

1-27-2021

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### Recommended Citation

Ghali, Fadia and Abd El Moteleb, M. (2021) "Stability Region Estimation of Hybrid Multi-Machine Power System.," *Mansoura Engineering Journal*: Vol. 26 : Iss. 1 , Article 1.

Available at: <https://doi.org/10.21608/bfemu.2021.143583>

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## STABILITY REGION ESTIMATION OF HYBRID MULTI-MACHINE POWER SYSTEM

تحديد منطقة العمل المستقرة للنظم الكهربية المركبة المحتوية على وحدات متعددة

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### خلاصة :

يتجه العالم اليوم إلى استغلال كل أنواع الطاقة المتاحة لسد احتياجات جميع القطاعات سواء الصناعية أو الزراعية أو المجتمعات السكنية من الطاقة الكهربية مع المحافظة على المعايير الاقتصادية والبيئية ومن هنا تمثل الطاقة الكامنة في الرياح أحد المصادر النظيفة المتجددة للطاقة ، وقد نشأ ما يعرف بمزارع الرياح التي تعمل متصلة على الشبكات الكهربية الموحدة . وينشأ من ربط وحدات طاقة الرياح بالشبكات عدة نقاط تدرج تحت قطاع نوعية الطاقة المعقدة من حيث القيمة المضافة والتكلفة ومدى استقرار النظام عند مختلف نقاط أو ظروف التشغيل وهو ما يستدعي تحديد منطقة العمل المستقرة للنظام . ويقدم هذا البحث أسلوب جديد لتحديد منطقة العمل المستقرة للأنظمة الكهربية كبيرة الحجم المتضمنة مزارع الرياح باستخدام طريقة التقسيم والتجميع متلافية ما قدمته الطرق السابقة من عيوب وذلك عن طريق تقسيم النظام المركب إلى عدة أنظمة فرعية ثلاثية التركيب عددها  $\lfloor (n-1)/2 \rfloor$  في حالة النظم فردية عدد الوحدات وعدد  $\lfloor (n-2)/2 \rfloor$  ثلاثي التركيب بالإضافة إلى نظام ثنائي التركيب في حالة النظم زوجية عدد الوحدات ، ينتج عن هذا التقسيم ست دوال غير خطية لكل نظام فرعي وهو ما يعتبر نظام حر . وتم تكوين مصفوفة التجميع واستخدامها في تحديد منطقة العمل المستقر للنظام الكلي . تم تطبيق هذه الطريقة الحديثة على نظام مركب يحتوي على سبع وحدات متصلة بشبكة ربط تمثل إحدى هذه الوحدات مزرعة الرياح وقد تم تغير معدل ضخ الطاقة المولدة من المزرعة في الشبكة الموحدة وفي كل حالة تم حساب مدى استقرار النظام الكلي ، وقد أثبت أن زيادة معدل طاقة الرياح المولدة يؤدي إلى اتساع المجال المستقر للنظام الكلي وهو ما تعبر عنه دالة لياپونوف .

### Abstract

The concept of producing electric energy utilizing the energy embedded in wind is a real practice on a wide scale. At different locations where wind potential is so promising, the energy planner is talking about wind parks connected to existing utilities. The extension of interconnected utilities with non-conventional plants should be investigated regarding different topics; among which is the stability of the hybrid system under expected operating conditions.

In this paper, the stability problem is investigated using the decomposition aggregation technique. Attempts to overcome some of the drawbacks given in other methods have led to the application of the new technique based on Bellman's concept of vector Lyapunov function. An advanced approach is proposed in this work; triple-wise decomposition-aggregation for multi-machines hybrid power system considering the transfer conductance and uniform damping. For this algorithm the system is decomposed into  $(n-1)/2$  three-machine subsystems for odd number of machines, or  $(n-2)/2$  three-machine plus one two-machine subsystems for even number of machines. Six non-linearities are considered for each free subsystem. The domain of attraction is estimated to study the effect of introducing non-conventional energy source.

### 1. Introduction

Today the concept of producing electric energy utilizing the energy embedded in wind is a real practice on a wide scale; few kW to MW-rated wind energy conversion systems, autonomous as well as grid connected systems. At different locations where wind potential is so promising, the energy planner is talking about wind farms connected to existing utilities.

Basically, a design phenomenon is chosen to select the optimum configuration of the system components for each field of application. One of the standard configurations of wind energy conversion systems (WECS) is considered in this study. It consists of a variable speed wind turbine (WT) connected to synchronous generator, equipped with a gearbox and frequency converter. Variable speed is considered because it can increase the energy captured by the turbine and it also reduces some loads. Moreover, the frequency converter can control the generator power and thereby reduce the demands

on efficient damping. On the average, the variable speed systems are as efficient as the directly grid connected systems, because both the generator and gearbox no-load losses are much reduced. [1] [2] [3]

Extension of interconnected utilities with non-conventional plants should be investigated regarding the added value, efficiency, cost and availability of the system, and stability of the hybrid system under expected abnormal operating conditions. For power systems, the stability problem is concerned with the property that enables the synchronous machines to respond to a disturbance so as to move from one to another stable operating condition.

In this paper, the stability problem of hybrid power system is investigated using the triple-wise decomposition aggregation technique. This technique is proposed to overcome some of the drawbacks given in other methods; e.g. in the pair-wise scheme of work the complex power system is decomposed into (n-1) two-machine interconnected subsystems, where two non-linearities are considered for each free subsystem. Although this technique is more powerful than some old techniques due to the increased number of non-linearities, it fails to produce stable aggregation matrix for large-scale power systems. Therefore, the pair-wise scheme is applicable to medium scale systems and should be performed in such a way to assure decomposing the complex system into weakly coupled subsystems. One of the attempts to overcome such drawbacks has led to the application of the new triple-wise decomposition aggregation technique based on Bellman's concept of vector Lyapunov function. [4]

## 2. System Description and Modeling

The hybrid power system under consideration consists mainly of conventional synchronous generating plants, wind park composed of certain number of wind turbines coupled with synchronous generators and finally interconnected transmission lines. To carry out the stability study, each of these components will be modeled.

### 2.1 Wind turbine model

The mechanical power  $P_m$  produced by a wind turbine is given by:

$$P_m = 0.5 \rho C_p A U^3 \quad (1)$$

where

$\rho$  air density,  $U$  instantaneous wind speed,  $A$  rotor swept area,  $C_p$  power coefficient

The mechanical rotational speed  $\omega$  of the wind turbine is expressed by the swing equation:

$$\frac{d\omega}{dt} = \frac{\omega'}{2H} (T_m - T_e - \frac{D}{\omega'} \omega) \quad (2)$$

where

$T$  torque,  $m$  mechanical,  $e$  electrical,  $T_m = P_m / S_s (\omega'/\omega)$ ,  $S$  KVA rating.

### 2.2 Utility model

For a hybrid power system the differential equations describing the dynamics of the system are:

$$M_i \dot{\delta}_i + D_i \dot{\delta}_i = P_{mi} - P_{ei} \quad , i=1,2,\dots,n \quad (3)$$

Where  $P_{ei} = \sum E_i E_j Y_{ij} \cos(\delta_i - \theta_{ij})$

It is also assumed that for all machines the damping to inertia ratio is constant, that is:

$$D_i / M_i = \lambda \quad , i=1,2,\dots,n \quad (4)$$

Selecting the  $n$ th machine to be the reference one, and introducing  $2(n-1)$  state vector  $x$  as:

$$x = (\omega_1, \dots, \omega_{n-1}, \delta_1 - \delta_n, \dots, \delta_{n-1} - \delta_n)^T$$

Where

$$\delta_n = \delta, \quad \delta_i - \delta_n = \delta_i - \delta, \quad \omega_n = \omega, \quad \omega_i - \omega_n = \delta_i \quad (5)$$

## 3. Decomposition Aggregation Control Technique

The domain of attraction of the equilibrium point is the set of all points such that trajectories initiated at these points eventually converge to the origin. Since the stability question of how the non-conventional renewable energy systems would affect existing utilities should be accurately investigated, a powerful tool is to be implemented so as not to disregard a number of operating points as unstable. One of the most powerful tools is the decomposition aggregation technique, which is based on Bellman's concept of vector Lyapunov functions. It consists of decomposing a large-scale system into a set of subsystems. The stability properties for the disconnected free subsystems are derived, again aggregated to describe the domain of attraction of the complex system.

Two approaches could be proposed in estimating the domain of attraction of large-scale power systems; pair-wise and triple-wise decomposition aggregation techniques. In the pair-wise scheme of work the complex power system is decomposed into (n-1) two-machine interconnected subsystems where two non-linearities are considered for each free subsystem. Although this technique is more powerful than some old techniques due to the increased number of non-

linearities, it fails of producing stable aggregation matrix for large-scale power systems. Therefore, it is applicable to medium scale systems and should be performed in such a way to assure weakly coupled subsystems. [18][paper1] In the next section, the triple-wise technique, which is a step forward in the application of the decomposition aggregation technique, will be in detail discussed.

#### 4. Triple-wise Decomposition Aggregation Technique

##### 4.1 Power System Decomposition

stability considered for the triple-wise algorithm the interconnected system is decomposed into  $(n-1)/2$  three- machine subsystems for odd number of machines, or  $(n-2)/2$  three-machine plus one two-machine subsystems for even number of machines with six non-linearities order to obtain the largest asymptotic for each free subsystem. Figure (1) shows the schematic diagram of the decomposed system.

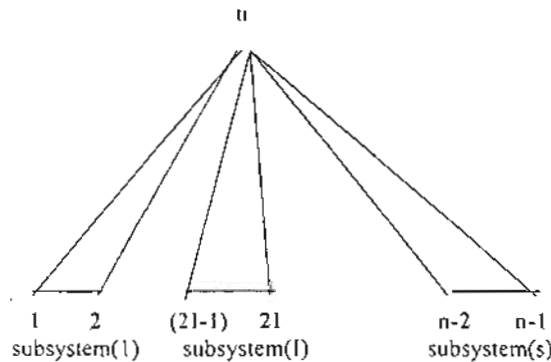


Figure (1) Schematic diagram of the decomposed system

Adopting the state vector  $x_I$  as

$$x_I = [\omega_{(2I-1),n} \quad \omega_{2I,n} \quad y_{(2I-1),n} \quad y_{2I,n}]^T \tag{6}$$

$$= [x_{I1} \quad x_{I2} \quad x_{I3} \quad x_{I4}]^T, \quad I = 1, 2, \dots, s$$

where  $(2I-1), 2I$  are the elements of the set  $J_I$ , the whole system is decomposed into  $s=(n-1)/2$  fourth-order interconnected subsystems. Each of the  $s$  subsystems may be written in the general form

$$\dot{x}_I = P_I x_I + B_I \Phi_I(y_I) + h_I(x) \tag{7}$$

$$y_I = C_I^T x_I, \quad I = 1, 2, \dots, s$$

where  $P_I, B_I, C_I^T$ , and  $h_I(x)$  are defined as follows

$$P_I = \begin{bmatrix} -\lambda & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad C_I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 & -1 \end{bmatrix}$$

$$B_I = \begin{bmatrix} -M_{2I-1}^{-1} A_I & 0 & -M_{2I-1}^{-1} \bar{A}_I & 0 & M_n^{-1} A_I & M_n^{-1} \bar{A}_I \\ 0 & -M_{2I}^{-1} \bar{A}_I & 0 & -M_{2I}^{-1} \bar{A}_I & M_n^{-1} A_I & M_n^{-1} \bar{A}_I \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$h_I(x) = \begin{bmatrix} \sum_{j=2I}^{n-1} (-M_{2I-1}^{-1} A_{(2I-1)j} \phi_{(2I-1)j}(y_{(2I-1)j}) + M_n^{-1} A_{nj} \phi_{nj}(y_{nj})) \\ \sum_{j=2I}^{n-1} (-M_{2I}^{-1} A_{2Ij} \phi_{2Ij}(y_{2Ij}) + M_n^{-1} A_{nj} \phi_{nj}(y_{nj})) \\ 0 \\ 0 \end{bmatrix}$$

In domain estimate for the considered power system, the subsystem of equation (5) is decomposed of non-linearities, i.e. the vector  $\Phi_I(y_I)$  is defined such that the free subsystem contains the largest number as (equ. 8):

$$\Phi_I(y_I) = [\phi_{11}(y_{11}), \phi_{12}(y_{12}), \phi_{13}(y_{13}), \phi_{14}(y_{14}), \phi_{15}(y_{15}), \phi_{16}(y_{16})]^T$$

where the six non-linearities  $\phi_{II}(y_{II})$  are defined as (equ. 9)

$$\phi_{11}(y_{11}) = \cos(y_{(2I-1)2n} + \delta_{(2I-1)2n} - \theta_{(2I-1)2n}) - \cos(\delta_{(2I-1)2n} - \theta_{(2I-1)2n})$$

$$\phi_{12}(y_{12}) = \cos(y_{2I,n} + \delta_{2I,n} - \theta_{2I,n}) - \cos(\delta_{2I,n} - \theta_{2I,n})$$

$$\phi_{13}(y_{13}) = \cos(y_{(2I-1)2I} + \delta_{(2I-1)2I} - \theta_{(2I-1)2I}) - \cos(\delta_{(2I-1)2I} - \theta_{(2I-1)2I})$$

$$\phi_{14}(y_{14}) = \cos(y_{2I,(2I-1)} + \delta_{2I,(2I-1)} - \theta_{(2I-1)2I}) - \cos(\delta_{2I,(2I-1)} - \theta_{(2I-1)2I})$$

$$\phi_{15}(y_{15}) = \cos(y_{n,(2I-1)} + \delta_{n,(2I-1)} - \theta_{(2I-1)n}) - \cos(\delta_{n,(2I-1)} - \theta_{(2I-1)n})$$

$$\phi_{16}(y_{16}) = \cos(y_{n,2I} + \delta_{n,2I} - \theta_{2I,n}) - \cos(\delta_{n,2I} - \theta_{2I,n})$$

Now, we can decompose each of these subsystems into a free (disconnected) subsystem and interconnections. The free subsystem has the general form (equ. 10):

$$\dot{x}_I = P_I x_I + B_I \Phi_I(y_I)$$

$$y_I = C_I^T x_I, I = 1, 2, \dots, s$$

#### 4.2 Free Subsystem Analysis

For the free subsystem equ. (8), we adopt a Lyapunov function in the form "quadratic form plus sum of integrals of the six non-linearities (equ. 11)

$$V_I(x_I) = x_I^T H_I x_I + \sum_{j=1}^6 d_{II} \int_0^{x_I} \phi_{II}(y_{II}) dy_{II}, \quad I = 1, 2, \dots, s$$

In this expression,  $H_I$  is a fourth-order symmetric positive definite matrix,  $d_{II}$  are positive numbers. The time derivative of  $V_I(x_I)$  of the free subsystem is derived as (equ. 12):

$$\dot{V}_I(x_I)_{(4,9)} = x_I^T (-G_I) x_I + 2 \Phi_I^T B_I^T H_I x_I + \sum_{I=1}^6 d_{II} \phi_{II}(y_{II}) \cdot \dot{y}_{II}$$

and equ. 13:

$$-G_I = P_I^T H_I + H_I P_I$$

It is computed in the form

$$G_I = \begin{bmatrix} 2(\lambda h_{11}' - h_{13}') & 2\lambda h_{12}' - h_{23}' - h_{14}' & \lambda h_{13}' - h_{33}' & \lambda h_{14}' - h_{34}' \\ 2\lambda h_{12}' - h_{23}' - h_{14}' & 2(\lambda h_{22}' - h_{24}') & \lambda h_{23}' - h_{34}' & \lambda h_{24}' - h_{34}' \\ \lambda h_{13}' - h_{33}' & \lambda h_{23}' - h_{34}' & 0 & 0 \\ \lambda h_{14}' - h_{34}' & \lambda h_{24}' - h_{34}' & 0 & 0 \end{bmatrix}$$

Selecting  $\rho_I$  as an arbitrary positive number, the following relations will yield a  $G$  positive definite matrix, where (equ. 14):

$$h'_{11} = \frac{(1 + \rho_1)}{\lambda} h'_{13}, \quad h'_{22} = \frac{(1 + \rho_1)}{\lambda} h'_{24}$$

$$h'_{33} = \lambda h'_{13}, \quad h'_{44} = \lambda h'_{24}$$

Under the conditions  $h'_{13}, h'_{24} > 0$  the corresponding matrix  $H_l$  is positive definite and given as ( equ. 15) :

$$H_l = \begin{bmatrix} \frac{(1 + \rho_1)}{\lambda} h'_{11} & 0 & h'_{13} & 0 \\ 0 & \frac{(1 + \rho_1)}{\lambda} h'_{22} & 0 & h'_{24} \\ h'_{13} & 0 & \lambda h'_{13} & 0 \\ 0 & h'_{24} & 0 & \lambda h'_{24} \end{bmatrix} :$$

Selecting the constants  $d_{ll}$  as (equ. 16) :

$$d_{11} = 2 M_{2l-1}^{-1} A_l h'_{33}, \quad d_{12} = 2 M_{2l}^{-1} \bar{A}_l h'_{44}, \quad d_{13} = 2 M_{2l-1}^{-1} \bar{A}_l h'_{33}$$

$$d_{14} = 2 M_{2l}^{-1} \bar{A}_l h'_{44}, \quad d_{15} = 2 M_n^{-1} A_l h'_{33}, \quad d_{16} = 2 M_n^{-1} \bar{A}_l h'_{44}$$

equation (10) becomes (equ. 17):

$$\begin{aligned} \dot{V}_l(x_l)_{(4,5)} = & -2k_l h'_{13} x_{l1}^2 - 2k_l h'_{24} x_{l2}^2 - 2M_{2l-1}^{-1} A_l h'_{13} \phi_{l1}(y_{l1}) \cdot x_{l3} \\ & - 2M_{2l}^{-1} \bar{A}_l h'_{24} \phi_{l2}(y_{l2}) \cdot x_{l4} - 2M_{2l-1}^{-1} \bar{A}_l \phi_{l3}(y_{l3})(h'_{13} x_{l3} + h'_{33} x_{l2}) \\ & - 2M_{2l}^{-1} \bar{A}_l \phi_{l4}(y_{l4})(h'_{24} x_{l4} + h'_{44} x_{l1}) \\ & + 2M_n^{-1} A_l \phi_{l5}(y_{l5})(h'_{13} x_{l3} + h'_{24} x_{l4} + h'_{33} x_{l2}) \\ & + 2M_n^{-1} \bar{A}_l \phi_{l6}(y_{l6})(h'_{13} x_{l3} + h'_{24} x_{l4} + h'_{33} x_{l1}) \end{aligned}$$

Now, let us introduce the positive constants  $\epsilon_{ll} \in \{0, \xi_{ll}\}$ , which satisfy the following condition

$$y_{ll} \phi_{ll}(y_{ll}) \geq \epsilon_{ll} y_{ll}^2, \quad l = 1, 2, \dots, 6 \quad (18)$$

on a compact interval  $U_{ll}$  of  $y_{ll}$ ,

$$U_{ll} = [\underline{U}_{ll}, \bar{U}_{ll}], \quad l = 1, 2, \dots, 6 \quad (19)$$

where  $\underline{U}_{ll}, \bar{U}_{ll}$  are respectively the negative and positive solutions of the following equation

$$\phi_{ll}(y_{ll}) = \epsilon_{ll} y_{ll}, \quad l = 1, 2, \dots, 6 \quad (20)$$

Based on inequality equ (18) and by adding to the right-hand side of equation (17) the non-negative expression

$$2M_{2l-1}^{-1} \bar{A}_l h'_{13} [y_{l3} \phi_{l3}(y_{l3}) - \xi_{l3}^{-1} \phi_{l3}^2(y_{l3})]$$

$$+ 2M_{2l}^{-1} \bar{A}_l h'_{24} [y_{l4} \phi_{l4}(y_{l4}) - \xi_{l4}^{-1} \phi_{l4}^2(y_{l4})]$$

where  $\xi_{l3}, \xi_{l4}$  are determined as

$$y_{ll} \phi_{ll}(y_{ll}) \geq \epsilon_{ll} y_{ll}^2, \quad l = 1, 2, \dots, 6 \quad (21)$$

We obtain (equ. 22)

$$\dot{V}_l(x_l) \leq -\psi_l \|x_l\|^2 \quad \forall l = 1, 2, \dots, s$$

where  $\psi_l$  is the minimum eigenvalue of the sixth-order symmetric positive definite matrix  $M_l$ . [5]

### 4.3 Power System Aggregation

The desired domain of attraction for the overall hybrid power system is generated using aggregation matrix of order  $(n/2)$  for triple-wise technique. The stability criterion of the equilibrium  $x = 0$  of the overall system is based on the construction of an aggregation matrix  $W = [w_{jk}]$ , whose elements (real numbers) obey the following inequality (equ. 23):

$$\dot{V}_I(x)_{(5)} = [\text{grad}V_I(x_t)]^T (P_I x_t + B_I \Phi_I(y_t) + h_I(x)) \leq \sum_{k=1}^s w_{Ik} u_I(x_t) u_k(x_k), \quad \forall I = 1, 2, \dots, s$$

where  $\dot{V}_I(x)_{(5)}$  is the time derivative of the function  $V_I$  of the decomposed subsystem of equation (5), and  $u_I$  are positive definite functions:

$$u_I(x_t) = \|x_t\| = (x_t^T x_t)^{1/2} \quad \forall I = 1, 2, \dots, s$$

Thus  $\dot{V}_I(x)_{(5)}$  can be expressed as (equ. 24):

$$\dot{V}_I(x)_{(5)} = \dot{V}_I(x_t)_{(8)} + [\text{grad}V_I(x_t)]^T h_I(x) \quad \forall I = 1, 2, \dots, s$$

where  $\dot{V}_I(x_t)_{(8)}$  is the total derivative of  $V_I$  along motions of the free subsystem.

Using the following relations (equ. 25):

$$|\phi_{(2I-1),I}(y_{(2I-1),I})| \leq \xi_{(2I-1),I} (|x_{I2}| + |x_{I3}|)$$

$$\xi_{(2I-1),I} = \sin(\theta_{(2I-1),I} - \delta_{(2I-1),I}^o)$$

$$|\phi_{2I,I}(y_{2I,I})| \leq \xi_{2I,I} (|x_{I2}| + |x_{I3}|)$$

$$\xi_{2I,I} = \sin(\theta_{2I,I} - \delta_{2I,I}^o)$$

We get (equ. 26):

$$[\text{grad}V_I(x_t)]^T h_I(x) \leq \bar{\psi}_I \|x_t\|^2 + 2 \sum_{k=1}^s Z_{Ik} \|x_t\| \|x_k\|$$

where

$$\begin{aligned} Z_{Ik} &= Z_2(\bar{Z}_{Ik}, \bar{\bar{Z}}_{Ik}) \quad (27) \\ \bar{Z}_{Ik} &= Z_2 \{ (M_n^{-1} A_{n,(2k-1),I} \xi_{n,(2k-1),I} + M_{2I-1}^{-1} A_{(2I-1),(2k-1),I} \xi_{(2I-1),(2k-1),I}) Z_2(h'_{I3}, h'_{I1}); \\ &\quad (M_n^{-1} A_{n,2k-1} \xi_{n,2k-1} + M_{2I}^{-1} A_{2I,(2k-1),I} \xi_{2I,(2k-1),I}) Z_2(h'_{21}, h'_{22}) \} \\ \bar{\bar{Z}}_{Ik} &= Z_2 \{ (M_n^{-1} A_{n,2k} \xi_{n,2k} + M_{2I-1}^{-1} A_{(2I-1),2k} \xi_{(2I-1),2k}) Z_2(h'_{I3}, h'_{I1}); \\ &\quad (M_n^{-1} A_{n,2k} \xi_{n,2k} + M_{2I}^{-1} A_{2I,2k} \xi_{2I,2k}) Z_2(h'_{24}, h'_{22}) \} \end{aligned}$$

and  $\bar{\psi}_I$  is the maximum eigenvalue of the fourth-order symmetric matrix  $Q_I$ , whose elements are defined as (equ. 28):

$$q'_{11} = q'_{22} = q'_{33} = q'_{44} = q'_{12} = q'_{14} = q'_{23} = q'_{34} = 0$$

$$q'_{13} = M_{2I-1}^{-1} h'_{I1} \sum_{k=1}^s (A_{(2I-1),(2k-1),I} \xi_{(2I-1),(2k-1),I} + A_{2I-1,2k} \xi_{2I-1,2k})$$

$$q'_{24} = M_{2I}^{-1} h'_{22} \sum_{k=1}^s (A_{2I,(2k-1),I} \xi_{2I,(2k-1),I} + A_{2I,2k} \xi_{2I,2k})$$

Hence, the elements of the  $s \times s$  aggregation matrix  $W$  can be defined in the following form (equ. 29):

$$w_{Ik} = \begin{cases} -(\psi_I - \bar{\psi}_I), & k = I \\ 2Z_{Ik} & k \neq I \end{cases} \quad \forall I, k = 1, 2, \dots, s$$

Therefore, the stability criteria of the whole system can be defined as the equilibrium state  $x = 0$  [equation (5)] is asymptotically stable if the aggregation matrix  $W$  [equation (29)] has eigenvalues with negative real parts.



#### 4.4 Stability Domain Estimate For The Complex System

In order to determine a stability domain estimate for the complex system, we proceed the following systematic step. Assuming that the complex system is asymptotically stable having domain  $\mathcal{E}$  which is to be estimated as:

$$\mathcal{E} = \{x: V(x) \leq \gamma\}$$

where the Lyapunov function for the complex system is

$$V(x) = \sum_{k=1}^s \beta_k V_k(x_k) \quad (30)$$

$\beta = \beta^T$  is a matrix (with zero off diagonal) chosen such that  $W^T \beta + \beta W$  is negative definite matrix, equ. 31.32.

$$\gamma = \min\{\beta_k V_k^o: k = 1, 2, \dots, s\}$$

$$V_k^o = \min_{m=1, \dots, 6} \min_{x_i^m \in (x_i^m, x_i^m)} \left\{ (x_i^m)^T H_i x_i^m + \sum_{l=1}^6 d_{li} \int_0^{y_i^m} \phi_{li}(y_{li}) dy_{li} \right\}$$

$$\forall i = 1, 2, \dots, s$$

$V_k^o$  is the estimate of asymptotic stability domain for the  $k$ th free subsystem.

#### 5. Verification of the control algorithm

The triple-wise decomposition aggregation technique is applied to estimate the domain of attraction of a hybrid power system containing a wind park. As a start point, the operation of wind parks is to be analyzed.

**First**, a WECS supplies electrical power in the electrical network as long as the wind velocity lies within its operating range. If the wind velocity falls out the cut-in / cut-out speed range, the WECS is to be disconnected, which is an extreme condition. For a wind park containing a number of WECS's located on a wide area, the diversity of wind speed would minimize the possibility of the extreme of disconnecting all WECS's due to falling outside the operating range. Moreover, the WECS's are sectionalized. Thus the wind park can be theoretically considered as one bus-bar (B.B.), but practically connected to the utility through more than one B.B.

**Second**, based on equation (32), the analysis of the decomposition aggregation technique has come to the following conclusion: The triple-wise decomposition for odd number of machines is more powerful than for even number, since the estimate of the two machines sub-system will dictate the domain of attraction of the whole system. Therefore, combining these two conclusions establishes the basis of estimation the domain of attraction of the hybrid system; incrementally increasing the power injected by the wind park between minimum and maximum available power, keeping an odd number of B.B.s. The new approach is verified using one of the IEEE standard systems[6]. The parameters of the system under study are given in tables (1), (2).

The power generated on B.B.#2 by the wind park is increased in steps. Figure (2) shows the incremental change in each step considering case #1 as the base case. The incremental change reached 26.7% of the base value. In figure (3), wind park generation (WG) is given in percentage of the whole system generation, starting at 13.24% to 17.22%. For each case, the corresponding stability function  $V$  (domain estimate) is calculated, showing a monotone increasing trend. The contour representation of the  $V$  function is given in figure (4) showing non-intersected contours with increasing values outwards.

An interesting point to be mentioned; as B.B.#6 was tested to connect the WECS, the  $V$  function showed the same monotone increasing trend but slightly saturated in cases 5,6,7. This remark could lead to the possibility of using the technique to optimally locate new WECS's.

#### 6 Conclusion

As stated in previous work [4], estimation of the domain of attraction applying pair-wise decomposition aggregation technique implies that the decomposition of the system model should be performed in such a way that the resulting subsystems are weakly coupled. This means that, any strong interconnection between system machines (except a reference one) may lead to unstable aggregation matrix. However, the triple-wise decomposition scheme, proposed in this paper, is more suitable for real power systems than the pair-wise decomposition. This technique allows strong interconnections among machines to be included in the subsystems instead of exposing them as interconnections among subsystems. Real power systems are almost invariably composed of weakly connected groups of tightly interconnected machines. It is also clear that in the triple-wise case Lyapunov function of any subsystem contains six nonlinearities, while in the case of pair-wise decomposition it is a function of two nonlinearities only. This means that, larger stability domain estimates can be obtained by increasing the number of the nonlinearities included in each free subsystem. The second property of the triple-wise decomposition is; the technique is more powerful for odd number of machines than for even number, since the estimate of the two machines sub-system will dictate the domain of attraction of the whole system.



Hybrid power systems containing unconventional units require advanced techniques to evaluate the system operation under different conditions. Wind energy conversion systems imply dynamic conditions are set under study, utilizing the properties of the triple-wise decomposition aggregation technique. The work presented in this paper approved that the new technique is providing much better estimate for the domain of attraction, and the increased energy of the WECS also improves this domain. This technique can be applied to select the optimum allocation of planned WECS's.

### References

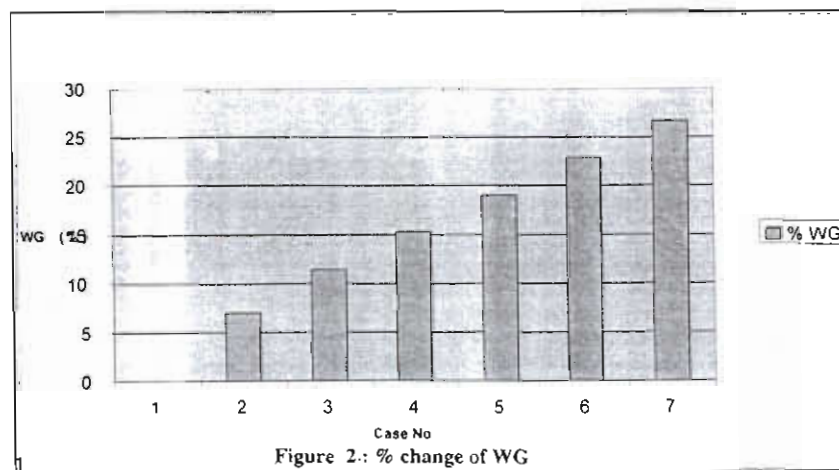
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Table (1) : System parameters

B.B	E (p.u.,deg)	M	Pm (p.u.)	Local Load
1	1.35 -5	0.42	0.117	0.6+j0
	1.35 11	0.4	0.825	0.24+j0.18
3	1.3 9	0.307	0.513	0.48+j0.36
4	1.4 14	0.316	0.755	0.34+j0.21
5	1.4 10	0.35	0.786	0.32+j0.24
	1.4 -6	0.32	0.252	0.65+j0
7	1.45 0	10	3.87	4041+j0.895

Table (2) : T.L. parameters (p.u.,deg)

Y12	0.75	-82
Y16	0.001	-82
Y17	0.62	-80
Y23	0.0008	-82
Y25	0.001	-82
Y27	0.61	-77
Y34	0.7	-81
Y37	0.65	-75
Y45	0.0008	-82
Y47	0.6	-30
Y56	0.55	-82
Y57	0.61	-77
Y67	0.5	-80



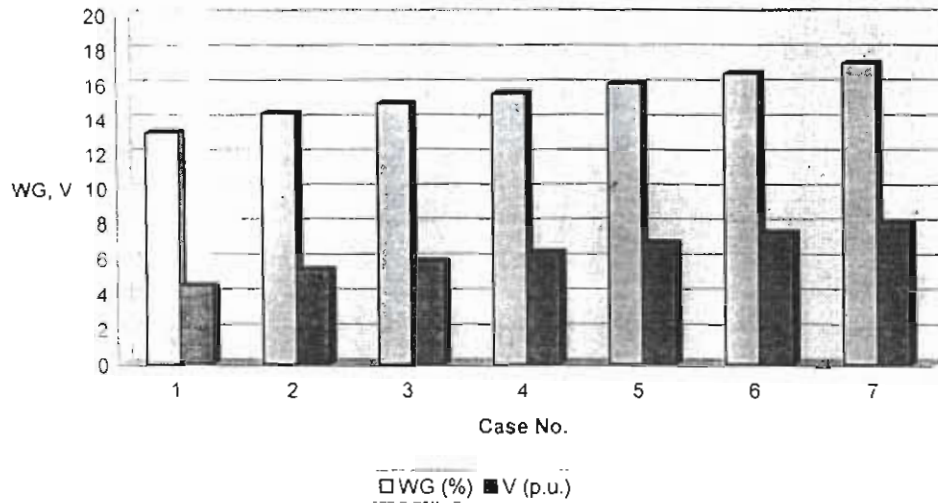


Figure 3 : Effect of changed WT on stability function V

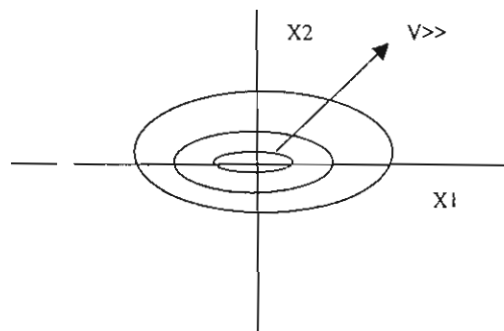


Figure (4) Contour representation of V function