[Mansoura Engineering Journal](https://mej.researchcommons.org/home)

[Volume 21](https://mej.researchcommons.org/home/vol21) | [Issue 1](https://mej.researchcommons.org/home/vol21/iss1) Article 9

3-1-1996

Surface Finish Improvement of Turned Surfaces via Tool Having a Novel Tool Nose Edge (Elliptical form).

Ibraheem Mohamed Ammar Industrial Production Engineering Department., Faculty of Engineering., El-Mansoura University., Mansoura., Egypt.

Follow this and additional works at: [https://mej.researchcommons.org/home](https://mej.researchcommons.org/home?utm_source=mej.researchcommons.org%2Fhome%2Fvol21%2Fiss1%2F9&utm_medium=PDF&utm_campaign=PDFCoverPages)

Recommended Citation

Ammar, Ibraheem Mohamed (1996) "Surface Finish Improvement of Turned Surfaces via Tool Having a Novel Tool Nose Edge (Elliptical form).," Mansoura Engineering Journal: Vol. 21 : Iss. 1, Article 9. Available at:<https://doi.org/10.21608/bfemu.2021.151794>

This Original Study is brought to you for free and open access by Mansoura Engineering Journal. It has been accepted for inclusion in Mansoura Engineering Journal by an authorized editor of Mansoura Engineering Journal. For more information, please contact mej@mans.edu.eg.

Surface Finish Improvement of Turned surfaces Via Tool Having a Novel Tool Nose Edge (Elliptical form)

تحسين درجة تشطيب الأسطح المشغلة بالدراطة باستخدام عدة قطع لما وقدوة جديدة (قطع باقع)

RY

I.M. Elewa

Assoc. Prof. Ind. Prod. Dept. Faculty of Eng., Mansoura Univ.

الملفس العريم

ترتبط أهمية درحة تشطيب الأسطح للشغلة بالغرض الوظيفي للدي يؤديه الجزء المتج ولقد دلست التجارب المعملية عشى أن السبطح المشعل تتحسن درجة نعرمته بريادة بصف قطر مقممة عدة القطع المشخدمة. كذلك نزله توجد نهاية ازيادة بصف القطر للقممة عدة القطبع بعدهنا بصبح ربادته غير مؤثر على درحة نعومة السطح. ولهذا يتعين أن تكون قيمة بصف قطر نقوس مقدمة عدة القطع كبيرة بالمسسة لقيمسة المتصديدة المستحدمة حتى يمكن تعادى علامات النغدية على السطح المشعل.

ويهدب هذا البحث إلى دراسة تأثير شكل مقدمة عدة القطع على الأسطح المشغلة بعملية اغراطة. وقد تم استخدم عدة قطع لها مقدسة ميصية الأول مرة كعاتم استناط العلاقات الرياصية المحتلفة والبئ تربط دلك الحكل الجديد انعاملات التشنطيب السنطحي ولقلد أجريت نحارب عملية ماستخدام عدد الفطع الجميدة وأجرى لها مقدمات حادة وكدلك أسطوانية. هدا ولقد دلت العلاقات النظرية المدعمة ماثجلارب الهملية علمي تصوق عدد قطع ذوات المقدمات الييضية على غيرها من العدد التقليدية.

Abstract

The importance of surface finish is evident for sliding or rolling elements of kinematic chains. The surface finish improves with the increase in nose radius value of cutting tool. This has been confirmed by many experimental studies. However, there are nose radii maxima beyond which no improvement in surface finish is likely. The nose radius must be sufficiently large in relation to feed so that feed marks of the tool on the workpiecc can be avoided. Surface finish improves with increased nose radius due to reduction in the saw-tooth effect of feed ridges

The effect of the nose shape of a single point tool on the surface roughness of the turned surface is investigated. A new model assuming that the tool nose of an elliptical form is employed in order to evaluate surface roughness parameter. An experimental work employing the new tool profile has been carried out. A comparative study between the experimental and theoretical results has been done using the new tool and the other tools having sharp and circular edges. Both theoretical and experimental results showed that the new tool edge gave a better surface finish center line average height, than the conventional types.

Kevword

Surface roughness. Turning process. Cutting variable. Center lathe. Tool nose.

1. Introduction

The control of surface roughness of a workpiece has become increasingly important during the last few decades. The importance of the surface finish and its influence on the life of the workpiece are well understood [1, 2, 3]. The nature of the interaction between any two

M.58 I.M. Elewa

surfaces will depend on the geometrical shape, their relative motion and the intensity of loading between them. It had been postulated that the variety of possible combinations of all these factors is infinite [4, 5, 6].

One of the major goals of researches in this area has been the development of models which can predict surface finish change upon the simultaneous variation of cutting factors such as speed, feed, nose radius, etc. The relative motion between cutter and workpiece is bound to effect the accuracy of the product as well as the roughness of the finished surface [7, 8].

In the past, in order to derive mathematical basis for surface roughness assessment, two common approaches had been employed. One method has been based on the assumption of a random distribution of asperity heights and spacing, in order to develope a statistical solution [9]. The other method assumes that the surface is composed of a series of regular shapes (hemisphere, semi cylinders) [10]. However, the surface finish, while accounting for the effect of feed rate and nose radius of the cutting tool, have considered the effect of cutting speed to be significant [11, 12, 13]. The nose radius and the feed rate are the most important consideration in obtaining the desired surface roughness [14]. The nose raduis should be three times the feed rate for good surface finish [15].

The theoretical roughness is perfectly determined whenever the geometrical shape of the tool and the feed values are known. The profiling cutting edge would be, theoretically reproducable upon the surface [16].

The significance of the tool edge, i.e. type, geometty on surface roughness in the machining operation has been noted by many authors [17, 18]. The complexity of the action of a rounded edge has been investigated by many researches [19, 20, 21].

2. Theoretical Analysis

A new theoretical model is developed under the assumption that the cutting tool edge is provided with an elliptical nose-shape as shown in Fig. (1).

This model is concerned with case where the tool nose has a major and minor diameter (ellipse) as shown in Fig. (1). In this case the feed $F < 2a$

This model is valid under the following assumptions :-

(a) Turning is vibration free.

- (b) A single point tool is employed to turn the surface.
- (c) Errors in the machine slide ways along which the tool moves have negligible effects or the shape and amount of feed.
- (d) The wave form of the surface profile is repetitive.

A relationship between the kinematics roughness and the proposed tool nose radius is seeked. Such analysis is based on the work of many investigators $[16, 10, 22]$ who idealized the surface machined by a single-point tools to take the form of radiused grooves. In this investigation it is assumed that the wear of the tool does not change the feed marks (see Fig. $2).$

It is required to establish the height and spacing of the asperities. The spacing is the feet rate of the machining process and the asperities height may be calculated knowing the tool tip radius and feed rate.

Since, the basic equation of the ellipse is as follows:

$$
X^2/a^2 + Y^2/b^2 = 1
$$
 (1)

Where:

$$
X = F/2, Y = b - R_m
$$

Therefore:

$$
R^2 \mathbf{m} - 2bR \mathbf{m} + F^2 b^2 / 4a^2 = 0 \tag{2}
$$

By solving the above quadratic equation, we get:

$$
R_m = b \pm (b^2 - F^2 b^2 / 4a^2) \frac{1}{2}
$$
 (3)

It is clear from the above equation that $R_{\rm m}$ has two possible values.

However, since $R_{\text{m}} \leq b$ then the only valid solution is when,

 $R_m = b - [b^2 - (F^2 b^2/4a^2)]$ 1/2

 (4)

٠

Fig. 1. The edge profile of the new proposed tool

Fig. 2. The kinematics roughness using the novel (elliptical) tool edge.

To evaluate the Arithmetic Average mean (R_a) it may be assumed that the surface produced by a single point tool is represented by a series of elliptical cusps of height b and the base F. The Arithmetic Average mean (R_2) is given by sum of the absolute values of all areas above and bellow the mean line divided by the sampling length as shown in Fig. (2) .

The area of the ellipse as follows:

$$
A = \int X \, dV
$$

Since: $X^2 = [a^2 - y^2 a^2/b^2]$ i.e. $X = a (1 - y^2/b^2)$ ^{1/2}

Then, the area above mean line is given by the following relation;

M.60 I.M. Elewa

$$
A_1 = 2a \int_{b-c}^{b-c} (1 - y^2 + b^2)^{1/2} dy
$$
 (5)

The area below the mean line is:

$$
A_2 = 2[F / 2(R_m - C) - a \int_{b-R_m}^{b-c} (1 - y^2 / b^2)^{1/2} dy]
$$
 (6)

The area below line A2 equal to the area above the mean line. Hence,

$$
R_a F = 2 A_1
$$

The integration of equations (5) and (6) is solved analytically and a computer program used to estimate the value of R_2 where

$$
R_{a} = [A_{1} \pm A_{2}] / F = 2 A_{1}/F
$$
 (7)

Equations (4) and (7) are used to evaluate the theoretical values of beak-to-valley height (R_{1n}) and the Arithmetic Average mean (R_a) for each specimen at the major and minor diameter and the value of feed rate corresponding to every equation, see Appendix (A)

3 Experimental Work

3.1 Specimens material selection and preparation

The material used in the experimental work is mild steel. The chemical composition and the mechanical properties of this steel are shown in Tables (1) and (2) respectively.

Fig. 3. Effect of major diameter on surface roughness parameter (R_m) $(b=1$ mm)

Fig. 4. The effect of Minor Radius on surface roughness parameter (R_m) $(2a=4$ mm $)$

The specimens were divided into three different groups according to the tool geometry used in the cutting process. All the specimens were first parted off on a lathe and their surfaces were ground with the same condition. This was mainly to unify the condition of specimen surface before producing the final finishing.

Table (1). Chemical composition of mild steel specimens

C %	Si %	Mn %	s %	P %
$0.15 - 0.45$	ົ່ V. IZ	$0.08 - 0.09$	0.05	0.05

Table (2). Mechanical properties of mild steel

4. Cutting Tool Material and Nose Dimensions

The cutting tool material has been chosen to be using a high speed steel. The geometry of both the sharp edge and the circular profile have been chosen according to the ISO specifications for machining the mild steel as indicated in Tables (4, 5).

A five new blank of H.S.S. material are used to produce five tools having different geometries (major diameter and minor diameter of the cllipse). The operation sequence for fortning the new tools was as follows

- 1. The required shape of every tool is obtained using a computer program on a blue-print paper.
- 2. the blue-print paper has been stocked on a small sheet of metal and formed to give the new profile shape using a tool grinding m/c.

 $M.61$

M 62 I.M. Elewa

- 3. The new blank of H.S.S. is clamped with the formed sheet for producing the required profile of the new tool utilizing a tool grinding m/c.
- 4. The new profile of the five tools which obtained has been measured employing a measurescope 10 in order to estimate the actual profile of each and compared with the suggested value.
- 5. The suggested dimensions of the five new tool profile were as follows :
	- The niajor radius (a) are 16, 14, 10, 6, 4 mm respectively The minor radius (b) are $8, 7, 5, 3, 2$ mm respectively

5. Measurement Technique

A Talysurf 4 instrument linked to PC processor has been employed in order to determine and record the surface roughness parameters R_a and R_m of all specimens. The workpieces were placed on two vee-blocks situated on a precise surface plate. The stylus of the talysurf was positioned to move across the lay. Five measurements were taken at various locations on each specimen. The results are listed in Tables (3, 4, 5).

6, Results and Discussion

The theoretical results obtained using equations(4) and (7) show the effects of major and minor radius of the ellipse on the surface roughness parameters $(R_a$ and R_m). From Fig. (3) it is found that the surface roughness values have decreased for high major radii at constant minor radius and using three different level of feed. Likewise the theoretical relationship between the minor radius of the ellipse and the surface roughens showed that the surface roughness value has increased when the minor radius of the ellipse was increased at constant major radius and using three different levels of feed rate. A comparative study between the theoretical values obtained using equation (A.2) and those of table (A.1) for sharp, circular and elliptical models has been carried out. It was found that the theoretical values of surface roughness obtained using the elliptical model are about six time less than those obtained using the circular model and more than 33 times less than the values obtained using the sharp model.

The machining operation generates surface when a process roughness is superimposed on the Kinematics roughness. An important problem to the analysis of the machined surface is needed to separate between the above two types of roughness.

An experimental work has been carried out using three different tool profile shapes viz; sharp, circular and elliptical. The obtained results are shown in Figs. (6-10)

It is clear from the different results obtained employing the tools having sharp, circular and elliptical nose that the new tool gave better surface finish at different levels of feed rate. It is obvious that when using the new tool (ellipse) the surface get rougher when its size a and b diminish. Also it was found that the experimental results of surface roughness values obtained using the elliptical model ($a = 16$ mm, $b = 8$ mm) are about (2.6-3.6) time less those for sharp model and about (2-2.7) time less than these obtained using the circular tools. When the geometry of the new tool changed to $(a = 4$ mm, $b = 2$ mm) the obtained results are about $(1.4-1.8)$ and about $(1-1.5)$ time less than the values obyained using sharp and circular tool respectively.

Table (3) Surface Finish Parameters Upon Employing Elliptically Nosed Tools

Specimen No.	Feed Rate mm/rev.	Ra (Experimental) μ m	Ra (Theoretical) μ m	Nose Dimensions mm	Remarks
1 2 3 4 $\overline{\mathbf{S}}$	0.08 0.11 0.14 0.18 0.23	1.40 2.02 2.52 2.91 3.20	0.0064 0.0121 0.0196 0.0325 0.0530	$a = 16$ $b = 8$	- Dry Cutting -Cutting Speed $v = 50$ mt/min
6 7 8 9 10	0.08 0.11 0.14 0.18 0.23	1.74 2.25 2.68 3.15 3.35	0.0073 0.0139 0.0225 0.0317 0.0606	$a = 14$ $h = 7$	- Dry Cutting -Cutting Speed $v = 50$ mVmin
11 12 13 4 15	0.08 0.11 0.14 0.18 0.23	1.82 2.93 3.74 3.82 3.91	0.0103 0.0194 0.0314 0.0520 0.0848	$a = 10$ $b = 5$	- Dry Cutting -Cutting Speed $v = 50$ m v min
16 17 18 19 20	008 0.11 0.14 0.18 0.23	2.58 3.53 3.90 4.23 4.34	0.0171 0.0323 0.0524 0.0860 0.1414	$a = 6$ $b = 3$	- Dry Cutting -Cutting Speed $v = 50$ mt/min
21 22 23 24 25	0.08 0.11 0.14 0.18 0.23	3,41 3.92 4.32 4.49 4.63	0.0257 0.0485 0.0786 0.1299 0.2121	$a = 4$ $b = 2$	- Dry Cutting -Cutting Speed $v = 50$ m v/m in

Table (4) Surface Finish Parameters Upon Employing Circular Nosed Tools (Depth of cut = 0.5 mm)

 $\frac{1}{2}$

l.

M.64 I.M. Elewa

Specimen No.	Feed Rate mm/rev.	Ra (Experimental) π π	Řа (Theoretical) πw	Nose Angle	Remarks
26 27 28 29 30	0.08 0.11 0.14 0.18 0.23	5.06 5.73 6.54 7.68 8.66	6.25 8.750 11.25 14.375	Approach. Angle = 750 End Cutting Edge = 150	- Dry Cutting -Cutting Speed $V = 50$ m V min

Table (5) Surface Finish Parameters Upon Employing Sharp Nosed Tools (Depth of cut = 0.5 mm)

7. Conclusions

Initial results revealed that the introduction of the newly tool edge form (elliptical) resulted in the workpiece thaving superior surface finish than those of sharp and circular tool edges.

The experimental work indicates that it will be an extremely long and difficult way toward the final goal. The investigation covers only surfaces obtained by one type of operation.

This it does in a very incomplete way and it also needs to cover a wide range of materials, tools, different cutting conditions and different surface roughness parameters.

Also, the paper at hand has dealt only with the surface roughness without any consideration to the aspect of the power consumed or physical properties of the surface layer. An investigation in that direction though might ment some consideration is beyond the scope of the present work.

Fig. 5. Theoretical surface roughness values for three compared tool nose shapes

Fig. 6. The effect of feed rate on Ra using sharp, circular, and ellipse models

Fig. 7. The effect of feed rate on Ra using sharp, circular, and ellipse models

M.65

M.66 I.M. Elewa

Fig. 8. The effect of feed rate on Ra using sharp, circular, and ellipse models

Fig. 9 The effect of feed rate on Ra using sharp, circular, and ellipse models

Fig. 10 The effect of feed rate on Ra using sharp, circular, and ellipse models

8. References

- 1. Vaipayee, S., "Fundamental approach to numerical assessment of surface roughens", Microtecnic, Vol. 27 n 7, Pp 360-361, 1973.
- 2. Witzke, F. "Surface finish and other specification", Surface Metrology, Semen, American Soc. for Tool and Manuf. Eng., Pp. 340, 1961.
- 3. Donaldson, C., Lecald, G. and Goold, V., "Tool Design", Tata McGraw-Hill Pub. Company Ltd., New Delhi, Third edition, Pp. 272-276, 1983.
- 4. Stats, T., "Surface finish in metal cutting", CIRP, Vol. 12, Pp. 190-194, 1964.
- 5. ANSI B64 (American National standard) 1-1978.
- 6. Vajpages, S., "Analytical study of surface roughness in turning", Wear, Vol. 70, Pp. 165-175, 1981.
- 7. Shiraishi M. and Sato S., "Dimensional and surface roughness controls in turning operation", Trans. of the ASME, Vol. 112, Pp. 78-83, 1990.
- 8. Thwaite, E. G., "Measurement and control of surface finish in manufacturing", Prec. Eng. Vol. 4, Pp. 207-217, 1984.
- 9. Buc, J. and Nowicki, B., "the measurement of the real area of contact between two metal surface", 8 th Int. Machine Tool Design Res. Conf. Manchester, 1967.
- 10. Schofield, R. and Thorn, R., "Mathematical expression of surface finish characteristics", Proc. Inst. mech. Engs, Vol. 182 part 3K, 1967-68.
- 11. Sundaram, R. and Lambert, B., "Mathematical models to predict surface finish turning of steel", Part I and II, Int. J. of Prod. Res., Vol. 19, Pp. 547-564, 1981.
- 12. Dickinson, G., " A survey of factors effecting surface finish", proceedings of Conference on properties and Methodology of Surfaces, Vol. 182, Part 3K, Pp. 135-147, 1978.

M 67

M.68 I.M. Elewa

- 13. Fisher, H. and Elrod, J., "Surface finish as a function of tool geometry and feed a theoretical approach", Microtechnic, Vol. 25, Pp. 175-178, 1971.
- 14. Shaw, M.S., "Study of machined surfaces", Proceedings DECD Conference, Sept. 1966.
- 15. Elewa., I.M., "An Investigation on the effect of indexable inserts nose radius on the surface roughness in turning operation", 4 th Int. Joint Conf. on Mech. Eng. Tech., Shoubra Faculty of Eng., Cairo, Egypt, 1989.
- 16. Field, M., "Plastically deformed debris and built up edge produced on surfaces by the chip removal and abrasive machining process", Annals of CIRP, Vol. 23, No. 1, 1974.
- 17. Albrecht, P., " A new development in the theory of metal cutting", J. Eng. of Int., Trans. ASME, Ser B, Vol. 82, 1960.
- 18. Rubenstein, C., "The influence of strain rate in orthogonal cuting", Int. J. Mach. Tool Des. and Res., Vol. 12, Pp. 19-, 1974.
- 19. El-Hakim, M. and Mahdy, M., "Some factors affecting the chatter behavior in turning", 2 nd Int. Conf. on Prod. Eng. Des. and Control, Univ. of Alex., Pp. 221-236, 1983.
- 20. Azzain, E. K., "The effect of tool surface finish and tool geometry on the wear of hard metal tools", 2 nd Int. Conf. on Prod. Eng., Design and Control, Univ. of Alex., pp. 271-285, 1983.
- 21 Selmy, A. L., An investigation into the effect of cutting variables on cutting forces and power in turning processes" 2 nd Int. Ain-Shams Univ. Conf. on Prod. Eng. & Des. for Development, pp. 685-694, 1987.
- 22. Londad, P.M., "Relationship between the process roughness and the kinematics roughness in turning surfaces", Annals of the CIRP, Vol. 25, No. 11, pp. 455-459, 1976.

Appendix A

From Fig. (A 1) the different relationship between parameters can be deduce as follows:

$$
F_1 p = \sqrt{\left[\frac{1}{2}F - (-C)\right]^2 + \left[(b - R_m) - 0\right]^2}
$$

$$
F_2 p = \sqrt{\left[\frac{1}{2}F - C\right]^2 + \left[(b - R_m) - 0\right]^2}
$$

Where

$$
L = b - R_{m}
$$

$$
\therefore \sqrt{(\frac{1}{2}F + C)^{2} + L^{2}} + \sqrt{(\frac{1}{2}F - c)^{2} + L^{2}} = 2a
$$
 (A.1)

From eqn (1) it can be found that :

$$
L^{2} = [(a^{2} - c^{2}) + \frac{F^{2}}{4}(\frac{c^{2}}{a^{2}} - 1)]
$$

Where $c = \sqrt{a^{2} - b^{2}}$

$$
\therefore L^{2} = [b^{2} - \frac{F^{2}b^{2}}{4a^{2}}]
$$

$$
\therefore L = b_1 \sqrt{1 - \frac{F^2}{4a^2}}
$$

\n
$$
Rm = b - L = b - b_1 \sqrt{1 - \frac{F^2}{4a^2}}
$$

\n
$$
Rm = \sqrt{1 - \frac{F^2}{4a^2}}
$$
\n(A.2)

Where $F \leq 2a$ From Figs (A.2, A.3) it can be deduce the following The area above the mean line= A_1

A₁ =
$$
2 \int_{b-c}^{b} X \, dy
$$

\nA₂ = $\int_{c}^{b} \frac{a}{b} \sqrt{b^2 - y^2} \, dy$
\nA₃ = $\frac{b}{a} \left[\frac{a}{b} \sqrt{b^2 - y^2} \, dy \right]$
\nA₄ = $\frac{a}{b} \left[(b - c) \sqrt{2bc - c^2} + b^2 \arcsin \frac{b - c}{b} \right]$ (A.4)

The area below the mean line = A_2 as shown in Fig. (A4).

$$
\frac{b}{2}K_2 = \frac{b}{2}F(Rm - c) - \int_{0}^{b} \sqrt{b^2 - y^2} dy
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2}]_{0}^{b-a} = 0
$$

\n
$$
V_2 = F(Rm - c) - \frac{b}{2}[\sqrt{b^2 - y^2} + b^2 \arcsin \frac{y}{2
$$

or
$$
\sqrt{2bR_m - R_m^2} = \frac{bF}{2a}
$$
 (A.6)

By substituting from eq. (A.6) in eq $p(A.5)$
 $Az = F(R_m - c) - \frac{a}{b}[(b - c)\sqrt{2bc - c^2} + b^2 \arcsin \frac{b - c}{b}]$

$$
\sqrt{2bc - c^2 + b^2} \arcsin \frac{b}{b} +
$$

$$
\frac{a}{b}[(b - R_m)\frac{bF}{2a} + b^2 \arcsin \sqrt{1 - \frac{F^2}{4a^2}}]
$$

The area above and below the mean line are equal when the wave form is repetitive. or $A_1 = A_2$

$$
\therefore ab \arcsin 1 = F(R_m - c) + \frac{a}{b} \left[(b - R_m) \frac{bF}{2a} + b^2 \arcsin \sqrt{1 - \frac{F^2}{4a^2}} \right]
$$

M.69

M.70 IM. Elewa

Fc = FRm + $\frac{1}{2}$ bF $-\frac{1}{2}$ FRm - ab{arcsin 1 - arcsin $\sqrt{1 - F^2/4a^2}$ }
c = $\frac{1}{2}$ (Rm + b) - $\frac{ab}{P}$ arcsin $\frac{F}{2a}$

The R_a value is given by

$$
Ra = \frac{A_1| + A_2|}{F} = \frac{2A_1}{F}
$$

$$
Ra = \frac{2a}{F}[b \arcsin \sqrt{\frac{2bc - c^2}{b}} - \frac{b - c}{b} \sqrt{2bc - c^2}]
$$

Where

$$
c = b\left[1 - \frac{1}{2}\sqrt{1 - \frac{F2}{4a2}} - \frac{a}{F}\arcsin\frac{F}{2a}\right]
$$

Fig. A.1 The basic property of the ellipse

 $(A.7)$

Fig. A.2 The machined surface using the new model

Fig A.3 The area above the mean line

Tlable A.1

Fig. A.4. The area below the mean line

