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GEOMETRY OF PLAIN SQUARE FABRIC WOVEN FROM FLEXIBLE YARNS

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عنوان البحث : هندسة القماش السادة متماثل الخواص للسداء واللحمة
والمنسوج من خيوط لاتقاوم الإنثناء

ملخص البحث

يقدم البحث دراسة نظرية لهندسة النسيج السادة لقماش تتماثل فيه خواص خيوط السداء واللحمة (قماش مربع) ويقدم البحث صيغا دقيقة للربط بين المتغيرات الإنشائية للقماش ذو النسيج السادة . ولما كان من المعتاد قياس كثافة الخيوط فى القماش المنسوج (عدد الخيوط/ سم) وكذلك قطر الخيط ، فإنه من الممكن تقدير نسبة تغطية الخيوط في القماش ، أما إذا كانت نمرة الخيط معلومة فيمكن بمعلومية كثافة الخيط (جرام/ سم٣) إيجاد قطر الخيط ، وقد أمكن التعبير عن زاوية النسيج . نسبة تشريب الخيوط ، كثافة التعبئه كدوال فى نسبة تغطية الخيوط ، وأمكن التعبير عن وزن القماش (جرام / متر٢) بدلالة قطر الخيط ، نسبة تغطية الخيوط ، كثافة الخيط (جرام / سم٣) وأمكن التعبير عن كثافة القماش (جرام / سم٣) بدلالة كثافة الخيط (جرام / سم٣) ونسبة تغطية الخيوط ، أما كثافة التعبئه للقماش فأمكن التعبير عنها بدلالة نسبة تغطية الخيوط .

Abstract:

This paper presents a theoretical study of geometry of plain square fabric woven from flexible yarns . Accurate formulae could be obtained to relate different parameters of fabric geometry . It's a usual procedure to measure yarn density (yarns/cm in the fabric) and yarn diameter . From these two parameters yarn cover ratio can be calculated . If yarn count is known it is needed to measure yarn density to be able to estimate yarn diameter . Weave angle, yarn crimp ratio and packing density of yarn into the fabric could be expressed as functions of yarn cover ratio. Fabric weight (g/m^2) could be expressed in terms of yarn diameter, yarn cover ratio , and yarn density (g/cm^3) . Fabric density (g/cm^3) could be expressed in terms of yarn density (g/cm^3), and yarn cover ratio . Packing density of yarn into the fabric could be expressed in terms of yarn cover ratio .

Introduction :

Recent investigations have suggested geometrical models that can be used to determine relationships between fabric parameters. Peirce was perhaps the earliest and is probably the best known [1] .These investigators used models based on neat geometrical ideas such as straight lines joined to circular arcs and so on. Fig.(1) shows Peirce's Flexible Thread Model .

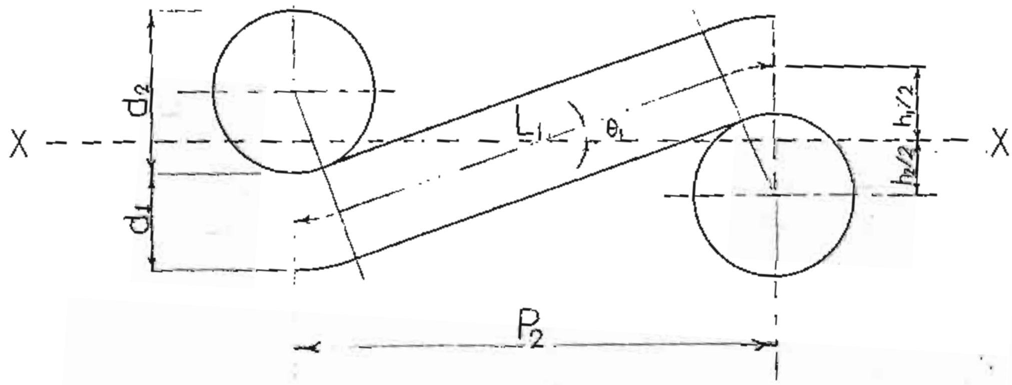

Fig. (1)

(Subscripts 1 and 2 for warp and weft respectively. d = yarn diameter. L = modular length h/2 = max. displacement of the yarn axis from the cloth plane. p = yarn spacing. X = cloth plane. θ = angle between yarn axis and cloth plaine

Peirce [2] gave the most detailed mathematical analysis, assuming a thread without any stiffness. This assumption gave circular cross-sections and a yarn path made up of a straight portion between circular arcs wrapping around the other crossing yarn .

Peirce derived 7 formulae from the configuration shown in Fig.(1) with "D" the sum of two diameters , as a scale unit .

$$C_1 = \left(L_1 / P_2 \right) - 1 \qquad (i)$$

$$C_2 = \left(L_2 / P_1 \right) - 1 \qquad (ii)$$

$$P_2 = \left(L_1 - D\theta_1 \right) \cos\theta_1 + D \sin\theta_1 \qquad (iii)$$

$$P_1 = \left(L_2 - D\theta_2 \right) \cos\theta_2 + D \sin\theta_2 \qquad (iv)$$

$$h_1 = \left(L_1 - D\theta_1 \right) \sin\theta_1 + D \left(1 - \cos\theta_1 \right) \qquad (v)$$

$$h_2 = \left(L_2 - D\theta_2 \right) \sin\theta_2 + D \left(1 - \cos\theta_2 \right) \qquad (vi)$$

$$D = h_1 + h_2 \qquad (vii)$$

Peirce derived some approximations which seemed excessive in some cases. In order to derive the formula $h = \left(\frac{4}{3}\right) P \sqrt{C}$ he assumed that (θ) is small, but this is true only in very open structure [3].

Yarn diameter is not easily measured, so various investigators have obtained formulae for its calculation. It's aimed to derive more general and accurate formulae to specify fabric geometry. This work is the first step towards this object. It's beginned with plain square fabric as the simplest woven fabric structure and it's dealed with flexible threads as the case in most textile yarns.

Mathematical Model of Plain Square Fabric Woven from Limp Yarns:

Fig.(2) shows the model of plain square fabric woven from limp threads.

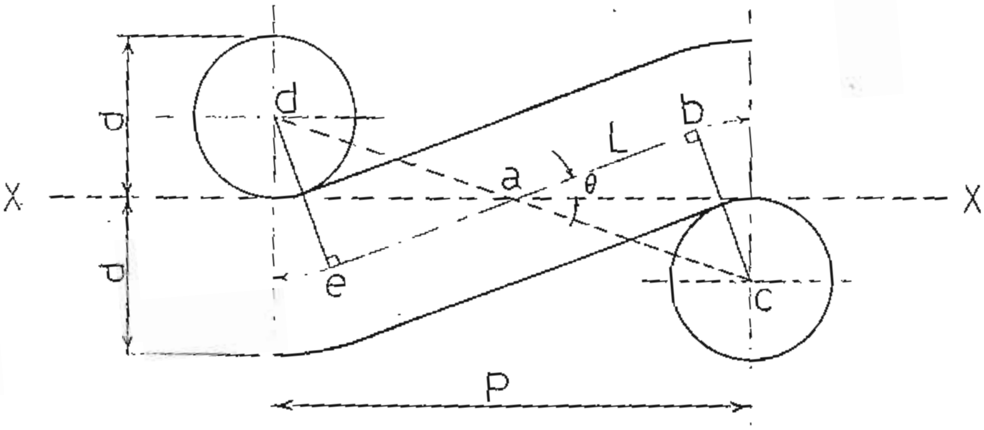

Fig. (2)

P = yarn spacing cm, d= yarn diameter (cm), θ = weave angle (deg).
L = modular length cm, X = cloth plane.

Mathematical Relationships:

The following mathematical relationships could be derived :

$$d = 1 / \sqrt{25 \pi \rho_y N_m} \quad (1)$$

$$P = 1 / N \quad (2)$$

[illegible] $= d / P$ (3)

Where :

ρ_y =specific yarn density, Nm = metric yarn count, N = number of threads / cm

Three variables are given : yarn count Nm, specific yarn density ρ_y, and yarn sett N.

Yarn cover ratio K can be expressed as follows :

$$K = N \Big/ \sqrt{25\ \pi\ \rho_y\ N_m} \qquad (4)$$

An expression for weave angle θ could be derived as follows :

$$\tan\theta = \frac{2-\sqrt{1-3K^2}}{(1/K)\sqrt{1-3K^2}+2K} \qquad (5)$$

Crimp ratio C is expressed as follows :

$$C = \frac{\pi K\,\theta}{90} + \sqrt{1-3K^2} - 1 \qquad (6)$$

or

$$C = \frac{\pi K}{90}\tan^{-1}\frac{2-\sqrt{1-3K^2}}{\left(\frac{1}{K}\right)\sqrt{1-3K^2}+2K} + \sqrt{1-3K^2} - 1 \qquad (6)$$

Fabric weight in g/m^2 can be expressed as follows:

$$W = 5000\ \pi\ (1+C)\rho_y\ Kd \qquad (7)$$

or

$$W = 500\ \pi\rho_y\ Kd\left(\frac{\pi K}{90}\ \tan^{-1}\frac{2-\sqrt{1-3K^2}}{\frac{1}{K}\sqrt{1-3K^2}+2K} + \sqrt{1-3K^2}\right) \qquad (7)$$

Fabric density in g/cm^3 can be expressed as follows :

$$\rho_f = \pi K\ \rho_y\ (1+C)\ /4 \qquad (8)$$

or

$$\rho_f = \frac{\pi K\ \rho_y}{4}\left(\frac{\pi K}{90}\ \tan^{-1}\frac{2-\sqrt{1-3K^2}}{\frac{1}{K}\sqrt{1-3K^2}+2K} + \sqrt{1-3K^2}\right) \qquad (8)$$

Fabric packing density P.D. can therefore be obtained since

$$P.D = \rho_f\ /\rho_y = \pi\ K\ (1+C)\ /\ 4 \qquad (9)$$

or

$$P.D = \frac{\pi K}{4}\left(\frac{\pi K}{90}\ \tan^{-1}\frac{2-\sqrt{1-3K^2}}{\frac{1}{K}\sqrt{1-3K^2}+2K} + \sqrt{1-3K^2}\right) \qquad (9)$$

It's worth mentioning that calculating cover ratio K, weave angle θ, crimp ratio C need only two variables d and P. In other words we need to measure the value of d and P and then cover ratio, weave angle and crimp ratio can be calculated using equations (3 , 5 , 6).

Substituting in equations (5 , 6 , 7 8 , 9) for K where $0 < K \leq (1/\sqrt{3})$ we obtain table (1):

Table (1): Effect of yarn Cover Ratio (K) on weave Angle (θ). Crimp Ratio (C), Packing density (P.D.), Fabric Weight (W), and Fabric Density (ρ_f) in plain square fabric woven from limp threads.

K	θ	C	P . D	W	ρ_f
0.0100	0.5730	0.00005	0.00785	1.5708	0.00392
0.0300	1.7200	0.00045	0.02357	4.7145	0.01178
0.0500	2.8700	0.00125	0.03932	7.8638	0.01966
0.1000	5.7680	0.00502	0.07893	15.7868	0.03996
0.2000	11.784	0.02040	0.16028	32.0568	0.08000
0.3000	18.379	0.04686	0.24666	49.3321	0.12333
0.4000	26.167	0.08600	0.34120	68.2350	0.17060
0.5000	36.870	0.14350	0.44910	89.8103	0.22450
0.5500	45.734	0.18200	0.51059	102.120	0.25529
0.5700	60.000	0.21000	0.54834	109.670	0.27417

Yarn density ρ_y (g cm^3) must be Known beside yarn cover ratio (K)
[ρ_y is taken 0.5 g cm^3] .

Yarn density and either yarn diameter d (cm) or yarn spacing P (cm) must be known beside yarn cover ratio{ρ_y = 0.5 g cm^{-3} and d = 0.02 cm}..

Effect of Yarn Cover Ratio (K) on Weave Angle (θ) :

Fig.(3) shows the effect of yarn cover ratio K on weave angle θ . It is clear that θ increases with an increasing rate of change as K increases . The maximum weave angle for square plain fabric is 60 (*Jamming Condition*). This occurs when K reaches about 0.577 .

Effect of Yarn Cover Ratio on Yarn Crimp Ratio :

As shown in Fig.(4) yarn crimp ratio C increases with an increasing rate of change as yarn cover ratio K increases . Maximum yarn crimp ratio is 0.21 (*Jamming Condition*) .

Effect of Yarn Cover Ratio on Packing Density of Yarn into the Fabric:

Fig.(5) shows that increasing yarn cover ratio K causes packing density of yarn into the fabric to increase with an increasing rate of change . Maximum packing density is about 0.548 (*Jamming Condition*).

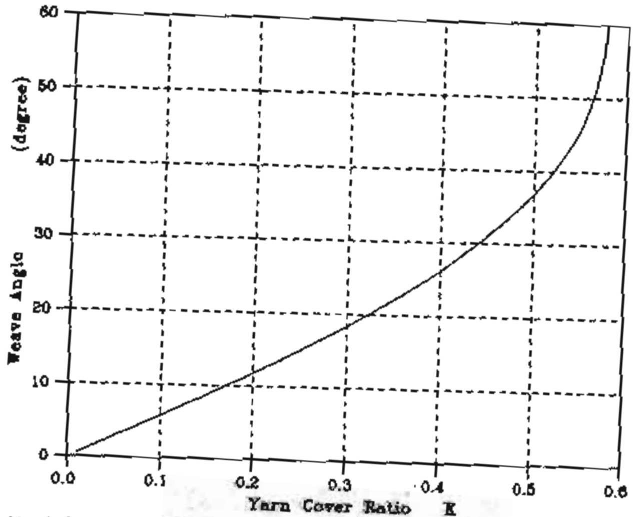

Fig.(3): Relationship between Yarn Cover Ratio and Weave Angle in Plain Square Fabric Woven from Flexible Yarns

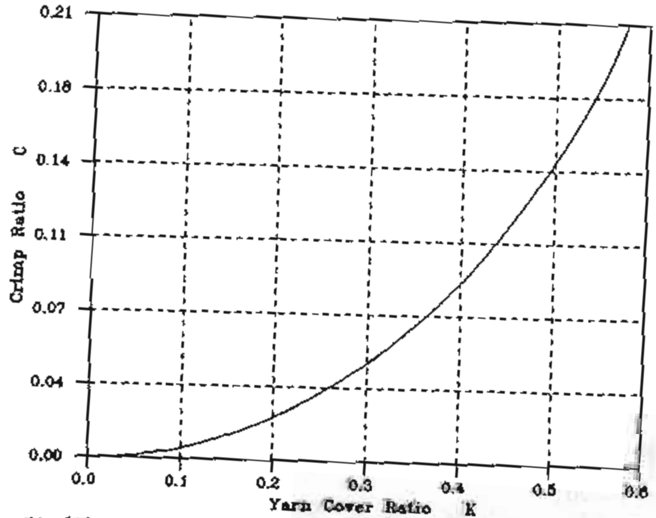

Fig.(4): Relationship between Yarn Cover Ratio and Yarn Crimp Ratio in Plain Square Fabric Woven from Flexible Yarns

Effect of Yarn Cover Ratio on Fabric Weight (g/m^2) :

Fabric weight (g/m^2) increases at a rate greater than that of yarn cover ratio . This is shown in Fig.(6) . When the yarn density is 0.5 g/cm^3 and yarn diameter is 0.02 cm maximum fabric weight is 109.67 g/m^2 It is worth saying that fabric weight increases linearly with yarn density (g/cm^3).

Effect of Yarn Cover Ratio on Fabric Density (g/cm^3):

It is clear from that Fabric density (g/cm^3) increases at a rate greater than that of yarn cover ratio. This is shown in Fig.(7) for a fabric made from yarns of density 0.5 g/cm^3 which reaches a density of about 0.274 g/cm^3 at *jamming condition*..

CONCLUSION :

Geometry of square plain fabric made from circular yarns could be analytically described . If a square plain woven fabric is wanted to be woven from certain yarns on certain weaving machines , yarn cover ratio is easy to be estimated . Knowing yarn cover ratio helps us to predict fabric weave angle , yarn crimp ratio , fabric weight , fabric density or packing density of yarn into the fabric. This helps predict the total yarn length required to weave a certain fabric length . This helps in mill organization and reducing yarn waste and down time. Formulae derived in this paper are not only accurate but also suitable for square fabrics of any cover ratio. These are different from Peirce's formulae which are approximate and suitable for only very open structures .

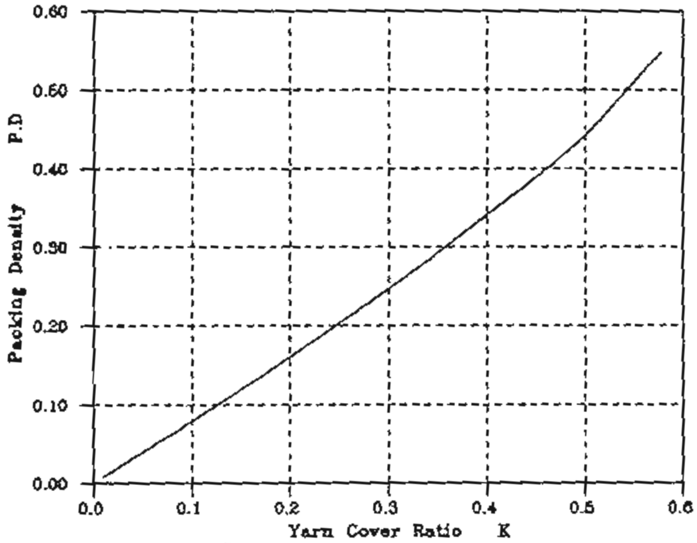

Fig.(5): Relationship between Yarn Cover Ratio and Yarn Packing Density in Plain Square Fabric Woven from Flexible Yarns

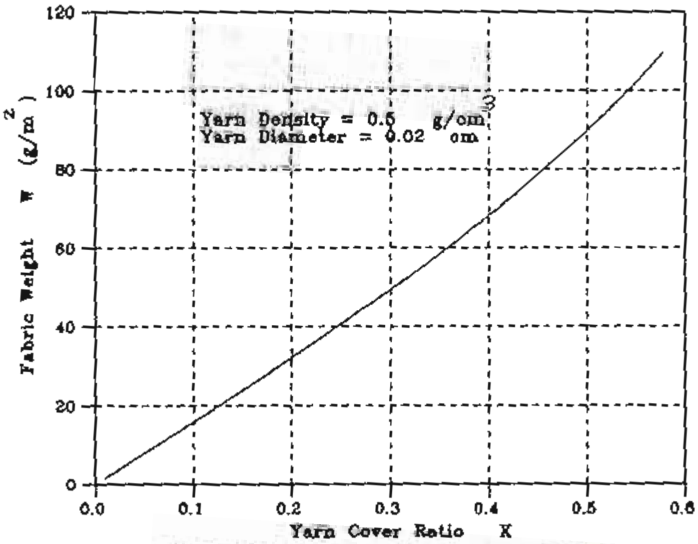

Fig.(6): Relationship between Yarn Cover Ratio and Fabric Weight in Plain Square Fabric Woven from Flexible Yarns

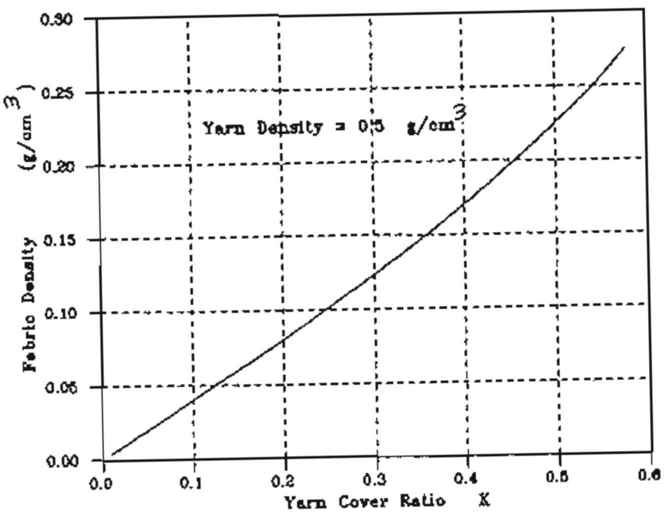

Fig.(7): Relationship between Yarn Cover Ratio and Fabric Density in Plain Square Fabric Woven from Flexible Yarns

APPENDIX

from the model shown in Fig. (2):

Projection vertically gives :

$$P = (L - 2d\theta) \cos\theta + 2d \sin\theta \quad (a)$$

Projection horizontally gives :

$$a = (L - 2d\theta) \sin\theta + 2d (1 - \cos\theta) \quad (b)$$

In triangle abc we get :

$$(\overline{ab})^2 = (\overline{ac})^2 - (\overline{bc})^2 \quad (c)$$

$$\therefore (eb/2)^2 = (dc/2)^2 - (\overline{bc})^2 \quad (d)$$

$$(\overline{eb})^2 = (\overline{dc})^2 - 4(\overline{bc})^2 \quad (e)$$

$$\text{but } (\overline{dc})^2 = P^2 + d^2 \text{ and } bc = d \quad (f)$$

$$\therefore (\overline{eb})^2 = P^2 - 3d^2 \quad (g)$$

$$(L - 2d\theta) = \sqrt{P^2 - 3d^2} \quad (h)$$

Equations (a , b , h) are three equations in 3 unknowns L , P *and* θ as d must be given.

Dividing both equation (b and a) by P after substituting for $(L - 2d\theta)$ by $(P^2 - 3d^2)^{1/2}$ gives :

$$K = \sqrt{1 - 3K^2} \sin\theta + 2K(1 - \cos\theta) \quad (i)$$

$$1 = \sqrt{1 - 3K^2} \cos\theta + 2K \sin\theta \quad (j)$$

Dividing equation (i) by K and adding it to equation (j) gives :

$$\left\{ \frac{1}{K}\sqrt{1 - 3K^2} + 2K \right\} \sin\theta + \left\{ \sqrt{1 - 3K^2} - 2 \right\} \cos\theta = 0$$

$$\therefore \tan\theta = \frac{2 - \sqrt{1 - 3K^2}}{\frac{1}{K}\sqrt{1 - 3K^2} + 2K} \quad (k)$$

$$\text{as } (L - 2d\theta) = \sqrt{P^2 - 3d^2} \quad (l)$$

$$\text{and } L = P(1 + C) \quad (m)$$

$$\therefore C = \sqrt{1 - 3K^2} + 2K\theta - 1 \quad (n)$$

where θ is in radian

$$\text{or } C = \sqrt{1 - 3K^2} + \frac{\pi K \theta}{90} - 1 \quad (o)$$

where θ is in degrees.

REFERENCES :

[1] P. Ellis . Woven Fabric Geometry Past and Present Part I. Textile Institute and Industry. August 1974 . 244-247.

[2] F.T Peirce. Journal of Textile Institute. 1937. 28. T 45.

[3] P. Ellis . Woven Fabric Geometry Past and Present Part II . Textile Institute and Industry. August 1974 . 303- 306 .

[4] P. Ellis . Woven Fabric Geometry Past and Present Part III . Textile Institute and Industry. August 1974 . 339- 342.