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SINGLE AND DOUBLE PLANES OF CABLES IN **HARP CABLE STAYED BRIDGES** Bv

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الكبار ي ذات الكابلات احادية وثنائية المستو ي.فيثار ية الشكل

الغلاصة:

إن هف هذا البحث الرئيسي هو الوصول الى التمثيل الرياضي الامثل لاجراء التحليل الاستاتيكي للكباري ذات مستويين من الكابلات المتوازية(قيثارية الشكل) والمربوطه على طول ارتفاع الجزء العلوى فوق مستوى كمرات سطح الكوبر ي والموصلة الى كمرات الأ سطح الخاصة بحركة المارة والسيارات حيث تع افتراض حالتين من الحل الرياضي لإجراء التحليل الخاص بهذة الكباري اولاهما افتراض الكابلات أحادية المستوى والأخرى ثنانية العستوى. وقد تم اجراء هذا البحث لكباري ذات ثلاثة بحور باطوال 140 متر لكل من البحرين الخارجين و280 متر للبحر الداخلي وبطول اجمالي 560 متر للكوبري بالكامل مفترضا ثمانية حالات تحميل مغتلفة و حالتين خاصتين باحمال الرياح احداهما في الانجاة الطولمي للكوبري والاخرى في الانجاة العرضبي وقد تم الاخذ في الاعتبار تأثير اربعة انواع شائعة الاستخدام من الوصلات بين الأبر اج وكمرات سطح الكبارى وقد تم إجراء التحليل الاستاتيكي لهذه الكباري باستخدام طريقة الطاقة المبنية على تصغير طاقة الوضم باستخدام طريقة الانحدار ات المتبادلة ۖ وقد قام الباحث بإنشاء جميع بر امج الحسوب المستخدمة في هذا البحث مع تدوين أهم النتائج

Abstract

The main object of this research is concerned about the best choice of the mathematical model to carry off the static analysis for cable stayed bridges. The static analysis of cable stayed bridges having three spans with considering single or double planes of cables in harp shape is carried out. Eight cases of loading which include the most symmetric and asymmetric traffic loads are considered. In addition, two special cases of loading (wind loads in both longitudinal and transverse directions) are investigated. To take the influence of connections between pylons and floor beams into consideration, four common cases are presented. The static analysis is carried out taking into consideration geometric and material nonlinearities. The own weight of all structural elements, and traffic load including impact are taken into account. In the static analysis, the energy method, based on the minimization of the total potential energy of structural elements, via conjugate gradient technique is used. The procedure is carried out using the iterative steps to acquire the final configurations. All prepared computer programs in FORTRAN language for this work are written by the author. The major conclusions, which have been drawn from the present work, are outlined.

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1- Introduction

Cable stayed bridges, in which the deck is elastically supported at points along its length by inclined cable stays, are now entering a new era, reaching to medium and long span lengths (with a rang of 400m to 1500 m of center span]) [1]. Three elements, namely girders, pylons and inclined cable stays are the main principal components of this type of bridges.. The most common cable stayed bridges may be classified as harp, radiating and fan depending on the of cables arrangements and their connections with pylons. This paper is concerned with the bridges having cables in harp shape. The own weight of all structural elements with all various considered cases of cauivalent traffic loads including impact on the girders are considered. To get the best mathematical model for the required analysis, two cases as single and double planes of cables are considered. In both cases, the analysis is carried out considering cable and space frame elements for cables and pylons and floor beams, respectively. Also, the study is carried out considering four common methods of connection between pylons and floor beams.

The Energy method is a unifying approach to the analysis of both linear and non-linear structures. It is an indirect method of analysis and valid for both small

and large structures. The energy method is applied to the analysis of general pin-ended truss and cable structures. Both geometric and material nonlinearities are directly incorporated within the formulation, thereby accounting for large displacement and strains as well as configuration changes due to the structural response [2]. A summary together with a step-by-step iterative procedure is presented. Main sources of knowledge about this method are given in[3], [4], [5], [6] and [7]. The obtained numerical results for all cases are discussed and compared. maior Finally. the conclusions are presented.

2. Step- by --step static response analysis by minimization of the total potential energy.

The point at which W (total potential energy) is a minimum defines the equilibrium position of the loaded structure. Mathematically, the equilibrium condition in the *i* direction at joint *i* may be expressed as:

$$
\frac{\partial W}{\partial x_{\mu}} = [g_{\mu}] = 0 \qquad , i=1, 2 \text{ and } 3
$$
 (1)

The location of the position where W is a minimum is achieved by moving down the energy surface along a descent vector v a distance Sv until W is a minimum in that direction, that is, to a point where

$$
\frac{\partial W}{\partial S} = 0 \tag{2}
$$

Where: x_{ii} = the displacement of joint j corresponding to a particular degree of freedom in direction, i, and g_{ii} = the corresponding gradient of the

energy surface.

From this point a new descent vector is calculated and the above process is repeated. The method is mathematically expressing the displacement vector at the $(k+1)$ th iteration as:

 $[x]_{k+1} = [x]_k + S_k v_k$ (3) Where:

 v_k = the descent vector at the kith iteration from x_k in the displacement space, and

 S_k = the step length determining the distance along v_k to the point where W is a minimum

Summary of the iterative procedures

The main steps in the iterative processes required to achieve structural equilibrium by minimization of the total potential cnergy may be summarized as follows:

First, before the start of the iteration scheme

a) Calculate the tension coefficients for the pretension forces in the cables by:

$$
t_{jn} = \left[\left(T_0 + \frac{EA}{L_0} e \right) / L_0 \right]_{jn} \tag{4}
$$

Where: t_{in} = the tension coefficient of the force in member jn , and $e = the$ elongation of cables due to applied load.

 T_0 = initial force in a pin-jointed member or cable link due to pretension;

 $E =$ modulus of elasticity. $A = area of$ the cable element, and

 L_0 = the unstrained initial length of the cable link.

b) Assume the elements in the initial displacement vector to be zero.

c) Calculate the lengths of all the elements in the pretension structure using the following equation:

$$
L_0^2 = \sum_{i=1}^3 \left(X_m - X_{ji} \right)^2 \tag{5}
$$

Where: $X =$ element in displacement vector due to applied load only, and

d) If either the method of steepest descent or the method of conjugate gradients is used, calculate the elements in the scaling matrix, [8,9, and 10]:

$$
H = diag\{k_{11}^{-1/2}, k_{22}^{-1/2}, \dots, k_{nn}^{-1/2}\}
$$
 (6)

Where: $n =$ total number of degrees of freedom of all joints, and

 $k =$ the 12 x 12 matrix of the element in global coordinates.

The steps in the iterative procedure then are summarized as:

Step (1) Calculate the elements in the gradient vector of the TPE, using:

$$
g_m = \sum_{n=1}^{j_n} \sum_{r=1}^{12} (k_{nr} x_r)_n - \sum_{n=1}^{p_n} t_{jn(Nni + zni - Xj - xji) - Fni}
$$
\n
$$
\tag{7}
$$

Step (2) Calculate the Euclidean norm of the gradient vector, $R_k = [g_k^T g_k]^{1/2}$, and check if the problem has converged. If $R_k \leq R_{min}$ stop the calculations and print the results. If not, proceed to step $(3).$

Step (3) Calculate the elements in the descent vector, v using:

$$
[\nu]_{k+1} = -[H\llbracket g \rrbracket_{k+1} + \beta_k [\nu]_k \tag{8}
$$

 (9) , and

 $[v]_0 = -[g]_0$ Where

$$
\beta_{k} = \frac{[g]_{k+1}^{T}[H]^{T}[H][g]_{k+1}}{[g]_{k}^{T}[H]^{T}[H][g]_{k}} = \frac{[g]_{k+1}^{T}[\hat{K}][g]_{k+1}}{[g]_{k}^{T}[\hat{K}][g]_{k}}
$$

Step (4) Calculate the eoefficients in the step-length polynomial from:

$$
C_4 = \sum_{n=1}^{P} \left(E A a_3^2 / 2 L_0^3 \right)_n \tag{11-a}
$$

$$
C_3 = \sum_{n=1}^{P} \left(EAa_2a_3 / L_0^3 \right)_n \tag{11-b}
$$

$$
C_2 = \sum_{n=1}^{P} \left[l_0 a_3 + \mathcal{EA} \Big(a_2^2 + 2 a_1 a_3 \Big) / 2 L_0^3 \right]_n + \sum_{n=1}^{P} \sum_{i=1}^{12} \sum_{r=1}^{12} \left(\frac{1}{2} v_r k_{\mu} v_r \right)_n
$$

$$
C_1 = \sum_{n=1}^{P} \left[\ell_0 a_1 + E A a_1 a_1 / L_0^3 \right]_n + \sum_{n=1}^{P} \sum_{r=1}^{12} \sum_{r=1}^{12} \left(x_r k_{\infty} v_r \right)_n - \sum_{n=1}^{N} F_n v_n
$$

Where:

$$
a_{1} = \sum_{i=1}^{3} \left[(X_{m} - X_{n}) + \frac{1}{2} (x_{m} - x_{n}) \right] (x_{m} - x_{n})
$$

\n
$$
a_{2} = \sum_{i=1}^{3} \left[(X_{ni} - X_{ji}) + (x_{ni} - x_{n}) \right] (y_{ni} - y_{n})
$$

\n
$$
a_{3} = \sum_{i=1}^{3} \frac{1}{2} (y_{ni} - y_{ji})^{2}
$$
\n(12)

' ji J

Where:

 $f =$ number of flexural members.

 $P =$ number of pin-jointed members and cable links,

 $F =$ element in applied load vector, and

 K_{sr} = Element of stiffness matrix in global coordinates of a flexural element.

Step (5) Calculate the step-length S using Newton's approximate formula as:

$$
S_{k+1} = S_k - \frac{4C_4S^3 + 3C_3S^2 + 2C_2S + C_1}{12C_4S^2 + 6C_3S + 2C_2}
$$
\n
$$
\tag{13}
$$

Where: k is an iteration suffix and $S_{k=0}$ is taken as zero

Step (6) Update the tension coefficients using the following equation.

$$
(t_{ab})_{k+1} = (t_{ab})_k + \frac{EA}{(L_0^3)_{ab}} (a_1 + a_2s + a_3s^2)_{ab}
$$
 (14)

Step (7) Update the displacement vector using equation (4) .

Step (8) Repeat the above iteration by returning to step (1) .

3. Wind assumptions

It is convenient to express the wind velocity as the summation of the mean wind velocity in the long- wind direction and the fluctuating time-dependent velocity components. Because this paper is concerned about static analysis only, the fluctuating wind speed component is neglected, and the mean wind velocity is only considered. The most general law describing the way in which the

mean wind velocity U (Z) varies with height, Z, is the "logarithmic law" and is given by: $U(z) = 2.5$ u^{*} ln (Z /Zo) (15) Where:

 u^* = the shear velocity or friction wind velocity, and

 Zo = roughness length.

4. Geometry and properties of bridge

A three span cable stayed bridge having cables in harp shape considering single plane of cables (Fig. 1-a) and double planes of cables (Fig.1-b) are considered. The bridge has two equal exterior spans of 140m, each, and interior span of 280 m. The deck girder has a total span of 560 m. The bridge is symmetric and is composed of three major elements: (a) the deck girder, (b) the two pylons and (c) eleven cables on each side of pylon. The cables were 6x37 classes IWRC [11] of zinceoated bridge rope. All cables have an area of 61.94 cm², diameter of 10.16 cm, own weight of 48.96 kg/m, modulus $v \text{cm}^2$ elasticity of 1584 of and maximum breaking load of 925 tons. The initial tension in all cables was taken as 10 % of the maximum breaking load (92.5) tons). The pylon is designed as reinforced concrete with hollow rectangular uniform section having 3 m, width (parallel to xaxis) and 5 m depth(parallel to Y-axis) and the thickness of all walls is 0.40m.

The properties and shape of cross-section of pylon is given in Fig.(1-d). The modulus of elasticity of pylon is 300 $t/cm²$, and its own weight is 14.4 t/m. The deck was taken as steel box-girder in orthotropic plate shape with properties $Fig.(1-c).$ The given in modulus of elasticity is 2100 t/cm², and its own weight including asphalt for each main girder, is 5.78 ℓ m. The cross girders were taken as built I- section with an area of 0.12 m^2 , modulus of elasticity of 2100 $t/cm²$, polar moment of inertia of 0.0543517 m^4 , and moment of inertias about the principal axes as 0.01042 m^4 . and 0.05435 m^4 , respectively. Finally, the top transverse members between the two pylons and other points have a square cross reinforced concrete sections with dimensions 1x1 m. The width of the bridge is 24 m. In order to take into account the influence of connection types between pylon and floor beams, four cases [12] arc considered as shown in Fig. (2).

5. Analysis Considerations

The static analysis for cable staved bridge in harp shape with all mentioned geometry and properties is carried out by the energy method, which is based on the minimization of the total potential energy

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of the structural elements, via conjugate gradient technique. The program used in the analysis and all programs used for the generation of geometry and properties of bridges are developed by the author. The elements for decks and pylons were analyzed as space frame elements, while the cables are considered as space cable elements with the global system of coordinates given in Figs $(1-a)$ and $(1-b)$. Two mathematical models as shown in Figs. $(1-a)$ and $(1-b)$ (single and double planes of cables) are considered. respectively. The first model considering single plane of cables is carried out for the first four cases of loading as shown in Fig. (4). This model has 1302 and 1278 degrecs of freedom for both pairs of connection types (A and Bas well as C and D), respectively. The second model considering double planes of cables as shown in Fig $(l-b)$ is considered. This model has 2604 and 2556 degrees of freedom for the similar types \circ f connections, respectively. The analysis is carried out for all eight cases of loading shown in Fig (4). The total equivalent live load for both mathematical models including impact on the bridge with 24 m as road width is 5.28 t/m for half width of the bridge. The deck floor is at a level of 10m above the ground with mean wind speed as 108 km/hour.

The shear velocity of wind and the roughness length are taken as 2.85734 and 0.15m, respectively. The area exposed to wind for cables is taken as 0.1061 m $\frac{2}{m}$ /m and the drag coefficient varies between 0.9 and 1.2 according to flow air regime. The area exposed to wind for pylon in longitudinal and transverse wind are 5 m^2 /m and 3 m² /m, respectively .Also the area exposed to wind for decks in longitudinal and transverse wind are zero and 3.15 m² /m, respectively. The drag coefficients for both pylon and floor beam is considered as 2. The initial tension in cables in all cases is taken as 92.5 tons (10) % of maximum breaking load). In addition to the eight mentioned cases of loading. two combined cases arc Both wind loads in the considered. longitudinal and transverse directions of the bridge in addition to case of loading 1 are applied to carry out the complete static analysis.

Figures 5 to 35 show some of the obtained results for connection cases A and B. The sway along pylon height in the and lateral directions is longitudinal given in Figs $(5 \text{ and } 8)$. Figures 9 to 11 contain the variations of normal forces along pylon height, while Figs 12 to 15 contain the variation of bending moments. The deflections, normal forces, and

bending moments along the floor beams are given in Figs. (16-35). The variations of final tensions in cables 11 and 12 as examples with all connection types are given in Table (1). The variations of maximum responses in pylon and floor beams are given in Tables (2) and (3) , respectively. Finally the effect of transverse wind loads considering single and double planes of cables and cases of strut locations with pylons are given in Table (4) .

6. Analysis of Results

It may be concluded that:

1. During the analysis for asymmetric cases the computation will converge slowly and sometimes becomes difficult to eonverge when only the transverse wind loads with higher wind velocity and then the analysis by the energy method via gradients needs numerous conjugate iterations (Maximum number of iterations $=$ 5 to 6 times degrees of freedom).

2. From the numerical results it can be evidently seen that a complete coincident between the two considered mathematical models (a single and double planes of cables) for the first four cases of loading $(cases 1, 2, 3, and 4)$. Also, case of wind in the longitudinal direction of the bridge gave completely similar results for both cases of mathematical models.

3. The analysis cannot be executed for the asymmetric cases of loading (5, 6, 7, and

8) considering the single plane of cables mathematical model.

4. The double planes of cables is an essential assumption to carry out the analysis of cable stayed bridges with asymmetric cases of loading.

5. Case of loading 2 covers the most cases of the biggest responses (the sway in the direction. longitudinal the bending moments in the pylons, and the deflection and the bending moment in the floor beams, and the final tensions in cables).

6. The variations of normal forces in the pylons, floor beams and the final tension in cables have insignificant difference in all cases of loading.

7. Case of wind in the transverse direction of the bridge causes an inductance in the lateral deformation and bending moments (out-of-plane displacement) for both pylons and floor beams. The values of these lateral deformations depend on the connection types between pylons and the floor beams and the number of struts between pylons as well as their locations.

7. Major Conclusions:

The major conclusions that have been drawn from the present work are:

The first mathematical model 1. considering single plane of eables(Fig.1a) is simple and valid for all cases of symmetrical gravity loads with small insignificant difference in the various

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responses compared with the second model of double planes of cables.

2. The second mathematical model of analysis considering double planes of cables is valid with various cases of symmetric and asymmetric loading, dimensions and cable arrangements. It gives good results similar to the first model.

3. In the analysis for wind in the transverse direction of the bridge, the double planes of cables with enough numbers of lateral struts must be considered.

4. The connection type between pylons and floor beam has a significant influence on all various responses. Type B in all phases of comparisons is the best choice.

5. Connection type D is better than connection type C in case of transverse wind (wind perpendicular to the longitudinal direction of the bridge)

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Cable	Connection	Case of loading							
Number	type	Case1	Case2	Case3	Case4	Case5	Case6	Case7	Case8
Member Number (11) $Fig.(1-a)$	Α	166	112	145	168	148	129	111	146
	B	167	113	145	168	140	127	107	140
	C	166	112	141	168	150	130	113	148
	D	165	104	148	166	149	128	109	147
Member Number (12) $Fig.(1-a)$	A	179	172	103	181	156	141	145	155
	B	178	169	105	180	148	139	139	148
	C	180	176	100	183	158	141	147	157
	Ð	180	176	100	183	159	142	143	157

Table (1): Final tensions in cables 11 and 12 (tons) with connection type and loading.

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Table (3): Variations of floor beam responses with connection types and loads.

• In normal force values: (-) means compression and no sign mean tension.

Table (4): Sway at top of pylon 1, and deformations at mid- beam due to wind, cm

- Maximum values for all cases and their corresponding.
- case and VI (case of I1 struts along pylon height at cable levels).

Fig. (1-a): Single plane of cables for three span cable stayed bridge in harp shape.

Fig.(2): Connection types between pylon and floor beam.

 $-0.16 -0.12 -0.08 -0.04$ $\pmb{0}$ -0.2 -0.15 -0.1 -0.05 0 $-0.16 -0.12 -0.08 -0.04$ $\mathbf 0$ -0.2 -0.15 -0.1 -0.05 Lateral sway, m

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 $\pmb{0}$

Bending moment, t.m.

0

1200

800

400

 $\pmb{0}$

-4001500

1000

500

Fig. (13): Bending moment in plane along pylon height in three dimensional.

-500 1200

800

400

 $\pmb{0}$

 $-400 - 500$

 $\pmb{0}$

500

1000

1500

