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# A Fuzzy Heuristic Approach to Unit Commitment.

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## A FUZZY HEURISTIC APPROACH TO UNIT COMMITMENT

طريقة منطقية باستخدام النظام المبهم لحل مسألة جدولة وحدات التوليد

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## خلاصة:

إن مسألة جدولة وحدات النَّوليد وكيفية إدارتها في مقابلة الأحمال الكهربية لــــهي إحــدي المســـالل الهامة في تشغيل نظم الڤوى الكهربية بكلا شفيه الاقتصادي والأمني. فمن المعلوم أن الأحمـــــال الكهربيـــة تختلف خلال أيام الأسبوع بل تتباين خلال ساعات اليوم الواحد والمطلوب تحديد أي الوحدات التسى يفضـــل فصلها إذا ما انخفض الحمل وأبها يفضل إدارتها عندما يزداد الحمل في الساعات النالية وذلك بهدف تحقيق التكلفة الاقتصادية في طل مجموعة من القيود التي تضمن أمان التشغيل. ولأهمية هذا الموضوع فقد تعددت الأبحاث وتنوعت بين نوع يعتمد كليا على الرياضيات وآخر بعتمد كليا على خبرة المئســغل وثـــالث بجمـــع بينهما بهدف الاستفادة من مزايا وتجنب عبوب كل منهما.

وهذا البحث هو من ذلك النوع الثالث إذ ابتكرت فيه طريقة جديدة تعتمد على نظرية النظام المبسهم وذلسك بنمذجة المسألة بتوصيف أجزالها الغير قطعية أو الغير معيارية مثل دالة الهدف أو فيد الاحتيــاطي أو فيـــد اتزان القدر 5 (وجميعها مرتبطة بالقيم الغير دقيقة عادة للحمل المتنبأ به) في صور 5 علاقات مبهمة. ثم تسم عمل هيكل استَدلالي لحل المسألة على أسس وقواعد هذه النظرية وقد تُمتَ صياغته في صورة برنامج كتب بلغة فورتران لاستخدامه على الحاسب الآلي، وقد روعي في هذه الطريقة استخدام تكنيك جديد بعمل علـــــي بَخَسيم الحمل وكذلك وحدات الَّنوليد إلى ثلاثٌ مستويات بحيث يتم فحص الحالات النَّاتجةَ عـــــن جــــز ء ســـن الموحدات وليس منظومة الوحدات كلها وبهذا تم إسراع الطريقة في إيجاد الحل. وقد تم تطبيق الطريقة على نظامين مختلفين وقورنت نتائجها مع النتائج التي تم الحصول عليها باستخدام طرق أخرى وأثبتت النتــــانـج فاعلية الطريقة الجديدة وأنها تمناز بالسرعة وكذلك ايجاد الحلول الأقل تكلفة.

#### Abstract:

This paper presents a new approach based on the application of artificial intelligent techniques to solve the unit commitment problem. The proposed approach uses the operator experience, some heuristics, and the application of fuzzy sets theory. In this approach, load demand, rescrve requirements, and production cost are expressed in fuzzy set notations. The "if - then" rules and fuzzy logic operations are used to find the optimal solution. The heuristics such as dividing the load and generating units into levels are used to speed the solution.

The approach is applied to two systems, one consists of four generating units. and the other consists of ten units. The results demonstrate the effectiveness of this approach and that it has two advantages: its short processing time and the total operating cost computed is low comparing with other methods.

## 1. Introduction

Unit commitment (UC) is aimed at devising a proper generator commitment schedule for an electric power system over a period of one day to one week [1-2]. The optimal solution for this problem which verifies a minimum operating cost over the study period through the constraints imposed on the system including power generation load balance, spinning reserve requirements, and other constraints on unit

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operation. To achieve the exact solution to such a complex combinatorial optimization problem, the global search techniques such as dynamic programming [3-4] or integer programming [5] methods are resorted. These methods worked efficiently and obtained optimal solutions only in small systems.

In the large systems however, the procedure became incompetent as additional constraints were imposed on power systems and problem size increases, the mentioned methods need too long computational time and sometimes fail to find a solution. To reduce the search space and the execution time, some heuristic approaches such as priority listing [6], neural network [7], and expert systems [8] approaches have been employed. Although being simple in concept and fast in computation, these heuristic methods sometimes fail to give the sub-optimal solution.

Between these two extremes of global search and highly heuristic methods, the Lagrangian relaxation (LR) method [9] is viewed as a desirable compromise which is efficient and well applicable to the large scale UC problem. But the application of this dual technique may find no feasible or still sub-optimal solutions [10].

Recently, some researches have aimed to apply the fuzzy systems for solving the unit commitment problem [11, 12, 14, 15, 16]. The use of fuzzy gives a suitable solution which verifies all constraints by representing a membership degree for each uncertainty one.

In the proposed approach the fuzzy sets theory and fuzzy logic operations are used to find the optimal solution. A heuristic technique based on dividing the load and generating units into levels is applied to minimize the computing time.

## Nomenclature



## 2. Fuzzy Sets Associated with Unit Commitment

## 2-1 Definitions of the fuzzy variables and quantities in U.C.

Since load demand depends on weather variables, social behavior of customers, ...... etc, there always exist errors in the forecasted system loads, i.e. the load demand is imprecise thus it can be described as a fuzzy quantity. Also, every variable associated with system load will be considered a fuzzy variable such as unit generation production cost and spinning reserve [12].

In this paper, to reach an optimal commitment schedule under the fuzzy environment; production cost, load demand equality constraint, and security (spinning reserve inequality relation) constraint are all expressed in fuzzy set notations. While the crisp quantities include: limits on unit outputs, minimum up/down times constraints ...........etc.

## 2.2 Definition of thresholds and membership functions for fuzzy variables [13]

The values of thresholds are obtained by last experiences of the operator or by empirical formulas as follows:

### 2-2-1 Load balance membership function:

The predicted system load is usually subject to  $\pm 2\%$  to  $\pm 5\%$  variation [17]. Thus, the thresholds for system demand are:

- Nominal demand: having the maximum degree of grade in membership function *i.e.* degree "one". The nominal value for demand equals to the mean value of the predicted demand.
- $\pm \Delta d(t)$ : denotes the maximum range of variation of the hourly predicted demand. It has the least value of membership degree i.e. degree "Zero". In this study,  $\Delta d(t)$  is taken equal to the maximum predicted deviation i.e. ±5%.

The membership function can be described by equation (1), which is illustrated  $by Fig. 1.$ 



Fig. (1) Load balance membership

## 2-2-2 Spinning reserve requirements membership function

There are two thresholds for the reserve variable:

Nominal reserve value, this value will verify the greatest degree of satisfaction, i.e. degree "One".

M.M. El-Saadawi, M. A. Tantawy & E. Tawfik

Minimum acceptable reserve (least value), this value will verify a completely unacceptable degree of satisfaction i.e. degree "Zero".

According to these thresholds, the membership function can be chosen as explained by equation 2, and Fig. 2.



Fig. (2): Reserve membership

## 2-2-3 Operating Cost

The thresholds of operating cost arc:

- Level represents the ideal cost  $(C_0)$ , it has the maximum grade of membership i.e. degree "One". Selecting this level may be subjective and dependent on specific practice. One good candidate for the ideal cost  $(C_0)$  is the minimum cost results from applying economic dispatch for every state of the crisp problem with nominal system demand and reserve requirements [17]
- The highest acceptable cost ( $C_0 + \Delta C$ ), has the least degree of membership i.e. degree "Zero". It can be determined by choosing  $\Delta C$  as a certain percentage of  $C_0$ . Due to the operator experience,  $\Delta C$  should be taken as a small value to reduce the number of studied states. However, in this study ∆C is taken equal to a large value of C<sub>0</sub>, so that more candidate states can be studied.  $\Delta C$  is taken as 20% of C<sub>0</sub>.

According to these definitions, the cost membership function can be chosen as



Fig. (3) Cost membership

## 3. Proposed Solution Strategy

The proposed solution strategy consists of three main stages:

#### Stage  $(l)$ :

Divide the load into three levels: base, medium, and peak loads:

- Ref. [8] indicated that, in general, between 60% and 90% of the maximum generation capacity supplies the medium load.
- In this paper, the base load level (Base) is generally taken as the value of minimum load during study periods.
- The medium load is approximately between the base level and 90% of the highest load during study period [8], so the medium level (Med) is taken as 90% of the biggest load.
- The peak load is approximately between 90% to 100% of the highest value during study period.

## Stage  $(2)$ :

At first a priority list of units based on the full load incremental costs is illustrated, then by using the thresholds calculated in stage(1), the units are divided to:

- Base units: these units have the characteristics, the highest starting cost, longest minimum up/down times, and lowest average production cost. These units will always be committed during study period.
- Peak units: these units have the characteristics of highest production cost and smallest minimum up/down times. A list of all possible states for these units is made.
- Medium load: the characteristics of these units are between the last two types i.e. medium incremental cost and medium minimum up/down times. A list of all possible states for these units is made.

### Stage  $(3)$ :

This stage consists of the following steps:

Step  $(1)$ : hour  $(h) = 1$ 

All base units must be run i.e. status "on".

Step  $(2)$ : check if (load  $(h) \leq$  Base) then, the state is only the base units are on. and go to step  $(6)$ .

Else if (load  $(h)$  > Med) go to step  $(7)$ .

Else: (Base < load (h)  $\leq$  Med), check if

$$
\sum_{i=1}^{N_b} P_{max} (i) \geqslant [load (h) + Reserve amount]
$$

Then the state is only the base units are on, and go to step  $(6)$ .

Else: call the states for medium units and go to step  $(3)$ .

Step  $(3)$ : For each state candidate only the state (k) which verify the condition

$$
\sum_{i=1}^{N_b} P_{\text{max}}(i) + \sum_{j=1}^{N_m} P_{\text{max}}(j) * U_s(j) \ge 0.95 * Load(h)
$$

Where the value (0.95) is assumed on basis of the deviation in forecasting load ( $\pm$  %5) to make fuzzy relations, so that the cases

#### M.M. El-Saadawi, M. A. Tantawy & E. Tawfik

where the generation is larger or equal to 95% of the predicted load are the candidates cases to be studied. Assume number of candidates is  $k_{\text{in}}$ . For each candidate (k) compute the following:

Step  $(4)$ :

- Load demand  $P_d$ , thus calculate  $\mu_d(k)$  using equation (1).
- Reserve amount, Res(k), then calculate  $\mu_{p}(k)$  using equation (2).
- If the reserve constraint is neglected put  $\mu_{\nu}(k) = 1$
- Production cost by an economic dispatch, then calculate  $\mu c(k)$  using equation (3).
- Overall membership degree  $\mu$ o(k) where,

$$
\mu_{\mathfrak{g}}(k) = \min \left( \mu_{R}(k), \mu_{d}(k), \mu_{c}(k) \right)
$$

Step  $(5)$ : Choose the best state which has the maximum overall membership degree  $\mu_{st}(h)$  to be the solution at hour (h) where.

$$
\mu_{st}(h) = \max (\mu_0(1), \mu_0(2), ..., \mu_0(k))
$$
  
Step (6):  $h = h + I$  if  $(h > 24)$  go to step (8)

else: go to step (2).

Step  $(7)$ : The base and medium units are "on" then, check if:

$$
\sum_{i=1}^{N_b} P_{\text{max}}(i) + \sum_{j=1}^{N_m} P_{\text{max}}(j) \ge \text{Load(h)} + \text{Rserve}
$$

Then, the state is: only the base and medium units are "on", and go to step $(6)$ .

Else: call all states for peak units, for each state candidate only the state  $k$  which verifies the condition:

$$
\left[\sum_{j=1}^{N_p} P_{\text{max}}(j) * U_s(j) + \sum_{i=1}^{N_b} P_{\text{max}}(i) + \sum_{i=1}^{N_m} P_{\text{max}}(i)\right] \ge 0.95 * \text{Load(h)}
$$

Assume the no. of candidates km, and go to step(4).

Step  $(8)$ 

Print the state determined for each hour (h) to be the sup-optimal schedule.

### 4. Testing Results

The proposed approach was implemented on Pentium, 900MHz personal computer with 128Mbyte local memory. The computer program was written using the FORTRAN language and applied to the following two test cases:

## 4-1 Case "One":

The proposed approach is applied to the system mentioned in Ref. [1, 12] to compare the results with that computed by Dynamic Programming (DP) and Fuzzy Logic Approach (FLA). The system consists of 4 units, over 8-hour periods load pattern, the complete data of the system as given in Rcf. [12] table 1&2 (in the Reference) are shown in appendix  $(A)$ . Unit  $(3)$  is taken as a base unit, whereas

unit (2) is medium unit, and unit (1)  $\&$  (4) are peak units. The given load is divided into three levels, under 280 MW (base load), between 280 and 540 MW (niedium load), and above 540 MW (peak load).

The method is applied once without taking reserve requirements into account which is the case tested by DP and FLA methods in Ref [12]. Another application will be carried out with taking 6% of load as spinning reserve into account to prove the effectiveness of the proposed method. A sample of the results for a 600 MW load and the final results for the first application (without reserve requirements) are shown in Table 1.

The total cost is compared with that calculated by both of the DP and FLA. The comparison results, illustrated in Table 2, show that the cost obtained by the proposed approach is lower than that obtained by FLA approach and close to that obtained by DP method. This, inherently, demonstrates the closeness of the overall results and the effectiveness of the proposed approach. Moreover, the proposed approach has an additional advantage, it can be applied with the spinning reserve constraint taken into account as illustrated by Table 3.

In Table 3-a, the state 2 has the minimum production cost but it violates the reserve constraints (so that  $\mu_R(2) = 0$  and hence  $\mu_0(2) = 0$ ). Meanwhile, the overall degree of state 3 is the maximum among the three states  $(u_0(3) = 0.704)$ . The best state is the state that has the maximum overall membership degree. Comparing that with the results in Table 1-a, where the reserve constraint is neglected, state 2 has the maximum overall degree  $\mu_0$  (2) = 0.898. So that state 2 is the best state in this case.

The results also show that the principles of this method can be expanded to consider more complicated cases with additional constraints.

Table I-a: Results for 600 MW load with neglecting spinning reserves -Case One

ᇦ ⋧	ᢛ Γē ಷ 0 Ö, z	Units	Candidate States							Chosen State		
ā 4 ∝		Status	μ <sub>R</sub>	μp	μc	$\mu_{\mathbf{O}}$	Prod. Cost(5)	State No.	<b>Hst</b>	Prod. Cost(S)		
600		10	$\overline{0}$ .	0.1	0.832	0.832	12611		0.898	12450		
			1.0	0.1	0.898	0.898	12450					
			0. ا	0.1	0.704	0.704	12923					



Table 1-b: Final results with neglecting spinning reserves - Case One

#### M.M. El-Saadawi, M. A. Tantawy & E. Tawfik E. 22



Table 2 A comparison between total cost calculated by three-methods - Case One

Table 3-a: Results for 600 MW load with reserve taken into account -Case One







## 4-2 Case "Two":

In this case, the proposed approach is applied to the 10 units system mentioned in  $[13]$  to compare the results with that obtained by Evolutionary Programming (EP) methodology. The system data is shown in appendix (B). The application was carried out with 10% of load as spinning reserve requirements taken into account [13].

In this study, units  $(1)$  and  $(2)$  are considered as base units, units  $(3)$  through  $(6)$ are medium units, and the units (7) through (10) are peak units. The given load is divided to three levels: 700MW (base load), between 700MW and 1500MW (Medium load), and above 1500MW (peak load). The final economical results for this case study are shown in Table 4.

The total cost calculated by the proposed method is then compared with the cost calculated by DP method and EP, as shown in Table 5. The comparison confirms that Mansoura Engineering Journal, (MEJ), Vol. 27, No. 4. December 2002.

the proposed method has also proved to be an efficient tool for solving the unit commitment problem. Another advantage of this method is its short computing time compared with EP method. The proposed method takes only 2 seconds using pentium's personal computer, whereas, the same system solved by EP method takes  $100$  seconds  $[13]$ .

		LOAD		Min.					
								Prod.	Total
H	Value	Type	Res.	μr	ļlp	μc	$\mu_0$	Cost	cost
	ΜW		℅					$(\mathsf{S})$	$(\mathbb{S})$
	700	BASE				Base units are only "on"		13683	013683
$\overline{2}$	750	MED				Base units are only "on"		14554	028237
$\overline{\mathbf{3}}$	850	MED				Base units are only "on"		16302	044540
4	950	MED	09.5	.825	1.0	.919	.825	19816.7	064356
5	1000	MED	12.0	1.00	1.0	.911	911.	20356.7	084713
6	1100	MED	13.6	1.00	1.0	.892	.892	22499	108212
$\overline{7}$	1150	MED	08.7	0.57	1.0	.835	.570	23242	131454
8	1200	MED	17.6	1.00	1.0	.718	.718	26412	157867
9	1300	MED	08.6	0.54	1.0	.887	.540	26589	184455
10	1400	PEAK	10.9	1.00	1.0	.801	.801	30675	215130
$\mathbf{1}$	1450	PEAK	10.8	1.00	1.0	.774	.774	31984	247114
12	1500	PEAK	10.8	1.00	1.0	.687	.687	34005	281119
13	1400	PEAK	10.9	1.00	1.0	.811	.811	30095	311214
14	1300	<b>MED</b>	08.6	0.54	1.0	.887	.540	26589	337803
15	1200	MED	11.0	1.00	1.0	.750	.750	24150	361953
16	1050	MED	26.9	1.00	1.0	.732	.732	21598	383551
17	1000	MED	33.0	1.00	1.0	.811	811	20758	404309
18	1100	MED	21.0	1.00	1,0	.899	.899	22442	426751
19	1200	MED	11.0	1.00	1.0	.750	.750	24150	450901
20	1400	РЕАК	12.6	1.00	1.0	.725	.725	31439	482340
$\overline{21}$	1300	MED	08.6	0.54	1.0	887	.540	26589	508929
22	1100	MED	16.5	1.00	1.0	1.00	1.00	22276	531205
$\overline{23}$	900	MED	10.0	1.00	1.0	.808	.808	17653	548858
24	800	<b>MED</b>		Base units are only "on"				15427	564285

Table 4 : Final results for the optimal scheduling, Case Two.

Table 5: A comparison between three methods-Case Two.

	DP method [13]	EP method [17] $\vert$	Proposed method
Total cost $(S)$	565825	565352	564285
Computing time (sec)	N/A	100	

### 5. Conclusions

A new heuristic method using fuzzy set notations has been developed for solving the unit commitment (UC) problem. The variables: load demand, cost and spinning reserve are represented by fuzzy membership functions. The key to implement this method is to be able to determine the thresholds for the last variables. The method also depends on dividing both the generating units and load pattern into base, medium, and peak levels using the operator's experience. The principles of this method can be expanded to consider more complicated cases with additional constraints.

A computer program was written and applied to two systems. With the 4-units system, the cost of the schedule obtained by the proposed method is \$74773. When the problem was solved by using two other methods the cost was \$75267 and \$74773 respectively. With the 10-units system, the cost of the schedule calculated by the proposed method is \$564285 during 2 seconds. The problem was solved by another method and the cost was \$565352 during 100 seconds. The results lead to the effectiveness of the method proposed to solve the UC problem. The results also show that the solution will be obtained with saving in the computing time, and saving in the total cost.

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## 7. APPENDICES

 $(A)$  4 - Units system data.

Table (A-1): Unit characteristics







(B) 10-Units system data

Table (B-1): Load Pattern

Time							
Load MW	700					750   850   950   1000   1100   1150   1200   1300   1400   1450   1500	



		$\frac{1}{5}$ -	Unit	Unit	i S	Unit Lη,	Unit	Unit	$U$ nit	$J_{\text{mix}}$	Unit
$P_{max}$	(MW)	455	455	130	130	162	80	85	55	55	55
$P_{min}$	(MW)	150	150	20	20	25	20	25	10	19	10
a*	(S/h)	1000	970	700	680	450	370	480	660	665	670
$b^{\pi}$	(S/MWh)	16.19	17.26	16.60	16.50	19.70	22.26	27.74	25.92	27.27	27.79
$c^*$	$(S/MW^2-h)$	0.00048	0.00031	0.002	0.00211	0.00395	0.00712	0.00079	0.00413	0.00222	0.00173
Min. up time (h)		S	8	$\overline{5}$	5	6	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	L	1	î
Min down time (h)		8	8	5	5	6	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{I}$	ī	ı
Hot start cost (\$)		4500	5000	550	560	900	170	260	30	30	30
Cold start cost (S)		9000	10000	1100	1120	1800	340	520	60	60	60
Cold start time (h)**		5	5	$\overline{4}$	$\ddot{ }$	$\overline{\mathbf{4}}$	$\overline{2}$	$\overline{2}$	$\bf{0}$	$\bf{0}$	$\bf{0}$
Initial status $(h)$ <sup>***</sup>		$\bf 8$	$\mathbf{g}$	5	5	6	$-3$	$-3$	-1	$-1$	-1

Table (B-2): Unit characteristics

\* a, b and c are the cost coefficients of a generating units, the generation cost of any unit (\$/h) is computed as:  $C = a_i + b_i P_g + c_i P_g^2$ 

\*\* Cold starting time: is the time needed (in hours) for starting a generation unit.

\*\*\* Initial Status: is the status of a generation unit before the studied duration, it is positive when the unit was committed (up) before the studied period (8 hrs for unit 1 in the table) and it is negative when the unit was decomitted (shut down) before the studied period (3 hrs for unit 6 in the table)