

9-1-2021

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### Recommended Citation

Samir, Fatma; Ibrahim, Ayman; and Shaheen, Samir (2021) "Coarse Segmentation of Textured Images Using Variance Analysis.," *Mansoura Engineering Journal*: Vol. 22 : Iss. 3 , Article 7.

Available at: <https://doi.org/10.21608/bfemu.2021.150960>

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## Coarse Segmentation of Textured Images Using Variance Analysis

التجزئى البنىوى للصور باستخدام تحليل التباين

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### ملخص البحث

يتناول البحث تجزئة منظر يحتوى على عدة بنىات اى بنىات متمايزة مع عدم وجود معلومات مسبقة عن ١- عدد البنىات التى يحتوى عليها المنظر. ٢- خصائص واتجاهات هذه البنىات. ٣- أماكن المناطق التى تحتلها هذه البنىات . وأستخدمنا فى هذا البحث نماذج ماركوف للحقول العشوائيه الجاوسيه GMRF لتمذجه الصور وقد ابتكرنا طريقة جديدة لتقدير عدد البنىات وأماكنها. فنقسم الصور إلى عدد من الكتل المربعة غير المتداخلة مع إفتراض تجانس كل كتلة منها ( أى تكويها من بنىة واحدة ) ويستخدم مقيس جديد للمسافات بناء على تحليل التباين بفرض تجميع الكتل المتماثلة

وبمقارنه الطريقة الجديدة للتجزئى بالطرق المعروفة والمنشوره تبين تفوق الطريقة الجديدة من حيث السرعة النسبيه وتحسين عمليه التجزئى للتقريبى ومن ثم زيادة دقة تقدير بارامترات النموذج المستخدم مما يعكس على دقة التجزئى البنىاتى

### ABSTRACT

This paper presents a novel approach for the segmentation of a textured scene. The algorithm is image-based, no specific model is assumed for the image. Also, no *a priori* knowledge about the different texture regions, neither their number, nor their behavior is assumed. The algorithm partitions the image into small disjoint square windows, and the variance for each window data is calculated. Then the *K-means* clustering algorithm is applied upon these windows. Together with the *K-means* algorithm, a new distance measure has been defined. This new distance measure was deduced from a statistical test known as *Bartlett's test* based on the variance of the windows data. The same statistical test has also been applied but in a different fashion to determine the number of different textures in the image. The new image-based distance measure has been tested and compared to a model-based *Euclidean* distance measure, with each window modeled by a *non-causal Gaussian Markov Random Field* (GMRF). The results of the comparison have shown that the new distance measure is much simpler and faster, while yielding to a still robust and effective segmentation.

Accepted July 30, 1997

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## 1. Introduction

The objective of texture segmentation is to separate an image into different regions of homogeneous behavior. It is an early processing task necessary to many applications such as: segmentation of scenes into distinct objects or regions, classification or recognition of surface materials, computation of surface shapes and orientation, motion analysis, and image data compression.

Textures are generally defined according to one of two different aspects, namely structural and statistical. From the structural point of view, textures are formed by micro structures or grains, which are primitives known as *texels*, and an accompanying grammar for the placement and reproduction of these texels. Obviously, this approach restricts to quite regular patterns, as it is highly difficult to determine either the texels, or the grammar for non-regular textures. However, from the statistical point of view, texture is defined as a pattern represented by a set of statistical measurement features. Because of the random nature of natural textures, many researchers focused on the latter approach and applied the stochastic models for the image representation. Special interest was directed towards the *Simultaneous Auto Regressive models (SAR)* [1-7] and the *non-causal Markov Random Field models (MRF)* [2-13]. Also, the *Gibbs Distribution (GD)* received much attention [8].

This interest towards random field models encouraged many researchers to base their texture segmentation algorithms upon these models. *Derin and Elliott* [14], and [15] made use of the Gibbs distribution, while [5, 16-19] used the MRF models and [5, 20, 21] exploited the SAR models.

To achieve good segmentation using the above models, the segmentation should be initiated with sufficient information about the number of textures in the image, and their model parameters. This problem can be solved if an algorithm is defined to coarsely segment the image (segmentation at a low resolution). With a coarsely segmented image, we can define the number of different textures in the image, their occupied locations and therefore the model parameters. Obviously, this algorithm should be simple and fast to initiate a fine segmentation algorithm with a minimized overhead.

*Fung et al.* [16] proposed an algorithm that performs coarse segmentation. Their algorithm is constructed within the framework of a split and merge paradigm. It is based on the GMRF together with a likelihood ratio test to guide the split and merge process. The algorithm proposed by *Fung et al.* leads to good results, but is too exhaustive to be used prior to a fine segmenter.

Since many problems of image analysis are related to human visual perception, it will be quite useful to review a model of human texture perception. Texture research has been mostly driven by *Julesz'* conjecture that *the human visual system cannot discriminate between textures differing only in third and higher-order statistics* [22]. Although *Gagalowicz* [1] presented several counterexamples to the *Julesz'* conjecture, he pointed-out that these counterexamples are very difficult to generate, and that they are only faintly discriminable, and thus concluded that the *Julesz'* conjecture seems to provide a useful bound for human visual texture discrimination. As a direct consequence

of this conjecture, second-order statistics are frequently used as effective features for texture classification and discrimination. The second order moment (or simply the sample variance) is obviously one of the simplest second order statistics that can be used.

This paper presents a novel approach to coarsely segment a textured scene in an unsupervised fashion. The algorithm starts by partitioning the image into a number of relatively small disjoint windows, and the variance is calculated for the window data. Because of their small size, most of the windows are homogeneous. We apply a well-known vector quantization algorithm known as the *K-means* clustering algorithm upon the windows to cluster them in  $K$  different classes (or textures). Together with the *K-means* algorithm, a new distance measure has been defined. This new distance measure was deduced from a statistical test known as *Bartlett's test* [23] based on the variance of the windows data. The same statistical test has been applied but in a different fashion to determine the number of different clusters formed.

The algorithm results in a coarse segmented image, where the smoothness of the boundaries depends on the size of the used windows.

The coarse segmentation can be used in applications where it is required to estimate the texture locations, which can also be used to model the textures according to some appropriate models. The algorithm also provides a good initiation for finer segmentation algorithms with a minimized overhead.

This paper involves (1) The coarse segmentation of textured scenes in an unsupervised environment, and (2) The definition of a new distance measure based on a statistical test to be used to discriminate between textures.

The organization of the paper is as follows : Section II introduces the new distance measure. Section III defines the proposed segmentation algorithm. Section IV presents a comparison of the time complexity of the proposed algorithm with other similar algorithms. Section V gives the segmentation and comparison results. Section VI is the conclusion with the recommended perspectives.

## II. The New Distance Measure

To be useful, a distortion measure must be tractable, so that it can be analyzed and computed, and subjectively relevant, so that differences in distortion values can be visually discriminative. From [24], any *distance function*  $d(x, y)$  can be accepted as having the qualities of a distance measure if it satisfies the following rules:

$$\begin{aligned} d(x, x) &= 0, \\ d(x, y) &> 0 \quad \text{for } x \neq y, \\ d(x, y) &= d(y, x), \\ d(x, y) + d(y, z) &\geq d(x, z) \end{aligned} \quad (1)$$

Our new distance measure is applied on scalar data. It is based on the variances of the compared data sets. It is often important to know with some degree of certainty whether the variances of two data sets are the same, i.e. they do not differ significantly; such variances are said to be *homogeneous*. We made use of a statistical test known as *Bartlett's test* [23]. This test is applied to test the homogeneity of two or more variances.

*Bartlett's test* is a special application of the well-known  $\chi^2$  test, in which we compare the difference between the total number of degrees of freedom times the natural logarithm of the pooled estimate of variance and the sum, extended over all samples, of the product of the degrees of freedom and the natural logarithm of the estimate of variance. Thus, if  $n_i$  is the sample size,  $s_i^2$  is the estimate of variance from sample  $i$ ,  $k$  is the number of samples, and  $\bar{s}^2$  is the pooled estimate of variance, then the *Bartlett's test* requires the calculation of

$$x^2 = \frac{1}{c} \left[ v \ln \bar{s}^2 - \sum_{i=0}^{k-1} (v_i \ln s_i^2) \right] \quad (2)$$

where

$$s_i^2 = \frac{1}{n_i - 1} \sum_0^{n_i-1} (x - \bar{x}_i)^2 \quad \text{and} \quad \bar{x}_i = \frac{1}{n_i} \sum_0^{n_i-1} x \quad (3)$$

$$v = \sum_{i=0}^{k-1} v_i \quad (4)$$

where  $v_i$  is the number of degrees of freedom of sample  $i$  ( $v_i = n_i - 1$ )

$$\bar{s}^2 = \frac{1}{v} \sum_{i=0}^{k-1} v_i s_i^2 \quad (5)$$

and

$$c = 1 + \frac{1}{3(k-1)} \left[ \sum_{i=0}^{k-1} \left( \frac{1}{v_i} \right) - \frac{1}{v} \right] \quad (6)$$

The distance measure we are proposing will be the  $\chi^2$  test, calculated for two variances ( $k=2$ ) such that

$$d(x, y) \equiv \chi^2 \Big|_{s_0^2 = s_x^2, s_1^2 = s_y^2, v_0 = n_x - 1, v_1 = n_y - 1, k=2} \quad (7)$$

It can be easily shown that this new distance measure complies well with the required properties in (1).

### III. The Segmentation Procedure

The proposed segmentation algorithm begins by partitioning the image into a number of relatively small disjoint windows. The variance for each window data is calculated. Then, we apply the *K-means algorithm*, with the new distance measure defined in (7).

As previously discussed, the proposed algorithm works in an unsupervised environment, in which the number of textures in the image is unknown. Therefore, when applying the *K-means algorithm*, we are faced with the problem of determining the number of clusters  $K$  we want to be formed.

To determine the number of clusters (textures) in the image, we begin the segmentation process by assuming the image to be composed of only two textures, and apply the *K-means algorithm*. Then, we apply a statistical test based on the  $\chi^2$  test defined in (2) to determine whether the formed clusters are homogeneous or not to some *pre-defined level of significance*. If the clusters are found to be not homogeneous, we repeat the *K-means algorithm* while enabling it to form one more cluster. This procedure is repeated until we first find that two (or more) of the formed clusters are homogeneous. Finding that some of the formed clusters are homogeneous gives an indication that the number of formed clusters is greater than the actual number of textures in the image. Thus we can determine that the actual number of textures in the image is equal to the number of clusters formed at the previous stage.

#### The Statistical Test Used to Guide the K-Means Algorithm

##### A. Determining The Homogeneity of Two Clusters

To determine whether two clusters  $i$  and  $j$  are homogeneous or not, we are going to test the homogeneity of their variances using the *Bartlett's test* by computing the  $\chi^2$  measure defined in (2). We adopt the classical procedure for testing statistical hypotheses [23]:

- 1- Define the null hypothesis  $H_0$  and the alternative hypothesis  $H_a$ 
  - $H_0$  : the two variances are not significantly different (homogeneous).
  - $H_a$  : the two variances are significantly different.
- 2- Calculate the test statistic ( $\chi^2$  measure) for the pre-defined number of degrees of freedom ( $k=2$ , then degrees of freedom = 1)

- 3- Define the significance level  $\alpha$  for the test ( $P[\chi^2 > \chi_\alpha^2] = \alpha$ ), and accordingly determine from the  $\chi^2$  tables the value  $\chi_\alpha^2$  for the specified number of degrees of freedom (in our case, number of degrees of freedom =1).

The significance level  $\alpha$  is a measure of the probability that the difference between the two variances is due to chance alone.

- 4- Perform the test by comparing  $\chi^2$  and  $\chi_\alpha^2$ .

If the calculated value of  $\chi^2$  exceeds  $\chi_\alpha^2$ , then, the probability that the difference between the two variances is due to chance alone is even lower than the significance level  $\alpha$ , and we are justified in rejecting the null hypothesis (and accepting the alternate hypothesis that the two variances are significantly different).

#### B. The Proposed Algorithm

- 1- Partition the image into a number of relatively small disjoint square windows, and calculate the variance for each window data.
- 2- Initialize the *number of clusters* to 2.
- 3- Calculate the *cluster centers*.
- 4- Apply the *K-means* algorithm using the distance measure defined in (7).
- 5- Test the homogeneity of the variances for the formed clusters for a pre-defined significance level. If they are not significantly different go to 7.
- 6- Store the clustering results (windows classes), increment the current *number of clusters* by 1 and go to 3.
- 7- Actual *number of clusters* = current *number of clusters* - 1, retrieve the last clustering results.
- 8- *Segmentation* is done.

#### IV. Time Complexity and Storage of the Proposed Algorithm:

In this section, we are going to compare our image-based distance measure to a model-based distance measure based on the *Gaussian Markov Random Field Model* (GMRF), from the point of view of the *required computations* and *storage*. We have chosen the GMRF model as it is extensively used in the recent literature. The 4<sup>th</sup> order GMRF has shown to be robust for the synthesis of natural textures, and therefore we are going to use it in our comparison.

As previously stated, the image is partitioned into small  $M \times M$  disjoint square windows.

Let  $\Omega$  denote the set of grid points of the  $M \times M$  square window such that:

$$\Omega = \{s = (i, j): 0 \leq i, j \leq M - 1\}$$

and let  $\{y(s)\}$  denote a random field with  $y(s)$  the field at point  $s$  representing the zero-mean gray level.

Using the 4<sup>th</sup> order GMRF, it is needed to calculate for each window, a feature vector  $\Phi = (\hat{\Theta}, \hat{\sigma}^2, \hat{\mu})$  where  $\hat{\mu}$  is the sample mean, and  $\hat{\Theta}$  and  $\hat{\sigma}^2$  are respectively the GMRF parameters and the mean-square error and are defined as follows

$$\text{Let } Q(s) = \text{col}[y(s+r) + y(s-r), r \in N_s] \quad \text{and}$$

$$N_s = \{(0, 1), (1, 0), (1, 1), (-1, 1), (0, 2), (2, 0), (1, 2), (2, 1), (-1, 2), (-2, 1)\}$$

then the *least-square estimates*  $\hat{\Theta}$  of the parameters are defined as :

$$\hat{\Theta} = \left[ \sum_{s \in \Omega} Q(s) Q^T(s) \right]^{-1} \left[ \sum_{s \in \Omega} Q(s) y(s) \right]$$

and the mean square error  $\sigma^2$  due to the model parameters is defined as

$$\sigma^2 = \frac{1}{M^2} \sum_{s \in \Omega} [y(s) - \hat{\Theta}^T Q(s)]^2$$

**A. Required Computations**

Factor	Computations Needed ( $m$ = multiplication, $d$ = division, $a$ = addition)	
	New Method	Traditional Method
Features Calculation	$W \{M^2 a + M^2 m + 1 d\}$	$W \{(232 M^2 + 100) a + (211 M^2 + 100) m + 2 d + 10 \times 10 \text{ matrix inversion}\}$
$n$	0	$W \{12 a + 12 d\}$
<i>K-means</i>	$I \{3 a + 4 d + WK \text{ (distance measure)}\}$ $= I \{3 a + 4 d + WK (3 a + 2 m + 1 d)\}$	$I \{14 a + 15 d + WK \text{ (distance measure)}\}$ $= I \{14 a + 15 d + WK (19 a + 10 m)\}$
Total Com	$\{(6 M^2 + 6)W + (27 + 19 WK) I\} a$	$\{(1287 M^2 + 696 + 10 \times 10 \text{ matrix inversion}) W + (104 + 69 WK) I\} a$

Table 1. Computations needed to perform segmentation



The proposed algorithm achieves a very large speedup over a model-based segmenter using a 4<sup>th</sup> order GMRF. This speedup is due to several factors. These factors are the features calculation, the features normalization, and the K-means algorithm.

$$\text{segmentation time} = \text{features calculation} + \text{features normalization} \\ + K\text{-means (using some distance measure)}$$

If the image is partitioned into  $W$  disjoint windows, and composed of  $K$  textures, then the computations needed for the segmentation are as shown in Table I. In the table,  $I$  represents the number of iterations needed to perform *K-means*. The last row of the table shows the total number of computations needed in terms of *additions* by considering that typically

$$\text{Time of one multiplication} = \text{at least Time of 5 additions}$$

$$\text{and Time of one division} = \text{at least Time of 6 additions}$$

$$\text{speedup} = \frac{(1287M^2 + 696)W + (104 + 69WK)I + 10 \times 10 \text{ matrix inversion}}{(6M^2 + 6)W + (27 + 55WK)I}$$

For typical values of a  $512 \times 512$  image, with 6 textures, and choosing a  $16 \times 16$  window, assuming the worst value of  $I = 20$ , the speedup is found to be **88.54!** And for a  $256 \times 256$  image, with 4 textures,  $I = 10$ , the speedup is **144.56!**

### B. Required Storage

For the proposed distance measure, we only need to store the variance while for the GMRF, we need to store 12 parameters for each window.

## V. Results

In this section, we present the segmentation results performed on some images of real textures. All test images were digitized into  $64 \times 64$  sample images of 256 gray levels. The digitized images have been used with no pre-processing. Each image was subdivided into non-overlapping windows. We used  $16 \times 16$  and  $8 \times 8$  windows. The sample variance was computed for each window. Also, the 4<sup>th</sup> order GMRF parameters were calculated, to help perform a comparison between the proposed distance measure and a model-based distance measure.

The *K-means* algorithm was applied in both, the image-based and the model-based cases. For the GMRF model, we applied the Euclidean distance measure with normalized parameters.

To determine the homogeneity of the clusters formed by the *K-means*, we apply the statistical test defined in section IV.A. We used for this test a significance level  $\alpha = 5\%$ , which equivalently corresponds to  $\chi^2$  values of  $\chi_{\alpha}^2 = 3.841$ . Thus if the computed value of  $\chi^2$  for any two clusters is found to be larger than 3.841, these two clusters can be considered to be significantly different. Figures 1 through 4 show the original image followed by the segmentation performed using the GMRF parameters and finally by that

based on the proposed distance measure. The first row shows the experiments carried out using  $16 \times 16$  windows, while the second row shows the experiments carried out using  $8 \times 8$  windows.

The results have shown to be consistent for values of  $\alpha$  ranging from 0.5% to 5%. But if the textures are very similar a value of  $\alpha = 10\%$  is more appropriate.

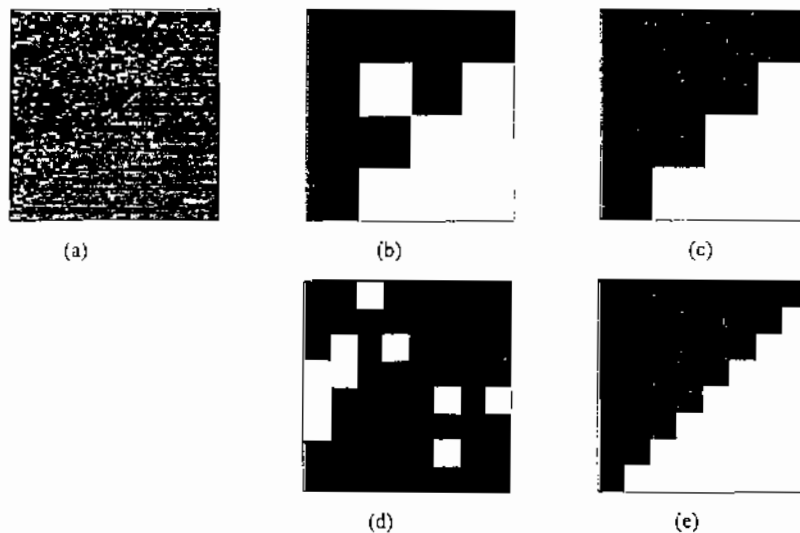


Fig. 1. (a) Original image composed of "Flowers and Tiles" (b) Coarse seg. using GMRF, Euclidean distance, and K-means,  $16 \times 16$  window (c) Coarse seg. using new algorithm,  $16 \times 16$  window (d) Coarse seg. using GMRF, Euclidean distance, and K-means,  $8 \times 8$  window (e) Coarse seg. using new algorithm,  $8 \times 8$  window

## VI. Conclusion

In this paper, we have presented a new algorithm for the coarse segmentation of a textured scene using an image-based approach. The algorithm works in an unsupervised fashion, with no required a priori knowledge about the number of textures in the image nor their behavior. The segmentation was performed using the *K-means* algorithm with a newly defined distance measure. The algorithm was compared to algorithms using a model-based approach. The GMRF was used in the comparison. The results indicate that the proposed algorithm is much simpler and faster while using less storage and achieving

better segmentation. The new algorithm also gives good results even with small block sizes (giving less coarse segmentation). The main problem that faces the proposed algorithm is the choice of the blocks sizes. If these blocks are chosen too large, regions of differing textures may be lost. If the blocks are chosen too small, discriminating among similar textures may be difficult. Also, if the textures in the image are of significantly

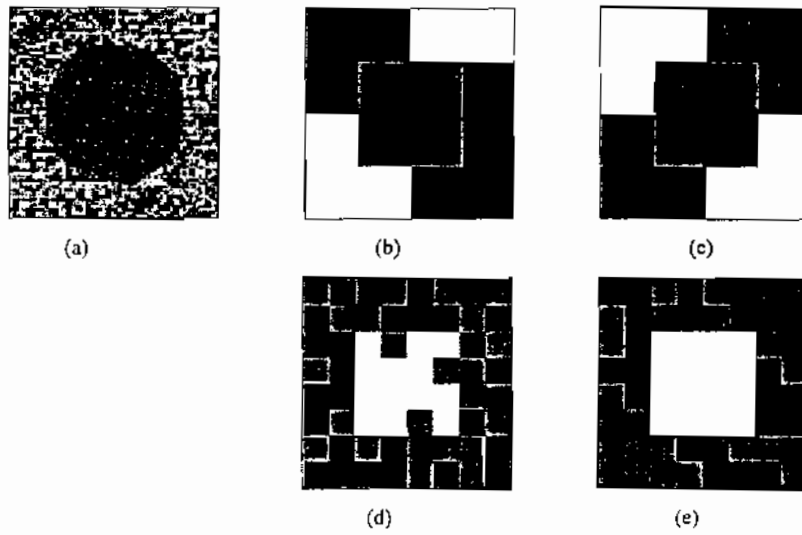


Fig. 2. (a) Original image composed of "Flowers, Water, and Sand" (b) Coarse seg. using GMRF, Euclidean distance, and K-means, 16x16 window (c) Coarse seg. using new algorithm, 16x16 window (d) Coarse seg. using GMRF, Euclidean distance, and K-means, 8x8 window

(e) Coarse seg. using new algorithm, 8x8 window

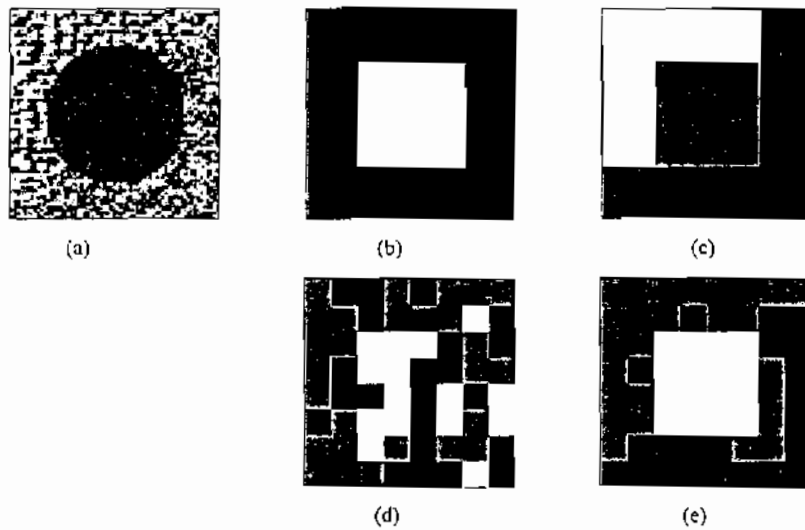


Fig. 3. (a) Original image composed of "Raffia, Grass, and Rocks" (b) Coarse seg. using GMRF, Euclidean distance, and K-means, 16x16 window (c) Coarse seg. using new algorithm, 16x16 window (d) Coarse seg. using GMRF, Euclidean distance, and K-means, 8x8 window

(e) Coarse seg. using new algorithm, 8x8 window

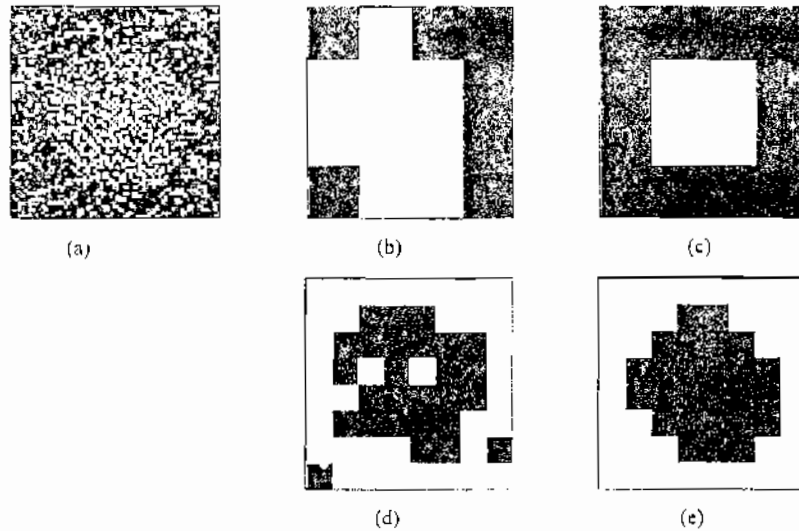


Fig. 4. (a) Original image composed of "Rocks and Stucco" (b) Coarse seg. using GMRF, Euclidean distance and K-means, 16x16 window (c) Coarse seg. using new algorithm, 16x16 window (d) Coarse seg. using GMRF, Euclidean distance, and K-means, 8x8 window

(e) Coarse seg. using new algorithm. 8x8 window

different degrees of smoothness or coarseness, discrimination between them is a difficult task. Therefore, some method should be applied to take into account the grain size of the different textures

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