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## SHEAR BEHAVIOR OF HIGH-STRENGTH FIBER REINFORCED CONCRETE DEEP BEAMS

دراسة سلوك القص في الكمرات العميقة من الخرسانة عالية المقاومة المسلحة بالألياف

Ahmed M. Yousef<sup>1</sup> and Y. I. Agag<sup>2</sup>

### خلاصة:

يتناول هذا البحث دراسة تأثير إضافة ألياف من الصلب على سلوك القص ومقاومة الكمرات الخرسانية العميقة العالية المقاومة المصممة بحيث تنهار في القص والتي تصل مقاومة الضغط المميزة للخرسانة بها إلى حوالي ٩٠ ميجاباسكال. وقد تم اختبار عدد ١٤ كمرات عميقة بسيطة الارتكاز بدون تسليح جز عى تحت تأثير حملين مركزيين متساويين على السطح العلوي للكمرات. وبداية فإن المتغيرات التي تم دراستها هي مقاومة الضغط للخرسانة عالية المقاومة ونسبة الألياف المضافة للخلطة بالإضافة إلى نسبة بحر القص إلى العمق الفعال للكمرة. وقد أوضحت مقارنة النتائج المعملية أن جساءة الكمرات وحمل التشريح في القص وكذلك الحمل الأقصى للكمرات التي تحتوى على ألياف من الصلب كانت أعلى من مثيلتها التي لا تحتوى على ألياف من الصلب. كما وجد أن تقليل نسبة بحر القص إلى العمق الفعال ينتج عنه زيادة مقاومة التشريح في القص وكذلك زيادة مقاومة القص القصوى للكمرات المحنوية على الألياف. زيادة نسبة الألياف ينتج عنها نقص ملحوظ فى اتساع الشروخ القطرية وكذلك زيادة مقاومة القص. وقد تم استخدام نتائج هذه التجارب فى اقتراح معادلة لحساب مقاومة القص القصوى للكمرات العميقة من الخرسانة عالية المقاومة التي تحتوى على الألياف بنسب مختلفة.

### ABSTRACT

The effect of inclusion of steel fibers on the shear behavior of High-Strength concrete (HSC) deep beams with a characteristic compressive strength generally approaching 90 MPa is investigated. Fourteen HSC simply supported deep beams without web reinforcement were tested under two-point symmetric top loading. The primary variables considered in this study were the characteristic compressive strength, the amount of steel fibers and shear span-to-depth ratios ( $a/d$ ). Comparing the behavior of identical beams with and without steel fibers showed that High-Strength Fiber Reinforced Concrete (HSFRC) deep beams have higher stiffness, higher initial diagonal cracking and higher ultimate load. Reduction of  $a/d$  ratio increases both the diagonal cracking and ultimate shear strength of HSFRC deep beams. Increasing the amount of steel fibers considerably reduces the crack width and increase the shear strength. Depending on the results of this investigation, an empirical equation (similar to the equation used by the Egyptian code for predicting the shear strength of deep beams without steel fibers) was proposed for predicting the shear strength of HSFRC deep beams.

**Keywords:** Fiber Reinforced Concrete; Deep Beams; High-Strength Concrete; Crack Width; Shear Strength.

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## INTRODUCTION

Considerable efforts are still being made in every part of the world to develop new construction materials. Fiber-reinforced concrete is one of the most promising new construction materials. Many studies have been carried out to explore the mechanical properties and strength characteristics of fiber reinforced concrete [1-3]. In addition, many tests have been reported on shear in fiber reinforced concrete shallow and deep beams [4-8]. These tests were conducted on beams constructed from ordinary strength concrete with compressive strength generally less than 50 MPa. Recently, Khuntia et. al. [9] summarize all available previous shear tests on fiber reinforced concrete shallow beams with and without web reinforcement. Steel fibers have some advantages over vertical stirrups or bent-up flexural steel. First, the fibers are randomly distributed through the volume of the concrete at much closer spacing than can be obtained by the smallest reinforcing rods. Secondly, the first-crack tensile strength and the ultimate tensile strength are increased by the steel fibers. The first-crack strength is increased by the crack arrest mechanism of closely spaced wires. The ultimate tensile strength is increased because additional energy is needed to pull or to strip the fibers out of the concrete, if the fibers were not broken off during initial cracking. The fibers also increase the shear-friction strength of ordinary strength concrete.

Another potential area of steel fiber use is in High-Strength Concrete (with compressive strength of 50 MPa or more). Recently, the use of High-Strength Concrete (HSC), which is growing rapidly, becomes attractive for tall building structures as well as for earthquake resistant structures where a reduction of the mass is of great importance [10]. However, the maximum potentiality of HSC can not be realized fully in structures due to its relative brittleness and lack of ductility. This drawback can be overcome by addition of steel fibers in HSC mix.

The main objective of this experimental investigation is to study the shear behavior of High-Strength Fiber Reinforced Concrete (HSFRC) deep beams and to propose an empirical equation for predicting the ultimate shear strength of this type of beams.

## EXPERIMENTAL PROGRAM

### Details of Test Specimens

The details of the test program are given in Table 1. The test specimens included 14 simply supported deep beams without web reinforcement and with constant cross section of total height 400 mm and width 80 mm. All the specimens were tested with shear span-to-depth ratios ( $a/d$ ) less than 1.0. The effective span of the beams  $l_p$  is equal to 750 mm and the distance  $c$  between the two loads varied to achieve the desired  $a/d$  ratio. The tested beams satisfy the requirements of ACI-ASCE Committee 326 [11] which defines the deep beams as beams with  $0.0 \leq a/d < 1.0$ . The ratio  $l_p/d$  of the tested beams is approximately equal to 2.0 and consequently, these beams are also considered as deep beams according to the requirements of ACI 318-99 building code [12]. The shear span-to-depth ratio varies from 0.50 to 0.88. The details of the beams are shown in Fig. 1. At locations of loading or support points thin steel plates, as shown in Fig. 1, were used to avoid the premature crushing at these points. The test specimens were divided into five groups of similar shear span-to-depth ratio ( $a/d$ ). Within each group, the steel fibers percent by volume  $F_p$  was varied. Each group includes one HSC specimen without steel fibers served as a control. Specimen DB1 constructed with ordinary strength concrete for comparison purposes.

## Materials

Three concrete mixes were used to cast the fourteen deep beam specimens. The first mix was ordinary strength concrete (used for specimen DB1) and had a specified 28-days compressive strength (cube 150x150x150 mm) of 35 MPa. The second and third mixes were HSC with 28-days compressive strength of 60 MPa and 85 MPa respectively. Ordinary Portland cement was used in conjunction with natural sand with a fineness modulus of 2.80. Details regarding the various mix constituents and quantities are given in Table 2.

The actual characteristic compressive strength  $f_{cu}$  of the HSC mixes 2 and 3 without and with different percent of steel fibers are given in Table 3, in addition to the splitting cylinder tensile strength  $f_{sp}$  (based on 150x300 mm cylinder) and the flexural strength  $f_r$  (based on 100x100x500 mm beams). It can be seen from Table 3 that, increasing the fiber contents from 0.0 to 1.5 percent slightly increased the characteristic compressive strength  $f_{cu}$  by 5% and 9% for mix 2 and mix 3, respectively. Table 3 shows that, addition of steel fibers to the concrete mixes considerably enhances the flexural strength  $f_r$  and the splitting cylinder tensile strength  $f_{sp}$ . Increasing the fiber contents from 0.0 to 1.5 percent increased the flexural strength of mix 2 and mix 3 by 63% and 65%, respectively, while the splitting strength was increased by 78% and 70%, respectively.

The cube compressive strength  $f_{cu}$  of each specimens in Table 1 was obtained on the same day of testin after 28 days of casting. The beams were cast horizontally on a vibrating table using a wooden mould. The beams and the control specimens were cast and cured under similar conditions. The specimens were kept covered with polyethylene sheets until 48 hours before testing to prevent the loss of moisture.

In the testing program, 16 mm high grade deformed steel bars having 380.6 MPa yield strength were used as main longitudinal tensile reinforcement. To provide adequate anchorage, these bars were welded to a thick steel plate at each end as shown in Fig. 1. The steel fibers used in the present study was hooked-ended straight fiber of a type available in the Egyptian market (called HAREX type). The length and equivalent diameter of the fibers were 24.3 mm and 0.76 mm, respectively, and thus the aspect ratio was 32. The minimum tensile strength of the fiber was 534 MPa. The fiber contents were varied from 0.0% to 1.5% as shown in Table 2.

## Test Procedure

The beams were simply supported and tested in a loading frame under two point loading. Special bearing assemblies (roller, bearing blocks, etc.) were designed to facilitate the application of loads to the test specimens. Dial gauges were mounted at the bottom face of the beams at mid-span and under the loading points. All the beams were tested 28 days after casting. Two days before testing the beams were allowed to dry and painted with white color to facilitate crack detection. Each beam specimen was instrumented with electrical strain gages on the main longitudinal reinforcing bars at mid-span. Small fraction of the pre-determined failure load of the specimens was applied slowly and then removed in order to exercise the deformation instruments. Load was then applied in small increments and all deformation readings were recorded at the end of each load increment. The initiation and propagation of cracks were marked and widths of all the cracks within the shear span were measured by using a hand-held microscope with an accuracy of 0.005 mm. The mode of failure was noted after final collapse.

Table (1): Details of test specimens.

Beam	$f_{cu}$ MPa	$a/d$	Fiber percent by volume ( $F_p$ %)	Longitudinal bars	$\rho_l$ % ( $A_l/b.d$ )
DB1	35.3	0.75	-	2 $\phi$ 16	1.37
DB2	61.8	0.75	-	2 $\phi$ 16	1.37
DB3	63.3	0.75	0.50	2 $\phi$ 16	1.37
DB4	83.9	0.75	-	4 $\phi$ 16	2.73
DB5	85.4	0.75	0.50	4 $\phi$ 16	2.73
DB6	89.3	0.75	1.00	4 $\phi$ 16	2.73
DB7	83.9	0.60	-	2 $\phi$ 16	1.37
DB8	85.4	0.60	0.50	2 $\phi$ 16	1.37
DB9	89.3	0.60	1.00	2 $\phi$ 16	1.37
DB10	91.2	0.60	1.50	2 $\phi$ 16	1.37
DB11	86.8	0.50	-	2 $\phi$ 16	1.37
DB12	91.2	0.50	1.50	2 $\phi$ 16	1.37
DBS13	88.8	0.88	1.00	4 $\phi$ 16	2.73
DBS14	86.8	0.88	-	4 $\phi$ 16	2.73

Table (2): Details of Concrete Mixes.

	Mix1	Mix 2	Mix 3
$f_{cu}$ (MPa)	35	60	85
Coarse Aggregate (kg)	1215	1152	1152
Gravel (M.N.S.) mm	19	19	12
Natural Sand (kg)	654	620	586
Ordinary Portland Cement (kg)	400	450	475
Water (kg)	160	135	123.5
W/C (ratio)	0.40	0.30	0.26
Silica Fume (kg)	-	22.5	71.25
Superplasticizer (kg)	-	9.0	14.25
Slump (mm)	110	70	80

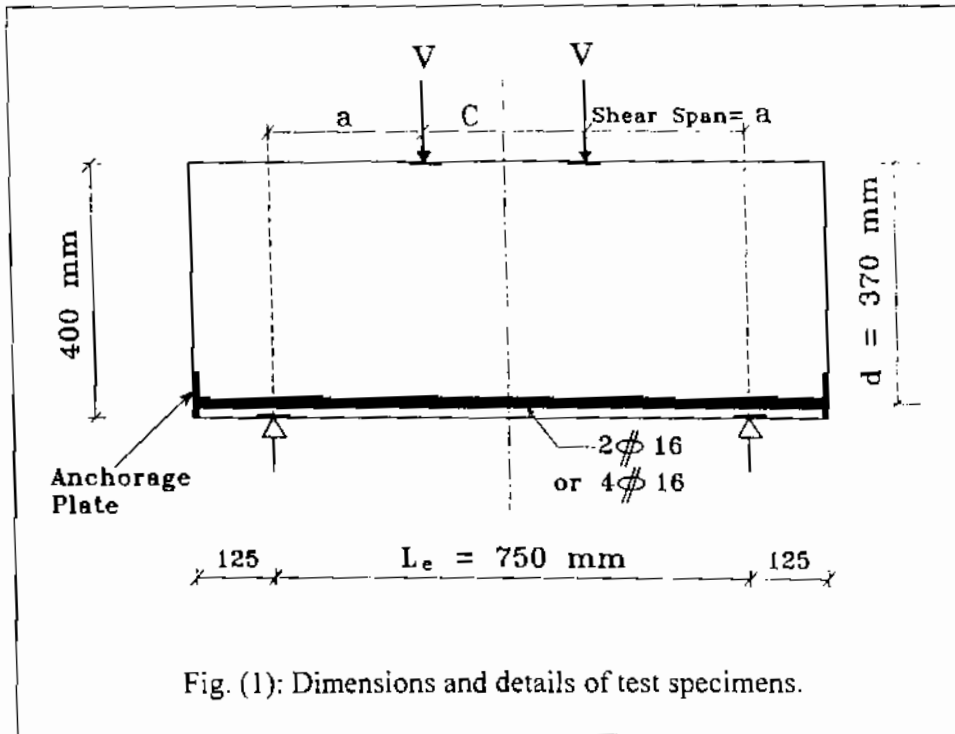


Fig. (1): Dimensions and details of test specimens.

Table (3): Strength Characteristics of HSFRC mixes.

	$f_{cu}$ (0.0%) (MPa)	$f_{cu0.5}$ (0.5%) (MPa)	$f_{cu1.0}$ (1.0%) (MPa)	$f_{cu1.5}$ (1.5%) (MPa)	$f_r$ (0.0%) (MPa)	$F_{r0.5}$ (0.5%) (MPa)	$f_{r1.0}$ (1.0%) (MPa)	$f_{r1.5}$ (1.5%) (MPa)	$f_{sp}$ (0.0%) (MPa)	$f_{sp0.5}$ (0.5%) (MPa)	$f_{spl.0}$ (1.0%) (MPa)	$f_{spl.5}$ (1.5%) (MPa)
Mix 2	61.8	62.7	63.3	64.9	6.45	7.25	8.30	10.50	3.60	4.45	5.95	6.40
$(f_{fiber}/f)^*$	-	1.015	1.024	1.05	-	1.13	1.29	1.63	-	1.24	1.65	1.78
Mix 3	83.9	85.4	89.3	91.2	8.05	9.35	10.60	13.30	4.60	5.75	7.25	7.80
$(f_{fiber}/f)^*$	-	1.018	1.064	1.09	-	1.16	1.32	1.65	-	1.25	1.58	1.70

\*  $(f_{fiber}/f)$  = The ratio between the strength properties (compressive, flexural and splitting) of HSC specimens with steel fibers and the HSC specimens without fibers

## ANALYSIS OF TEST RESULTS

### Load-Deflections behavior

The mid-span deflection curves for specimens with different shear span-to-depth ratio ( $a/d$ ) and different percent of steel fibers ( $F_p$ ) are shown in Figs. 2.a to d. In early stages of loading, the beams behaved in a truly elastic manner. It is clear from Fig. 2.a and b that for the same percent of steel fibers, increasing the  $a/d$  ratio leads to a decrease in the stiffness of the HSFRC beams. As can be seen in Fig. 2.a, HSFRC deep beam DB9 with  $F_p$  equal to 1% and  $a/d$  ratio equal to 0.6 is more rigid than HSFRC beam DB13 with the same  $F_p$  and with  $a/d$  equal to 0.88. For the same  $a/d$  ratio, increasing the steel fiber percent in the tested specimens slightly enhances the stiffness of the beams. As can be seen from Fig. 2.c, beam DB10 with  $a/d$  equal to 0.6 and with  $F_p$  equal to 1.5% is stiffer than the tested beams with the same  $a/d$  ratio and with less fiber percent.

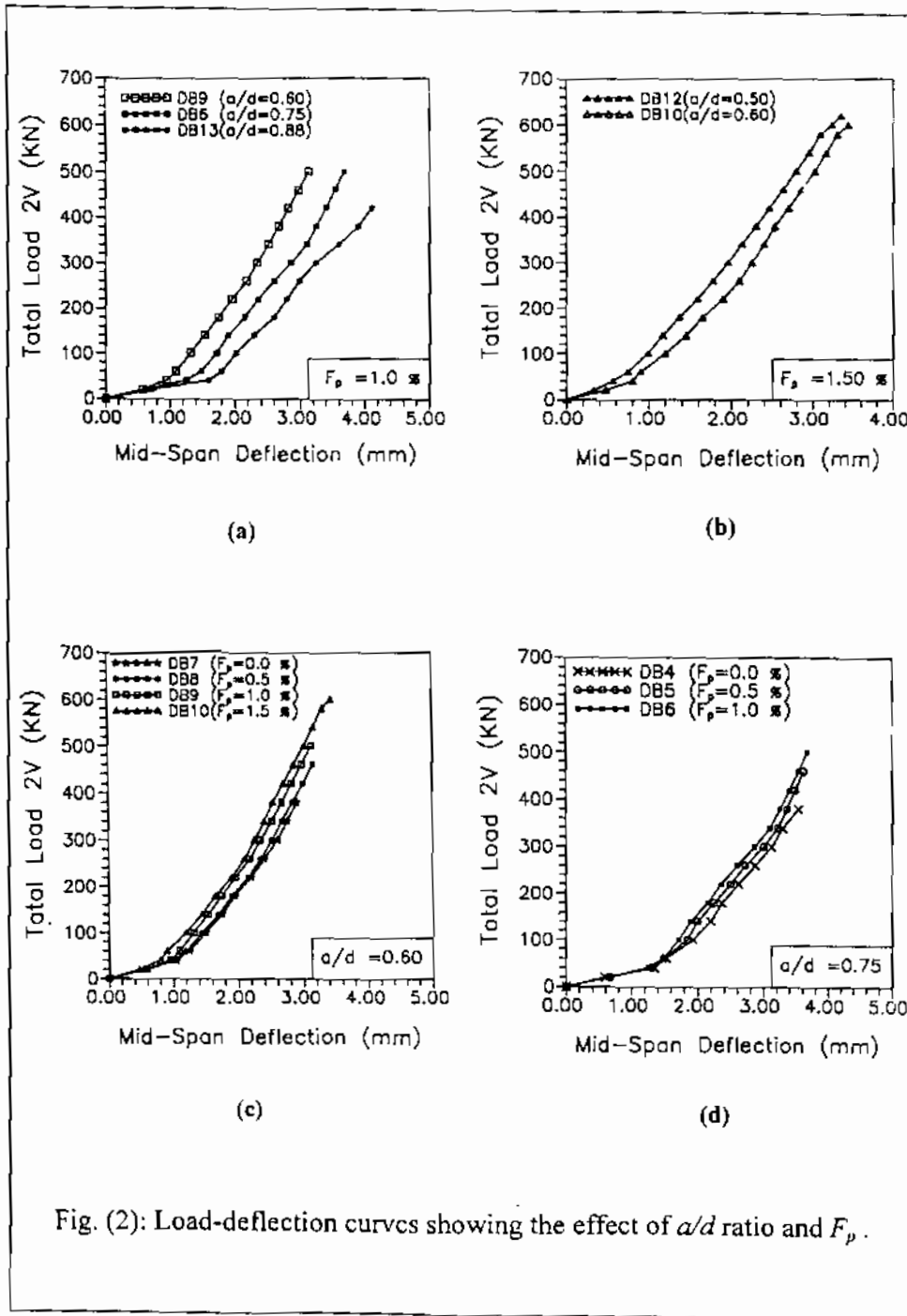
### Longitudinal Steel Strains

Records of the longitudinal steel strain at the mid span of sample of the tested beams are plotted in Fig. 3 versus the total applied load (twice the shear  $V$ ). The results showed that strains in the region of maximum bending moment are almost uniform at every load level. Failure of the tested specimens occurred before yielding of the longitudinal bars. Tensile steel strains increase approximately at a constant rate. Formation of inclined cracks has no effect on the strain readings. For the same  $a/d$  ratio, that the strain readings in the longitudinal bars of HSFRC specimens (specimens DB6 and DB13) are less than that of ISC specimens without steel fibers (specimens DB4 and DB14, respectively). This indicates that addition of steel fibers to HSC deep beams increases the stiffness of these beams.

### Cracking Behavior and Ultimate Loads

All the tested specimens failed in shear. The span of the specimens collapsed due to excessive destruction of concrete in the shear span. Photographs of sample of test specimens that show typical observed cracking patterns and failure mode are shown in Fig. 4. The numbers written along the cracks on the photographs indicate the termination of cracks observed at the end of a particular load stage. These crack loads on the photographs are in tons since the loading jack indicator is divided into tons. The first flexural cracks were formed in the region of maximum bending moment.

Table 4 presents the total measured flexural cracking strength ( $2V_{crf}$ ), diagonal cracking strength ( $2V_{crsh}$ ) and ultimate strength ( $2V_{uexp}$ ) of the 14 deep beams tested in this study. For the HSFRC specimens, between 34% and 43% of the ultimate load, a sudden major inclined crack formed. This was a diagonal shear crack that usually originated suddenly in the middle of the shear span and propagated toward the support and loading point from a subsequent increase in applied load. Inclined crack angle with respect to the horizontal plane was about 45 to 55 deg for the tested specimens. With further increase in the applied load, the existing diagonal shear cracks propagated very slowly while a few numbers of new inclined cracks were formed. An almost stable position of all existing cracks was observed at approximately 70 percent of ultimate load and after this level, the diagonal crack width increased. Finally, shear failure occurred suddenly by fracture of the concrete along the inclined crack.





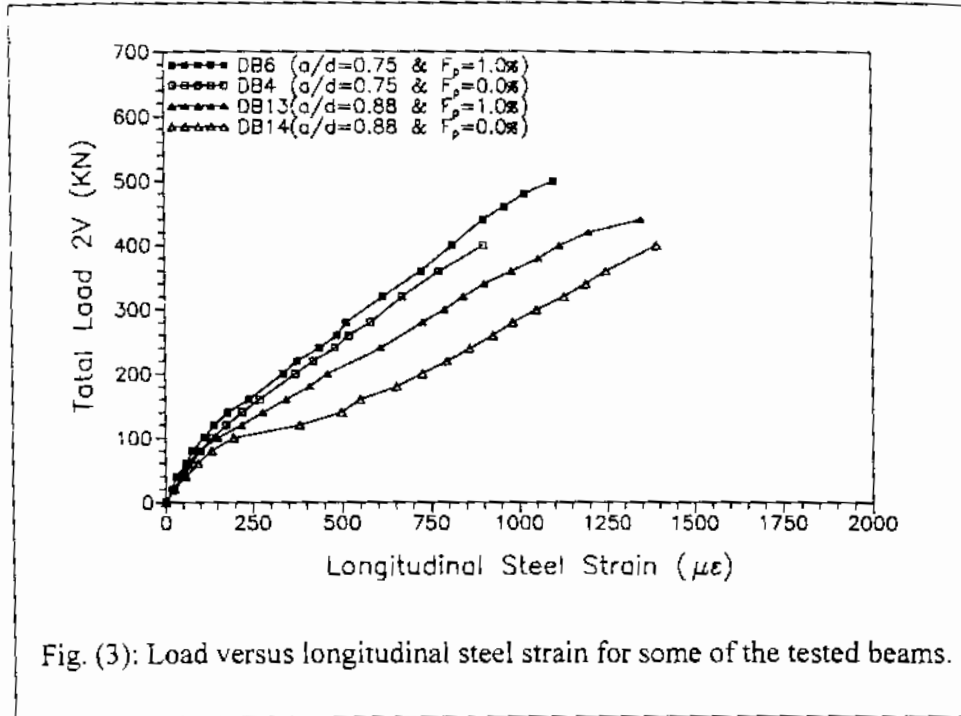
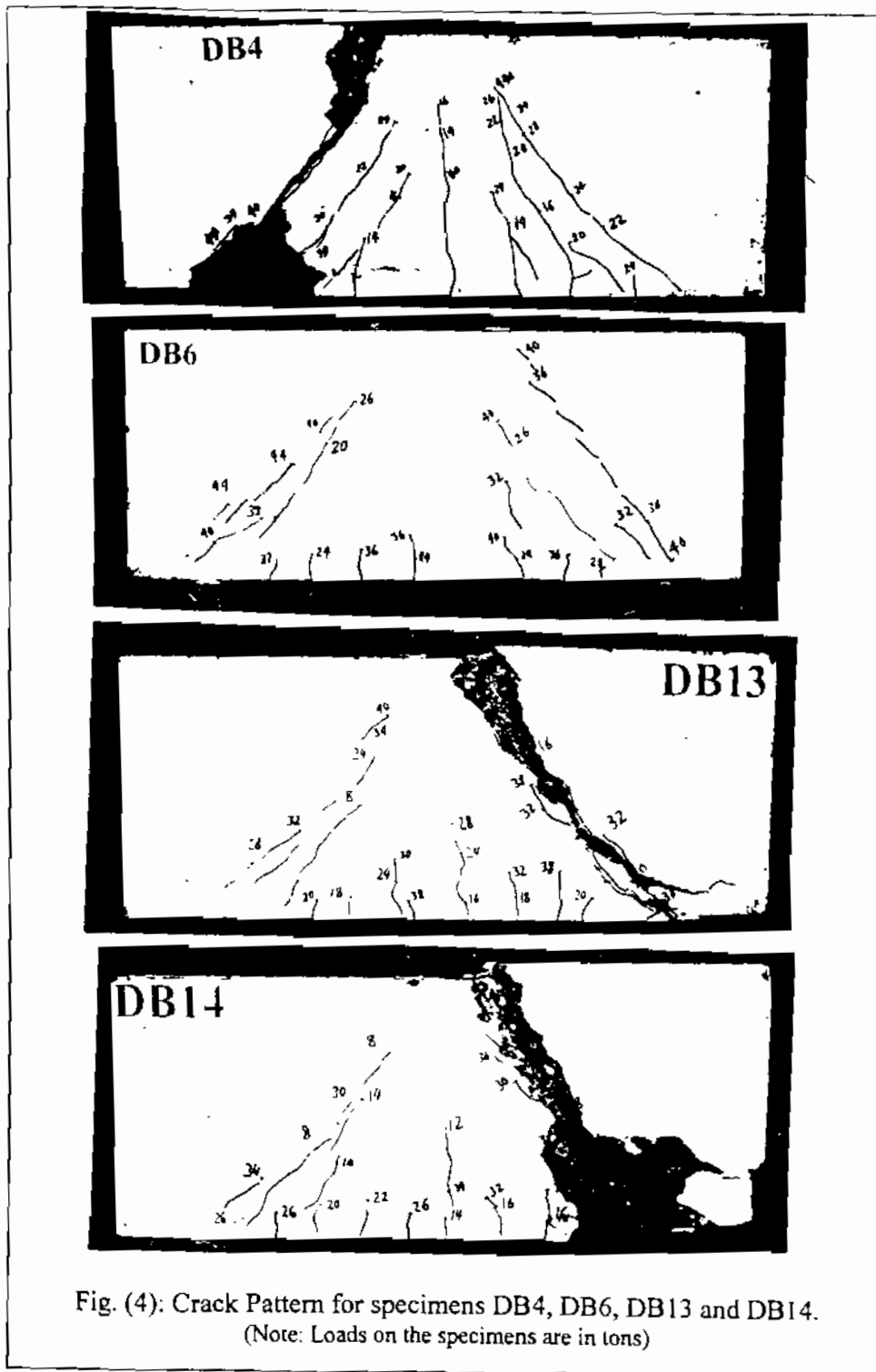


Fig. (3): Load versus longitudinal steel strain for some of the tested beams.

Table (4): Summary of test results.

Beam	$f_{cu}$ MPa	$2V_{crfl}$ (flexure) KN	$2V_{crsh}$ (shear) KN	$2V_{uexp}$ KN	$\left(\frac{V_{crsh}}{V_{uexp}}\right)$	$\frac{2V_{crsh}}{b \cdot d \sqrt{f_{cu}}}$	$\frac{2V_{uexp}}{b \cdot d \sqrt{f_{cu}}}$
DB1	35.3	65	100	195	0.51	0.57	1.11
DB2	61.8	100	150	335	0.45	0.65	1.44
DB3	63.3	115	195	450	0.43	0.83	1.91
DB4	83.9	110	160	435	0.37	0.59	1.60
DB5	85.4	105	195	505	0.39	0.71	1.85
DB6	89.3	140	200	525	0.38	0.72	1.88
DB7	83.9	110	160	430	0.37	0.59	1.59
DB8	85.4	115	180	490	0.37	0.66	1.79
DB9	89.3	140	200	535	0.37	0.72	1.91
DB10	91.2	180	220	630	0.35	0.78	2.23
DB11	86.8	140	190	450	0.42	0.69	1.63
DB12	91.2	220	220	645	0.34	0.78	2.28
DB13	88.8	160	160	445	0.36	0.57	1.60
DB14	86.8	120	140	380	0.37	0.51	1.38



The HSFRC deep beams failed with ample warning by gradual failure while the HSC deep beams without fibers failed with explosive shear failure mode. Explosive herein refers to a sudden release of stored energy, accompanied by a deafening sound. The cracking patterns of the inclined cracks shown in Fig. 4, gave the appearance of a tied-arch system to the beams, with the tension reinforcement acting as the tie rod and portions of the beams outside the inclined crack as compression struts.

In order to eliminate the effect of the little differences in concrete strength when comparing the diagonal cracking strength and the ultimate strength of the tested beams with and without fibers, the total normalized stresses ( $2V/(b d \sqrt{f_{cu}})$ ) can be used as given in Table 4. It is clear that inclined tension cracking load for HSFRC beams was less than that of beams without fibers. A plot of the variation of the total normalized ultimate stresses versus the different values of  $a/d$  ratio is shown in Fig. 5. It can be seen that the ultimate shear stress of the beams increases remarkably as  $a/d$  decreases. This increase in ultimate shear strength is mainly attributed to the increase in arch action that seems to be increased with decreasing  $a/d$  ratio. The normalized ultimate shear stress of HSFRC deep beams are generally greater than that of beams without fibers, i. e., increasing the percentage of fibers considerably increase the normalized ultimate shear stress.

### Crack Width

Diagonal cracks are almost uniform on both sides of the beam. Maximum crack widths along the major diagonal crack in the shear span occurred almost at middepth of the beam. Plots of the total applied load (twice the shear  $V$ ) versus the observed maximum diagonal crack width for the tested deep beams with different  $a/d$  ratio and different fiber contents  $F_p$  are shown in Fig. 6.a to d.

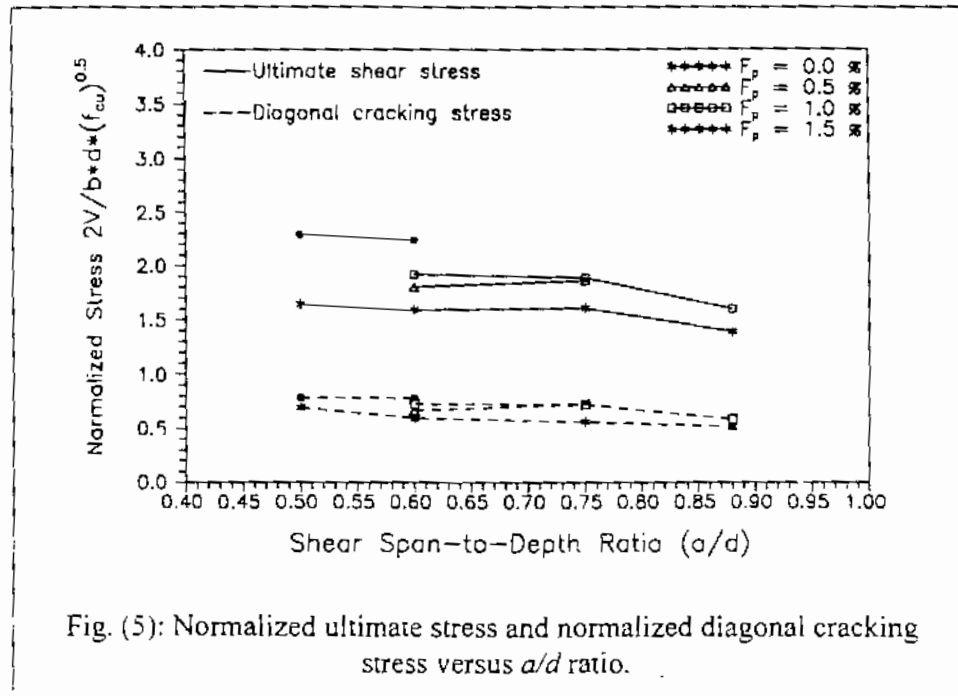


Fig. (5): Normalized ultimate stress and normalized diagonal cracking stress versus  $a/d$  ratio.

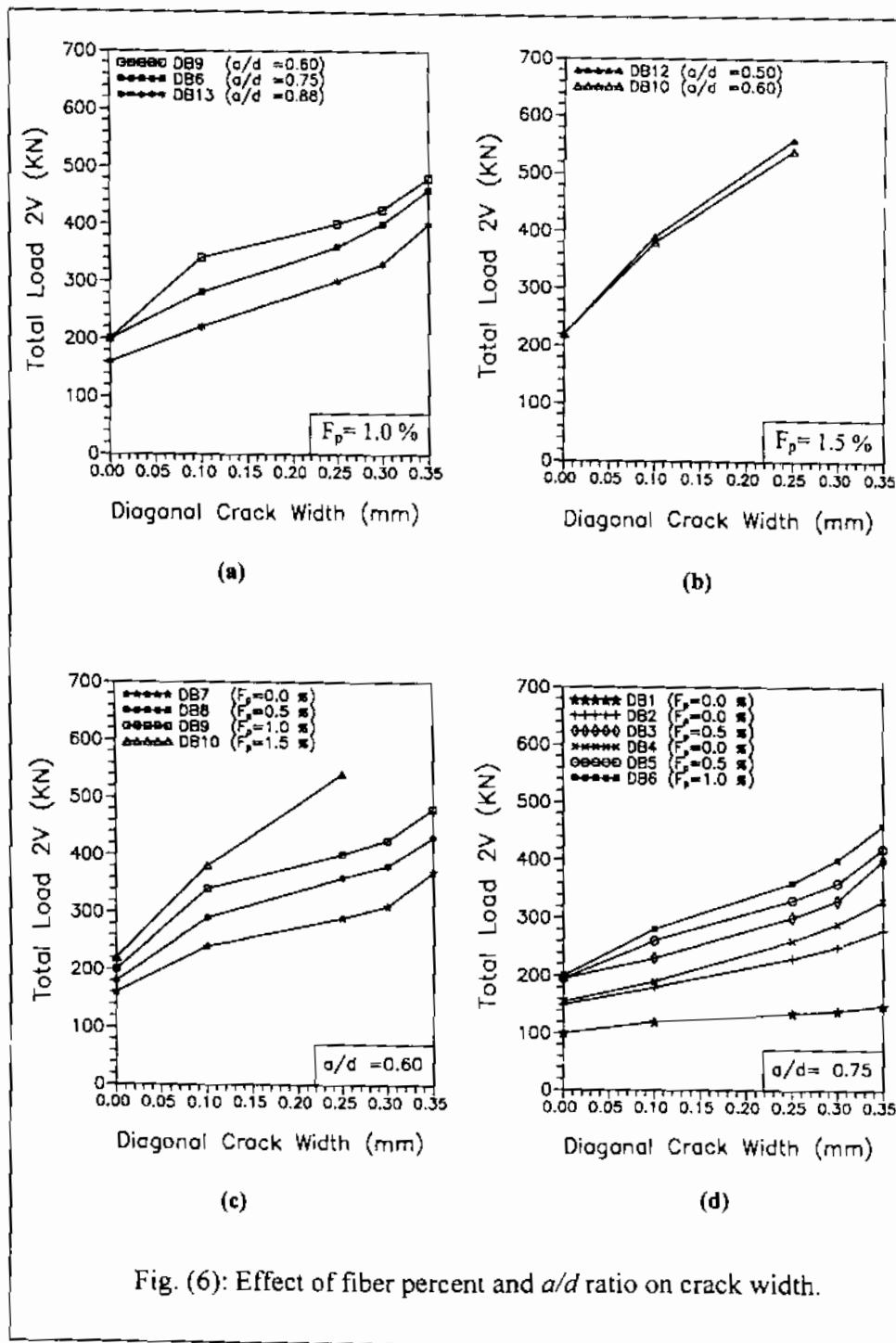


Fig. (6): Effect of fiber percent and  $a/d$  ratio on crack width.

Tests indicate that the diagonal cracks tend to increase with load. It can be seen from Fig. 6.a and b that the development of the major diagonal crack for HSFRC beams containing the same fiber percent is faster for beams with higher  $a/d$  ratio. For example, at the same load, the diagonal crack width for beam DB13 with fiber percent equal to 1% and with  $a/d$  ratio equal to 0.88 was considerably more than that of beam DB9 with the same fiber percent but with  $a/d$  ratio equal to 0.60. Figure 6.d shows that, for  $a/d$  ratio equal to 0.75, the development of the major diagonal crack of HSFRC beam DB6 with fiber percent 1.0% is slower than that of the beams without fibers or beams with smaller fiber content. This indicates that, for beams with the same  $a/d$  ratio, increasing the fiber content restricts the diagonal cracks development.

### PROPOSED DESIGN EQUATION FOR ULTIMATE SHEAR STRENGTH OF HSFRC DEEP BEAMS

The development of a simple expression to predict the ultimate shear strength of HSFRC deep beams is very important for successful practical application of steel fibers. In order to design the deep beams with and without shear reinforcement in shear, many of the available codes [12-14] use the standard method which assumes that the total design shear strength,  $v_{ud}$ , is taken as the sum of the shear carried by the concrete,  $v_c$ , and that carried by the shear reinforcement,  $v_s$ . It should be noted that the ultimate shear force  $V_{ud}$ , can be calculated by multiplying the ultimate shear strength  $v_{ud}$  by  $(b \cdot d)$ . The same method can be used to predict the ultimate shear strength of HSFRC deep beams,  $v_{udfr}$ , which can be expressed as

$$v_{udfr} = v_c + v_{fr} \quad (1)$$

Where,  $v_c$ , is the shear strength carried by the concrete and  $v_{fr}$ , is the shear strength carried by the steel fibers. The equations used in the draft of the Egyptian Code for design of reinforced concrete structures [14] for shear design of deep beams are the same as these used in the 1995 Code [13], except that the units are changed to be S.I. units. According to the draft of the Egyptian Code for design of reinforced concrete structures [14], the design shear strength of normal strength concrete can be calculated from the following expression:

$$v_c = 0.24 \delta_{dc} \sqrt{f_{cu}} / \gamma_c \quad (2)$$

And

$$\delta_{dc} = 3.5 - 2.5 (M_u / V_u d) \quad (3)$$

where  $f_{cu}$  is the cube (150x150x150 mm) compressive strength in MPa,  $\gamma_c$  is the material safety factor for concrete which is taken equal to 1.5,  $M_u$  is the ultimate moment at the critical section and  $V_u$  is the ultimate shear force at the critical section.

From the results of the present experimental investigation it is clear that the main factors affecting the part of the ultimate shear strength that carried by steel fibers  $v_{fr}$  is the quantity of the steel fibers in the concrete mix in the form of fiber percent by volume ( $F_p$ ). The percent increase in strength of the beams containing steel fibers to that of the same group

Table (5): Summary of test results.

Beam	$f_{cu}$ MPa	$F_p$ %	$V_{uexp}$ KN	Percent increase in strength due to fiber	$V_c$ KN (Eq. 2)	$V_{fr}$ KN (Eq. 4)	$V_{udfr}$ KN (Eq. 5)	$\left(\frac{V_{udfr}}{V_{uexp}}\right)$
DB1	35.3	-	97.5	-	55.7	-	55.7	0.57
DB2	61.8	-	167.5	-	74.1	-	74.1	0.44
DB3	63.3	0.50	225.0	34	75.0	13.5	88.5	0.39
DB4	83.9	-	217.5	-	86.4	-	86.4	0.40
DB5	85.4	0.50	252.5	16	87.1	15.6	102.7	0.41
DB6	89.3	1.00	262.5	21	89.1	32.0	121.1	0.46
DB7	83.9	-	215.0	-	106.2	-	106.2	0.49
DB8	85.4	0.50	245.0	14	107.2	15.6	122.8	0.50
DB9	89.3	1.00	267.5	24	109.6	32.0	141.6	0.53
DB10	91.2	1.50	315.0	47	110.8	48.5	159.3	0.51
DB11	86.8	-	225.0	-	121.6	-	121.6	0.54
DB12	91.2	1.50	322.5	43	124.6	48.5	173.1	0.54
DB13	88.8	1.00	222.5	17	71.1	31.9	103.0	0.46
DB14	86.8	-	190.0	-	70.3	-	70.3	0.37

and without steel fibers are given in Table 5. It can be seen that, the increase in strength of the beams containing the same fiber percent and tested at different  $a/d$  ratio were approximately the same (for example DB6 and DB9). This showed that the  $a/d$  ratio had not considerable effect on  $V_{fr}$ . The fiber shape and fiber aspect ratio affect also the ultimate strength of reinforced concrete beams [7]. However, in this study, these last two factors were constant because only one type of steel fibers (HAREX type) was studied.

As discussed before and as it is well known [12,13,14], the measures of tensile strength of plain concrete, such as the flexural strength  $f_r$  and the cylinder splitting strength  $f_{sp}$  are considered to be proportional to  $\sqrt{f_{cu}}$ . It can be assumed that  $v_{fr}$  is also proportional to  $\sqrt{f_{cu}/\gamma_c}$  as used in the Egyptian code for normal strength concrete. Thus, interpolation of the values of the ultimate shear strength of HSFRC deep beams tested in this study results in the following expression to predict the values of  $V_{fr}$  for different percents of steel fiber ( $F_p$ ):

$$V_{fr} = (0.14 F_p \sqrt{f_{cu} / \gamma_c}) b d \quad (4)$$

It should be noted that, in getting the expression given in Eq. 4, it was intended to maintain.

approximately, the same level of conservatism required by the Egyptian code equation for design of reinforced concrete deep beams without fibers (Eq. 2 in this work) as indicated by the tested beams without fibers. In order to change the level of conservatism in Eq. 4, the value of 0.14 can be reduced or increased. The value of the fiber percent  $F_p$  should be expressed in Eq. 4 as 0.5, 1.0, 1.5, ... etc.

Then, the values of  $V_{udfr}$  can be calculated using the following equation.

$$V_{udfr} = (0.24 \delta_{dc} \sqrt{f_{cu} / \gamma_c} + 0.14 F_p \sqrt{f_{cu} / \gamma_c}) b d \quad (5)$$

The calculated ultimate shear force that carried by concrete ( $V_c$ ) and the calculated ultimate shear force that carried by steel fibers ( $V_{fr}$ ) are given in Table 5, in addition to the ratio between the calculated ultimate shear force ( $V_{udfr}$ ) using Eq. 5 and the measured shear force of the tested beams ( $V_{uexp}$ ). It can be seen that the values of the ultimate shear strength of the tested beams without fibers ( $F_p = 0.0\%$ ) predicted by the proposed equation (i.e., the Egyptian code equation) were very conservative. These beams were constructed from normal strength concrete (beam DB1) and HSC (beams DB2, DB4, DB7, DB11, DB14). This showed that the equation of the Egyptian code for calculating the shear strength of deep beams without web reinforcement can be safely applied to HSC deep beams. It is clear also from Table 4 that, the values of the ultimate shear strength of HSFRC deep beams ( $F_p = 0.5\%$ ,  $1.0\%$  and  $1.5\%$ ) predicted by the proposed equation were conservative for the beams tested in this study. This indicates that the proposed equation can be safely used to calculate the contribution of the steel fibers to the ultimate shear strength.

## CONCLUSIONS

Based on the results of the present experimental investigation on the shear behavior of HSC deep beams with and without steel fibers, the following conclusions can be drawn:

1. HSFRC deep beams with characteristic compressive strength approaching 90 MPa have higher stiffness, higher initial diagonal cracking load and higher ultimate load than that of the same HSC deep beams without steel fibers.
2. Reducing the shear span-to-depth ratio increases both the diagonal cracking strength and ultimate shear strength of HSFRC deep beams.
3. Increasing the amount of steel fibers up to 1.5% in HSFRC deep beams considerably reduces the crack width, provides better cracking control and increases the shear strength.
4. The equation of the Egyptian code for calculating the shear strength of deep beams can be conservatively and safely applied to HSC deep beams.
5. A simple empirical equation for conservatively predicting the contribution of the steel fibers in the ultimate shear strength of HSFRC deep beams without shear reinforcement was proposed. This equation is similar in shape and approximately maintains the same level of conservatism of the equation used by the Egyptian code for predicting the shear strength of deep beams without steel fibers. The two equations are used together for designing the shear strength of HSFRC deep beam.

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