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Semi (θ)-Compactness in Intuitionistic Fuzzy Topological Spaces.

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Semi θ– Compactness in Intuitionistic Fuzzy Topological Spaces

الاحكام من النوع -semi θ علي الفراغات التوبولوجية الفازية الحدسية

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الملخص العربي:

يهدف هذا البحث إلي إدخال مفهوم جديد من مفاهيم الإحكام هو semi θ-compactness وذلك علي الفراغات التوبولوجية الفازية الحدسية. كم تم تعريف الإحكام المحلي من النوع -semi θ وكذلك تم استنتاج بعض الخصائص لهذه المعرفات الجديدة كما تم دراسة علاقاتها ببعض مثيلاتها المعرفة من قبل وتأثير بعض الدوال عليها.

Abstract:
The purpose of this paper is to construct the concept of semi θ-compactness in intuitionistic fuzzy topological spaces. We give some characterizations of semi θ-compactness, locally semi θ-compactness. A comparison between these concepts and some other types of compactness in intuitionistic fuzzy topological spaces are established.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy topological space, Intuitionistic fuzzy semi θ-compact.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [11], and later Atanassov [1,2] generalized this idea to intuitionistic fuzzy sets. On the other hand, Coker [3] introduced the notions of intuitionistic fuzzy topological spaces, fuzzy continuity and some other related concepts. In this paper, we introduce the concepts of semi θ-compactness, locally semi θ-compactness in intuitionistic fuzzy topological spaces. We give some characterizations and basic properties for these concepts. For definitions and results not explained in this paper, we refer to the papers [1, 3, 5, 6, 8], assuming them to be well known. The words "neighbourhood", "continuous" and "irresolute" will be abbreviated as respectively "*nbd*", "*cont.*" and "i".

2. Preliminaries

First, we present the fundamental definitions.

Definition 2.1[2]. Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS, for short) U is an object having the form $U = \{\langle x, \mu_U(x), \gamma_U(x)\rangle : x \in X\}$ where the functions $\mu_U : X \to I$ and $\gamma_U : X \to I$ denote respectively the degree of membership (namely

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$\mu_U(x)$) and the degree of nonmembership (namely $\gamma_U(x)$) of each element $x \in X$ to the set U, and $0 \leq \mu_U(x) + \gamma_U(x) \leq 1$ for each $x \in X$.

The reader may consult [3, 4, 6] to see several types of relations and operations on IFS's, intuitionistic fuzzy points (IFP's, for short) and some properties of images and preimages of IFS's.

Definition 2.2[3]. Let X be a nonempty set and let the IFS's U and V be in the form $U = \{\langle x, \mu_U(x), \gamma_U(x)\rangle : x \in X\}$, $V = \{\langle x, \mu_V(x), \gamma_V(x)\rangle : x \in X\}$ and let $\{U_j : j \in J\}$ be an arbitrary family of IFS's in X. Then

(i) $U \leq V$ iff $\mu_U(x) \leq \mu_V(x)$ and $\gamma_U(x) \geq \gamma_V(x), \forall x \in X$;

$(ii) \overline{U} = \{\langle x, \gamma_U(x), \mu_U(x)\rangle : x \in X\}$;

$(iii) \cap U_j = \{\langle x, \wedge \mu_{U_j}(x), \vee \gamma_{U_j}(x)\rangle : x \in X\}$;

$(iv) \cup U_j = \{\langle x, \vee \mu_{U_j}(x), \wedge \gamma_{U_j}(x)\rangle : x \in X\}$;

(v) $\underset{\sim}{1} = \{\langle x, 1, 0\rangle : x \in X\}$ and $\underset{\sim}{0} = \{\langle x, 0, 1\rangle : x \in X\}$;

(vi) $\overline{\overline{U}} = U, \overline{\underset{\sim}{0}} = \underset{\sim}{1}$ and $\overline{\underset{\sim}{1}} = \underset{\sim}{0}$;

(vii) $[]U = \{\langle x, \mu_U(x), 1 - \mu_U(x)\rangle : x \in X\}$;

$(viii)$ $\langle\rangle U = \{\langle x, 1 - \gamma_U(x), \gamma_U(x)\rangle : x \in X\}$.

Definition 2.3[3]. An intuitionistic fuzzy topology (IFT, for short) on a nonempty set X is a family Ψ of IFS's in X containing $\underset{\sim}{0}, \underset{\sim}{1}$ and closed under finite infima and arbitrary suprema.

In this case the pair (X, Ψ) is called an intuitionistic fuzzy topological space (IFTS, for short) and each IFS in Ψ is known as an intuitionistic fuzzy open set (IFOS, for short) in X. The complement $\overline{U}$ of an $IFOS$ U in an $IFTS$ (X, Ψ) is called an intuitionistic fuzzy closed set (IFCS, for short), in X.

Proposition 2.4[3]. Let (X, Ψ) be an IFTS on X. Then, we can constract the following two IFTS's:

(i) $\Psi_{0,1} = \{[]U : U \in \Psi\}$;

(ii) $\Psi_{0,2} = \{\langle\rangle U : U \in \Psi\}$.

Definition 2.5[8]. Let X, Y be nonempty sets and $U = \langle x, \mu_U(x), \gamma_U(x)\rangle$, $V = \langle y, \mu_V(y), \gamma_V(y)\rangle$ IFS's of X and Y, respectively . Then $U \times V$ is an IFS of $X \times Y$ defined by: $(U \times V)(x, y) = \langle (x, y), \min(\mu_U(x), \mu_V(y)), \max(\gamma_U(x), \gamma_V(y))\rangle$.

Definition 2.6[8]. Let$(X, \Psi), (Y, \Phi)$ be IFTS's and $A \in \Psi$, $B \in \Phi$. We say that (X, Ψ) is product related to (Y, Φ) if for any IFS's U of X and V of Y , whenever ($\overline{A} \not\geq U$ and $\overline{B} \not\geq V$) $\Rightarrow (\overline{A} \times \underset{\sim}{1} \cup \underset{\sim}{1} \times \overline{B} \geq U \times V$), there exist $A_1 \in \Psi, B_1 \in \Phi$ such that $\overline{A_1} \geq U$ or $\overline{B_1} \geq V$ and $\overline{A_1} \times \underset{\sim}{1} \cup \underset{\sim}{1} \times \overline{B_1} = \overline{A} \times \underset{\sim}{1} \cup \underset{\sim}{1} \times \overline{B}$.

Definition 2.7[9]. An IFP $c(a, b)$ is said to be intuitionistic fuzzy *θ-cluster* point(IF*θ-cluster* point, for short) of an IFS U iff for each $A \in N_\varepsilon^\theta(c(a, b)), cl(A)$ q U.

The set of all $IF\theta$*-cluster* points of U is called the intuitionistic fuzzy θ –closure, of U and denoted by $cl_\theta(U)$. An IFS U will be called $IF\theta$*-closed*(IFθCS, for short) iff $U = cl_\theta(U)$. The complement of an $IF\theta$*-closed* set is $IF\theta$*-open*(IFθOS, for short).

Lemma 2.8.[10] Let X,Y are $IFTS$'s such that X is product related to Y. Then the product $U \times V$ of $IF\theta OS$ U of X and $IF\theta OS$ V of Y is an $IF\theta OS$ of $X \times Y$.

Definition 2.9. An IFS U of an IFTS X is called $\varepsilon - nbd$[4]($\varepsilon\theta - nbd$) [10]of an IFP $c(a, b)$, if there exists an $IFOS$ ($IF\theta OS$) U in X such that $c(a, b) \in U \leq U$.

The family of all *ε-nbd* (*εθ-nbd*) of an IFP $c(a, b)$ will be denoted by $N_\varepsilon(N_\varepsilon^\theta)(c(a, b))$.

Definition 2.10.[7] An IFS U of an IFTS X is said to be an IFsemiopen(IFSOS, for short)(IFpreopen(IFPOS, for short)) iff $U \leq cl(int(U))(U \leq int(cl(U)))$.

Definition 2.11. Let (X, Ψ) and (Y, Φ) be two IFTS's. A function $f : X \to Y$ is said to be.

(i) IF-cont.[3](IFsemi-cont.(IFS-cont., for short)[7]) if the preimage of each IFOS in Y is IFOS(IFSOS) in X.

(ii) IFi (IFsuper i) function if the preimage of each IFSOS in Y is IFSOS (IFOS) in X[10].

(iii) IFstrongly θ-(resp. IFθ-, IFfaintly, IFSθ-)cont. if the preimage of each IFOS(resp. IFθOS, IFθOS, IFSOS) of Y is IFθOS(resp. IFθOS, IFOS,IFθOS) in X[9,10].

(iv) IFweakly cont.[7] if for each IFOS V of Y, $f^{-1}(V) \leq int(f^{-1}(cl(V)))$.

(v) IF-[10](resp. IFsemi-[10], IFpre-, IFsuper semi-, IFθ-, IFfaintly-[10])open if the image of each IFOS(resp. IFOS, IFOS, IFSOS, IFθOS, IFθOS) of X is IFOS(resp. IFSOS, IFPOS, IFOS, IFθOS, IFOS) in Y.

Definition 2.12. An IFS U of an IFTS$(X, \Psi)$is said to be an IF[3](IFθ-)compact relative to X iff every an IF(θ-)open cover of U has a finite subcover.

Definition 2.13. An IFTS (X, Ψ) is called :

(i) IFcompact[3](resp. IFS-compact, IFλ-compact, IFθ-compact) iff every an IFopen (resp. semiopen, λ-open, θ-open) cover of X has a finite subcover which covers X.

(ii) Locally IF(θ-)compact if for each IFP $c(a, b)$ in X, there is $U \in N_c(c(a, b))$ such that $\mu_U(c) = 1$, $\gamma_U(c) = 0$ and U is an IF(θ-)compact relative to X.

(iii) IF-submaximal if each dense subset of X is IFOS.

(iv) IFS-closed iff every an IFsemiopen cover of X has a finite subfamily whose closures cover X.

(v) IF-regular iff for each $U \in \Psi$, $U = \vee\{U_j : U_j \in \Psi, cl(U_j) \leq U\}$.

Lemma 2.14. Let $f : X \to Y$ be an IFS-cont. and IFpreopen function, then $f^{-1}(V)$ is an IFSOS in X for each an IFSOS V in Y.

Proof. Let V be an IFSOS in Y, then there exists an IFOS U of X such that $U \leq V \leq cl(U)$. Now, $f^{-1}(U) \leq f^{-1}(V) \leq f^{-1}(cl(U))$, since f is an IFpreopen function we have, $f^{-1}(U) \leq f^{-1}(V) \leq f^{-1}(cl(U)) \leq cl(f^{-1}(U))$. Since f is an IFS cont., $f^{-1}(U)$ is an IFSOS in X, implies there is an IFOS G of X such that $G \leq f^{-1}(U) \leq f^{-1}(V) \leq cl(f^{-1}(U)) \leq cl(G)$. Hence $f^{-1}(V)$ is an IFSOS in X.

3. Semi θ-compactness in IFTS's

Definition 3.1. (i)A family $\{\langle x, \mu_{U_j}, \gamma_{U_j}\rangle : j \in J\}$ of IFSOS's(IFθOS's) in X such that $\vee\{\langle x, \mu_U(x), \gamma_U(x)\rangle : x \in X\} = 1$, is called an IFsemi($\theta$-)open cover of X.

(ii)A finite subfamily $\{\langle x, \mu_{U_j}, \gamma_{U_j}\rangle : j = 1, 2, ..., n\}$ of an IFsemi(θ-)open cover, which is also a semi(θ-)open cover, i.e. $\overset{n}{\underset{j=1}{\vee}}\{\langle x, \mu_{U_j}, \gamma_{U_j}\rangle\} = 1$, is called a finite subcover of $\{\langle x, \mu_{U_j}, \gamma_{U_j}\rangle : j \in J\}$.

Definition 3.2. A family $\{\langle x, \mu_{U_j}, \gamma_{U_j}\rangle : j \in J\}$ of IFS's satisfy the θ-finite intersection property (θ-FIP, for short) iff for every finite subfamily $\{\langle x, \mu_{U_j}, \gamma_{U_j}\rangle : j = 1, 2, ..., n\}$ of the family, we have $\overset{n}{\underset{j=1}{\wedge}}\{\langle x, \mu_{U_j}, \gamma_{U_j}\rangle : j \in J\} \neq 0$.

Definition 3.3. An IFTS(X, Ψ) is called fuzzy semi θ-compact(IFSθ-compact, for short) iff every an IFsemiopen cover of X has a finite subcollection(subcover)of IFθOS's, which covers X.

Definition 3.4. An IFS U of an IFTS$(X, \Psi)$is said to be an IFSθ-compact relative to X if for every family $\{U_j : j \in J\}$ of IFSOS's in X such that $U \subseteq \underset{j\in J}{\vee} U_j$, there is a finite subfamily $\{U_j : j = 1, 2, ..., n\}$ of IFθOS's such that $U \subseteq \overset{n}{\underset{j=1}{\vee}} U_j$.

Remark 3.5 . From the above definition and some other types of IF compactness, one can illustrate the following implications:

IFSθ-compact $\Rightarrow$IFS-compact $\Rightarrow$IFλ-compact $\Rightarrow$IF-compact $\Rightarrow$IFθ-compact

Theorem 3.6. (X, Ψ) is an IFSθ-compact iff every family $U = \{U_j : j \in J\}$ of IFSCS's in X having the θ-FIP, $\underset{j\in J}{\wedge} U_j \neq \underset{\sim}{0}$.

Proof. $(\Longrightarrow:)$ Let $U = \{U_j : j \in J\}$ be a family of IFSCS's in X having the θ-FIP. Suppose that $\underset{j\in J}{\wedge} U_j = \underset{\sim}{0}$, then $\underset{j\in J}{\vee} \overline{U_j} = \underset{\sim}{1}$. From the IFS$\theta$-compactness and $\{\overline{U_j} : j \in J\}$ is IFSOSs, there is a finite subfamily $\{\overline{U_j} : j = 1, 2, ..., n\}$ of IFθOS's such that $\overset{n}{\underset{j=1}{\vee}} \overline{U_j} = \underset{\sim}{1}$. Then $\overset{n}{\underset{j=1}{\wedge}} U_j = \overline{\overset{n}{\underset{j=1}{\vee}} \overline{U_j}} = \underset{\sim}{0}$, which is a contradiction to the θ-FIP. Hence $\underset{j\in J}{\wedge} U_j \neq \underset{\sim}{0}$.

$(\Longleftarrow:)$ Let $U = \{U_j : j \in J\}$ be an IFsemiopen cover of X. Hence $\{\overline{U_j} : j \in J\}$ is a family of IFSCSs having the θ-FIP. Then from the hypothesis, we have $\underset{j\in J}{\wedge} \overline{U_j} \neq \underset{\sim}{0}$ which implies $\underset{j\in J}{\vee} U_j \neq \underset{\sim}{1}$ and hence a contradiction with that $\{U_j : j \in J\}$ is an IFsemiopen cover of X.

Theorem 3.7. An IFTS(X, Ψ) is an IFSθ-compact iff $(X, \Psi_{0,1})$ is an IFSθ-compact.

Proof. $(\Rightarrow)$ Let $\{[]U_j : j \in J\}$be an IFsemiopen cover of X in $(X, \Psi_{0,1})$. Hence $\vee([]U_j) = \underset{\sim}{1} \Longrightarrow \vee\mu_{U_j} = \underset{\sim}{1}, \wedge\gamma_{U_j} = \underset{\sim}{1} - \vee\mu_{U_j} = \underset{\sim}{0}$. Since (X, Ψ) is an IFSθ-compact, there is $\{U_j : j = 1, 2, ..., n\}$of IF$\theta$OS's such that $\overset{n}{\underset{j=1}{\vee}} U_j = \underset{\sim}{1}$. Now we have, $\overset{n}{\underset{j=1}{\vee}} \mu_{U_j} = \underset{\sim}{1}$ and $\overset{n}{\underset{j=1}{\wedge}} (\underset{\sim}{1} - \mu_{U_j}) = \underset{\sim}{1} - \overset{n}{\underset{j=1}{\vee}} \mu_{U_j} = \underset{\sim}{0}$. Hence $\{[]U_j : j \in J\}$ has a subcover of IFθOS's and then $(X, \Psi_{0,1})$ is an IFSθ-compact.

$(\Leftarrow)$ Let $\{U_j : j \in J\}$be an IFsemiopen cover of X in (X, Ψ). Since $\vee U_j = \underset{\sim}{1}$, we have $\vee\mu_{U_j} = \underset{\sim}{1}, \wedge\gamma_{U_j} = \underset{\sim}{1} - \vee\mu_{U_j} = \underset{\sim}{0}$. Since $(X, \Psi_{0,1})$ is an IFSθ-compact, there is a subfamily $\{U_j : j = 1, 2, ..., n\}$of IF$\theta$OS's such that $\overset{n}{\underset{j=1}{\vee}} ([]U_j) = \underset{\sim}{1}$.i.e. $\overset{n}{\underset{j=1}{\vee}} \mu_{U_j} = \underset{\sim}{1}$ and $\overset{n}{\underset{j=1}{\wedge}} (\underset{\sim}{1} - \mu_{U_j}) = \underset{\sim}{0}$. Hence $\mu_{U_j} = \underset{\sim}{1} - \gamma_{U_j} \Longrightarrow \overset{n}{\underset{j=1}{\vee}} \mu_{U_j} = \overset{n}{\underset{j=1}{\vee}} (\underset{\sim}{1} - \gamma_{U_j}) \Longrightarrow \underset{\sim}{1} = \underset{\sim}{1} - \overset{n}{\underset{j=1}{\wedge}} \gamma_{U_j} \Longrightarrow \overset{n}{\underset{j=1}{\wedge}} \gamma_{U_j} = \underset{\sim}{0} \Longrightarrow \overset{n}{\underset{j=1}{\vee}} U_j = \underset{\sim}{1}$ i.e. $\{U_j : j \in J\}$ has a finite subcover of IFθOS's. Hence (X, Ψ) is an IFSθ-compact.

Theorem 3.8. An IFTS(X, Ψ) is an IFSθ-compact iff $(X, \Psi_{0,2})$ is an IFSθ-compact.

Proof. Similar to the proof of Theorem 3.7.

Theorem 3.9 . Every an IFSθ-compact space X which is submaximal regular is an IFS-closed.

Proof. Let $\{U_j : j \in J\}$be an IFsemiopen cover of X . Then $cl(U_j) = cl(H_j)$ where H_j is an IFOS in X. Since X is submaximal regular space, then $\{cl(U_j) : j \in J\}$ is an IFopen cover of X and consequently an IFsemiopen cover of X. Since X is an IFSθ-compact, then there is a subfamily $\{cl(U_j) : j = 1, 2, ..., n\}$of IF$\theta$OS's such that $\overset{n}{\underset{j=1}{\vee}} cl(U_j) = \underset{\sim}{1}$. Hence X is an IFS-closed.

Theorem 3.10. Every an IFθ-compact space X which is submaximal regular is an IFSθ-compact.

Proof. Let $\{U_j : j \in J\}$ be an IFsemiopen cover of X. Since every an IFSOS in an IFsubmaximal regular X is an IFθOS, then $\{U_j : j \in J\}$ is an IFθ-open cover of X. Since X is IFθ-compact, then there is a subfamily $\{U_j : j = 1, 2, ..., n\}$ of IFθOS's such that $\overset{n}{\underset{j=1}{\vee}} U_j = 1$. Hence X is an IFSθ-compact.

Corollary 3.11 . Every an IF-(IFS-, IFλ-)compact space X which is submaximal regular is an IFSθ-compact.

Lemma 3.12. If U is an IFθCS of an IFTS(X, Ψ) and $c(a,b) \notin U$, then there is an IFOS V of X such that $c(a,b) \in cl(V)$, for each $a, b \in (0,1)$.

Proof. Let U be an IFθCS and $c(a,b) \notin U$. Hence $c(1-a, 1-b)q\overline{U}$, for each $a, b \in (0,1)$. From the definition of IFθOS $\overline{U}$, there is an IFOS $V = \langle x, \mu_V, \gamma_V \rangle$ of x such that $c(1-a, 1-b)qcl(V) \leq \overline{U}$, where $cl(V) = \langle x, \wedge\mu_{G_j}, \vee\gamma_{G_j} \rangle$ and $\{\langle x, \mu_{G_j}, \gamma_{G_j} \rangle : j \in J\}$ is the family of IFCS's containing V. Hence $1 - a > \vee\gamma_{G_j}$ or $1 - b < \wedge\mu_{G_j}$ which implies $a < \wedge\mu_{G_j}$ and $b > \vee\gamma_{G_j}$. Hence $c(a,b) \in cl(V)$.

Theorem 3.13. If U is an IFθ-closed of an IFSθ-compact space X, then U is an IFSθ-compact relative to X.

Proof. Let $V = \{V_j : j \in J\}$ where $V_j = \{\langle y, \mu_{v_j}, \gamma_{v_j} \rangle : j \in J\}$, be an IFsemiopen cover of U. For $x(a,b) \notin U$ and by Lemma 3.12, there is an IFOS G of X such that $x(a,b) \in G$. Hence $\{V_j : j \in J\} \vee \{cl(G(x)) : x \in \overline{U}\}$ is an IFsemiopen cover of X. Since X is IFSθ-compact, there is a finite subfamily $\{V_j : j = 1, 2, ..., n\} \vee \{cl(G(x_i)) : i = 1, 2, ..., m\}$ of IFθOS's which covers X and consequently $\{V_j : j = 1, 2, ..., n\}$ covers U. Hence U is an IFSθ-compact relative to X.

Theorem 3.14. If (X, Ψ_1) and (Y, Ψ_2) are IFSθ-compact spaces and (X, Ψ_1) is product related to (Y, Ψ_2), then the product $X \times Y$ is IFSθ-compact.

Proof. Let $\{U_j \times V_j : j \in J\}$ be an IFsemiopen cover of $X \times Y$, where U_j's and V_j's are IFSOS's in X and Y, respectively. Then $\{U_j : j \in J\}$ and $\{V_j : j \in J\}$ are IFsemiopen covers of X and Y, respectively. Thus there exist subfamilies $\{U_j : j = 1, 2, ..., n\}$ and $\{V_j : j = 1, 2, ..., n\}$ of IFθOS's such that $\overset{n}{\underset{j=1}{\vee}} U_j = 1_X$ and $\overset{n}{\underset{j=1}{\vee}} V_j = 1_Y$. From the product related of X, Y and Lemma 2.8, we have $\underset{j \in J_1 \vee J_2}{\vee} U_j \times V_j \equiv \underset{j \in J_1 \vee J_2}{\vee} U_j \times \underset{j \in J_1 \vee J_2}{\vee} V_j = 1_X \times 1_Y$. Thus $X \times Y$ is IFSθ-compact.

4. Functions and IFSθ-compact space

Theorem 4.1. If $f : X \to Y$ is an IFS-cont. surjection function and U is an IFSθ-compact relative to X, then $f(U)$ is an IF-compact relative to Y.

Proof. Let $V = \{V_j : j \in J\}$ where $V_j = \{\langle y, \mu_{v_j}, \gamma_{v_j} \rangle : j \in J\}$, be an IFopen cover of $f(U)$. Since f is IFS cont., then $\{f^{-1}(V_j) : j \in J\}$ is an IFsemiopen cover of U. Since U is IFSθ-compact, there is a finite subfamily $\{V_j : j = 1, 2, ..., n\}$ of IFθOS's such that $U \subseteq \overset{n}{\underset{j=1}{\vee}} V_j$, which implies that $f(U) \subseteq \overset{n}{\underset{j=1}{\vee}} ff^{-1}(V_j) = \overset{n}{\underset{j=1}{\vee}} V_j$. Hence $f(U)$ is an IF-compact relative to Y.

Corollary 4.2. If $f : X \Rightarrow Y$ is an IFS-cont. surjection function and X is an IFSθ-compact , then Y is an IF-compact.

Corollary 4.3. If $f : X \to Y$ is an IF-cont. surjection function and U is an IFSθ-compact relative to X, then $f(U)$ is an IF-compact relative to Y.

Corollary 4.4. If $f : X \to Y$ is an IF-cont. surjection function and X is an IFSθ-compact, then Y is an IF-compact.

Theorem 4.5. Let $f : X \to Y$ be an IFSθ-cont. and IFθ-open function. If U is an IFθ-compact relative to X, then $f(U)$ is an IFSθ-compact relative to Y.

Proof. Let $V = \{V_j : j \in J\}$ where $V_j = \{\langle y, \mu_{V_j}, \gamma_{V_j}\rangle : j \in J\}$, be an IFsemiopen cover of $f(U)$. Since f is IFSθ-cont., then the family $\{f^{-1}(V_j) : j \in J\}$ of IFθOS's covers U [Note θ-open $\Rightarrow$open $\Rightarrow$semiopen]. Since U is an IFθ-compact, there is a finite subfamily $\{f^{-1}(V_j) : j = 1, 2, ..., n\}$of IF$\theta$OS's such that $U \subseteq \overset{n}{\underset{j=1}{\vee}} f^{-1}(V_j)$. Since f is an IFθ-open, we have $f(U) \subseteq f(\overset{n}{\underset{j=1}{\vee}} f^{-1}(V_j)) \equiv \overset{n}{\underset{j=1}{\vee}} ff^{-1}(V_j) \equiv \overset{n}{\underset{j=1}{\vee}} V_j$. Hence $f(U)$ is an IFSθ-compact relative to Y.

Corollary 4.6. Let $f : X \to Y$ be an IFSθ-cont. and IFθ-open function. If U is an IFSθ-compact relative to X, then $f(U)$ is an IFSθ-compact relative to Y.

Corollary 4.7. Let $f : X \to Y$ be an IFSθ-cont. and IFθ-open function. If X is an IFθ-compact , then Y is an IFSθ-compact.

Corollary 4.8. Let $f : X \Rightarrow Y$ be an IFSθ-cont. and IFθ-open function. If X is an IFSθ-compact , then so is Y.

Theorem 4.9. If $f : X \to Y$ is an IFfaintly cont. function and U is an IFSθ-compact relative to X, then $f(U)$ is an IFθ-compact relative to Y.

Proof. Similar to the proof of Theorem 4.1.

Corollary 4.10. If $f : X \to Y$ is an IFfaintly cont. function and X is an IFSθ-compact, then Y is an IFθ-compact.

Theorem 4.11. If $f : X \to Y$ is an IFsuper i function and U is an IFcompact relative to X, then $f(U)$ is an IFSθ-compact relative to Y.

Proof. Similar to the proof of Theorem 4.1.

Corollary 4.12. If $f : X \to Y$ is an IFsuper i function and X is an IFcompact, then Y is an IFSθ-compact.

Theorem 4.13. Let $f : X \to Y$ be an IFsuper semiopen and IFstrongly θ-cont. bijective function. If Y is an IF-compact, then X is an IFSθ-compact.

Proof. Let $\{U_j : j \in J\}$ be an IFsemiopen cover of X. Since f is an IFsuper semiopen, then the family $\{f(U_j) : j \in J\}$ is an IFopen cover of Y. Since Y is an IF-compact, there is a subfamily $\{f(U_j) : j = 1, 2, ..., n\}$of IFOS's which cover Y. Now, $\{U_j : j = 1, 2, ..., n\} = \{f^{-1}(f(U_j)) : j = 1, 2, ..., n\}$ is an IFθ-open cover in X (since f is IFstrongly θ-cont. bijective function). Hence X is an IFSθ-compact.

Corollary 4.14. Let $f : X \to Y$ be an IFsuper semiopen and IFstrongly θ-cont. bijective function. If V is an IFcompact relative to Y, then $f^{-1}(V)$ is an IFSθ-compact relative to$X$.

Theorem 4.15. Let $f : X \to Y$ be an IFsemiopen and IFfaintly cont. surjection function. If Y is an IFSθ-compact, then X is an IFcompact.

Proof. Similar to the proof of Theorem 4.13.

Corollary 4.16. Let $f : X \to Y$ be an IFsemiopen and IFfaintly cont. surjection function. If V is an IFSθ-compact relative to Y, then $f^{-1}(V)$ is an IF-compact relative to X.

Corollary 4.17. Let $f : X \to Y$ be an IFsemiopen and IFfaintly cont. surjection function. If V is an IFSθ-compact relative to Y, then $f^{-1}(V)$ is an IFθ-compact relative to X.

Theorem 4.18. Let $f : X \to Y$ be an IFfaintly open and IFθ- cont. surjection function. If Y is an IFSθ-compact, then X is an IFθ-compact.

Proof. Similar to the proof of Theorem 4.13.

Corollary 4.19. Let $f : X \to Y$ be an IFfaintly open and IFθ- cont. surjection function. If V is an IFSθ-compact relative to Y, then $f^{-1}(V)$ is an IFθ-compact relative to X.

Theorem 4.20. Let Y be an IF-submaximal regular space and $f : X \to Y$ be an IFpreopen surjection function. If f is an IFS-cont. and X is an IFSθ-compact, then Y is so.

Proof. Let $\{V_j : j \in J\}$ be an IFsemiopen cover of Y. Since f is an IFS-cont.and IFpreopen, then by Lemma 2.14 the family $\{f^{-1}(V_j) : j \in J\}$ is an IFsemiopen cover of X. Since X is an IFSθ-compact, there is a subfamily $\{f^{-1}(V_j) : j = 1, 2, ..., n\}$of IF$\theta$OS's which covers X. Now, $\{V_j : j = 1, 2, ..., n\} = \{ff^{-1}(V_j) : j = 1, 2, ..., n\}$ is an IFθ-open cover in Y, since Y is IF-submaximal regular space. Hence Y is an IFSθ-compact.

Corollary 4.21. Let Y be an IF-submaximal regular space and $f : X \to Y$ be an IFi function. If X is an IFSθ-compact, then so is Y.

Corollary 4.22. Let Y be an IF-submaximal regular space and $f : X \to Y$ be an IFi function. If U is an IFSθ-compact relative to X, then $f(U)$ is an IFSθ-compact relative to Y.

5. Locally IFSθ-compact

Definition 5.1. An IFTS(X, Ψ) is said to be locally IFSθ-compact if for each an IFP $c(a, b)$ in X, there is $U \in N_\epsilon(c(a, b))$ such that $\mu_U(c) = 1$, $\gamma_U(c) = 0$ and U is an IFSθ-compact relative to X.

Remark 5.2. Every an IFSθ-compact space is locally IFSθ-compact but the converse may not be true.

Example 5.3. An infinite discrete IFTS is locally IFSθ-compact but not IFSθ-compact.

Remark 5.4. Every locally IFSθ-compact space is locally IF-compact but the converse may not be true

Theorem 5.5. Let Y be an IF-submaximal regular space and $f : X \to Y$ be an IF-open surjection function. If f is an IFi function and X is locally IFSθ-compact, then so is Y.

Proof. Let $y(m, n)$ be an IFP in Y. Then $y(m, n) = f(x(a, b))$ for some $x(a, b)$ in X. Since X is locally IFSθ-compact, there is $U \in N_\epsilon(x(a, b))$ such that $\mu_U(x) = 1$, $\gamma_U(x) = 0$ and U is an IFSθ-compact relative to X. Since f is an IF-open function, $f(U) \in N_\epsilon(y(m, n))$ with $(f(U))(y) = \bigvee_{x \in f^{-1}(y)} U(x) = 1$ and by Theorem 3.19, $f(U)$ is an IFSθ-compact relative to Y. Hence Y is locally IFSθ-compact space.

Corollary 5.6. Let Y be an IF-submaximal regular space and $f : X \to Y$ be an IF-open surjection function. If f is an IFsuper i function and X is locally IFSθ-compact, then so is Y.

Proof. Since every an IFsuper i function is an IFi and from Theorem 5.5, the proof be obtained.

Theorem 5.7. Let $f : X \to Y$ be an IF-cont. and IF-open surjection function. If X is locally IFSθ-compact, then Y is locally IF-compact.

Proof. Let $y(m,n)$ be an IFP in Y. Then $y(m,n) = f(x(a,b))$ for some $x(a,b)$ in X. Since X is locally IFSθ-compact, there is $U \in N_\varepsilon(x(a,b))$ such that $\mu_U(x) = 1$, $\gamma_U(x) = 0$ and U is an IFSθ-compact relative to X. Since f is an IF-open function, $f(U) \in N_\varepsilon(y(m,n))$ with $(f(U))(y) = \underset{x \in f^{-1}(y)}{\vee} U(x) = \underset{\sim}{1}$ and by Corollary 4.3, $f(U)$ is an IF-compact relative to Y. Hence Y is locally IF-compact space.

Corollary 5.8. Let $f : X \to Y$ be an IF-cont. and IF-open surjection function. If X is locally IFSθ-compact, then Y is locally IFθ-compact.

Proof. Obvious, since every locally IF-compact is locally IFθ-compact.

Corollary 5.9. Let Y be an IF-regular space and $f : X \to Y$ be an IF-open surjection function. If f is an IFweakly function and X is locally IFSθ-compact, then Y is locally IF-compact.

Proof. It is follows from the above Theorem and the fact that every an IFweakly cont. function is an IF-cont. in an IF-regular space.

Theorem 5.10. Let X be an IF-regular space and $f : X \to Y$ be an IFθ-open bijective function. If f is an IFSθ- cont. and X is locally IFSθ-compact, then so is Y.

Proof. Using Corollary 4.6, the proof similar to the proof of Theorem 5.5.

Theorem 5.11. Let $f : X \to Y$ be an IFsuper semiopen and IFstrongly θ-cont. surjection function. If Y is an locally IFcompact, then X is an locally IFSθ-compact.

Proof. Let $x(a,b)$ be an IFP in X. Since f is surjective, there is $y(m,n)$ such that $f(x(a,b)) = y(m,n)$. Since Y is locally IFcompact, there is $V \in N_\varepsilon(y(m,n))$ such that $\mu_V(y) = 1$, $\gamma_V(y) = 0$ and V is an IF-compact relative to Y. Using Theorem 4.13, $f^{-1}(V)$ is an IFSθ-compact relative to$X$. Since f is an IFstrongly θ-cont, then $f^{-1}(V) \in N^\theta_\varepsilon(x(a,b))$ and hence $f^{-1}(V) \in N_\varepsilon(x(a,b))$ .Therefore $f^{-1}(V)(x) = V(f(x)) = V(y) = \underset{\sim}{1}$. Hence for $x(a,b)$ in X, there is $f^{-1}(V) \in N_\varepsilon(x(a,b))$ such that $f^{-1}(V)(x) = \underset{\sim}{1}$ and $f^{-1}(V)$ is an IFSθ-compact relative to$X$. Hence X is an locally IFSθ-compact.

Corollary 5.12. Let $f : X \to Y$ be an IFsuper semiopen and IFstrongly θ-cont. surjection function. If Y is locally IFcompact, then X is locally IFcompact.

Theorem 5.13. Let $f : X \to Y$ be an IFsemiopen and IFfaintly cont. surjection function. If Y is locally IFSθ-compact , then X is locally IFcompact.

Proof. Using Corollary 4.17, the proof is smiliar to proof of Theorem 5.5.

Theorem 5.14. Let $f : X \to Y$ be an IFfaintly open and IFθ-cont. surjection function. If Y is locally IFSθ-compact , then X is locally IFθ-compact.

Proof. Using Corollary 4.19, the proof is smiliar to proof of Theorem 5.5.

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