Mansoura Engineering Journal

Volume 27 | Issue 4 Article 6

1-25-2021

Fundamental Natural Frequencies for Simply Supported Trusses.

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FUNDAMENTAL NATURAL FREQUENCIES FOR SIMPLY SUPPORTED TRUSSES

" الترددات الطبيعية الأساسية للجمالونات البسيطة الارتكاز "

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الخلاصة : هذا البحث يتناول في البداية عرضا مبسطا لطرق حساب الترددات الطبيعية وأهمية إستخدامها في تحليل المنشآت خاصة التحليل الديناميكي, ثم تم عمل التحليل الإستاتيكى لعدد كبير جدا من الجمالونات ذات الارتكاز البسيط لنوعية هامة وكثيرة الاستخدام في المنشآت ,نوع على شكل Warren والآخر على شكل حرف N مع تغيرات عديدة في عدد البواكى لكل نوع وطول الباكية والارتفاع الداخلي والخارجي وكذلك ميول الأسقف وكثافة الأحمال المنتظمة المؤثرة على المنشأ .ثم تم بعد ذلك تصميم كل نوع من الجمالونات طبقا للكود المصري للمنشآت المعدنية ثم تم حساب الترددات الطبيعية وتركزت النتائج حول التردد الأول والثاني لكل نوع وتم عرض النتائج .وقد تم الوصول بهذه النتائج إلى علاقات تقريبية لاستخدامها في الحصول على الترددات المطلوبة لمثل هذا النوع من المنشآت ثم عمل مقارنات بين القيم المحسوبة باستخدام هذه العلاقات والقيم الحقيقية ونسبة الخطأ في كل حالة .

ABSTRACT

The free vibration mode shapes have certain special properties, which are very useful in structural analysis, especially in the dynamic analysis. A brief review of eigenvalues eigenvector (natural frequencies at corresponding mode shapes) is presented The static analysis for a numerous number of N-Types and Warren system of trusses with variable parametric studies such as, intensity of loads, number of panels, panel length, interior and exterior heights, types of materials, ... and so on are carried out using stiffness method The eigenproblem equation is solved by common methods with condensation if required The final results for first and second frequencies are fitted to multiple regression programs to get the final empirical formulae. Finally, comparisons between exact and computed values of frequencies are demonstrated.

Accepted December 15, 2002.

1-INTRODUCTION

In many engineering problems, such as linearized buckling or vibrations, it is needed to know the eigenvalues (natural frequency), and the eigenvector (mode shapes) for structures. In static analysis of structures, the eigenproblem solution is reserved to know the critical loads. A complete dynamic analysis and the application of the response spectrum for seismic design of structures require the determination of eigenvalues eigenvector. The natural frequencies of a structure indicate whether or not the structure is likely to respond dynamically. The shapes of the modes will indicate in which way the structure is likely to respond and the best position for placing artificial dampers if required.

Numerous techniques of structure having few and large numbers of degrees of freedom are used. For a system having a few degrees of freedom, the direct and Viavello-Stodoala methods are applicable to use and easy to handle calculations. The most common iterative method is the Jocobi's method [1]. Also, iteration methods [2 and 3] have a wide use in computation techniques for structures with a large number of degrees of freedom .It is often necessary to reduce the problem size by simplifying the structure (the mathematical model) or by condensing the stiffness and mass matrices. There are a lot of methods in this field [4, 5, 6, 7, 8, 9, 10 and 11]. A brief review about famous and interesting methods [12] are mentioned.

Because only the first few lower frequencies are of interest in application of structural dynamic [12]; this work is focused about the computation of first and second natural frequencies by obtained empirical formulae with a large varieties of properties and dimensions. Finally, A comparison between computed frequencies using obtained empirical formulae and exact values of frequencies with a percentage of errors is demonstrated.

-Methods of computation eigenproblem:

If it is assumed that the natural frequencies and mode shapes are not significantly affected by the amplitude of vibration, then both eigenvalues and eigenvector may be found by the eigenproblem equation:

$$\left| [K] - \omega^2 [M] \right| \{a\} = \{0\} \quad \ldots(1)$$

Where

$[K]=$ The tangent stiffness matrix of structure at the static equilibrium position;

$[M]=$ The lumped or consistent mass matrix;

$\{a\}=$ The corresponding mode shapes ; and

$\omega^2 =$ The N*N diagonal matrix of the square of the natural frequencies.

The formulation of Eq (1) is an important mathematical problem for eigenproblem and its nontrivial solution requires that, the determinant of the matrix factor of {a} be equal to zero as:

$$\left| [K] - \omega^2 [M] \right| = \{0\} \quad \ldots(2)$$

2-STEP BY STEP ANALYSIS

This work is carried out for N-system and Warren simply supported trusses for many variations of geometry of strucures and their properties. This analysis is achieved through the following steps: -

1) The static analysis of Warren and N-systems of simply supported trusses shown in Figs (1) and (2), respectively is carried out, using the stiffness method FORTRAN program and the Egyptian code. First author constructs this FORTRAN program code. All types of analysis are proposed with the following parametric studies for both types of simply supported trusses.
   - Number of panels ranges between 6 and 14.
   - The panel lengths range from 1.5 to 3 m, with step interval of 0.25 m.
   - The uniform distributed loads on the upper chord members are taken as 0.5 to 4 t/m with 0.5 t/m step intervals.
   - The exterior height of a truss, h1, is taken as 10% of its span, while the interior height, h2, ranges from 10% to 15% of its span.
   - The roof slopes were taken as 0 % to 10 % with steps of 2 %
   - The used material was mild steel with intensity of 7 85 t/m^3 and modulus of elasticity of 2000 t/cm^2.
2) Each type of truss with its properties and data assumptions is designed according to Egyptian code [13] with the following limitations and considerations: -
   - Each Warren truss is designed as six groups of members such as one group for each upper chord, lower chord, exterior verticals and interior verticals, and two groups for diagonals.
   - Each N-system of truss is designed as five groups of members such as, one group for each upper chord, lower chord, interior verticals, exterior verticals, and diagonal members.
   - The tension members are designed considering the allowable tension stress as 1 t/cm^2, while for compression members, the allowable buckling stress was taken as 0.8 t/cm^2.
   - The construction conditions are satisfied with minimum section of two angle 60x 60x 6 mm
3) The solution of eigenproblem equation for each case of truss is carried out using a FORTRAN program proposed by first author taking into consideration the lumped and consistent mass matrices.
4) The final obtained results throughout the multiple regression [14] FORTRAN program with accuracy up to ten parameters to get the empirical formulae used to compute the fundamental frequencies for simply supported trusses.

3- OBTAINED RESULTS

Some samples of results are drawn to show the variation of frequencies Considering Warren and N-system of trusses with 2m panel length, 6, 8, 10, 12, and 14 panels, interior and exterior heights of 10 % of truss span, and the variation of the uniform distributed loads acting on upper chord members starting with 0.5 t/m and ending with 4 t/m, the results are drawn in Figs. (3) and (4), respectively. Also, the corresponding mode shapes for first and second natural frequencies for Warren and N-trusses with intensity of load of 1t/m are shown in Fig (5) and (6), respectively. These results showed that: -

- Increasing the intensity of the uniform distributed loads decreases the frequency values.
- Increasing the span of truss decreases the frequency values.
- The value of each frequency in case of N-system of truss is smaller than the corresponding value in case of Warren- truss. This difference is small. The use of consistent mass matrix instead of lumped mass matrix gave remote changes in frequency.
- The corresponding mode shapes shown in Figs. (5) and (6), and other modes for higher frequencies (not drawn) showed that these two frequencies (first and second frequencies) are enough. They can happen whereas other higher modes do not happen for these types of trusses.

4-EMPIRICAL FORMULAE

For a simply supported Warren and N-system of trusses, this work leads to the following approximate expression for the first and second natural frequencies in cycle per second (c.p.s).

$$\omega_i = R_l . R_m . R_h . R_e . R_\gamma \qquad ., i = 1, \text{ and } 2 \qquad (3)$$

Where
i = is the frequency number ;

- R_l is given by:

$$R_l = C_o + \Sigma C_i . a^i \qquad (4\text{-a})$$

Where i = 1,2,3,4 and 5.

n which a is the panel length of truss by meter; and coefficients C_o and C_i for first and second frequencies in both types of trusses (Warren trusses with W symbol, and N-system of trusses with N symbol) are given in Tables (1) and (2), respectively.

- R_m is the coefficient of effect of the uniform distributed loads (w t/m) on upper chord members.

The values of this coefficient for both types of trusses with panel lengths of 1.5, 2, 2.5 and 3m are given in Tables (3), (4), (5), and (6), respectively.

- R_h is the coefficient used to take the effect of variation of roof slopes

The values of this coefficient with all considered variations of roof slopes with panel lengths of 1.5, 2, 2 5, 3m for both types of trusses with all variations of number of panels are given in Tables (7, 8, 9 and 10), respectively.

- R_e is the coefficient for the effect of modulus of Elasticity.

Considering the modulus of Elasticity used in this work as 2000 t/m^2, the value of Re with modulus Ec is equal to $\sqrt{\dfrac{Ec}{2000}}$ (4-b)

- R_γ is the coefficient used to take the effect of intensity of material used in the design of truss sections. Considering the material used in this work is mild steel with an intensity of $7.85 t/m^3$ and total mass of truss γ_{ts}, the value of $R\gamma$ with other material with total mass of truss of γ_{tc} is given by:

$$R\gamma = \sqrt{\frac{\gamma_{ts}}{\gamma_{tc}}} \quad \ldots \qquad (4\text{-c})$$

γ_{tc} = Span of trusses • Intensity of loads $+ \sum A_i \cdot \gamma_i$.

where $i = 1,2,3 \ldots \ldots\ldots, NM$

NM = Total number of members

A_i = Area of members.

γ_i = Intensity of reference material as steel ($7.85 t/m^3$).

γ_{tc} = The same equation with design material instead of steel.

The following notes must be taken into considerations:

1- Total mass = total weight / gravity acceleration.

2- All coefficients given in Tables (1) and (2) are computed with a uniformly distributed load of $1 t/m^2$.

3- The values of Rm and Rh in Eq. (3) are given as unity in case of a uniform load of $1t/m^2$.

4- For any panel length not mentioned in all tables, linear interpolation technique for just upper and lower values to required value is used.

Finally to examine the empirical formulae mentioned in the expression (3) and (4), for computation of the first and second natural frequencies for both types of simply supported truss are given in Tables (11) and (12).

5-COMPUTED EXAMPLE

With reference to Tables (11), and (12) for case number (10), with N-system of truss, 8 panels, 2m panel length, h_1= 1.6 m, h_2=1.92 m, slope 4%, Ec= 1800 t/cm², intensity of loads = 3t/m, Eq. (4 - a) using Table (1), R1 for first frequency is:

$$R_1 = 16.32886 - 17.99504 \cdot (2) + 10.44851 \cdot (2)^2 - 3.105822 \cdot (2)^3 + 0.434465 \cdot (2)^4 - 0.020434 \cdot (2)^5$$

$= 3.584$ (Table 1)

$R_m = 0.918$, (Table 4)

$R_h = 1.07$ (Table 8)

$$R_e = \sqrt{\frac{1800}{2000}} = 0.949 \ , \qquad R_\gamma = \sqrt{\frac{\gamma_{ts}}{\gamma_{tc}}} = \sqrt{\frac{5.07}{4.95}} = 1.012 \ ,$$

$\omega_c = 3.584 \cdot 0.918 \cdot 1.07 \cdot 0.949 \cdot 1.012 = 3.38$ (c.p.s)

6-CONCLUSION

This work is concentrated about producing the empirical formulae to calculate the first and second natural frequencies for simply supported Warren and N- system of trusses. It is concluded that the formula given in (3) gave a good accuracy and easy computations for frequencies. A comparison between exact results using computer programs and corresponding computed frequencies using the formula given in (3) showed that the difference didn't exceed 0.5%. Also, we can conclude that this formula given in (3) is valid with all variable dimensions (spans, slopes, loads, heights...etc) for both types of trusses.

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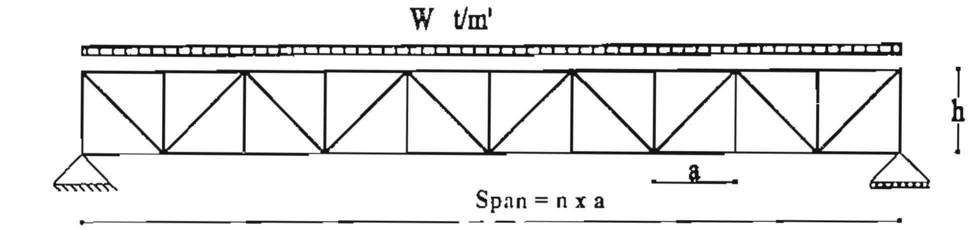
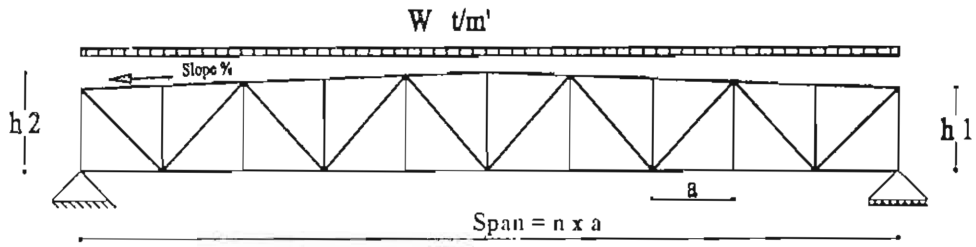
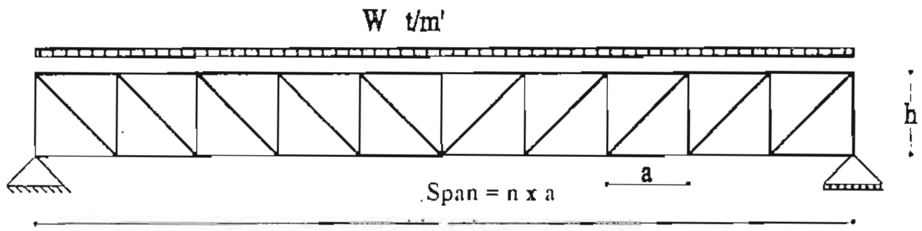


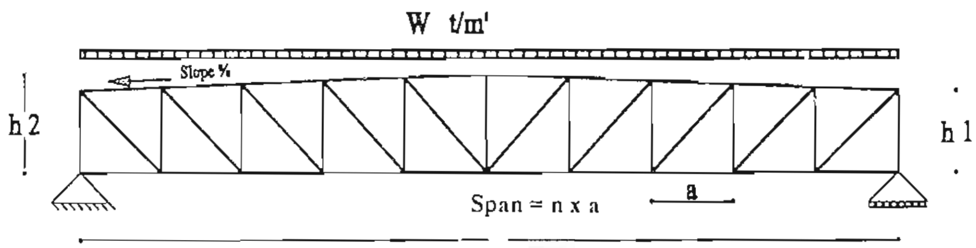
Fig(1-a) : Warren - system of truss with parallel chords



Fig(1-b) : Warren - system of truss with inclined upper chord



Fig(2-a): N- system of truss with parallel chords



Fig(2-b) : N - system of truss with inclined upper chord

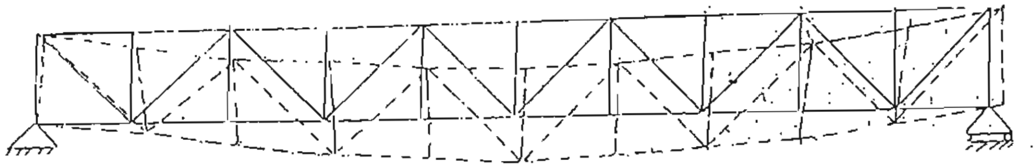


Fig (5-a) first mode shape with 1t/m as uniformly distributed load (Warren truss)

ω_1 = 3.143 c.p.s

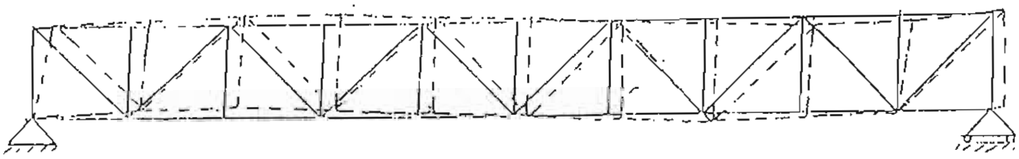


Fig (5-b) second mode shape with 1t/m as uniformly distributed load (Warren truss)

ω_2 = 7.895 c p s

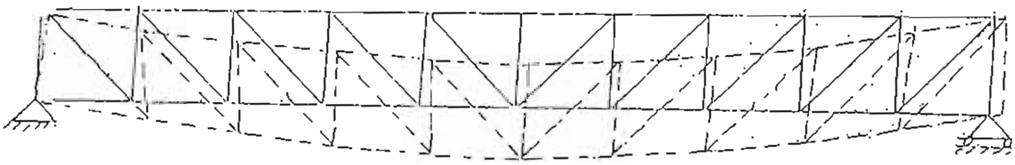


Fig (6-a) first mode shape with 1t/m as uniformly distributed load (N-system truss)

ω_1 = 3.095 c p s

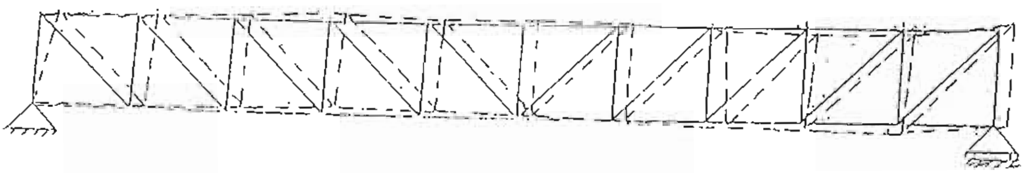


Fig (6-b) second mode shape with 1t/m as uniformly distributed load (N-system truss)

ω_2 = 7 296 c p s

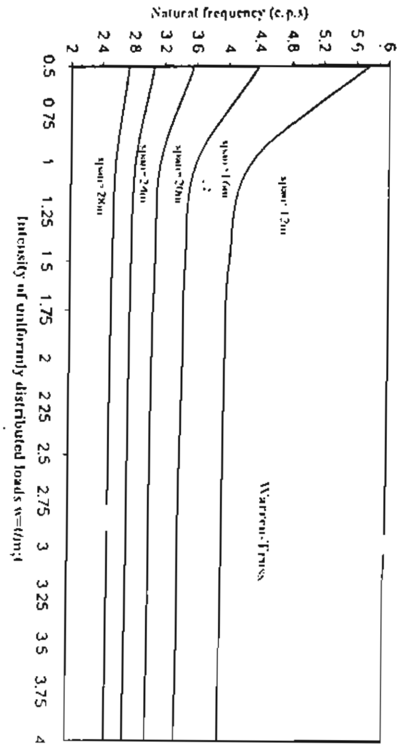


Fig.(3.a): Variation of first frequency with loads

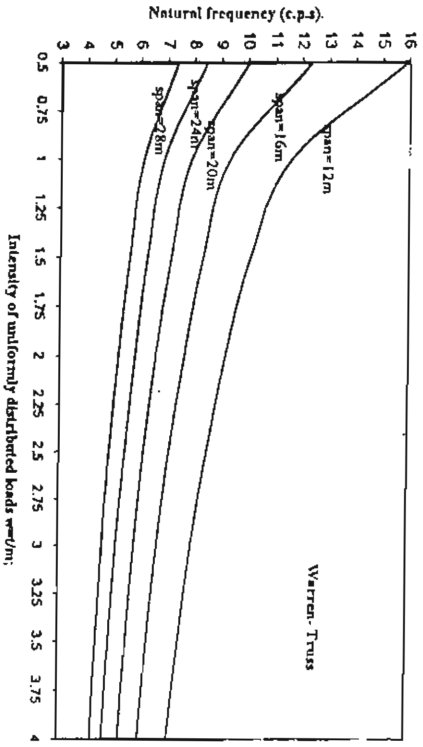


Fig.(3.b): Variation of second frequency with loads

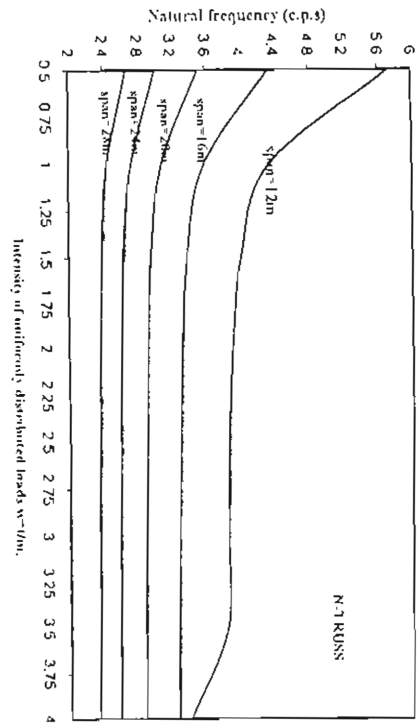


Fig.(4.a): Variation of first frequency with load

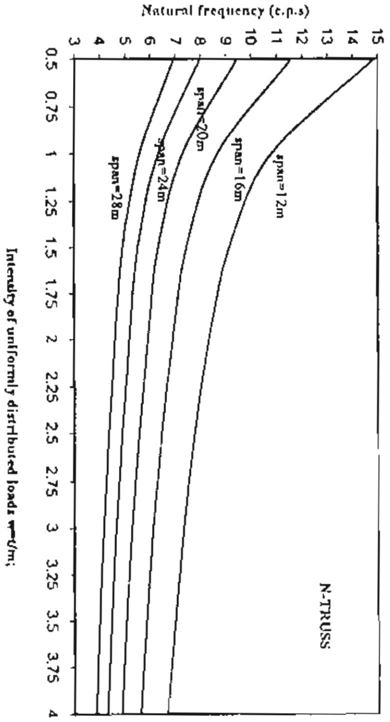


Fig.(4.b): Variation of second frequency with loads

Table (1) :Coefficients C_i for first natural ferquency in both types of trusses

NO OF PANELS → / TYPE OF TRUSS ↓		6	8	10	12	14
C_0	W	23.854390	15.699460	9.655907	8.936804	9.846342
	N	23.952910	16.328860	11.750310	10.094640	8.651513
C_1	W	-27.886040	-16.592610	-6.789466	-6.934849	-10.777110
	N	-27.025300	-17.995040	-11.927240	-10.480030	-8.224527
C_2	W	16.932540	9.410130	2.474560	2.941155	7.022154
	N	14.581520	10.448510	7.160754	6.807590	4.741590
C_3	W	-5.659229	-2.809595	-0.391738	-0.589959	-2.541217
	N	-3.707807	-3.105822	-2.418480	-2.597470	-1.566111
C_4	W	1.015320	0.416274	0.018457	0.043812	0.482124
	N	0.361263	0.434465	0.432958	0.541365	0.281348
C_5	W	-0.077267	-0.023338	0.000000	0.000000	-.037538
	N	0.000000	-0.020434	-0.032083	-0.047410	-0.021571

Table(2):Coefficients C_i for second natural ferquency in both types of trusses

NO OF PANELS → / TYPE OF TRUSS ↓		6	8	10	12	14
C_0	W	62.052980	42.715350	36.433390	31.619800	30.430050
	N	58.696020	48.274230	37.155740	29.505750	26.230690
C_1	W	-67.715020	-39.960520	-36 102580	-32.210950	-35.135620
	N	-64.799760	-57.972850	-42.144690	-30.892920	-28.520930
C_2	W	38.352600	18.089090	17.676560	16.388490	21.043580
	N	36.742140	36.767170	25.470560	17.131070	16.838300
C_3	W	-12.036810	-3840161	-4.114121	-3.975774	-6.527968
	N	-11.306630	-12.863410	-8.430673	-5.070652	-5.452091
C_4	W	2.040227	0 305943	0.365729	0.369076	0.982516
	N	1.814331	2.375100	1.455364	0.749359	0.915127
C_5	W	-0.148838	0.000000	0.000000	0.000000	-0.054548
	N	-0.120331	-0.182286	-0.103262	-0.042555	-0 062481

Table (3) :Coefficients R_m with panel length 1.50 m.

Weight t/m'	NO OF PANELS → / TYPE OF TRUSS ↓	FIRST FREQUENCY (c.p.s) 6	8	10	12	14	SECOND FREQUENCY (c.p.s) 6	8	10	12	14
0.5	W	1.384	1.322	1.236	1.159	1.115	1.389	1.363	1.327	1.288	1.262
	N	1.384	1.320	1.235	1.165	1.130	1.389	1.370	1.337	1.304	1.281
1.5	W	0.875	0.929	0.949	0.964	0.973	0.842	0.870	0.890	0.903	0.912
	N	0.892	0.934	0.944	0.951	0.958	0.837	0.858	0.870	0.878	0.884
2.0	W	0.841	0.909	0.939	0.952	0.965	0.764	0.803	0.828	0.843	0.855
	N	0.847	0.899	0.918	0.930	0.941	0.745	0.771	0.786	0 798	0 807
2.5	W	0.830	0.904	0.931	0.948	0.961	0.710	0.752	0.777	0 796	0 809
	N	0.829	0.886	0.908	0.923	0.934	0.691	0.723	0.743	0.758	0 769
3.0	W	0.829	0 900	0.927	0.946	0.957	0.668	0.709	0.736	0 758	0 769
	N	0.819	0.880	0.903	0.917	0.929	0.653	0.688	0.709	0 724	0 736
3.5	W	0.827	0.895	0.925	0.944	0.955	0.633	0.673	0 709	0 724	0.737
	N	0.814	0.875	0.899	0.915	0.927	0.623	0.658	0.681	0.697	0 709
4.0	W	0 826	0.892	0.924	0.942	0.954	0 603	0.642	0.674	0.695	0 709
	N	0 812	0 871	0.896	0.912	0.912	0.598	0.632	0.657	0 673	0.673

Table (4) :Coefficients R_m with panel length 2.0 m.

Weight t/m'	NO OF PANELS → / TYPE OF TRUSS ↓	FIRST FREQUENCY (c.p.s) 6	8	10	12	14	SECOND FREQUENCY (c.p.s) 6	8	10	12	14
0.5	W	1.340	1.222	1.134	1.085	1.064	1.374	1 20	1.267	1 231	1.209
	N	1.317	1 210	1.140	1 106	1.092	1 376	1 333	1.294	1 2658	1 246
1.5	W	0.963	0.965	0.979	0.983	0.986	0.870	0.893	0.904	0.910	0.915
	N	0.926	0 947	0 959	0.967	0.974	0.849	0.862	0.870	0.876	0.882
2.0	W	0.924	0.959	0.969	0.979	0.982	0.791	0.820	0.833	0.845	0.851
	N	0.903	0.932	0.946	0.958	0 966	0.772	0.795	0.809	0.819	0.827
2.5	W	0.922	0 953	0.966	0.976	0.978	0 733	0.762	0.779	0.794	0.800
	N	0 892	0.924	0 941	0 953	0.961	0 721	0.747	0.764	0.776	0.784
3.0	W	0.920	0.947	0.964	0.973	0.976	0.687	0.716	0.738	0.750	0.759
	N	0.889	0.918	0 936	0.950	0.959	0.682	0.708	0.726	0.739	0.749
3.5	W	0 918	0.946	0.962	0.970	0.974	0.648	0.679	0.702	0.714	0.727
	N	0 885	0 961	0 933	0.947	0.956	0.648	0.676	0.694	0.708	0.718
4.0	W	0 915	0.944	0.961	0 968	0 973	0.615	0.648	0.672	0.684	0.699
	N	0 883	0 914	0 932	0.945	0.945	0 619	0.649	0.668	0 683	0.683

Table (5) :Coefficients R_m with panel length 2.50 m.

Weight t/m'	NO OF PANELS → / TYPE OF TRUSS ↓	FIRST FREQUENCY (c.p.s)					SECOND FREQUENCY (c.p.s)				
		6	8	10	12	14	6	8	10	12	14
0.5	W	1.259	1.142	1.085	1.063	1.051	1.335	1.268	1.226	1.201	1.185
	N	1.239	1.145	1.101	1.083	1.073	1.352	1.304	1.270	1.248	1 234
1.5	W	0.968	0.989	0.988	0.990	0.995	0.875	0.982	0.897	0.904	0.909
	N	0.951	0.967	0975	0.981	0.985	0.862	0.877	0.887	0.894	0.900
2.0	W	0.965	0.981	0.983	0.987	0.990	0.794	0.812	0.824	0.835	0.841
	N	0.935	0.957	0.967	0.974	0979	0.790	0.811	0.824	0 834	0.840
2.5	W	0.963	0 946	0.980	0.983	0.988	0.733	0 752	0.770	0.780	0.789
	N	0.931	0.951	0.962	0.969	0.976	0.737	0.759	0.774	0.786	0.794
3.0	W	0.960	0.974	0.979	0.980	0.987	0.684	0 706	0.726	0.735	0.749
	N	0.927	0 948	0.959	0.966	0 972	0.694	0.718	0.735	0.746	0.755
3.5	W	0 957	0.972	0.977	0 978	0.985	0.643	0.668	0.689	0.700	0 715
	N	0.924	0.946	0 957	0 965	0 972	0.658	0.684	0.702	0.716	0.726
4.0	W	0.954	0.969	0.975	0.976	0.983	0.610	0.636	0.657	0.669	0.685
	N	0.921	0.944	0.956	0.964	0.964	0.627	0.655	0.674	0 689	0.689

Table (6) :Coefficients R_m with panel length 3.0 m.

Weight t/m'	NO OF PANELS → / TYPE OF TRUSS ↓	FIRST FREQUENCY (c.p.s)					SECOND FREQUENCY (c.p.s)				
		6	8	10	12	14	6	8	10	12	14
0.5	W	1.181	1.094	1.059	1.044	1.029	1 299	1.237	1.200	1.178	1.161
	N	1.173	1.105	1.083	1.069	1.056	1.334	1.289	1.263	1.246	1.231
1.5	W	0.987	0.992	0.990	0.996	0.994	0.875	0.889	0.891	0.900	0.901
	N	0.968	0.980	0.985	0.988	0.990	0 878	0 894	0.903	0.910	0.913
2.0	W	0.984	0 984	0.986	0.992	0.991	0.790	0.802	0.817	0.926	0.831
	N	0.961	0.971	0 978	0.984	0.986	0.804	0.822	0.835	0.845	0.851
2.5	W	0.981	0.981	0 984	0.988	0.989	0 726	0 743	0.760	0.769	0.781
	N	0.956	0 967	0.975	0.980	0.982	0.747	0.769	0.783	0.794	0.800
3.0	W	0.977	0.979	0.982	0.986	0.987	0.676	0.695	0.714	0.725	0.738
	N	0.953	0.965	0.973	0.978	0.981	0.701	0.726	0.743	0.755	0.764
3.5	W	0.973	0.976	0 979	0 983	0.985	0.635	0 656	0.676	0.689	0.702
	N	0.949	0 963	0.971	0.977	0.980	0.663	0.689	0.708	0.723	0.732
4.0	W	0.969	0.973	0.977	0 981	0.983	0.660	0.623	0.643	0.657	0.671
	N	0.946	0.961	0.969	0 975	0.975	0.630	0 658	0.677	0.693	0.693

Table (7) :Coefficients R_h with panel length 1.50 m.

Slope	NO OF PANELS / TYPE OF TRUSS	FIRST FREQUENCY (c.p.s)					SECOND FREQUENCY (c.p.s)				
		6	8	10	12	14	6	8	10	12	14
2%	W	1.064	1.042	1.036	1.033	1.031	1.006	1.001	1.002	0.999	0.998
	N	1.064	1.046	1.035	1.033	1.032	1.004	1.001	0.998	0.996	0 994
4%	W	1.126	1.090	1.069	1.063	1.060	1.010	1.000	1.000	0.996	0 994
	N	1.127	1.092	1.067	1.064	1.061	1.008	1.001	0.994	0.990	0 987
6%	W	1.186	1.135	1.099	1.090	1.086	1.011	0.997	0.997	0.991	0.989
	N	1.188	1.136	1.101	1.092	1.088	1.010	1.000	0 990	0.984	0 979
8%	W	1.243	1.181	1.130	1.116	1.110	1.011	0.994	0.992	0 984	0 982
	N	1.246	1.181	1.135	1.117	1.112	1.011	0.999	0.986	0.976	0 969
10%	W	1.298	1.231	1.167	1.140	1.131	1.011	0.993	0.987	0.976	0 974
	N	1.301	1.232	1.175	1.143	1.135	1.012	0.999	0.982	0.969	0 960

Table (8) :Coefficients R_h with panel length 2.0 m.

Slope	NO OF PANELS / TYPE OF TRUSS	FIRST FREQUENCY (c.p.s)					SECOND FREQUENCY (c.p.s)				
		6	8	10	12	14	6	8	10	12	14
2%	W	1.054	1.034	1.032	1.029	1.030	1.004	0.995	0.996	0.993	0 993
	N	1.048	1.036	1.034	1.033	1.032	1.002	0.997	0.996	0.994	0 992
4%	W	1.105	1.067	1.061	1.056	1.056	1 004	0.990	0.990	0 985	0 986
	N	1.093	1.070	1.065	1 662	1.061	1.002	0.994	0.990	0 987	0 983
6%	W	1.154	1 102	1.088	1.081	1.080	1.002	0.987	0.982	0 977	0.977
	N	1.136	1.104	1.093	1.089	1.087	1.001	0 992	0 984	0 979	0 974
8%	W	1 204	1.136	1.112	1.104	1.101	1.000	0.982	0.974	0 967	0.967
	N	1 184	1.35	1.119	1.114	1.111	1.001	0.988	0.977	0.970	0.964
10%	W	1.256	1.171	1 138	1.125	1.120	0.999	0 977	0.969	0 957	0 956
	N	1.237	1.166	1.144	1 136	1.132	1.001	0.984	0.971	0.962	0 953

Table (9) :Coefficients R_h with panel length 2.50 m.

Slope ↓	NO OF PANELS / TYPE OF TRUSS	FIRST FREQUENCY (c.p.s)					SECOND FREQUENCY (c.p.s)				
		6	8	10	12	14	6	8	10	12	14
2%	W	1.039	1.034	1.031	1.030	1.031	0.996	0.994	0.995	0.993	0.993
	N	1.041	1.036	1.033	1.032	1.031	0.999	0.998	0.996	0.994	0.992
4%	W	1.075	1.066	1.059	1.058	1.059	0.990	0.987	0.988	0.985	0.985
	N	1.078	1.069	1.064	1.061	1.060	0.997	0.995	0.991	0.988	0.984
6%	W	1.108	1.094	1.084	1.082	1.083	0.983	0.978	0.980	0.976	0.976
	N	1.114	1.100	1.062	1.088	1.086	0.994	0.990	0.986	0.980	0.975
8%	W	1.152	1.121	1.108	1.104	1.105	0.980	0.969	0.971	0.967	0.966
	N	1.153	1.128	1.118	1.113	1.109	0.993	0.986	0.979	0.972	0.965
10%	W	1.194	1.145	1.129	1.124	1.124	0.976	0.960	0.962	0.956	0.956
	N	1.191	1.154	1.140	1.135	1.129	0.991	0.980	0.972	0.964	0.954

Table (10) :Coefficients R_h with panel length 3.0 m.

Slope ↓	NO OF PANELS / TYPE OF TRUSS	FIRST FREQUENCY (c.p.s)					SECOND FREQUENCY (c.p.s)				
		6	8	10	12	14	6	8	10	12	14
2%	W	1.035	1.031	1.032	1.032	1.029	0.994	0.993	0.995	0.994	0.992
	N	1.038	1.036	1.033	1.033	1.031	1.000	0.999	0.997	0.995	0.992
4%	W	1.067	1.060	1.060	1.060	1.055	0.986	0.985	0.989	0.986	0.983
	N	1.073	1.068	1.1064	1.062	1.059	0.998	0.996	0.993	0.989	0.984
6%	W	1.098	1.087	1.085	1.085	1.079	0.978	0.976	0.981	0.977	0.973
	N	1.107	1 097	1.092	1.088	1.084	0.996	0.993	0.988	0.983	0.975
8%	W	1.126	1.111	1.080	1.108	1.101	0.970	0.967	0.972	0.968	0.964
	N	1.138	1.124	1.117	1.112	1.106	0.992	0.988	0.982	0.975	0.965
10%	W	1 155	1.133	1.127	1 128	1.120	0.962	0.956	0.963	0.958	0.953
	N	1.169	1.148	1.140	1.132	1.127	0.989	0.983	0.976	0.967	0.955

Table (11) :Data assumed for trusses to check empirical formulae

Case no:	Type of truss	No: of panels	Panel (a) (m)	Span (m)	R_1 (m)	R_2 (m)	Slope %	Ec t/cm2	Load t/m	γts	γtc
1	W	10	2	20	2	2	0	2000	1 0	2 16	2 16
2	W	10	2	20	2	2	0	2000	1 0	2 16	2.16
3	W	10	2	20	2	2	0	2000	1.0	2.16	2.08
4	W	10	2	20	2	2	0	2000	3.0	6.39	6 39
5	W	10	2	20	2	2.4	4	2000	3.0	6.37	6.37
6	W	14	2 5	35	3 5	3.5	0	2000	1.0	3.89	3 89
7	W	12	1.5	18	1.8	1.8	0	2000	2.0	3 84	3 84
8	N	6	1 5	9	0.9	0 9	0	1800	2.0	1.88	1.88
9	N	6	1.5	9	0.9	0.9	0	1800	2.0	1 88	1.88
10	N	8	2	16	1.6	1.92	4	1800	3 0	5.07	4 95

Table(12) : First and second Frequencies for trusses used in table (11)

Case No	First Frequency								Second Frequency							
	R_1	R_m	R_h	R_e	R_γ	ω computed	ω exact	Error %	R_1	R_m	R_h	R_e	R_γ	ω computed	ω exact	Error %
1	3.137	1	1	1	1	3.137	3 143	0 2	7 873	1	1	1	1	7.873	7.891	0 22
2	3.137	1	1	0.3162	1	0.992	0.994	0.2	7 873	1	1	0 3162	1	2 489	2 497	0 30
3	3.137	1	1	1	1.019	3.20	3.209	0.28	7 873	1	1	1	1 019	8 023	8 039	0 20
4	3.137	0.964	1	1	1	3.024	3.029	0.16	7.873	0.738	1	1	1	5 81	5 823	0 22
5	3.137	0.964	1.061	1	1	3.202	3.208	0.2	7.873	0.788	0 99	1	1	5 752	5 737	0 26
6	2.253	1	1	1	1	2 253	2.250	0.12	5 166	1	1	1	1	5 166	5 158	0 16
7	3 383	0.952	1	1	1	3 220	3.220	0.0	8 627	0.843	1	1	1	7 273	7 265	0 11
8	5 538	0.847	1	1	1	4 690	4.674	0 36	14 277	0 745	1	1	1	10.637	10.619	0 17
9	5.538	0 847	1	0 30	1	1.407	1.402	0.37	14.277	0 745	1	0.3	1	3.19	3.186	0 15
10	3.584	0.918	1.07	0.949	1.012	3.380	3.365	0.47	8 658	0.708	0.994	0.9419	1.012	5 84	5 828	0 20