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HYDRAULIC JUMP IN CIRCULAR OPEN CHANNELS

القفزة الهيدروليكية فى القنوات المكشوفة ذات القطاع الدائرى

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خلاصة

تتكون القفزة الهيدروليكية عند انتقال سريان ذى عمق تحت الحرج الى سريان ذى عمق فوق الحرج دون وجود عوائق. وللقفزة الهيدروليكية استخدامات عديدة فى شبكات الرى والصرف خاصة خلف المنشآت الهيدروليكية وكذلك فى المجارى المائية المستخدمة فى محطات تنقية مياه الشرب ومحطات معالجة مياه الصرف الصحى. ولحل مسألة القفزة الهيدروليكية يتم تطبيق معادلة كمية الحركة بدلا من معادلة الطاقة حيث ان الظاهرة تمثل فقدانا داخليا غير معلوم للطاقة.

فى هذا البحث تم تطبيق معادلة كمية الحركة على قفزة هيدروليكية متكونة فى قناة ذات ميل أفقى وقطاع دائرى والسريان بها ذا سطح حر وتم الحصول على معادلة ذات متغيرات لابعدية تعبر عن تلك الظاهرة. وتم وضع الحل فى صورة جداول ومنحنيات تشمل كل العناصر المختلفة للقفزة الهيدروليكية ووجد من الدراسة ان هذه العناصر تعتمد على العمق الإبتدائى للقفزة والتصرف المار وقطر الأنبوب. وبواسطة المعادلات والجداول والمنحنيات المستنتجة تم حل نوعين من المسائل:

١. عندما يكون معلوما التصرف وأحد عمقى المياه (العمق الابتدائى أو العمق التالى) وقطر الأنبوب فانه يمكن الحصول على باقى العناصر الأخرى للقفزة المائية (العمق الآخر للسريان والغير معلوم - مقدار الفاقد فى الطاقة بواسطة القفزة - ارتفاع القفزة).

٢. عندما يكون معلوما التصرف ومقدار الفاقد فى الطاقة بواسطة القفزة وقطر الأنبوب فانه يمكن الحصول على باقى العناصر الأخرى للقفزة.

ABSTRACT

The hydraulic jump is a transitional state from an upstream supercritical to downstream subcritical flow, and, for a given set of flow conditions, it has a fixed position length. The phenomenon of the hydraulic jump has been widely studied because of its frequent occurrence in nature, and because of its uses in many practical applications.

In the present study the momentum principle is used to derive an equation expressed the hydraulic jump occurred in a short horizontal reach of a circular open channel. The derived equation indicates that the initial water depth and sequent water depth (conjugate depths) are functions of the critical water depth.

In the present study the various elements of the hydraulic jump are expressed as dimensionless ratios, by dividing with the diameter of the circular channel. The procedure of dimensionless ratios described in the present paper can be used to determine the various elements of hydraulic jump in a circular channel when either both the discharge and the relative initial depth (or sequent depth) are known or the discharge and the relative dissipated energy are known. The methods of solution, analytical, graphical or by using tables are illustrated step by step in this paper.

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INTRODUCTION

Hydraulic jump is formed due to transition from supercritical flow to subcritical flow in open channel. It has been classified as rapidly varied flow and local phenomenon. In hydraulic jump transition, water surface rises abruptly, intense mixing occurs, surface rollers are formed, air is contained and energy is dissipated. Practical applications of the hydraulic jump are many; it is used (1) to dissipate energy (2) to recover head or raise the water level on the downstream side (3) to increase weight on an apron to reduce uplift on it (4) to increase discharge of a sluice gate by holding back tail-water (5) to indicate special flow conditions (6) to mix chemicals used for purification (7) to aerate water for city water supplies (8) to remove air pockets from water supply lines.

The hydraulic jump first investigated by Bidone, an Italianin, in 1818. Then, several studies have been undertaken to obtain the hydraulic jump characteristics, location and the length of the hydraulic jump, the length of the rollers, pressures at bed and the amount of energy dissipated.

Early studies on hydraulic jump characteristics made by Belanger (1828), Safranez (1929), Bakhmetef and Matzke (1936). Bradley and Peterk (1957), Rouse et al. (1959). Silvester (1964), Rajaratnam (1967-1968), Garge and Sarma (1971), Leutheusser and Kartha (1972), and Sarma and Newnham (1973).

Early studies on hydraulic jump in sloping channel made by Riegal and Beebe (1917) followed by Ellms (1928-1932). Later investigation made by Bakhmateff and Matzhe (1938), and Yarnell and Kindsvater (1938-1944). Modi and Seth (1968) demonstrated solution of the hydraulic jump in rectangular and trapezoidal channels in dimensionless forms. Abbott et al. (1969) used the finite difference method and Katapodes (1984) used the finite element method to solve the St. Venant equations numerically, the location of the hydraulic jump is automatically computed as part of the solution.

Au-Yeung, Y. (1972), presented two simple solution charts for obtaining the change in depth of water passing through a hydraulic jump in circular, triangular, trapezoidal and rectangular channels. Mehrotra, S. C. (1976), reported a first analytical treatment of the length of hydraulic jump. Anderson (1978) studied undular hydraulic jump.

McCorquodale and Khalifa (1983), used the strip-integral method to compute the jump length, water surface profile, and pressures at bed. Hager (1983), developed design charts and empirical relationships between various flow parameters of the hydraulic jump.

Recently, the hydraulic jump in sloping channel studied by Rajaratnam (1974), Hager and Bretz (1986), Hager (1989), Kawagoshi and Hager (1990) and Ohtsu and Yasuda (1990-1991). Wansochek and Hager (1987-1989) demonstrated a solution of the hydraulic jump in triangular and trapezoidal channels.

Other studies on the hydraulic jump performed by Hoyt and Sellin (1989) Gharangik and Chaudhry (1991), and Long et al. (1991).

THEORETICAL ANALYSIS

When the rapid change in the depth of flow occurs from a supercritical state to a subcritical one, the result is usually an abrupt rise of water surface, this local phenomenon is known as the hydraulic jump shown in Fig. (1). For simplicity, it is assumed that the flow is uniform in the channel except in the distance of the jump length. The momentum equation is well suited for the analysis of hydraulic jump owing to the initially unknown energy loss dissipated by the hydraulic jump.

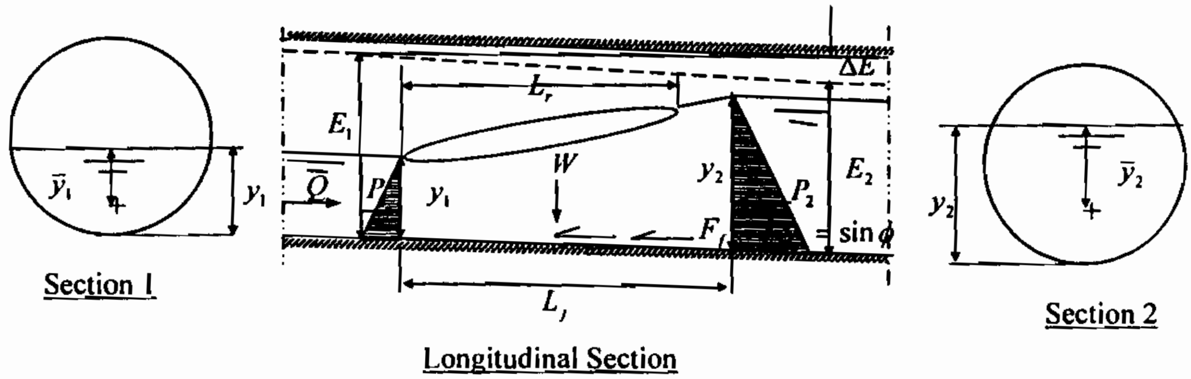

Fig. (1): Definition Sketch for a Hydraulic Jump in a Circular Open Channel.

The following assumptions are made in the present analysis:

1. The portion of the hydraulic jump is considered as the control volume.
2. The flow before and after the jump formation is uniform.
3. The pressure distribution is hydrostatic.

In the analysis of hydraulic jump in sloping open channels, it is essential to consider the weight of water in jump; in horizontal or gentle slope channels the effect of this weight is negligible.

I. Hydraulic jump in sloping circular channel:

Considering all effective forces parallel to the channel bed, the momentum equation may be written as follows:

$$\rho Q(\beta_2 V_2 - \beta_1 V_1) = P_1 - P_2 + W\sin\phi - F_f \qquad (1)$$

where ρ = water density, Q = discharge, β_1 = momentum coefficient at section 1, β_2 = momentum coefficient at section 2, V_1 = mean flow velocity at section 1, V_2 = mean flow velocity at section 2, P_1 = hydrostatic force at section 1, P_2 = hydrostatic force at section 2, W = weight of water within the jump, ϕ = the bed slope, and F_f = friction force.

The momentum equation in detail can be expressed as follows:

$$\frac{\gamma\beta_2 Q^2}{gA_2} - \frac{\gamma\beta_1 Q^2}{gA_1} = \gamma A_1\bar{y}_1 - \gamma A_2\bar{y}_2 + \frac{1}{2}\gamma K L_j (A_1 + A_2)\sin\phi - F_f \qquad (2)$$

or

$$\frac{\beta_2 Q^2}{gA_2} - \frac{\beta_1 Q^2}{gA_1} = A_1\bar{y}_1 - A_2\bar{y}_2 + \frac{1}{2} K L_j (A_1 + A_2)\sin\phi - \frac{F_f}{\gamma} \qquad (3)$$

where γ = specific weight of water, d_o = diameter the circular channel, A_1 = initial cross-sectional area, A_2 = sequent cross-sectional area, $\bar{y}_1$ = centroid depth of flow of the segment area of circle from the free water surface at section 1, $\bar{y}_2$ = centroid depth of flow of the segment area of circle from the free water surface at section 2, K = factor of correction to the shape of water surface at the jump, L_j = length of the jump, and g = gravitational acceleration.

II. Hydraulic jump in a horizontal circular channel:

In horizontal or gentle slope channels the effect of the weight component is negligible. Also the length of the jump is so small that the losses due to friction on the wall are negligible. Hence, in accordance with the momentum equation can be written as:

$$\frac{\beta_1 Q^2}{gA_1} + A_1\bar{y}_1 = \frac{\beta_2 Q^2}{gA_2} + A_2\bar{y}_2 \qquad (4)$$

For $\beta_1 \cong \beta_2 \cong 1.0$

$$\frac{Q^2}{gA_1} + A_1\bar{y}_1 = \frac{Q^2}{gA_2} + A_2\bar{y}_2 = F \qquad (5)$$

where F = specific force per unit weight of water.

Dividing both the sides of Eq. (5) by (d_o^3) the dimensionless specific force F^* $(= F/d_o^3)$ can be expressed as:

$$F^* = \frac{F}{d_o^3} = \frac{Q^2}{gd_o^5\left(A/d_o^2\right)} + \frac{A\bar{y}}{d_o^2 d_o} \qquad (6)$$

where F^* = dimensionless specific force $(= F/d_o^3)$.

Right side of Eq. (6) consists of two terms. The first term is the dimensionless momentum of the flow passing through the circular channel. The second term is the dimensionless hydrostatic force. They can be explained as follows:

Dimensionless momentum of the flow passing through the circular channel:

The Froude number in a circular partially filled pipe may be express as (Hager 1991):

$$F_r \cong \frac{Q}{\sqrt{gd_o^5 (Y)^2}} \qquad (7)$$

where F_r = Froude number ; and Y = relative water depth (y/d_o).

Applying Eq. (7) for critical flow, $F_r = 1.0$ and $y = y_c$ yields:

$$Y_c^4 = \left(\frac{y_c}{d_o}\right)^4 = \frac{Q^2}{gd_o^5} \qquad (8)$$

where Y_c = relative critical water depth (y_c/d_o), and y_c = critical water depth.

The cross-sectional area, A, of partially filled circular channel can be expressed in dimensionless form as:

$$\frac{A}{d_o^2} = \frac{(\theta - \sin\theta)}{8} \qquad (9)$$

where θ = central angle of cross-sectional area of partially filled circular channel as shown in Fig. (2).

The segment area, A, of a circular channel of partially filled conduit may be approximated as (Hager 1989):

$$\frac{A}{d_o^2} = \frac{4}{3}(Y)^{3/2}\left(1 - \frac{1}{4}(Y) - \frac{4}{25}(Y)^2\right) \qquad (10)$$

Dimensionless hydrostatic force:

The relative centric depth of the segment area from the free surface, $\bar{y}/d_o$, may be written as follows, Fig. (2):

$$\frac{\bar{y}}{d_o} = \frac{2}{3}\left(\frac{\sin^3(\theta/2)}{\theta - \sin\theta}\right) - \frac{\cos(\theta/2)}{2} \qquad (11)$$

or $$\frac{\bar{y}}{d_o} = \frac{2}{3}\left(\frac{\sin^3(\theta/2)}{8A/d_o^2}\right) - \frac{\cos(\theta/2)}{2} \qquad (12)$$

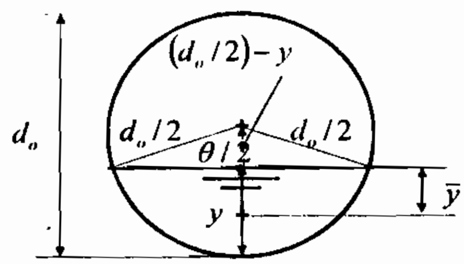

Fig. (2): Definition Sketch of Partially Filled Circular Channel.

From Fig. (2)

$$\cos(\theta/2) = \frac{d_o - 2y}{d_o} = 1 - 2Y \qquad (13)$$

And

$$\sin(\theta/2) = \frac{2\sqrt{d_o y - y^2}}{d_o} = 2\sqrt{Y - Y^2} \qquad (14)$$

Substituting Eqs. (13) and (14) into Eq. (12) gives:

$$\frac{\bar{y}}{d_o} = \frac{2}{3}\frac{\left(Y - (Y)^2\right)^{3/2}}{\left(A/d_o^2\right)} - \left(\frac{1}{2} - Y\right) \qquad (15)$$

or

$$\frac{\bar{y}}{d_o} = \frac{(1-Y)^{3/2}}{\left(2 - \frac{1}{2}(Y) - \frac{8}{25}(Y)^2\right)} - \left(\frac{1}{2} - Y\right) \qquad (16)$$

Herein, the dimensionless hydrostatic force affecting the jump may be written as:

$$\frac{A}{d_o^2}\frac{\bar{y}}{d_o} = \frac{2}{3}\left(Y - (Y)^2\right)^{3/2} - \frac{4}{3}(Y)^{3/2}\left(1 - \frac{1}{4}(Y) - \frac{4}{25}(Y)^2\right)\left(\frac{1}{2} - Y\right) \qquad (17)$$

Eqs. (9), (15) and (17) are tabulated for different values of relative water depth Y as shown in Table (1). Hence, curves of $\bar{y}/d_o$ and $A\bar{y}/d_o^2 d_o$ are plotted in Fig. (3).

Dimensionless specific force:

From equations (8), (10) and (17), the dimensionless specific force in Eq. (6) can be written as:

$$F^* = \frac{(Y_c)^4}{\frac{4}{3}(Y)^{3/2}\left(1 - \frac{1}{4}(Y) - \frac{4}{25}(Y)^2\right)} + \frac{2}{3}\left(Y - (Y)^2\right)^{3/2} - \frac{4}{3}(Y)^{3/2}\left(1 - \frac{1}{4}(Y) - \frac{4}{25}(Y)^2\right)\left(\frac{1}{2} - Y\right) \qquad (18)$$

where Y_c = relative critical water depth ($= y_c/d_o$).

Equation (18) gives dimensionless specific force F^* as a function of dimensionless ratios of water depth Y and critical depth Y_c. Hence, the values of F^* for different values of Y and Y_c can be tabulated as in Table (2). Also a set of curves between F^* and Y for different values of Y_c can be plotted as shown in Fig. (4).

Energy dissipated by the jump:

The specific energy equation for open channel can be written as:

$$E = d\cos\phi + \alpha\frac{V^2}{2g} \tag{19}$$

where E = specific energy, d = normal water depth, ϕ = bed slope angle, α = energy coefficient, V = mean flow velocity, and g = gravitational acceleration.
For a horizontal or small slope channels and α=1.0, Eq. (19) becomes:

$$E = y + \frac{V^2}{2gA^2} \tag{20}$$

If the discharge Q is given, then Eq. (20) may be written as:

$$E = y + \frac{Q^2}{2gA^2} \tag{21}$$

Dividing both the sides of Eq. (21) by (d_o) the dimensionless specific energy E^* ($= E/d_o$) can be expressed as:

$$E^* = \frac{E}{d_o} = \frac{y}{d_o} + \frac{Q^2}{2gd_o^5\left(A/d_o^2\right)^2} \tag{22}$$

or

$$E^* = \frac{y}{d_o} + \frac{\left(y_c/d_o\right)^4}{2\left(A/d_o^2\right)^2} = Y + \frac{(Y_c)^4}{2\left(A/d_o^2\right)^2} \tag{23}$$

The energy dissipated by the jump can be computed as:

$$\Delta E^* = \frac{\Delta E}{d_o} = E_1^* - E_2^*$$

$$= \left(Y_1 + \frac{(Y_c)^4}{2\left(A_1/d_o^2\right)^2}\right) - \left(Y_2 + \frac{(Y_c)^4}{2\left(A_2/d_o^2\right)^2}\right) \tag{24}$$

where ΔE^* = dimensionless energy dissipated by the jump ($=\Delta E/d_o$); E_1^* = relative initial specific energy; E_2^* = relative sequent specific energy; Y_1 = relative initial specifie water depth; and Y_2 = relative sequent water depth.
Using relative initial water depth Y_1 and relative sequent depth Y_2, the relative height of the jump can be computed as:

$$H_j^* = Y_2 - Y_1 \tag{25}$$

where H_j^* = relative height of the jump.
The values of E^*, ΔE^* and H_j^* for different values of relative critical depth Y_c computed and tabulated, Table (3). Set of curves of ΔE^* for different values of Y_c is plotted in Fig. (5). Also, a relationship between H_j^* and Y_1 for different values of Y_c is plotted in Fig. (6).

RESULTS AND ANALYSIS

To design a hydraulic jump structure in which a hydraulic jump is formed, it is necessary to know the location, the length, the surface profile of the jump and the amount of energy dissipated. Knowledge of the surface profile would be useful in the economic design of walls of a hydraulic jump structure. Then, to know the above parameters, it is necessary to know the discharge Q, the diameter d_o, the initial water depth Y_1 and the sequent water depth Y_2. The momentum equation is well suited for the analysis of hydraulic jump owing to the initially unknown energy loss dissipated by the jump.

The method described here can be used to determine the various elements of the hydraulic jump formed in a horizontal channel with circular cross-section. Two cases are studied in the present paper:

1. <u>case I</u>:
   a- Discharge, diameter and relative initial depth are known.
   b- Discharge, diameter and relative sequent depth are known.
2. <u>case II:</u>
   Discharge and relative dissipated energy are known.

In the two cases it is necessary to find the unknown variables in the problem. To solve any of the two cases, it can be used the derived equations (analytical solution) or the developed graphs (graphical solution) or the prepared tables (solution by using tables) as follows:

Analytical solution:

- The relative segment of a circular area, A/d_o^2, the relative centroid depth of the segment area below the free water surface, $\bar{y}/d_o$, and the hydrostatic force per unit weight affected on the wetted cross section, P/γ, can be expressed in terms of the depth of flow, y, and the diameter of the channel, d_o. These expressions are expressed in Eqs. (10), (15) and (17).
- Eq. (8) declares that the relative critical depth is function of the discharge and the diameter of the cross-sectional of channel.
- Eq. (18) enables the determination of the relative sequent depth if the relative initial depth is given or vice versa for a known relative critical depth.
- By using Eq. (24) the dimensionless energy dissipated by the jump can be determined.
- By using Eq. (25) the dimensionless height of the jump can be computed.

Graphical solution:

From the dimensionless specific energy curves, Fig. (4), it can be determined directly the relative sequent depth if the relative initial depth is given or vice versa for a known relative critical depth. From Fig. (5), it can be determined directly the relative specific energy dissipated by the jump if the relative initial depth is given or vice versa for a known relative critical depth. Finally Fig. (6) can be used to determine directly the relative height of the jump if the relative initial depth is known.

Solution by using tables:

From table (3) it can be determined directly the relative sequent depth if the relative initial depth is given or vice versa for a known relative critical depth. At the same time, it can be determined the relative specific energy dissipated by the jump and the relative height of the jump.

PROPOSED METHODS

The methods of solution, analytical, graphical or by using tables, are illustrated by the following steps:

1. **Case I:**

Given variables are: the discharge, Q, the diameter of the circular channel, d_o, and the initial water depth, y_1. The first step of the solution is to compute the relative critical depth, Y_c, by using equation (8). Then using any one of the following accepted solutions:

1. Analytical Solution

a. Using Eqs. (10), (15) and (18), compute the dimensionless specific force, F_1^*, corresponding to the relative initial depth Y_1 and relative critical depth Y_c.

b. Assume a value of relative sequent depth Y_2, and determined the corresponding dimensionless specific force, F_2^*.

c. Check the two values F_1^* and F_2^* as known that F_1^* corresponding to Y_1 is equal to F_2^* corresponding to Y_2.

d. Make other trials until the two values of dimensionless specific force are equal.

e. Compute the relative height of the jump H_j^* by using Eq. (25).

f. Determine the relative loss of energy in the jump ΔE^* by using Eq. (24) after computing the area corresponding to Y_1 and Y_2 from Eq. (9).

2. Graphical Solution

a. From the dimensionless specific curves Fig. (4), determine directly the relative sequent depth Y_2 corresponding to the relative initial depth Y_1 from the curve of the calculated relative critical depth Y_c.

b. Using the relative initial depth Y_1, and the relative critical depth Y_c, determine the corresponding value of relative loss of energy in the jump ΔE^* from the curves in Fig. (5).

c. Using the known relative initial depth Y_1, and the relative critical depth Y_c, determine the corresponding value of relative height of the jump H_j^* from the curves in Fig. (6).

3. Solution by Using Tables

From Table (3), determine directly the relative sequent depth, Y_2, the relative height of the jump, H_j^*, and the relative loss of energy in the jump, ΔE^*, corresponding to the relative initial depth, Y_1, and the calculated relative critical depth Y_c.

II. Case II:

The given data for this case are: the discharge, Q, the diameter of the circular channel, d_o, and the loss energy by the jump, ΔE.
The first step of the solution is computed the relative critical depth, Y_c, by using the equation (8). Then choose any accepted solution from:

1. Analytical Solution

a. Assume a trial solution of relative initial water depth $Y_1 < Y_c$.
b. Using Eq. (18), compute the dimensionless specific force F_1^*.
c. By trial and error determine the relative sequent $Y_2 > Y_c$, as known that F_1^* corresponding to (Y_1) is equal to F_2^* corresponding to (Y_2).
d. Compute the relative loss of energy in the jump ΔE^* from Eq. (24) after calculation of the area from Eq. (9).
e. If the calculated loss energy is not the same as given, repeat the above steps until the calculated value equals the given one.
f. Compute the relative height of the jump H_j^* by using Eq. (25).

2. Graphical Solution

a. From Fig. (5), determine the relative initial depth Y_1 corresponding to the dimensionless loss energy value ΔE^* from the curve of the calculated relative critical depth Y_c.
b. Using the relative initial depth Y_1, and the relative critical depth Y_c, determine the corresponding value of relative sequent depth Y_2 from the curves in Fig. (4).
c. Using the relative initial depth Y_1, and the relative critical depth Y_c, determine the corresponding value of relative height of the jump H_j^* from the curves in Fig. (6).

3. Solution by Using Tables

From Table (3), determine directly the relative initial depth, Y_1, the relative sequent depth, Y_2, and the relative height of the jump, H_j^*, corresponding to the relative loss of energy in the jump, ΔE^*, and the calculated relative critical depth Y_c.

ILLUSTRATIVE EXAMPLE

In order to illustrate the computational procedure, consider a circular pipe of diameter =1.25 m, passing a discharge =1.37 m^3/sec. at initial uniform flow water depth =0.40 m, it is required to compute the sequent depth, the height of the jump and the energy dissipated by the hydraulic jump.
The solution procedure is as follows:

Analytical solution:

1. Relative initial depth Y_1 = 0.32.
2. Relative critical flow depth Y_c = 0.50.

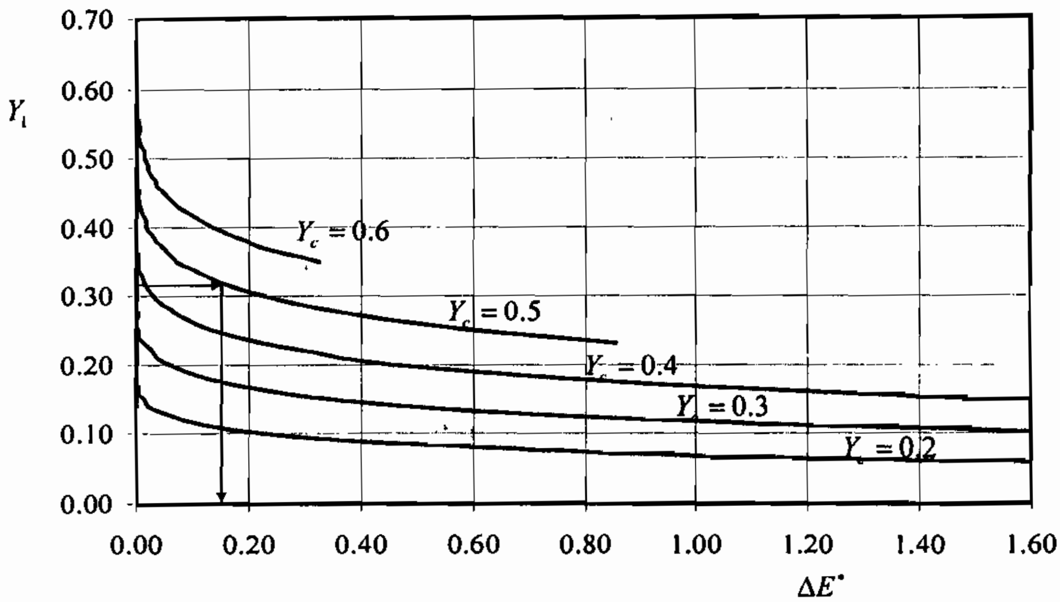

Fig. (5): Relationships Between Relative Initial Depth, $Y_1 = y_1/d_o$, and Relative Energy Loss $\Delta E^* = (E_1 - E_2)/d_o$ for Different Values of Relative Critical Depth $Y_c^* = y_c/d_o$.

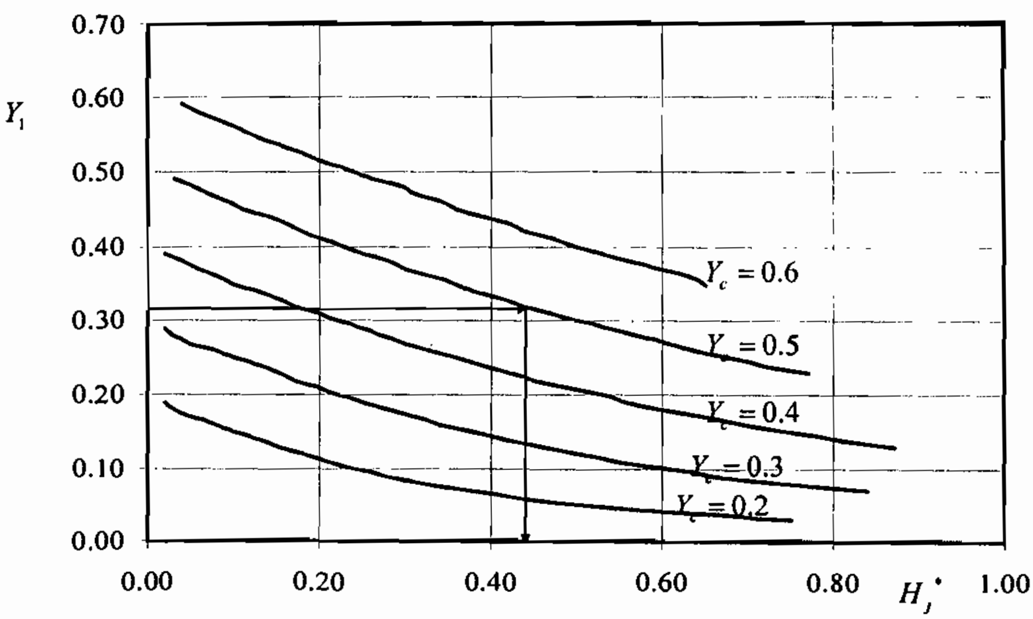

Fig. (6): Relationships Between Relative Initial Depth, $Y_1 = y_1/d_o$, and Relative Height of the Jump $H_j^* = (y_2 - y_1)/d_o$ for Different Values of Relative Critical Depth $Y_c = y_c/d_o$.

Table (1): Relative Area, A/d_o^2, Relative Centroid Depth, $\bar{y}/d_o$, and Relative first Moment of Water Area, $A\bar{y}/d_o^2 d_o$, as a Function of Relative water depth, y/d_o.

y/d_o	A/d_o^2	$\bar{y}/d_o$	$A\bar{y}/d_o^2 d_o$
0.02	0.0037	0.0144	0.0001
0.04	0.0105	0.0178	0.0002
0.06	0.0192	0 0251	0.0005
0.08	0.0294	0.0328	0.0010
0.10	0.0409	0.0401	0.0016
0.12	0.0534	0.0484	0.0026
0.14	0.0668	0.0569	0.0038
0.16	0.0811	0.0650	0.0053
0.18	0.0961	0.0734	0.0071
0.20	0.1118	0.0816	0.0091
0.22	0.1281	0.0899	0.0115
0.24	0.1449	0.0984	0.0143
0.26	0.1623	0.1067	0.0173
0.28	0.1800	0.1153	0.0207
0.30	0.1982	0.1237	0.0245
0.32	0.2167	0.1323	0.0287
0.34	0.2355	0.1409	0.0332
0.36	0.2546	0.1496	0.0381
0.38	0.2739	0.1583	0.0434
0.40	0.2934	0.1672	0.0490
0.42	0.3132	0.1759	0.0551
0.44	0.3328	0.1850	0.0616
0.46	0.3527	0.1940	0.0684
0.48	0.3727	0.2031	0.0757
0.50	0.3927	0.2122	0.0833
0.52	0.4127	0.2214	0.0914
0.54	0.4327	0.2307	0.0998
0.56	0.4526	0.2402	0.1087
0.58	0.4723	0.2497	0.1179
0.60	0.4920	0.2593	0.1276
0.62	0.5115	0.2690	0.1376
0.64	0.5308	0.2789	0.1480
0.66	0.5499	0.2889	0.1589
0.68	0.5687	0.2990	0.1700
0.70	0.5872	0.3093	0.1816
0.72	0.6054	0.3197	0.1935
0.74	0.6231	0.3303	0.2058
0.76	0.6404	0.3411	0.2184
0.78	0.6573	0.3521	0.2314
0.80	0.6736	0.3633	0.2447
0.82	0.6893	0.3748	0.2584
0.84	0.7043	C.3866	0.2723
0.86	0.7186	0.3988	0.2865
0.88	0.7320	0.4113	0.3010

Table (1): Relative Area, A/d_o^2, Relative Centroid Depth, $\bar{y}/d_o$, and Relative first Moment of Water Area, $A\bar{y}/d_o^2 d_o$, as a Function of Relative water depth, y/d_o. (continued)

y/d_o	A/d_o^2	$\bar{y}/d_o$	$A\bar{y}/d_o^2 d_o$
0.90	0.7445	0.4242	0.3158
0.92	0.7560	0.4376	0.3308
0.94	0.7662	0.4517	0.3461
0.96	0.7749	0.4665	0.3615
0.98	0.7816	0.4823	0.3770
1.00	0.7854	0.5000	0.3927

Table (2): Dimensionless Specific Force, F^*, and Relative Sequent Water Depth, Y_2, of Hydraulic Jump in a Circular Open Channel for Different Values of Relative Critical Depth, Y_c.

Y	$Y_c = 0.2$		$Y_c = 0.3$		$Y_c = 0.4$		$Y_c = 0.5$		$Y_c = 0.6$	
	F^*	Y_2	F^*	Y_2	F^*	Y_2	F^*	Y_2	F^*	Y_2
0.02	0.4325		2.1892		6.9190		16.8919		35.0271	
0.03	0.2320	0.78	1.1740		3.7102		9.0580		18.7827	
0.04	0.1526	0.64	0.7716		2.4383		5.9526		12.3430	
0.05	0.1091	0.55	0.5513		1.7418		4.2520		8.8166	
0.06	0.0838	0.49	0.4224		1.3338		3.2557		6.7505	
0.07	0.0668	0.44	0.3354	0.91	1.0585		2.5833		5.3560	
0.08	0.0554	0.40	0.2765	0.83	0.8717		2.1268		4.4091	
0.09	0.0470	0.37	0.2327	0.76	0.7327		1.7870		3.7041	
0.10	0.0408	0.34	0.1997	0.71	0.6276		1.5298		3.1703	
0.11	0.0361	0.32	0.1744	0.66	0.5468		1.3319		2.7595	
0.12	0.0325	0.30	0.1543	0.62	0.4820		1.1730		2.4296	
0.13	0.0298	0.28	0.1382	0.59	0.4298	1.00	1.0448		2.1632	
0.14	0.0278	0.27	0.1251	0.56	0.3870	0.95	0.9394		1.9439	
0.15	0.0261	0.25	0.1141	0.53	0.3509	0.90	0.8502		1.7582	
0.16	0.0250	0.24	0.1052	0.50	0.3209	0.86	0.7759		1.6033	
0.17	0.0242	0.22	0.0977	0.48	0.2954	0.82	0.7123		1.4705	
0.18	0.0237	0.21	0.0913	0.46	0.2734	0.78	0.6574		1.3556	
0.19	0.0234	0.21	0.0860	0.44	0.2544	0.75	0.6096		1.2554	
0.20	0.0234		0.0816	0.42	0.2381	0.73	0.5682		1.1683	
0.21	0.0236		0.0778	0.41	0.2238	0.70	0.5315		1.0912	
0.22	0.0240		0.0748	0.39	0.2114	0.67	0.4994		1.0232	
0.23	0.0246		0.0722	0.38	0.2004	0.65	0.4707	1.00	0.9623	
0.24	0.0253		0.0702	0.37	0.1909	0.63	0.4456	0.96	0.9087	
0.25	0.0262		0.0685	0.35	0.1825	0.61	0.4229	0.93	0.8601	
0.26	0.0272		0.0672	0.34	0.1750	0.59	0.4024	0.90	0.8158	
0.27	0.0283		0.0663	0.32	0.1686	0.57	0.3843	0.88	0.7764	
0.28	0.0296		0.0657	0.31	0.1630	0.55	0.3680	0.85	0.7407	

Table (2): Dimensionless Specific Force, F^*, and Relative Sequent Water Depth, Y_2, of Hydraulic Jump in a Circular Open Channel for Different Values of Relative Critical Depth, Y_c. (Continued)

Y	$Y_c = 0.2$		$Y_c = 0.3$		$Y_c = 0.4$		$Y_c = 0.5$		$Y_c = 0.6$	
	F^*	Y_2	F^*	Y_2	F^*		F^*	Y_2	F^*	Y_2
0.29	0.0311		0.0655	0.31	0.1580	0.54	0.3533	0.83	0.7083	
0.30	0.0326		0.0654		0.1537	0.52	0.3399	0.80	0.6784	
0.31	0.0343		0.0656		0.1500	0.51	0.3279	0.78	0.6514	
0.32	0.0360		0.0660		0.1468	0.49	0.3171	0.76	0.6267	
0.33	0.0380		0.0667		0.1442	0.48	0.3074	0.74	0.6043	
0.34	0.0400		0.0676		0.1419	0.47	0.2986	0.72	0.5835	
0.35	0.0421		0.0687		0.1401	0.45	0.2907	0.71	0.5646	1.00
0.36	0.0444		0.0699		0.1386	0.44	0.2836	0.69	0.5471	0.99
0.37	0.0467		0.0713		0.1376	0.43	0.2772	0.67	0.5312	0.97
0.38	0.0492		0.0729		0.1368	0.42	0.2716	0.66	0.5165	0.94
0.39	0.0518		0.0747		0.1364	0.41	0.2665	0.64	0.5031	0.92
0.40	0.0545		0.0767		0.1363		0.2621	0.63	0.4908	0.90
0.41	0.0573		0.0787		0.1365		0.2582	0.61	0.4795	0.88
0.42	0.0602		0.0810		0.1368		0.2547	0.60	0.4689	0.86
0.43	0.0632		0.0834		0.1376		0.2519	0.59	0.4597	0.85
0.44	0.0664		0.0859		0.1385		0.2494	0.58	0.4510	0.83
0.45	0.0696		0.0886		0.1396		0.2473	0.56	0.4430	0.81
0.46	0.0730		0.0914		0.1410		0.2456	0.55	0.4359	0.80
0.47	0.0764		0.0943		0.1426		0.2443	0.54	0.4293	0.78
0.48	0.0800		0.0974		0.1444		0.2434	0.53	0.4234	0.77
0.49	0.0836		0.1006		0.1463		0.2428	0.52	0.4181	0.75
0.50	0.0874		0.1040		0.1485		0.2425		0.4134	0.74
0.51	0.0913		0.1074		0.1509		0.2425		0.4091	0.72
0.52	0.0953		0.1110		0.1534		0.2428		0.4054	0.71
0.53	0.0993		0.1147		0.1561		0.2434		0.4022	0.70
0.54	0.1035		0.1186		0.1590		0.2443		0.3994	0.68
0.55	0.1078		0.1225		0.1621		0.2454		0.3970	0.67
0.56	0.1122		0.1266		0.1653		0.2468		0.3950	0.66
0.57	0.1167		0.1308		0.1686		0.2484		0.3935	0.65
0.58	0.1213		0.1351		0.1721		0.2503		0.3923	0.64
0.59	0.1260		0.1395		0.1758		0.2523		0.3915	0.63
0.60	0.1308		0.1440		0.1796		0.2546		0.3910	

Table (3): Relative Elements of Hydraulic Jump in a Circular Open Channel for Different Values of Relative Critical Water Depth, Y_c.

Y_1	$Y_c = 0.2$					$Y_c = 0.3$				
	E_1^*	Y_2	E_2^*	ΔE^*	H_j^*	E_1^*	Y_2	E_2^*	ΔE^*	H_j^*
0.02										
0.03	16.8332	0.78	0.7819	16.0513	0.75					
0.04	7.2962	0.64	0.6428	6.6534	0.60					
0.05	3.7522	0.55	0.5541	3.1981	0.50					
0.06	2.2301	0.49	0.4955	1.7346	0.43					
0.07	1.4360	0.44	0.4472	0.9888	0.37	6.9855	0.91	0.9172	6.0683	0.84
0.08	1.0055	0.40	0.4093	0.5962	0.32	4.7655	0.83	0.8383	3.9272	0.75
0.09	0.7431	0.37	0.3815	0.3616	0.28	3.3961	0.76	0.7699	2.6262	0.67
0.10	0.5782	0.34	0.3544	0.2238	0.24	2.5211	0.71	0.7214	1.7997	0.61
0.11	0.4722	0.32	0.3370	0.1352	0.21	1.9434	0.66	0.6734	1.2700	0.55
0.12	0.4005	0.30	0.3204	0.0801	0.18	1.5403	0.62	0.6355	0.9048	0.50
0.13	0.3522	0.28	0.3047	0.0475	0.15	1.2550	0.59	0.6074	0.6476	0.46
0.14	0.3193	0.27	0.2973	0.0220	0.13	1.0476	0.56	0.5798	0.4678	0.42
0.15	0.2965	0.25	0.2840	0.0125	0.10	0.8916	0.53	0.5527	0.3389	0.38
0.16	0.2816	0.24	0.2781	0.0035	0.08	0.7758	0.50	0.5263	0.2495	0.34
0.17	0.2721	0.22	0.2688	0.0033	0.05	0.6871	0.48	0.5092	0.1779	0.31
0.18	0.2666	0.21	0.2656	0.0010	0.03	0.6185	0.46	0.4926	0.1259	0.28
0.19	0.2641	0.21	0.2656	0.0000	0.02	0.5652	0.44	0.4766	0.0886	0.25
0.20						0.5240	0.42	0.4613	0.0627	0.22
0.21						0.4917	0.41	0.4541	0.0376	0.20
0.22						0.4668	0.39	0.4404	0.0264	0.17
0.23						0.4474	0.38	0.4340	0.0134	0.15
0.24						0.4329	0.37	0.4280	0.0049	0.13
0.25						0.4219	0.35	0.4175	0.0044	0.10
0.26						0.4138	0.34	0.4130	0.0008	0.08
0.27						0.4083	0.32	0.4062	0.0021	0.05
0.28						0.4050	0.31	0.4042	0.0008	0.03
0.29						0.4034	0.31	0.4042	0.0000	0.02
0.30										

Table (3): Relative Elements of Hydraulic Jump in a Circular Open Channel for Different Values of Relative Critical Water Depth, Y_c. (Continued)

Y_1	$Y_c = 0.4$					$Y_c = 0.5$				
	E_1^*	Y_2	E_2^*	ΔE^*	H_j^*	E_1^*	Y_2	E_2^*	ΔE^*	H_j^*
0.13	3.6856	1.00	1.0208	2.6648	0.87					
0.14	3.0085	0.95	0.9715	2.0370	0.81					
0.15	2.4938	0.90	0.9521	1.5417	0.75					
0.16	2.1061	0.86	0.8848	1.2213	0.70					
0.17	1.8043	0.82	0.8469	0.9574	0.65					
0.18	1.5660	0.78	0.8096	0.7564	0.60					
0.19	1.3757	0.75	0.7821	0.5936	0.56					
0.20	1.2241	0.73	0.7639	0.4602	0.53					
0.21	1.1004	0.70	0.7371	0.3633	0.49					
0.22	1.0000	0.67	0.7109	0.2891	0.45					
0.23	0.9170	0.65	0.6938	0.2232	0.42	1.9072	1.00	1.0507	0.8565	0.77
0.24	0.8496	0.63	0.6771	0.1725	0.39	1.7284	0.96	1.0120	0.7164	0.72
0.25	0.7932	0.61	0.6608	0.1324	0.36	1.5763	0.93	0.9839	0.5924	0.68
0.26	0.7459	0.59	0.6450	0.1009	0.33	1.4464	0.90	0.9564	0.4900	0.64
0.27	0.7072	0.57	0.6298	0.0774	0.30	1.3375	0.88	0.9383	0.3992	0.61
0.28	0.6751	0.55	0.6153	0.0598	0.27	1.2445	0.85	0.9117	0.3328	0.57
0.29	0.6483	0.54	0.6084	0.0399	0.25	1.1648	0.83	0.8943	0.2705	0.54
0.30	0.6258	0.52	0.5952	0.0306	0.22	1.0955	0.80	0.8689	0.2266	0.50
0.31	0.6076	0.51	0.5889	0.0187	0.20	1.0365	0.78	0.8523	0.1842	0.47
0.32	0.5926	0.49	0.5774	0.0152	0.17	0.9855	0.76	0.8362	0.1493	0.44
0.33	0.5806	0.48	0.5721	0.0085	0.15	0.9418	0.74	0.8205	0.1213	0.41
0.34	0.5708	0.47	0.5673	0.0035	0.13	0.9035	0.72	0.8053	0.0982	0.38
0.35	0.5632	0.45	0.5589	0.0043	0.10	0.8706	0.71	0.7979	0.0727	0.36
0.36	0.5575	0.44	0.5556	0.0019	0.08	0.8421	0.69	0.7835	0.0586	0.33
0.37	0.5534	0.43	0.5528	0.0006	0.06	0.8177	0.67	0.7699	0.0478	0.30
0.38	0.5506	0.42	0.5505	0.0001	0.04	0.7965	0.66	0.7633	0.0332	0.28
0.39	0.5491	0.41	0.5492	0.0000	0.02	0.7785	0.64	0.7509	0.0276	0.25
0.40						0.7630	0.63	0.7450	0.0180	0.23
0.41						0.7499	0.61	0.7341	0.0158	0.20
0.42						0.7386	0.60	0.7291	0.0095	0.18
0.43						0.7297	0.59	0.7244	0.0053	0.16
0.44						0.7222	0.58	0.7201	0.0021	0.14
0.45						0.7159	0.56	0.7126	0.0033	0.11
0.46						0.7112	0.55	0.7095	0.0017	0.09
0.47						0.7075	0.54	0.7069	0.0006	0.07
0.48						0.7050	0.53	0.7049	0.0001	0.05
0.49						0.7034	0.52	0.7035	0.0000	0.03
0.50										

Table (3): Relative Elements of Hydraulic Jump in a Circular Open Channel for Different Values of Relative Critical Water Depth, Y_c. (Continued)

Y_1	$Y_c = 0.6$				
	E_1^*	Y_2	E_2^*	ΔE^*	H_j^*
0.35	1.4296	1.00	1.1050	0.3246	0.65
0.36	1.3597	0.99	1.0954	0.2643	0.63
0.37	1.2983	0.97	1.0769	0.2214	0.60
0.38	1.2438	0.94	1.0504	0.1934	0.56
0.39	1.1957	0.92	1.0334	0.1623	0.53
0.40	1.1528	0.90	1.0169	0.1359	0.50
0.41	1.1149	0.88	1.0009	0.1140	0.47
0.42	1.0806	0.86	0.9855	0.0951	0.44
0.43	1.0515	0.85	0.9780	0.0735	0.42
0.44	1.0251	0.83	0.9634	0.0617	0.39
0.45	1.0014	0.81	0.9495	0.0519	0.36
0.46	0.9809	0.80	0.9428	0.0381	0.34
0.47	0.9626	0.78	0.9300	0.0326	0.31
0.48	0.9465	0.77	0.9239	0.0226	0.29
0.49	0.9324	0.75	0.9123	0.0201	0.26
0.50	0.9202	0.74	0.9069	0.0133	0.24
0.51	0.9096	0.72	0.8968	0.0128	0.21
0.52	0.9005	0.71	0.8922	0.0083	0.19
0.53	0.8927	0.70	0.8879	0.0048	0.17
0.54	0.8861	0.68	0.8804	0.0057	0.14
0.55	0.8808	0.67	0.8771	0.0037	0.12
0.56	0.8763	0.66	0.8743	0.0020	0.10
0.57	0.8729	0.65	0.8719	0.0010	0.08
0.58	0.8705	0.64	0.8700	0.0005	0.06
0.59	0.8687	0.63	0.8685	0.0002	0.04
0.60					

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NOTATION

The following symbols are used in this paper:

A = cross sectional area of flow; (L^2)
A_1 = initial cross-sectional area; (L^2)
A_2 = sequent cross-sectional area; (L^2)
d = normal water depth; (L)
d_o = internal diameter of circular channel; (L)
E = specific energy; (L)
E^* = dimensionless specific energy ($= E/d_o$);
E_1^* = initial dimensionless specific energy;
E_2^* = sequent dimensionless specific energy;
F = specific force per unit weight of water; (L^3)
F^* = dimensionless specific force ($= F/d_o^3$);
F_1^* = initial dimensionless specific force;
F_2^* = sequent dimensionless specific force;
F_f = friction force; (MLT^{-2})
F_r = Froude number;
g = gravitational acceleration; (LT^{-2})
H_j^* = relative height of jump;
K = factor of correction to the shape of water surface at the jump;
L_j = length of the jump; (L)
L_r = length of the rollers; (L)
P_1 = hydrostatic force at section 1; (MLT^{-2})
P_2 = hydrostatic force at section 2; (MLT^{-2})
Q = the discharge; (L^3T^{-1})
S_o = slope of the channel bed;
V = mean flow velocity; (LT^{-1})
V_1 = initial mean flow velocity; (LT^{-1})
V_2 = sequent mean flow velocity; (LT^{-1})
W = weight of water within the jump; (MLT^{-2})
Y = relative water depth;
Y_c = relative critical water depth;
Y_1 = relative initial water depth;
Y_2 = relative sequent water depth;
y = water depth; (L)
y_c = critical water depth; (L)
y_1 = initial water depth; (L)
y_2 = sequent water depth; (L)
$\bar{y}_1$ = centroid depth of flow of the segment area of circle from the free surface at section 1; (L)
$\bar{y}_2$ = centroid depth of flow of the segment area of circle from the free surface at section 2; (L)
α = energy coefficient;
β = momentum coefficient;
β_1 = momentum coefficient at section 1;
β_2 = momentum coefficient at section 2;
γ = specific weight of water; ($ML^{-2}T^{-2}$)
ΔE = energy dissipated by the jump;
ΔE^* = relative energy dissipated by the jump;
θ = central angle of cross-sectional area of partially filled circular channel;
ρ = density of water; (ML^{-3}) and
ϕ = channel bed slope.