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Free Vibrations of Cylindrical Shells by the Nodal Line Finite Difference Method-Part II-Numerical Examples.

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FREE VIBRATIONS OF CYLINDRICAL SHELLS BY THE NODAL LINE FINITE DIFFERENCE METHOD-PART II-NUMER ICAL EXAMPLES

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الإهتزاز الحر للقشريات الإسطوانية باستخدام طريقة الفروق المحددة لخطوط التقسيم-الجزء الثانى- أمثلة عددية

الخلاصة
فى هذا البحث يتم التحقق من كيفية تطبيق طريقة الفروق المحددة لخطوط التقسيم فى تحليل القشريات الإسطوانية الرقيقة للإهتزاز الحر وذلك باستخدام عدة أمثلة عددية، كما يتم أيضا توضيح استخدام هذه الطريقة فى تحليل البلاطات الرقيقة بالاستفادة من طرق وبرامج التحليل الخاصة بالقشريات الرقيقة.

1. INTRODUCTION

The analysis of cylindrical shells for free vibrations using the nodal line finite difference method has been presented in a companion paper (part I) [2]. The reader may also refer to reference [1] for some more details. In this paper the application of the nodal line finite difference is verified by many numerical examples. In addition, the applicability of the nodal line finite difference method for the analysis of thin plates by making use of the degenerate theory developed by Gibson [4] is demonstrated.

2. NUMERICAL EXAMPLES FOR CYLINDRICAL SHELLS

Based on the formulation given in the companion paper [2] a computer program has been developed in Quick Basic to compute the nondimensional frequencies of thin circular cylindrical shells of an open cross section. The cylindrical shells are assumed to have simply supported curved edges, but their straight edges are arbitrary supported. The program also computes the normal modes, which are corresponding to the nondimensional frequencies. Using this program, some illustrative examples have been solved. The results are in good agreement with those which have been obtained by Koumousis and Armenakas [5].

Before presenting the numerical examples, the following remarks should be taken into consideration:

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1. For given boundary conditions along the straight edges, both the nondimensional frequencies and normal modes are functions of the quantities L/mR, R/h, φ_o and υ; i.e., $\Omega = F(L/mR, R/h, \varphi_o, \rho)$ where

   Ω the nondimensional frequency of the thin circular cylindrical shell,

   L the length of the thin circular cylindrical shell,

   R the mean radius of the thin circular cylindrical shell,

   h the thickness of the thin circular cylindrical shell,

   φ_o the total central angle in radians,

   υ Poisson's ratio of the cylindrical shell material, and

   m the axial wave number

2. If the thin circular cylindrical shell has identical homogeneous boundary conditions along its two straight edges, then each of its normal modes is either symmetrical or antisymmetrical with respect to the axis of symmetry of its cross section. In this case, the overall coefficient matrix $[k_m]$ is a symmetrical banded matrix with a band width equal to seven.
3. Only cylindrical shells with two symmetrical straight edges are to be considered because the subprogram used for solving the eigenvalue problem of the matrix $[k_m]$ is based on Jcobi's method [3,6], which is valid for only symmetrical matrices.
4. For a thin circular cylindrical shell with given boundary conditions, the nondimensional frequencies decrease as L/mR increases on condition that the other parameters (R/h, φ_o and υ) remain unchanged [5].

Using the above mentioned program, the following numerical examples have been solved. Furthermore, the results are compared with those obtained by Koumousis and Armenakas [5].

Example 1 is a thin cylindrical shell with L/R=4, R/h=100, $\varphi_o = \frac{2}{3}\pi$ and $\upsilon = 0.3$. The cylindrical shell has been assumed to have a hinged immovable support at each of its two longitudinal edges (see the companion paper [2]). Due to symmetry, only half of the cylindrical shell has been considered and the problem has been solved twice; once for obtaining the symmetrical modes and the other for obtaining the antisymmetrical ones. Moreover, for both symmetrical modes and antisymmetrical modes the problem has been solved twice: once using 46 nodal lines and the other using 58 nodal lines. The first six values of the nondimensional frequency which are corresponding to m=1 are contained in the following table, namely $\Omega_{1,1}, \Omega_{1,2}, \Omega_{1,3}, ...$ and $\Omega_{1,6}$

Table 1

Method	[NLFDM] 46 nodal lines	[NLFDM] 58 nodal lines	Reference [5]
1st mode	0.0959028	0.09149	0.0857
2nd mode	0.111889	0.105983	0.0964
3rd mode	0.180293	0.1733	0.16278
4th mode	0.206773	0.203546	0.20656
5th mode	0.266661	0.26105	0.2687
6th mode	0.332755	0.325567	0.31472

It should be mentioned that the first, third and six modes are symmetrical ones, but the remaining modes are antisymmetrical ones.

Example 2 is for a thin circular cylindrical shell with L/R=20, R/h=100, $\varphi_o = 2\pi/3$ and $\upsilon = 0.3$. The cylindrical shell has the same boundary conditions as the cylindrical shell of example 1. Only half of the cylindrical shell has been considered, and the problem has been solved twice; once for obtaining the symmetrical modes and the other for obtaining the antisymmetrical ones. For both the symmetrical and antisymmetrical modes, 58 nodal lines have been used. The first six values of the nondimensional frequency which are corresponding to m=2 are contained in the following table, namely $\Omega_{1,1}, \Omega_{1,2}, \Omega_{1,3}, ...$ and $\Omega_{1,6}$. For the this numerical example, L/mR=10 and R/h=100. It should be noticed that increasing the value of L/mR has result in lower values for the nondimensional frequency.

Table 2

Method	[NLFDM] 58 nodal lines	Reference [5]	Type of mode
1st mode	0.060307	0.054549	symmetrical
2nd mode	0.076544	0.076519	antisymmetrical
3rd mode	0.119941	0.119206	antisymmetrical
4th mode	0.167914	0.157014	symmetrical
5th mode	0.242098	0.233241	antisymmetrical
6th mode	0.323103	0.312582	symmetrical

Example 3 is for a thin circular cylindrical shell with L/R=4, R/h=100. $\varphi_o = 2\pi/3$ and $\upsilon = 0.3$. The two straight edges of the cylindrical shell have been assumed to be clamped

(rigidly fixed). The axial wave number has been assumed to be equal to one; i.e., m=1. The nondimensional frequencies and the corresponding normal modes have been computed using 58 nodal lines. The following table contains the first six nondimensional frequencies and their corresponding values obtained by Koumosis and Armenakas [5]

Table 3

Method	[NLFDM] 58 nodal lines	Reference [5]	Type of mode
1st	0.098419	0.091898	symmetrical
2nd	0.118267	0.110511	antisymmetrical
3rd	0.201662	0.192755	symmetrical
4th	0.203633	0.210338	antiymmetrical
5th	0.294052	0.295252	antisymmetrical
6th	0.367050	0.358362	symmetrical

For this example, L/mR=4, R/h=100 and $\upsilon = 0.3$. These values are the same one of example 1. Comparing the results of this example with those of example 1, it can be remarked that a circular cylindrical shell with clamped straight edges has natural frequencies higher than the natural frequencies of a circular cylindrical shell with immovable hinged supports along its straight edges on condition that the two cylindrical shells have the same values for the quantities L/mR, R/h, φ_o and υ. The first six normal modes of the above thin circular cylindrical shell are shown in Fig. 1.

It should be remarked that the results obtained by Komuousis and Armenakas [5] are sometimes upper bounds and sometimes lower bounds with respect to the results obtained by the nodal line finite difference method. This is due to the approximations made in Jacobi's subroutine [3,6].

Example 4 is for studying the effect of the value of the central angle (φ_o) on the nondimensional frequencies. For this purpose, a thin circular cylindrical shell with L/R=4, R/h=100, $\upsilon = 0.3$ and different values for φ_o has been considered. The cylindrical shell has been assumed to have simply supported longitudinal edges, and the axial wave number has been assumed to be equal to one. The central angle (φ_o) has been assumed to have the following values: 30, 40, 50, 60, 80 and 120. The following table contains the first six nondimensional frequencies corresponding to each value of the central angle. According to the results contained in table 4, it can be concluded that increasing the value of the central angle leads to lower nondimensional frequencies. The relationship between the nondimensional frequency and the central angle for the first three nondimensional frequencies are shown in Fig. 2.

Table 4

angle in degree	30 $\Delta\varphi = 1$	40 $\Delta\varphi = 1$	50 $\Delta\varphi = 1$	60 $\Delta\varphi = 1$	80 $\Delta\varphi = 1$	120 $\Delta\varphi = 1.03$
1st mode	0.42529	0.25133	0.18038	0.15287	0.13772	0.09149
2nd mode	0.78416	0.49629	0.32596	0.23074	0.14616	0.10598
3rd mode	1.08007	0.93621	0.60495	0.42535	0.25341	0.17330
4th mode	1.64678	0.95512	0.86007	0.63589	0.36679	0.20355
5th mode	2.5561	1.47075	1.03428	0.9363	0.53369	0.26105
6th mode	3.62801	2.07400	1.33933	0.96370	0.70848	0.32557

3. DEGENERATE SHELL THEORY FOR PLATES

After an extended theoretical and computer investigation Gibson [4] concluded that shell theory could equally well predict the structural behavior of plate structures. Thus in the case of cylindrical shell theory, by introducing an infinite value for the radius into the stress equilibrium equations they immediately reduce to those equilibrium equations relevant to plates. It would follow that a program developed for analyzing cylindrical shells should be equally capable of analyzing plate structures by introducing an infinite radius into the input data. The introduction of an infinite radius into the input data is clearly impossible but the use of a large radius coupled with a suitably modified central angle, readily demonstrated that plates could be analyzed by this method, termed by GIBSON the Degenerate Theory. It forms the link between shells and plates as far as computer methods are concerned.

Gibson [4] has concluded that a program developed for analyzing cylindrical shells can be used for analyzing a flat plate of width C by using the following input data:

the central angle $(\varphi_o) = 2^\circ$

the radius $(R) = C/(2\sin 1^\circ)$

The exact nondimensional frequencies of the plate have been determined by using the following relation:

$$\overline{\Omega}^2_{m,n} = \frac{P^2_{m,n}(1-\upsilon^2)R^2}{E} = \frac{\pi^4 h^2 (m^2+n^2)^2}{192\, R^2 \sin^4 1^\circ}, \quad m,n = 1,2,3,....$$

where $P_{m,n}$ is the natural circular frequency of the plate and $\overline{\Omega}_{m,n}$ is the nondimensional frequency of the plate. The above relation can easily be proved by making use of the following relation [7]:

$$P_{m,n}^2 = \pi^4 \left(\frac{m^2 + n^2}{a^2} \right)^2 \left(\frac{D}{\rho h} \right), \quad m,n = 1,2,3,.....$$

$$D = \frac{Eh^3}{12(1-\upsilon^2)},$$

$$a = 2R \sin 1^o$$

where a is the length of the plate side, h is the plate thickness, and ρ is the density of the plate material.

Example 5 is for analyzing a thin square plate with 3 meters length, 0.08 meter thickness and =0.15. In this case, $R = 3/(2\sin 1^o) = 85.948$ meters. The plate has been assumed to be simply supported at its four edges. Due to symmetry, one half of the plate has been considered. The nondimensional frequencies and normal modes have been computed using 20 nodal lines. The following table contains the first six nondimensional frequencies computed by the program and their corresponding exact values.

Table 5

Method	[NLFDM]	[Exact]	Mode Type
1st mode	4.44723	4.353340	symmetrical
2nd mode	10.8752	10.883350	antisymmetrical
3rd mode	21.6770	21.766699	symmetrical
4th mode	36.7153	37.003389	antisymmetrical
5th mode	55.8925	56.593418	symmetrical
6th mode	79.0897	80.536787	antisymmetrical

Table 6 contains the relative values of the first mode and the corresponding exact relative values. The exact modes can be determined from the relation [7]:

$$W = \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{a}) \cos(P_{m,n} t); \quad m,n = 1,2,3.....$$

The nodal line number 0 coincides with one edge of the plate, while the nodal line number 20 coincides with the axis of symmetry parallel to this edge as shown in Fig. 3. It should be mentioned that the exact relative values contained in the above table have been determined by multiplying $\sin(\pi y/3)$ by 81.5235, where y is the distance between the nodal line and the plate edge. The first six normal modes have been computed by the program and are shown in Fig. 4.

Table 6

Nodal Line No.	W [NLFDM]	W [Exact]
0	0.000000	0.000000
1	+0.638100E+01	+0.639626E+01
2	+0.127236E+02	+0.127531E+02
3	+0.189894E+02	+0.190313E+02
4	+0.251402E+02	+0.251921E+02
5	+0.311381E+02	+0.311977E+02
6	+0.369466E+02	+0.370109E+02
7	+0.425300E+02	+0.425959E+02
8	+0.478536E+02	+0.479183E+02
9	+0.528839E+02	+0.529453E+02
10	+0.575892E+02	+0.576458E+02
11	+0.619401E+02	+0.619909E+02
12	+0.659096E+02	+0.659539E+02
13	+0.694733E+02	+0.695102E+02
14	+0.726089E+02	+0.726380E+02
15	+0.752965E+02	+0.753179E+02
16	+0.775191E+02	+0.775334E+02
17	+0.792625E+02	+0.792710E+02
18	+0.805160E+02	+0.805198E+02
19	+0.812713E+02	+0.812722E+02
20	+0.815235E+02	+0.815235E+02

4. CONCLUSIONS

On the basis of the present work, the following conclusions can be drawn:

1. The technique of the nodal line finite difference method deals directly with the governing differential equations of thin circular cylindrical shells and does not require any assumptions or approximations which may affect the generality of these equations. Moreover, the technique is simple in concept and easy to program. When simple harmonic functions are used as basic functions, they uncouple the governing partial differential equations of thin circular cylindrical shells so that a system of three ordinary differential

equations can be obtained for each term of these basic functions. Thus, a term by term analysis can be carried out.

2. For symmetrical boundary conditions along the two straight edges of a thin circular cylindrical shell with an open cross-section and simply supported curved edges, the nodal line finite difference method yields a symmetrical banded matrix with a band width equal to seven.

3. Unlike the solution obtained by the finite element method, the solution obtained by the nodal finite difference method satisfies the exact partial differential equations of equilibrium of thin circular cylindrical shells; therefore, compatibility and equilibrium are satisfied at each point of each nodal line.

4. The orthogonality relationships, which are satisfied by the normal modes, obtained by the Nodal Line Finite Difference slightly differ from those obtained by the finite element method. The overall coefficient matrix $[k_m]$ of the developed solution technique [2] replaces the stiffness matrix, and any two different modes are orthogonal with respect to the unit matrix instead of the mass matrix.

5. Thin circular cylindrical shells with different dimensions and with the same values for $L/mR, R/h, \varphi_o$ and υ have the same natural frequencies and the same normal modes on condition that the boundary conditions along the straight edges remain unchanged.

6. For a thin circular cylindrical shell with an open cross-section, the frequencies decrease as the central angle increases on condition that the parameters $L/mR, R/h$ and υ remain unchanged.

7. The nodal line finite difference method can be used for the analysis of thin plates by making use of the degenerate theory, developed by Gibson [4].

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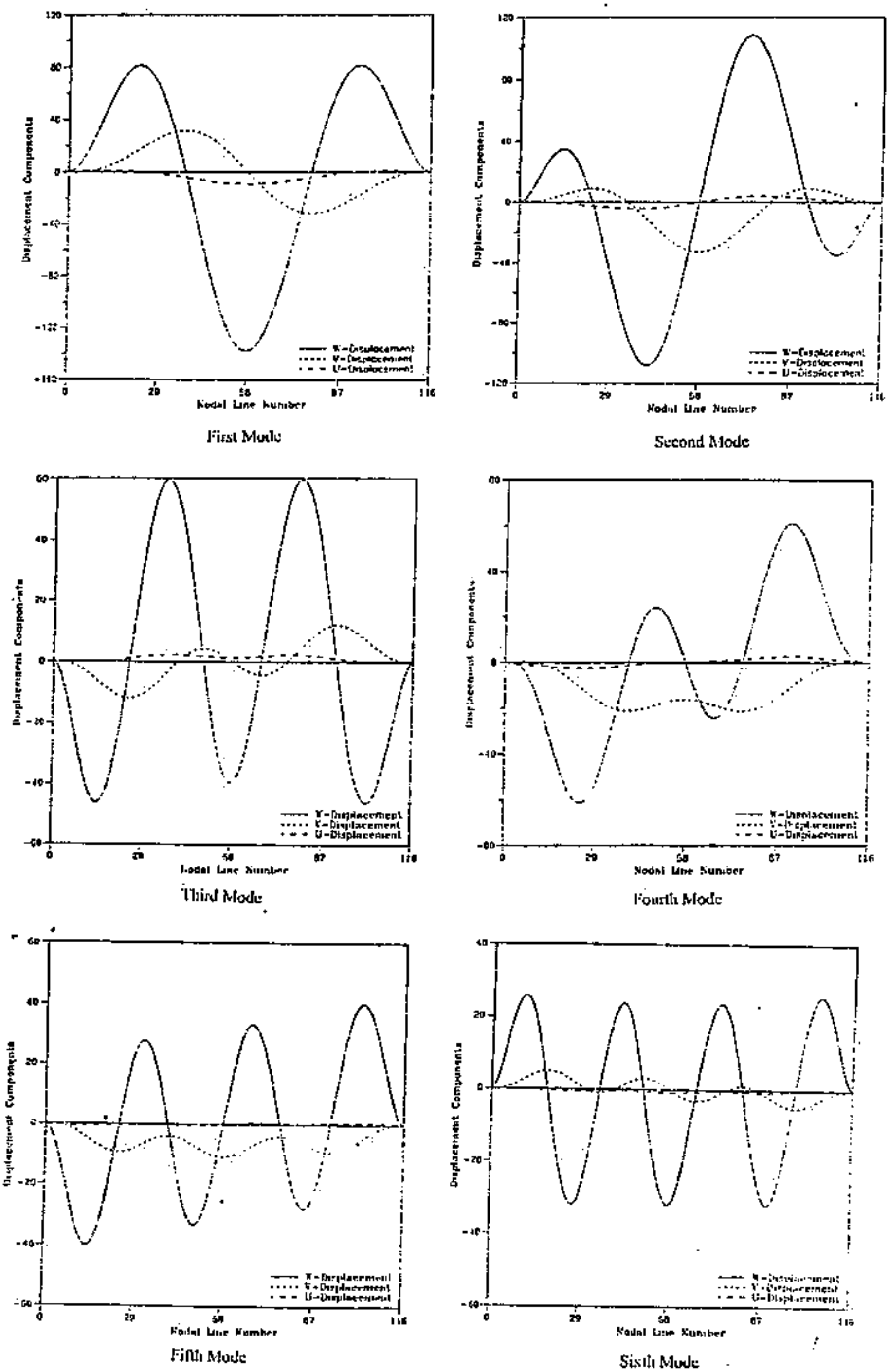


Fig. 1 Example 3, modes of vibration.

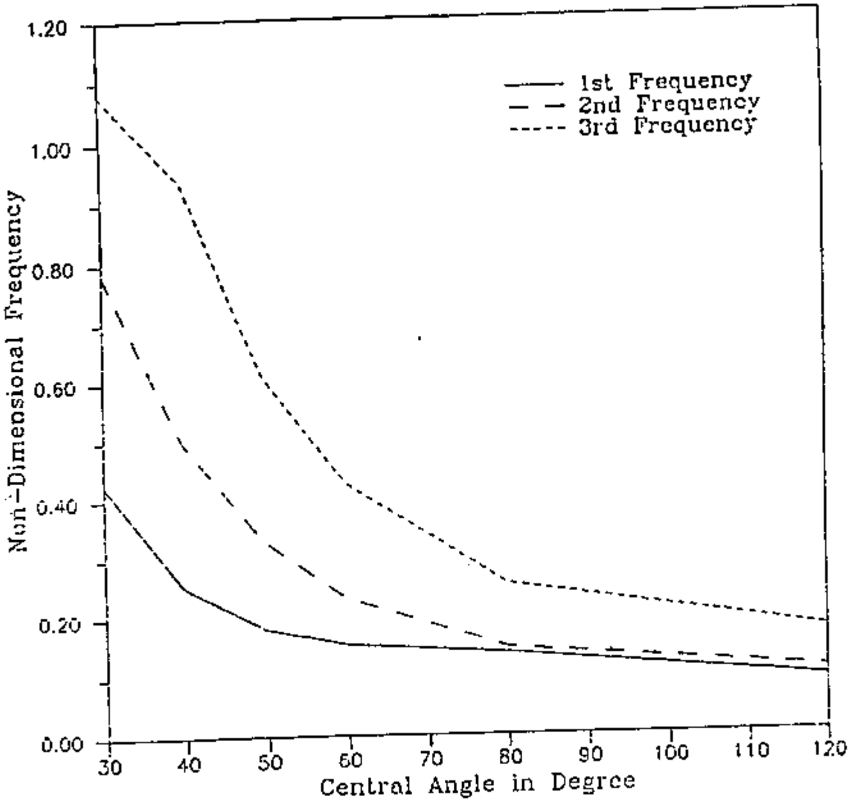


Fig. 2 Example 4, frequency vs. central angle.

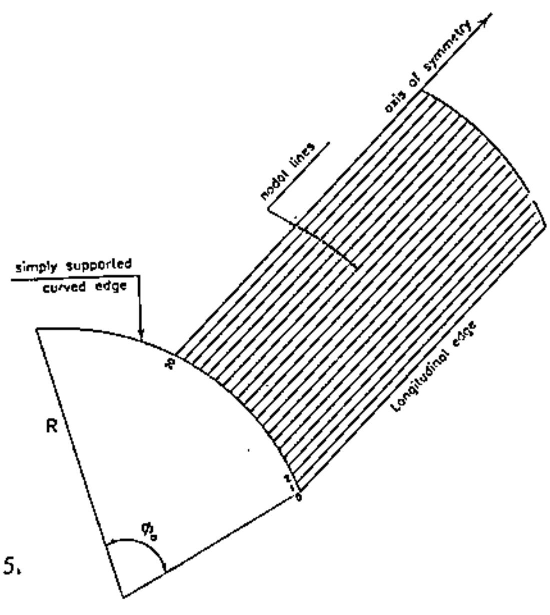


Fig. 3 Example 5.

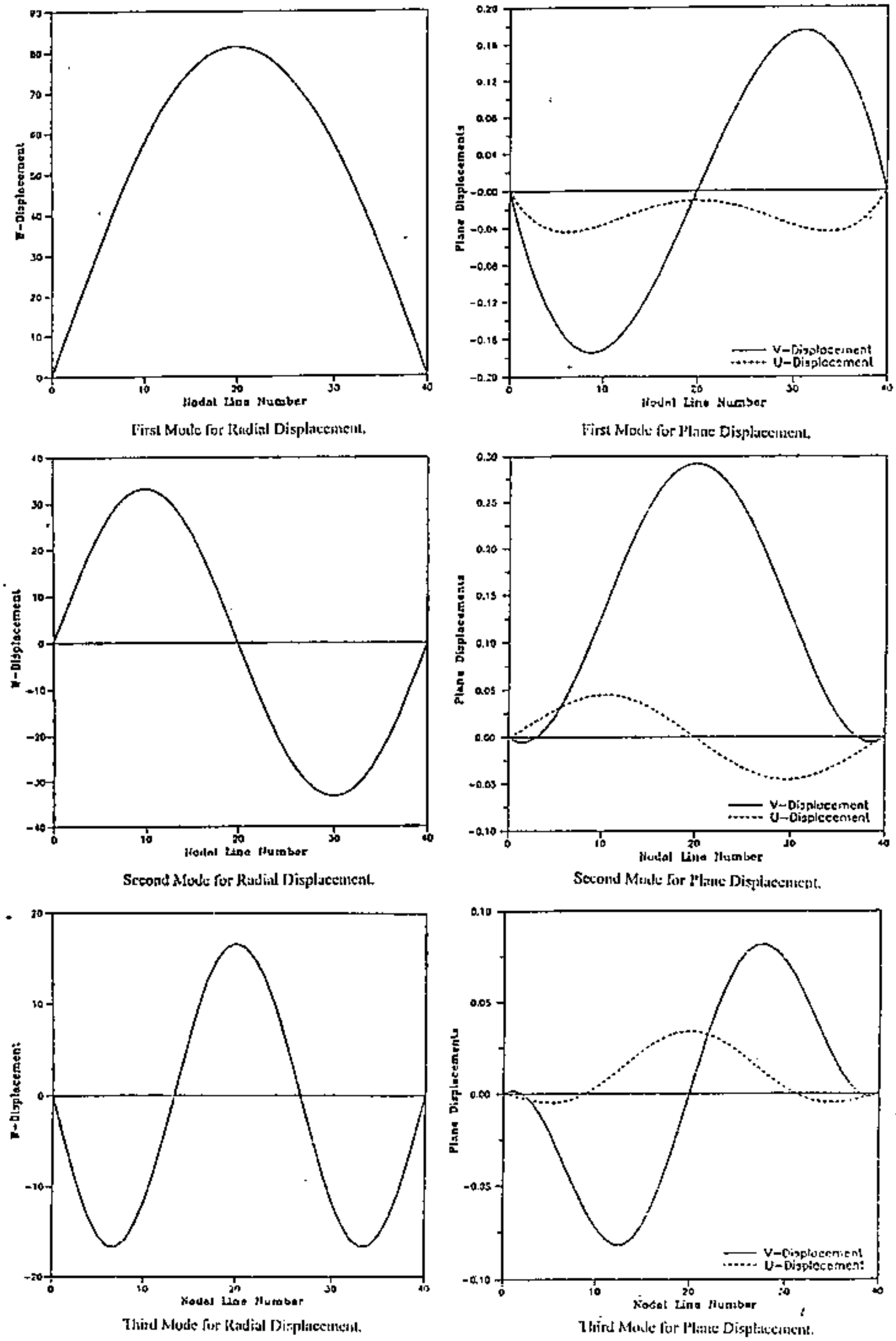
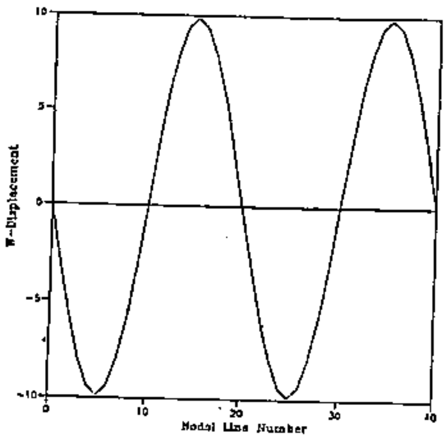
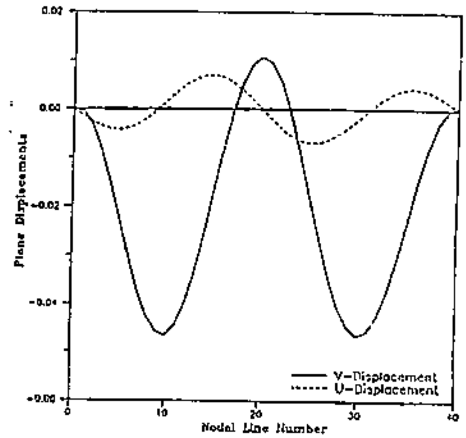


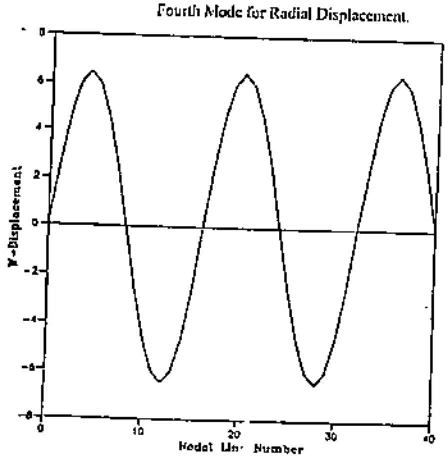
Fig. 4 Example 5, modes of vibration.



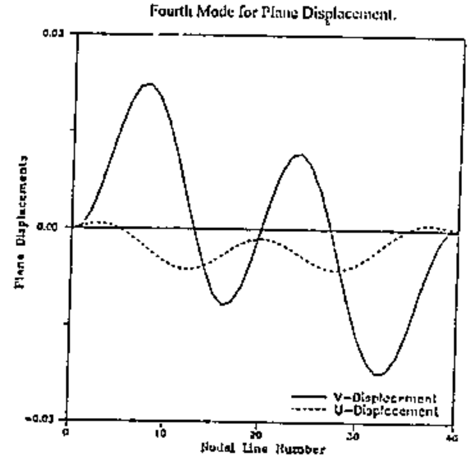
Fourth Mode for Radial Displacement.



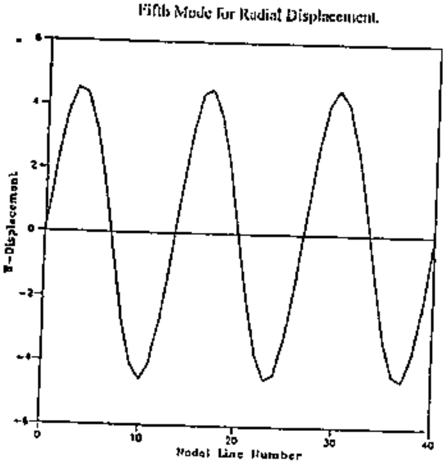
Fourth Mode for Plane Displacement.



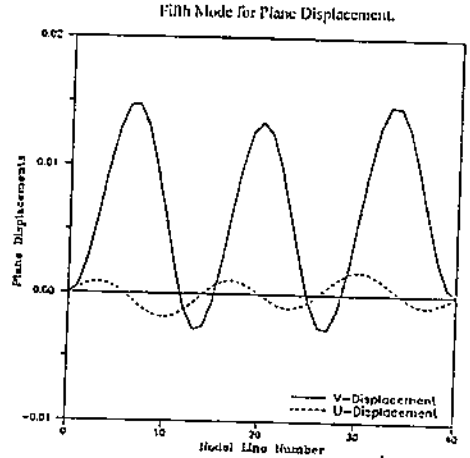
Fifth Mode for Radial Displacement.



Fifth Mode for Plane Displacement.



Sixth Mode for Radial Displacement.



Sixth Mode for Plane Displacement.

Fig. 4 Cont.