 \ , \ i \neq j} \qquad (13)$$

As functions of K_{ij} , complicated expressions for K'_{ij} are deduced for the general 3-dimensional case [7]. These expressions are much simplified when applying for our real system having $K_{13} = K_{23} = 0$:

$$\left.\begin{aligned} K'_{11} &= K_{11} - \frac{K_{12} K_{21}}{K_{22}} , & K'_{22} &= K_{22} - \frac{K_{21} K_{12}}{K_{11}} \\ K'_{12} &= K_{12} - \frac{K_{11} K_{22}}{K_{21}} , & K'_{21} &= K_{21} - \frac{K_{11} K_{22}}{K_{12}} \\ K'_{13} &= K'_{31} = K'_{23} = k'_{32} = \infty , & K'_{33} &= K_{33} \end{aligned}\right\} \qquad (14)$$

Thus:

$$K' = \begin{bmatrix} 0.0217 & -0.03827 & \infty \\ -0.0217 & 0.01148 & \infty \\ \infty & \infty & 8 \end{bmatrix}$$

5- Computing relative gain matrix (R. G. M.) μ , Where:

$$\mu = \left[\mu_{ij} \right] \qquad , \ \mu_{ij} = k_{ij} \ / \ K'_{ij} \qquad (15)$$

producing $$\mu = \begin{bmatrix} 1.4286 & -0.4286 & 0 \\ -0.4286 & 1.4286 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6- For stable interconnected system, the following properties must be fulfilled for the R.G.M.:

i- $\sum$ elements in any row or any column =1

ii- μ_{ij} must be positive, since negative values mean negative interactions, i.e. undesirable system from control point of view because loops fight each other.

iii- μ_{ij} = 1 means no interaction, and amount of deviation from unity represents amount of stability combination between controlled and manipulated variables.

Accordingly, it is clear that our original system is completely unstable one. Hence, design of computing - or biasing - relays which can be done easily from practical point of view, is a good idea for solving this problem. On the other side, it is notable that the only way for choosing the rates of these relays is long and unguaranteed trial and error prone.

As a final result for several trials, the chosen computing relays shown in figure (1) by double circled elements are tested and ensured to fulfil the necessary stability conditions. For this choice:

$$K_{modified} = \begin{bmatrix} 3.1 & 4.92 & 0 \\ -1.86 & 4.92 & 0 \\ -75.02 & -383.76 & 2000 \end{bmatrix}$$

$$K'_{modifed} = \begin{bmatrix} 4.96 & 13.12 & \infty \\ -4.96 & 7.872 & \infty \\ \infty & -383.76 & 2000 \end{bmatrix}$$

$$\mu_{modified} = \begin{bmatrix} 0.625 & 0.375 & 0 \\ 0.375 & 0.625 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

These results represent perfectly stable interconnected modified system. Resubstitution in the steps of deducing ARMA model produces the following modified interaction matrix:

$$B(z^{-1}) = \begin{bmatrix} 0.4 & 0.63 & 0 \\ -0.072 & 0.63 & 0 \\ -0.00968 & -0.0491 & 0.26 \end{bmatrix}$$

for which, roots of characteristic equation will be:

$$\left[Z_1 \, , \; Z_2, \; Z_3 \right] = \left[0.26 \; , \; 0.515+j0.1794, \; 0.515-j0.1794 \right]$$

Telling us that, this chosen modification satisfies reliable uncontracted stability conditions for the interconnected ARMA model.

III-POLE ASSIGNMENT SELF TUNING CONTROLLER:

An initial stage in multivariable design is to deduce interaction between input-output channels by self tuning on-line computer aided synthesis method [4,10,11]. The self tuning on-line nature requires intutive engineering judgements concerning sample rate, slew limiting and many other factors which are parts and parcel of digital controller implementation.

The pole placement technique [11] is one of the simplest direct self tuning design procedures. The key idea of this technique is to find a feedback law such that, the closed loop poles have the desired locations. The classical pole assignment method does not require that $B(Z^{-1})$ is minimum phase, and makes no constraints upon the channel time delay. Thus, it is a preferable technique for controlling our chemical process. For the general form of disturbed multivariable system:

$$\left[I + A(z^{-1}) \right] Y(t) = Z^{-k} B(z^{-1})\, U(t) + \left[I + C(Z^{-1}) \right] e(t) \qquad (16)$$

Where the noise corrupting the system e(t) is such that $[I + C(Z^{-1})]$ is inverse stable. To avoid restriction on $B(z^{-1})$, we choose the following control law:

$$\left[I + F(z^{-1}) \right] U(t) = -G(z^{-1}) \; Y(t) \qquad (17)$$

where : $G(Z^{-1}) = G_o + G_1 z^{-1} + \; \ldots + G_{ng} Z^{-ng}$

$F(Z^{-1}) = f_1 Z^{-1} + \; \ldots\ldots\ldots \; + f_{nf} Z^{-nf}$

F_j and G_j are p x p coefficient matrices,

$n_g = n_a - 1 \; , \; n_f = n_b + k - 1$

Shifting closed loop poles to the location (assigned poles) defined by the polynomial matrix $[I + T(z^{-1})]$, i.e.:

$$Y(t) = [I + F(z^{-1})] \, [I + T(z^{-1})]^{-1} \, e(t) \qquad (18)$$

where $n_t \le n_a + n_b + k - 1 - n_c$.

Substituting equations (17) and (18) into equation (16) producing:

$$\left[I + A(z^{-1}) \right] \left[I + F(z^{-1}) \right] + Z^{-k} B(z^{-1})\ G(z^{-1}) = \left[I + C(z^{-1}) \right] \left[I + T(z^{-1}) \right] \qquad (19)$$

from which, control parameters can be calculated by equating coefficients of similar powers of (Z^{-1}). Thus, the calculation procedure can be summarized in the following steps:

1- Reconstructing equation (16) to the form:

$$Y(t) = -A_1\ Y(t-1) - \ldots -A_{na}\ Y(t-n_a) + B_o\ U(t-k) + \ldots + B_{nb} U(t-k+n_b) + C_1\ e(t-1) + \ldots + C_{nc}\ e(t-n_c) \qquad (20)$$

and using Recursive Least Square (RLS) [7] method, estimated parameters $\hat{A}$ and $\hat{B}$ can be calculated.

2- Given $\hat{A}$ and $\hat{B}$ and pre-specified T, estimated $\hat{G}$ and $\hat{f}$ can be calculated by solving equation (19) at each sample.

3- Y(t) can be calculated from equation (18).

4- Control law U(t) is calculated from equation (17).
It is notable that, for non zero reference signals, Y(t) is replaced by [$Y(t) - Y_r(t)$] where $Y_r(t)$ is p-vector of nominally constant reference demand.

This procedure is applied on our modified interconnected ARMA model with substituting:

Setting values: $Y_{r1} = 0.7$, $Y_{r2} = 0.65$, $Y_{r3} = 0.8$
change in each set point = (0.05 or -0.05)

R. L. S. : Forgetting factor $\lambda = 0.99$
Covariance matrix element $p_o = 10^8$

Noise levels $e_1(t)$ changes from -0.15 to 0.0004
$e_2(t)$ changes from -0.15 to 0.0005
$e_3(t)$ changes from -0.15 to 0.0006
C = 0

Figures (2), (3) and (4) show samples of the obtained control performances, which prove that the pole assignment controller has been succeded for:
a) Regulating the system response, and keeping it equal to the reference signal variations for all the chosen poles locations (0.1 to 0.85).
b) The controller could reject random disturbances which corrupt the system output.

CONCLUSION :

Multivariable control strategy is designed for controlling performance of interconnected system which simulates a real chemical process in SEMADCO Urea- plant. Open loop system stability is ensured by designing appropriate computing relays for modifying the different interconnected static gains. Relative gain matrix procedure is used for this purpose.

On the other hand, closed loop controllers for the composite system are designed by using pole assignment technique. Good results are obtained which agree with the effectiveness of the designed control strategy.

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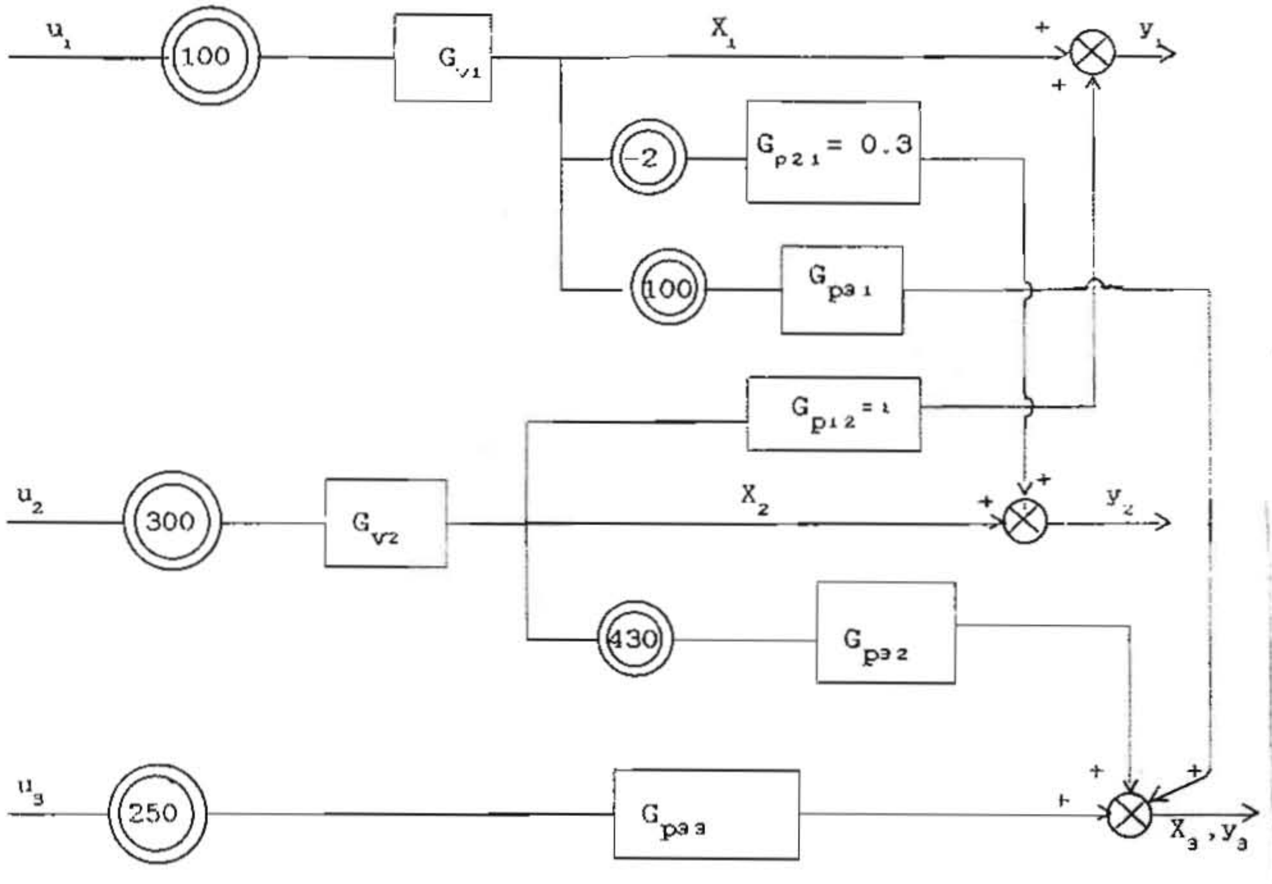


Fig. (1)

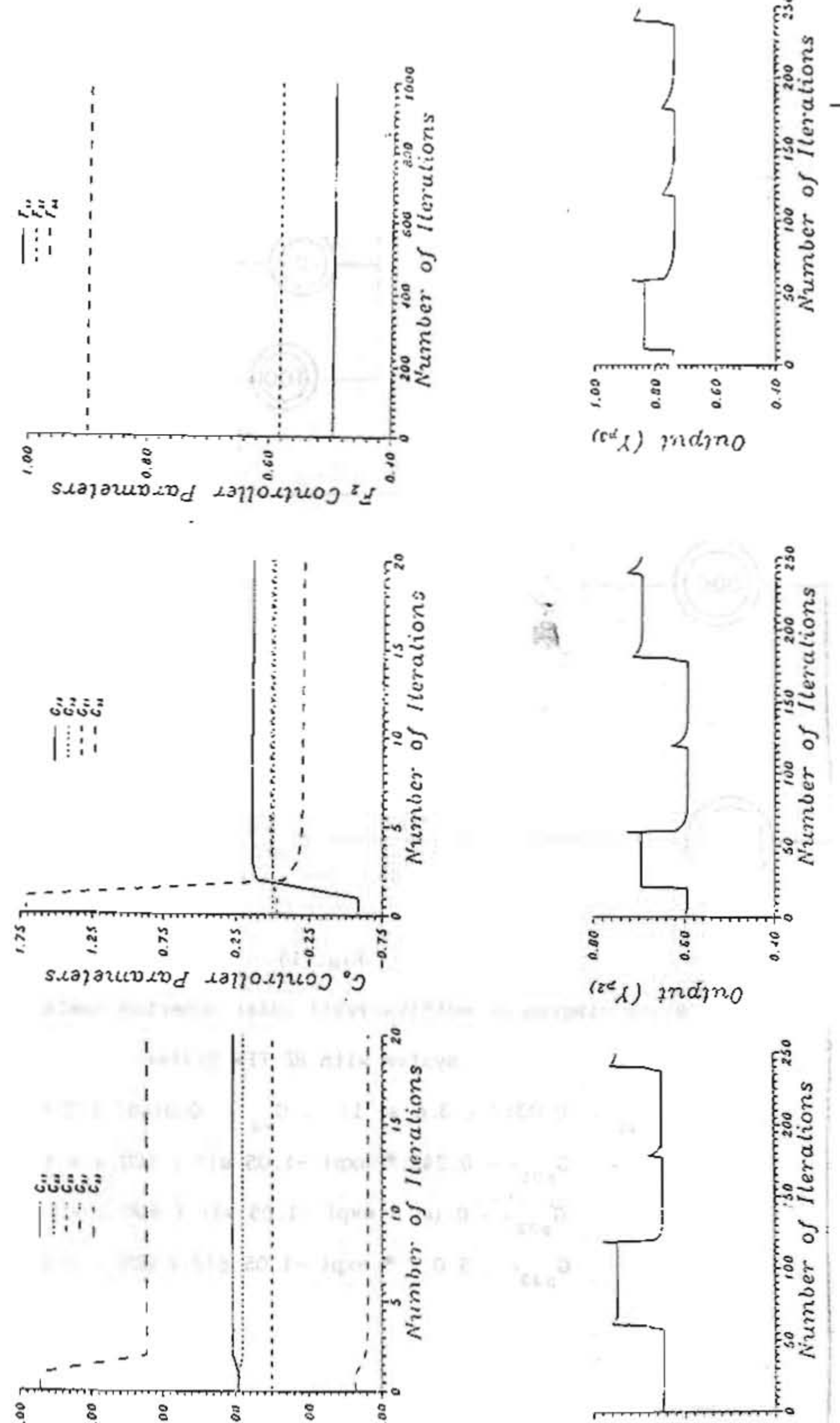
Block diagram of multivariable interconnected Aomina converter system with HP-PFW heater.

G_{v1} = 0.031/ (3.6 s+ 1) , G_{v2} = 0.0164/ (3.6 s+ 1) ,

G_{p31} = - 0.242 * exp(-1.05 s)/ (600 s + 1) ,

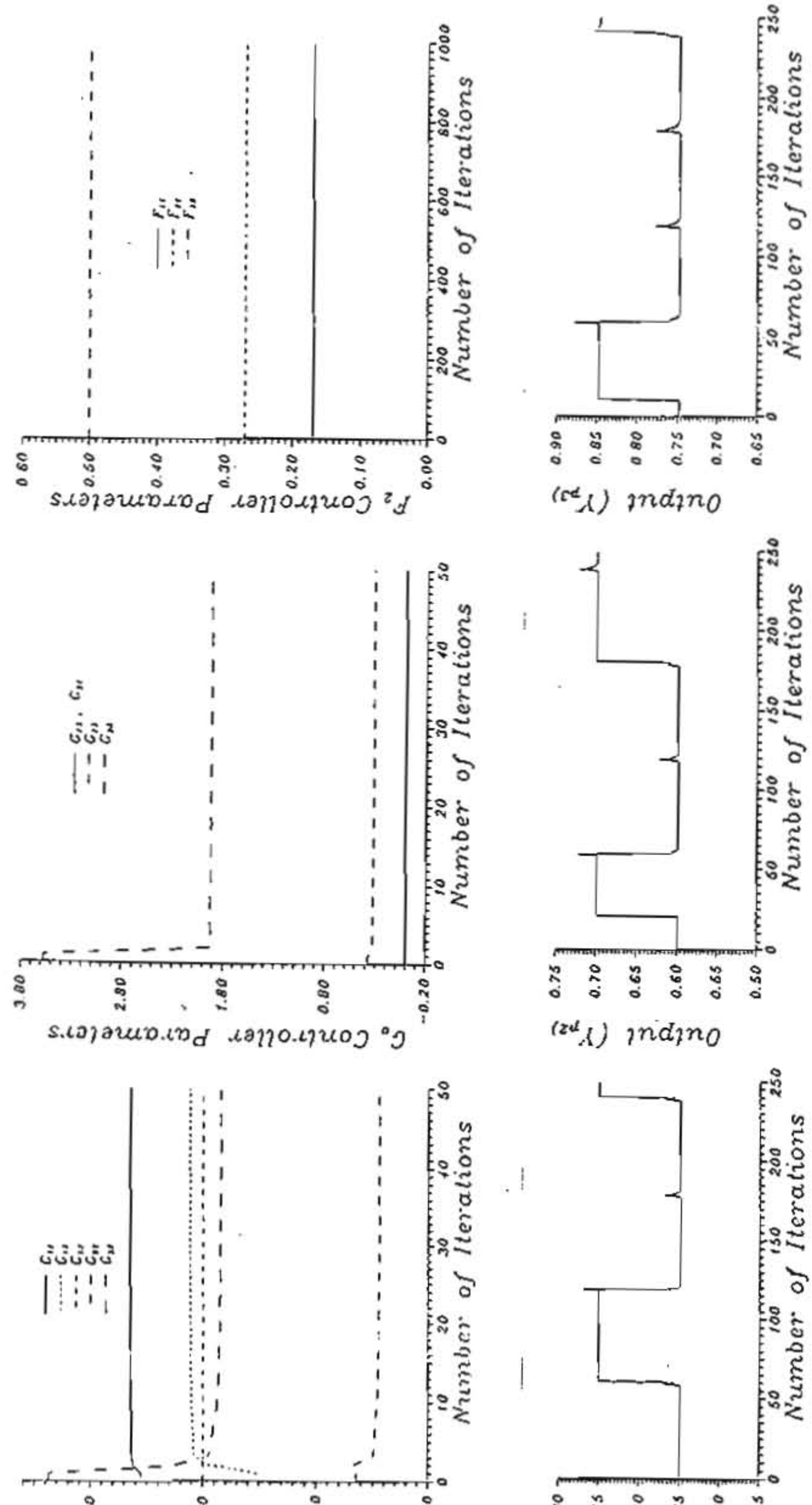
G_{p32} = - 0.181 * exp(-1.05 s)/ (600 s + 1) ,

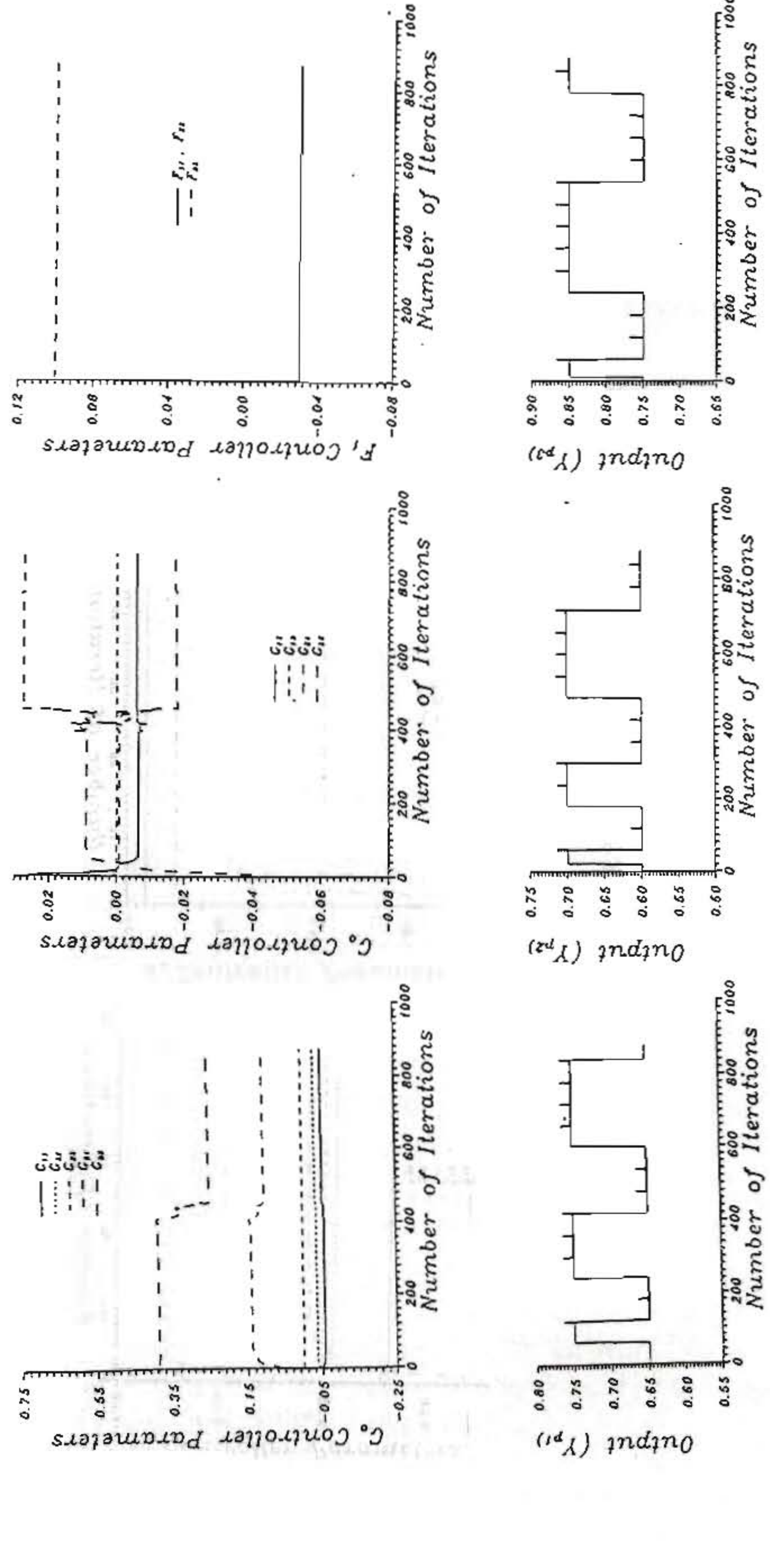
G_{p33} = 8.0 * exp(-1.05 s)/ (600 s + 1) , First Case : Poles are +0.3 , +0.2 , +0.1



Fig(3)

Second Case : Poles are +0.7 , +0.6 , +0.5





Fig(4)

Third Case : The Poles are +0.9 , +0.9 , +0.9