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Transient, Two-Dimensional Heat Conduction Model for Rewetting a Hot Vertical Plate Surface by a Falling Liquid Film.

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TRANSIENT, TWO-DIMENSIONAL HEAT CONDUCTION MODEL FOR REWETTING A HOT VERTICAL PLATE SURFACE BY A FALLING LIQUID FILM

نموذج نظرى للتوصل الحرارى الانتقالى ثنائى الابعاد للوح رأسى ساخن أثناء تبريد سطحه بفلم سائل

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ملخص: فى هذاالبحث تم وضع نموذج نظرى لمشكلة التوصل الحرارى الغير مستقرفى بعدى لوح رأسى ساخن يبرد من احد أسطحه الرأسية بفلم من السائل يسقط عليه. . فى هذا النموذج تم استبدال الاحداثيات الثابته بأحداثيات متحركة مع مقدمة فلم السائل؛ بذلك تم تحويل مسألة التوصيل الحرارى من النوع الانتقالى الى النوع المستقر الابسط فى الحل. النموذج الرياضى تم وضعه فى صورة لابعديه أمكن حلها عدديا (Numerical) ؛ أظهرت النتائج تأثير كل من رقم بويت (Boit Number) ودرجة الحرارة المبدئية للوح على سرعة الابتلال لسطح اللوح بالسائل (Rewetting Velocity) .كما اظهرت النتائج ان الحل ذو البعد الواحد غير دقيق، هذا وقد أمكن تمثيل النتائج العددية بدقة عاليه بمعادلة بسيطة وصريحه لحساب سرعة إبتلال فلم السائل لسطح اللوح الساخن ، وقد تبين من مقارنة الحسابات النظرية للمعادلة المستنبطة بقياسات معملية أن التجاوز فى الحسابات بسيط (أقل من ٥٪) بينما الخطأ فى الحل ذو البعد الواحد كبير (أكثر من ٣٠٪) .

Abstract

A transient, two-dimensional heat conduction model for rewetting a hot plate surface by a falling liquid film is developed. It is assumed that the plate thermal properties are independent on temperature, and the heat transfer coefficient is a constant value in the wet region and zero in the dry region. The mathematical model equations are transformed to a dimensionless form solved numerically . The results indicate that the heat conduction problem in the rewetting process is substantially two-dimensional one, not only for thick plates, i.e., a large Boit number, but also for a low initial wall temperature. An explicit simple formula for predicting the rewetting velocity is obtained by correlating the numerical results. Comparisons with other studies are made.

1. Introduction

The rewetting of a vertical surface by a falling liquid film is a heat transfer process of fundamental importance in many engineering applications such as refrigeration and distillation. In recent years, this process has had a great interest due to its importance to the emergency core cooling of nuclear water reactors in the event of postulated loss-of-coolant accidents (LOFCAs) [1].

The rewetting phenomenon refers to the establishing of a liquid contact with a hot solid surface at an initial temperature above the so called rewetting or sputtering temperature, which is defined as the temperature up to which the liquid may wet the solid

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surface [2].Simple schematic illustration for the rewetting of a hot surface is shown in Fig. 1. In this scheme, a hot vertical plate at an initial temperature T_w, above the rewetting temperature T_{wet}, is cooled by a falling liquid film at saturation temperature T_s. The film front. which is accompanied by violen boiling and sputtering, moves along the surface at approximately constant velocity. The plate surface temperature at the front point is equal to the rewetting temperature T_{wet}. Behind the front, in the wet region, the surface temperature drops rapidly to the film saturation temperature due to the high heat transfer rate in this region. In turn, ahead of the wet front, in the dry region, the solid surface experiences very poor heat transfer by vapour or vapour-mist flow. Therefore, the axial heat conduction in the wall from the hot dry to the cooled wet portion, at the film front, is very effective and plays an important role in the rewetting process [3]. This explanation is based on results of several previous experiential studies [1-4].

A large number of numerical and analytical models for predicting the rewetting velocity have been reported in the literature. The most common concept adopted to analyze the heat conduction problem in rewetting a hot surface is the dividing of the surface into two regions : the wet and the dry regions, separated by the moving film front.

Thompson [5] proposed two-dimensional-heat-conduction numerical model for the rewetting of the outer surface of a vertical hot tube. He assumed that the heat transfer coefficient in the wet region is proportional to the cube of wall superheat, and zero in the dry region. Dua and Tien [6] developed two-dimensional numerical model considering the poor heat transfer in the dry region. Their results showed a negligible effect of this consideration. Assuming that the heat transfer coefficient is a constant value in the wet region and zero in the dry region, Duffey and Porthouse [8] derived an analytical two-dimensional solution (briefly described in ref. [11]) for predicting the rewetting velocity. However, the complexity in calculating this implicit solution makes it impracticable.

Various one-dimensional models for predicting the rewetting velocity have been proposed in the literature. Elias and Yadigaroglu [9], and, recently, Frik [2] have developed one-dimensional numerical model. In such a model, the wet and dry regions are divided into several parts of constant heat transfer coefficients. Yamanouchi [10] derived an explicit analytical solution for the rewetting velocity assuming that the heat conduction in the wall is one-dimensional. Recently, Castiglia et. al. [11] have modified Yamanouchi's solution by introducing empirical formulas for calculating the rewetting temperature and wet-ragion heat transfer coefficient. The constant coefficients of those empirical relations were determined and adjusted so that the predictions coincide with experimental data. Four data sets from four different sources were used. Unfortunately, no general correlation for predicting all those data with acceptable accuracy could be recommended, however, four correlations; one for each data set, were proposed.

In this paper, two-dimensional heat conduction model for the rewetting of a hot vertical plate surface by a falling saturated liquid film is developed. It is assumed that the heat transfer coefficient is constant in the wet region and zero in the dry region. This model is solved numerically. The obtained results are correlated by a simple formula for calculating the rewetting velocity. The solution is compared with other studies results.

2. Physical Model

Consider an infinite-long hot plate of thickness Δ and at an initial temperature T_{iw}. This plate is cooled from one face by a falling saturated-liquid film. The back face is assumed insulated. The physical model is schematically described in Fig.1.

The mechanisms controlling the rate of rewetting the hot plate surface are the rate of heat conduction in the x-direction of the wall; from the dry to the wet portion, and the removal of this heat by the violent boiling in the narrow wet-front zone. Assuming constant thermal properties, the transient two-dimensional heat-conduction equation in the plate wall is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \qquad (1)$$

where x and y are the cartesian coordinates referred to an origin at the left corner of the upper plate end, τ is the time coordinate, and α is the thermal diffusivity of the plate material.

Equation (1) is assumed to be subject to the following boundary conditions :

$$\lim_{x \to -\infty} T(x,y) = T_s \qquad 0 \le y \le \Delta \qquad (2)$$

$$\lim_{x \to +\infty} T(x,y) = T_{iw} \qquad 0 \le y \le \Delta \qquad (3)$$

$$T(x_f,0) = T_{wet} \qquad (4)$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = \begin{cases} h(\,T(x,0) - T_s\,)/k & x \le x_f \\ 0 & x > x_f \end{cases} \qquad (5)$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=\Delta} = 0 \qquad -\infty < x < \infty \qquad (6)$$

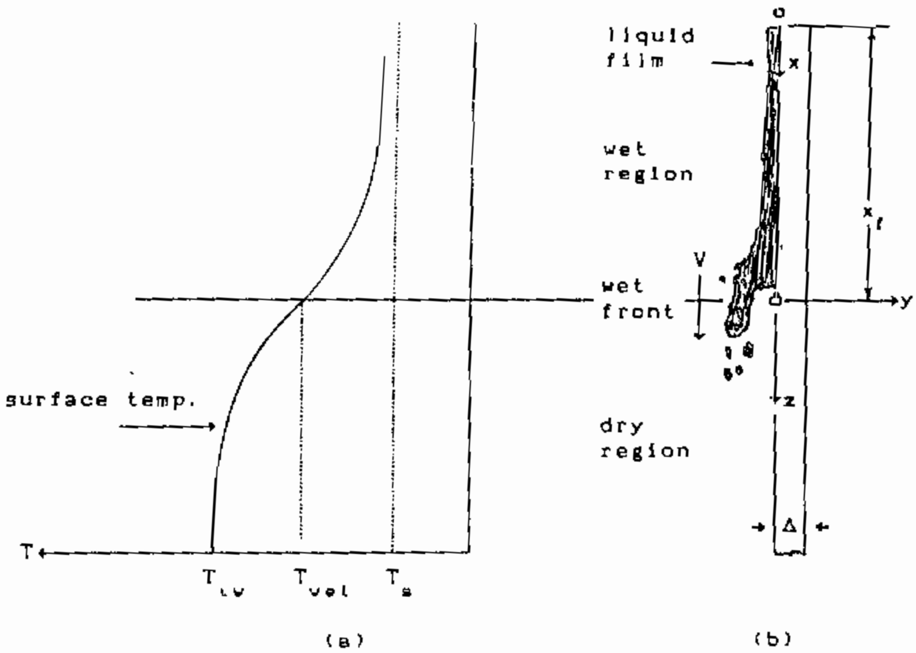

Fig. 1. Schematic illustration of rewetting a hot surface
(a) Temperature profile (b) wet-front moving

where T_{wet} is the rewetting temperature, k is the plate thermal conductivity and h is the wet-region heat transfer coefficient, which is considered to have a constant value. x_f is the wet front position related to the fixed coordinates x & y. The heat transfer coefficient in the dry region is neglected and assumed of zero value.

In several previous experimental studies [1-4], it has been observed that the wet front velocity, V, moves at approximately constant velocity. Utilizing this experimental result, the problem is simplified by relating the x and τ variables by a variable z so that a set of coordinates : y and z, move with the film front. The relation between x and z is defined by :

$$z = x - V\tau \quad (7$$

Using the above relation, the space and time gradients are related as :

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial^2 T}{\partial z^2} \quad (8); \qquad \frac{\partial T}{\partial \tau} = - V \frac{\partial T}{\partial z} \quad (9$$

Substituting relations (8) and (9) into equations (1)-(6), one gets :

$$\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2} = -\frac{V}{\alpha}\frac{\partial T}{\partial z} \qquad (10)$$

$$\lim_{z \to -\infty} T(z,y) = T_s \qquad 0 \le y \le \Delta \qquad (12)$$

$$\lim_{z \to +\infty} T(z,y) = T_{iv} \qquad 0 \le y \le \Delta \qquad (13)$$

$$T(\delta,0) = T_{vot} \qquad (14)$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = \begin{cases} h(T(z,0) - T_s)/k & z \le \delta \\ 0 & z > \delta \end{cases} \qquad (15)$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=\Delta} = 0 \qquad -\infty < z < \infty \qquad (16)$$

Consider the following dimensionless parameters :

$$z^+ = \frac{z}{\Delta},\quad y^+ = \frac{y}{\Delta},\quad Bi = \frac{h\Delta}{k},\quad V^+ = \frac{\Delta}{\alpha}V,\quad T^+ = \frac{T - T_s}{T_{vot} - T_s} \quad \text{and} \quad T^+_{iv} = \frac{T_{iv} - T_s}{T_{vot} - T_s} \qquad (17)$$

where Bi is the Biot number. Using these dimensionless parameters, equations (10) to (16) are respectively transformed to the following dimensionless forms :

$$\frac{\partial^2 T^+}{\partial z^{+2}} + \frac{\partial^2 T^+}{\partial y^{+2}} = -V^+ \frac{\partial T^+}{\partial z^+} \qquad (18)$$

$$\lim_{z^+ \to -\infty} T^+(z^+,y^+) = 0 \qquad 0 \le y^+ \le 1 \qquad (19)$$

$$\lim_{z^+ \to +\infty} T^+(z^+,y^+) = T^+_{iv} \qquad 0 \le y^+ \le 1 \qquad (20)$$

$$T^+(\delta,0) = 1 \qquad (21)$$

$$\left.\frac{\partial T^+}{\partial y}\right|_{y^+=0} = \begin{cases} Bi\ T^+(z^+,0) & z^+ \le \delta \\ 0 & z^+ > \delta \end{cases} \qquad (22)$$

$$\left.\frac{\partial T^+}{\partial y^+}\right|_{y^+=1} = 0 \qquad -\infty < z^+ < \infty \qquad (23)$$

Equations (18)-(23) are the mathematical formulation of the problem which has to be solved next.

3. Numerical Results and Discussion

For solving the above problem by the numerical finite difference method, it is impossible to use an infinite-long plate, and a finite length of $2L^+$ must be defined. This length has to be selected sufficiently long such that the axial heat conduction in the plate wall near its ends is negligible compared to that in the wet front region.

Dividing the plate wall along z^+- and y^+-directions, as it is shown schematically in Fig. 2, equation (18) in finite-difference form is written as :

$$\frac{1}{\Delta z^+}\left[\frac{T^+_{i+1,j}-T^+_{i,j}}{\Delta z^+}-\frac{T^+_{i,j}-T^+_{i-1,j}}{\Delta z^+}\right]+\frac{1}{\Delta y^+}\left[\frac{T^+_{i,j+1}-T^+_{i,j}}{\Delta y^+}-\frac{T^+_{i,j}-T^+_{i,j-1}}{\Delta y^+}\right]=-V^+\left[\frac{T^+_{i+1,j}-T^+_{i,j}}{\Delta z^+}\right] ; \quad (24)$$

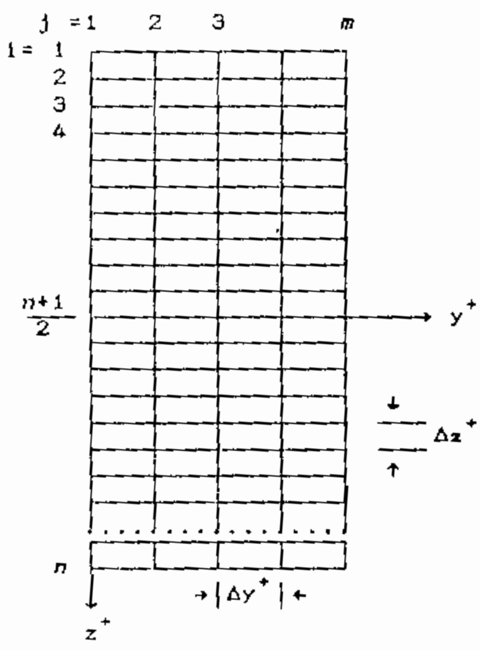

Fig. 2. Nodal net for 2-D. solution.

where i, j are the indices of the grid nodes in the z^+ - and y^+-directions, respectively. Equation (24) can be rearranged as

$$T^+_{i,j} = A\,T^+_{i-1,j} + B\,T^+_{i+1,j} + C\,(T^+_{i,j+1} + T^+_{i,j-1}); \quad (25)$$

where the constant coefficients A, B and C are defined by :

$$A=1/(2+2(\Delta z^+/\Delta y^+)^2+V^+\Delta z^+),\; B=A(1+V^+\Delta z^+) \text{ and } C=A(\Delta z^+/\Delta y^+)^2 \quad (26)$$

Equation (25) applies for the inner nodes (i,j) of $j = 2,3,..,m-1$ and $i = 2,3,....,n-1$.

The boundary conditions (19)-(21) are respectively written in the following finite difference forms :

$$T^+_{1,j} = 0; \qquad \text{for } j= 1,2,...., m \quad (27)$$

$$T^+_{n,j} = T^+_{iv}; \qquad \text{for } j= 1,2,...., m \quad (28)$$

$$T^+_{\frac{n+1}{2},1} = 1. \quad (29)$$

Calculating the heat balance for a boundary node (i,m) at the insulated surface of $y^+=1$, this yields

$$\left(\frac{\Delta z^+}{\Delta y^+}\right)^2\left[T^+_{i,m-1} - T^+_{i,m}\right] + \frac{1}{2}\left[T^+_{i-1,m} - T^+_{i,m}\right] + \frac{1}{2}\left[T^+_{i+1,m} - T^+_{i,m}\right] + \frac{\Delta z^+}{2}V^+\left[T^+_{i+1,m} - T^+_{i,m}\right]; \quad (30)$$

The above result can be formulated as :

$$T^+_{i,m} = AT^+_{i-1,m} + BT^+_{i+1,m} + 2CT^+_{i,m-1}; \quad \text{for } i = 2,3,....,n-1 \quad (31)$$

where the coefficients A, B and C are the same ones defined in equation (26). Similarly, calculating the heat balance for a wet-boundary node (i,1) gives

$$T^+_{i,1} = A_1T^+_{i-1,1} + B_1T^+_{i+1,1} + 2C_1T^+_{i,2}; \qquad i = 2,3,..,(n-1)/2 \quad (32)$$

where the coefficients A_1, B_1, and C_1 are defined by

$$A_1 = 1/(\;2 + 2(\Delta z^+/\Delta y^+)^2 + V^+\Delta z^+ - 2(\Delta z^+)^2/\Delta y^+\; Bi\;),$$

$$B_1 = A_1(1 + V^+\Delta z^+) \qquad \text{and} \qquad C_1 = A_1(\Delta z^+/\Delta y^+)^2 \quad (33)$$

Setting Bi = 0 in equation (32) yields the relationship for the dry-boundary nodes (i,1) of $i = (n+3)/2,(n+5)/2,....,(n-1)$.

So far, a system of linear algebraic equations has been obtained, which could be solved for given initial wall temperature, T_{iv}, and Biot number, Bi, to compute the rewetting velocity V^+. Alternatively, equation (29) is omitted and a guess on V^+ is made. If the resultant value of $T^+_{(n+1)/2,1}$ deviates from 1, a new guess on V^+ is made, and the procedure is repeated, until a sufficient accuracy is achieved. This approach has been followed in this paper. A computer progrem has been constructed, in which a subroutine called LEQT2F from the IMSL MATH/PC-LIBRARY for solving the linear system of equatione was introduced. Preliminary tests were performed to evaluate the effect of the grid dimensions on the accuracy and stability of solution. Finally, a square grid with $\Delta z^+ = \Delta y^+ = 0.1$ has been chosen. The number of nodes was 10 and 499 in the y^+- and z^+-directions, respectively. Numerical results for Boit number ranging from 0.1 to 200 and dimensionless initial wall temperature range from 1.03 to 5 have been obtained. These parameters renges are relevant to the rewetting conditions of a nuclear reactor core during postulated LOFCAs [2].

Examples of these numerical results are illustrated in graphs (3) to (4), in which the results are represented by fit or smooth curves. Figure (3) displays the behavior of the rewetting velocity, V^+, versus the initial wall temperature, T^+_{iv}; for five different values of Biot number, Bi. It is found that for a certain Biot number, the rewetting velocity decreases with the increase in the initial wall temperature and vice versa. In turn, the results of Fig. (4) indicate that the rewetting velocity is higher for larger Biot number. As a conclusion, the rewetting velocity is proportional to Biot number and inversely proportional to the initial wall temperature. These results are consistent with the findings of previous experimental studies [1-4].

Yamanouchi [10] derived an analytical, one-dimansional solution for a thin plate. This solution, in the form of the above-defined dimensionless parameters, reads as :

$$V^+ = \left[\frac{Bi}{T^+_{iv}(T^+_{iv}-1)} \right]^{0.5} \tag{34}$$

Comparison of the above one-dimensional solution with the current two-dimensional solution is shown in Fig. (5); for three different values of the dimensionless initial wall temperature, T^+_{iv}. The one-dimensional solution is represented by the dashed line in the graph. It is evident that for certain initial wall temperature, the difference betwsen the two solutions is bigger for larger Biot number, especially at low initial wall temperature. However, in most cases of Biot number > 0.1 and dimensionless initial wall temperature above 4.0, reasonable agreement between both solutions is achieved. This means that Biot number is not only the proper criterion which defines the boundary

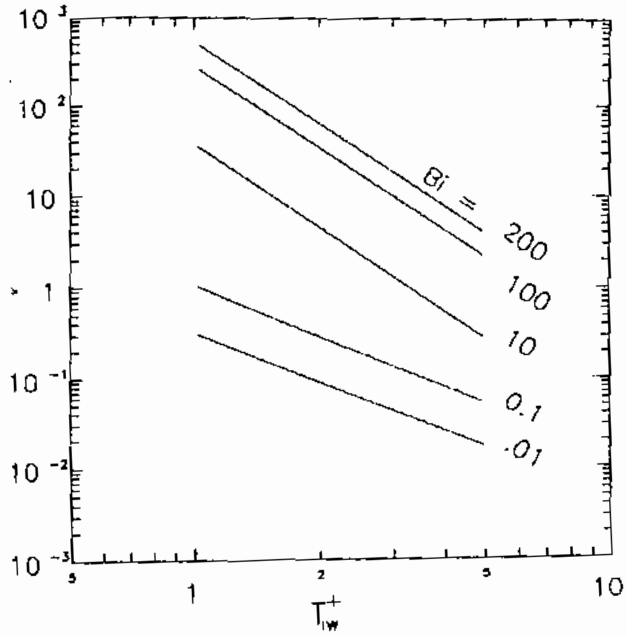

Fig. 3. Predicted rewetting velocity versus initial wall temperature.

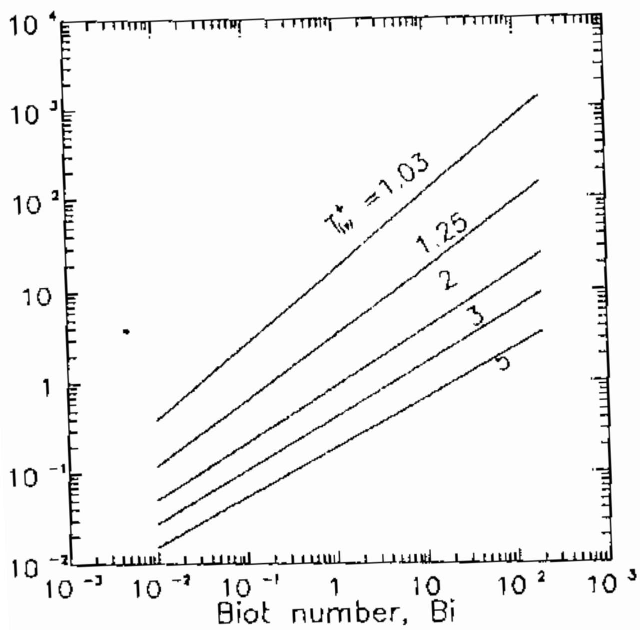

Fig. 4. The wet-front velocity versus Biot number.

between the one- and two-dimensional solutions. To clarify this as well as define this boundary between the two solutions, the obtained numerical results are plotted in Fig. 6; in terms of V^+ versus $[Bi/(T^+_{iw}(T^+_{iw}-1))]^{0.5}$, i.e., in terms of the two parameters of the one-dimensional solution. It is found that: for $Bi/(T^+_{iw}(T^+_{iw}-1))\leq 1$, one-dimensional solution (34) is appropriate and predicts the numerical results with relative rms error of ± 3 %. However, for $Bi/(T^+_{iw}(T^+_{iw}-1)) > 1$ the numerical results could be smoothed using the least squares procedure, by a fit formula with relative rms error of ± 5%. This formula is :

$$V^+ = 0.72 \frac{Bi^{0.85}}{(T^+_{iw}(T^+_{iw}-1))^{0.9}} \; ; \qquad \xi > 1 \qquad (35)$$

where $\xi = Bi/(T^+_{iw}(T^+_{iw}-1))$

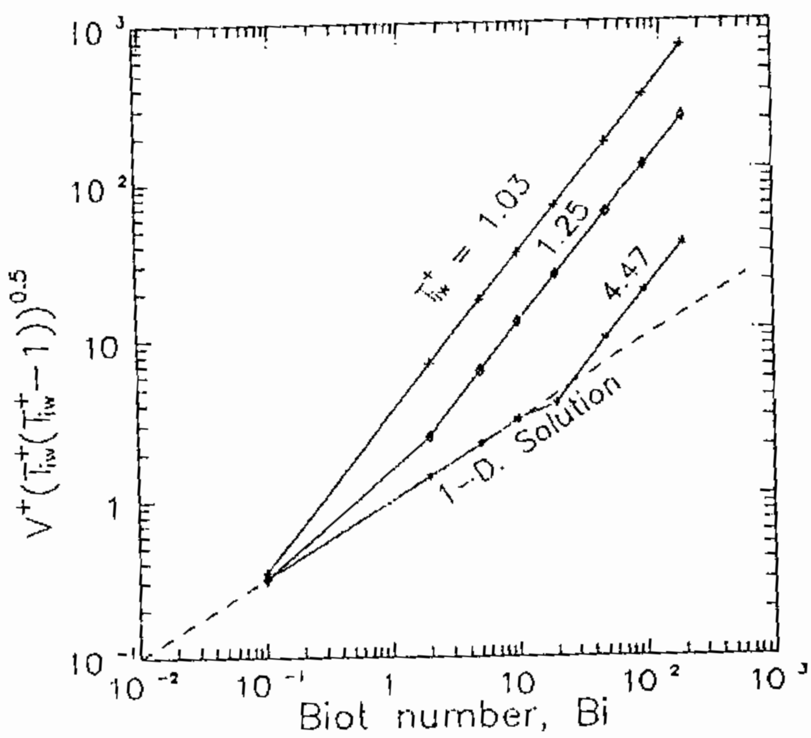

Fig. 5. Comparison of one- and two-dimensional solutions.

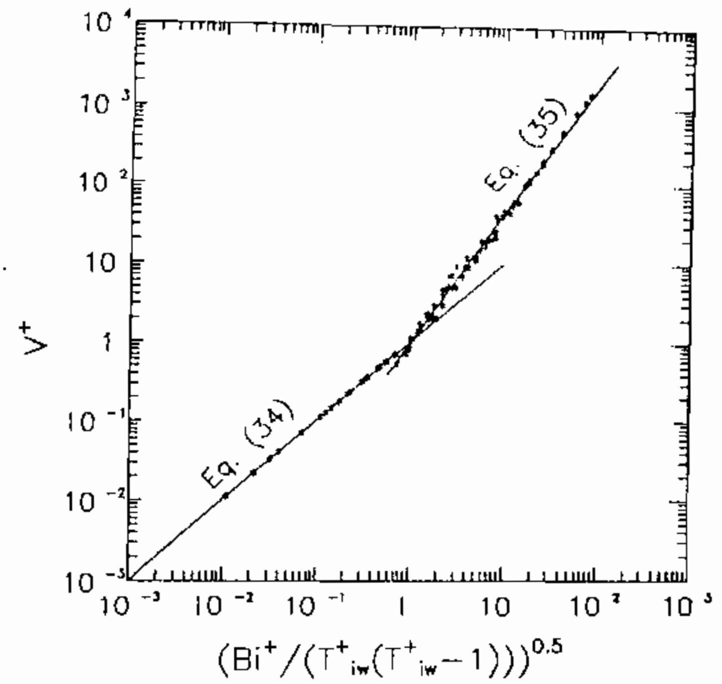

Fig. 6. Correlating numerical data predicted by the model.

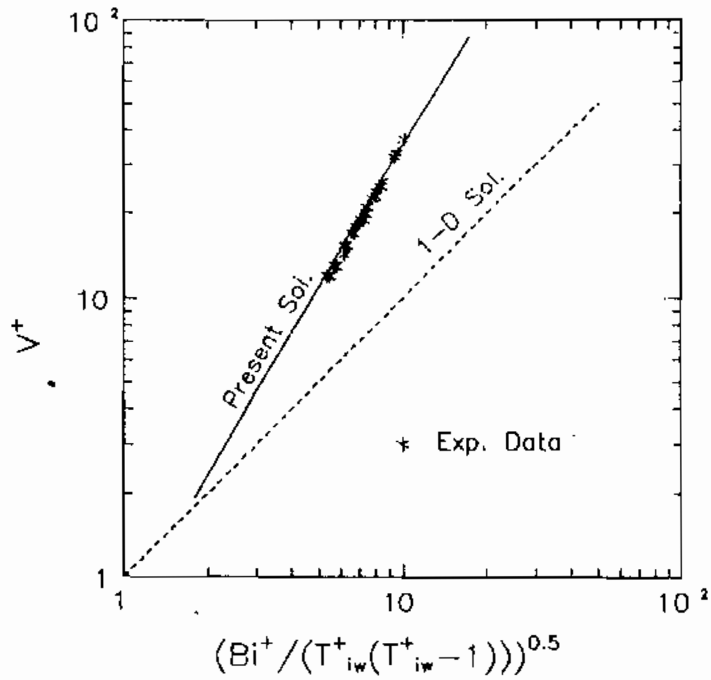

Fig. 7. Comparison of the present solution with experimental data from ref. [12].

Comparison of the present solution with measured data [12] is plotted in Fig. 7. These data (tabulated in ref. [5]) consists of 26 measured points representing 26 experimental runs, conducted on rewetting a hot vertical stainless steel tube (thickness =0.064" and outer diameter =.5"). by a falling water film. The comparison indicates that the present solution (35) predicts the experimental data with rms error of ± 6%. However, one-dimensional solution (34) predicts the same data with rms error of ± 310 %.

4. Main points

1 The two-dimensionless modelling of the transient heat conduction problem in a rewetting process is appropriate to highlight the effects of main relevant parameters affecting this phenomenon

2 The numerical data obtained, could be fitted by a simple explicit correlation for predicting the rewetting velocity with acceptable accuracy

3 One-dimensional modelling is inadequate to treat the problem.

Nomenclature

A,B & C	constants, see eq. (26)
A_1, B_1, C_1	constants, see eq. (33)
Bi	Biot number, $(h\Delta/k)$
i	nodes index in z^+-direction
j	nodes index in y^+-direction
k	thermal conductivity of plate material
L	half of plate length
L^+	half of dimensionless plate length, (L/Δ)
h	convective heat transfer coefficient
m	total number of nodes in y^+-direction
n	total number of nodes in z^+-direction
rms	root mean square error
T	temperature
T^+	dimensionless temperature, $(T - T_s)/(T_{wet} - T_s)$
T_{iw}	initial wall temperature
T^+_{iw}	dimensionless initial wall temp., $(T_{iw} - T_s)/(T_{wet} - T_s)$
T_s	saturation temperature
T_{wet}	rewetting temperature
V	rewetting velocity
V^+	dimensionless rewetting velocity, $(\Delta V/\alpha)$
x,y	fixed cartesian coordinates with the origion (0,0) at the left corner of the upper plate end
x_f	wet front position referred to fixed coordinates x,y
z,y	moving cartesian coordinates with the origion (0,0) at the wet fromt
z^+, y^+	dimensionless moving cartesian coordinates, $(z/\Delta,\ y/\Delta)$

Greek symbols

τ	time
α	thermal diffusivity of plate material
Δ	plate thickness
Δy^+	dimensional grid size in y-direction
Δz^+	dimensional grid size in z-direction

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