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VOLTAGE CONTROLLED-ELECTROOPTIC DIRECTIONAL COUPLER WITH PARALLEL ELECTRODES

التحكم بالجهد للروابط الكهروضوئية الموجهة مع الأقطاب المتوازية

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الخلاصة : في هذا الجوار تم تصميم الروابط الكهروضوئية الموجهة بين مادتين متشابهتين أو مادتين مختلفتين مع جهدين للتحكم. تم معالجة المادة الغير متماتلة كمادة متماتلة مع تعديل كلا من سمكها ومعامل انكسارها. تم حساب معامل الربط بخلط طريقة معامل الانكسار الفعال EIM مع نظرية ربط الحالات CMT. البارامترات التي تغير قيمة الفرق بين معاملي الانتشار لدلائل الموجة $\Delta\beta$ تؤثر أيضا في قيمة معامل الربط ولذلك القدرة المنقولة بين دلائل الموجة لا يمكن حسابها كدالة في $\Delta\beta$ فقط. اتضح أن معامل الربط يعتمد على معامل الانكسار بكل دليل موجة قبل وبعد جهد التحكم. في حالة المواد المتشابهة معامل الربط يزداد بزيادة فرق الجهد بين جهدي التحكم وتوضح أنه في هذه الحالة مادة KNbO_3 تعطى أكبر معامل ربط وفي هذه الحالة أيضا معامل الربط يزداد مع اختلاف توجيه المحاور للمادتين. ولذلك معامل الربط بين مادتين مختلفتين يزداد كثيراً حسب نوع المادتين. وقد وجد أن أكبر معامل ربط يحدث بين المادتين LiTaO_3 و BaTiO_3 . ونلاحظ أن معامل الربط بين مادتين مختلفتين يزداد أو يقل مع جهد التحكم على حسب المادتين. وقد تم دراسة تأثير كل البارامترات على معامل الربط و اتضح ان النتائج الرقمية كما هو متوقع.

ABSTRACT: An algorithm is presented to find the effect of controller voltages, orientation of the crystallographic axes (Z-cut, Y-cut and X-cut) and types of the two cores electrooptic materials on both coupling coefficient (C) and power transferred between two adjacent rectangular electrooptic waveguides. Coupling coefficient is calculated by mixing effective index method (EIM) with coupled mode theory (CMT). The electrooptic material is treated as isotropic material with very thin metal electrodes. The applied voltage induces a change of refractive index of waveguides cores and so, there are changes of both the coupling coefficient and the difference between propagation constants of the two waveguides ($\Delta\beta$). Coupling coefficient between two dissimilar electrooptic materials becomes very large than that between two similar electrooptic materials. Maximum coupling coefficient is occurred between LiTaO_3 and BaTiO_3 . Change of the coupling coefficient with the controller voltages depends on the type of the electrooptic material. The coupling coefficient depends upon the optical polarization (E_{pq}^x or E_{pq}^y) and the operating wavelengths (λ).

1- INTRODUCTION

Optical waveguide directional couplers are the building blocks of many integrated optic devices, such as signal dividers, wavelength division multiplexers, and modulators/switches [1-5]. When a voltage is applied as shown in Fig.1, there is a change in the refractive index due to the applied electric field. By implementing an appropriate design [5], phase mismatch ($\Delta\beta$) is induced in the optical fields of the two waveguides [6]. In conventional couplers, power switching is achieved [1] and the transferred power between the two waveguides are controlled by the applied voltage [7]. By adjusting the device parameters, most of the power can be

transferred back to guide 1[5]. The effect of the optical field on the change of refractive index is neglected. The directional coupler is composed of two diffused waveguides of an electrooptic crystal [8] as shown in Fig.1. There are six orientation cases between the crystallographic axes (a, b and c) and the system axes (x, y and z). For some orientation cases, the crystallographic axes must be rotated by an angle (θ) to eliminate the cross products terms of the index ellipsoidal (IE) as indicated in Appendix A. where the applied voltage is changed (in y-direction), the values of rotated angle (θ) becomes changed. Then for voltage controlled electrooptic directional coupler, cases which need an angle θ are neglected.

Although, the available orientation cases are, cases 1, 2 and 5 (for LiNbO₃ and LiTaO₃), cases 1 and 2 (for KNbO₃, BaTiO₃ and BaTiO₅). The orientation cases 1 and 2 are similar for uniaxial materials (LiNbO₃, LiTaO₃, BaTiO₃ and BaTiO₅) but cases 1 and 2 are dissimilar for biaxial material (KNbO₃) as indicated in Appendix A.

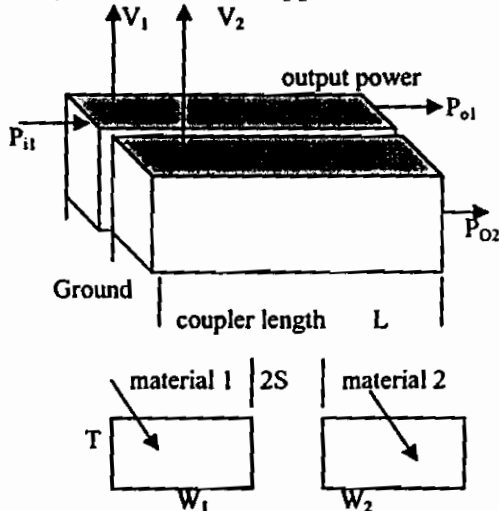


Fig.1 Electrooptic active directional coupler

This study is done with case 1. A study of the polarization dependence in the coupling coefficient of a directional coupler is done, but in this study, E_{pq}^x is used to avoid the propagation losses due to metal electrodes (where, losses of TM mode very large than that losses of TE mode). With very thin metal electrodes, anisotropic waveguide can be treated as isotropic wave guide [9] with modified the waveguide thickness and refractive index. In this study, coupling coefficient between two similar and dissimilar electrooptic materials with different orientations axes is calculated. Effect of applied voltages on C is estimated also, power transferred depends on both C and $\Delta\beta$.

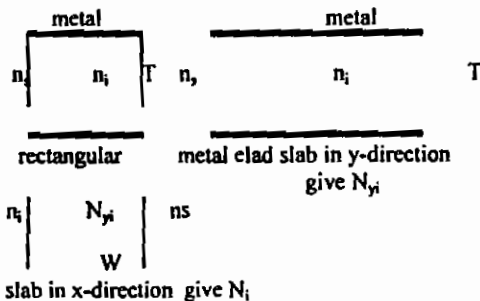


Fig. 2 Rectangular waveguide and its two slabs with EIM

2- MATHEMATICAL ANALYSIS

Coupling coefficient (C) and power transferred between guide 1 (P_1) and guide2 (P_2) are calculated with the following steps:

- 1- Calculation of the index change due to the applied voltage V, as explained in Appendix A.
- 2- Conversion the anisotropic core material into equivalent isotropic material using [9,10] modified waveguide thickness (T) and refractive index (n) as:

$$n_i = (n_{xi} n_{yi})^{0.5}, T_{iso} = (\epsilon_{xi} / \epsilon_{yi})^{0.5} T_{aniso} \quad (1)$$

- 3- Calculation of the propagation constant of each individual waveguide (β) by using effective index method (EIM). β is calculated by solving eigen value equation for each slab waveguide (Fig.2) [11];

$$\beta_i = k_0 N_i \quad (2)$$

where, $k_0 = 2\pi/\lambda$ and N_i is the effective refractive index of guide i and $i=1,2$.

- 4- Calculation of the coupling coefficient (C) by using EIM to convert two rectangulars directional coupler into two slabs directional coupler (Fig.3)[1,12]. Coupled mode method used to calculate coupling coefficient between the equivalent two slabs (Appendix B) [13-15].
- 5- Calculation of power transferred between two lossless rectangular waveguides defined as [16, 17]

$$P_2(z) = (C^2 / C_n^2) \sin^2 C_n z, \quad P_1(z) = 1 - P_2(z). \quad (3)$$

$$\text{where, } C_n^2 = C^2 + \Delta\beta^2 / 4, \quad \Delta\beta = \beta_1 - \beta_2$$

So, transferred power depends upon both C and $\Delta\beta$ not $\Delta\beta$ only, where, values of C depends on the parameters which effect on both β_1 and β_2 .

3- RESULTS AND DISCUSSION

The active directional coupler has two types of parameters. The first type is the structure parameters and they are, waveguide thickness (T), width of guide 1 (W_1), width of guide 2 (W_2), distance between the two waveguides (2S), material of guide 1 (M_1) material of guide 2 (M_2), orientation case of materials 1 and 2 and the

substrate material (n_s). The second type is the operating parameters and they are, operating wavelength (λ), mode numbers in x and y directions (p and q , respectively), applied voltages (V_1 and V_2) and polarization of the applied optical field (E_{pq}^x or E_{pq}^y). $M_1=1$ (LiNbO₃), $M_1=2$ (LiTaO₃), $M_1=3$ (KNbO₃), $M_1=4$ (BaTiO₃) and $M_1=5$ (BaTiO₅), where, $i=1$ material in guide 1 and $i=2$ material in guide 2. Data of main design example are $T=4\mu\text{m}$, $W_1=W_2=2\mu\text{m}$, $S=1\mu\text{m}$, material of guide 1 (LiNbO₃), material of guide 2 (LiNbO₃), orientation cases of materials 1 and 2 are case 1, $n_s=1.502$, $\lambda=0.633\mu\text{m}$, $p=q=0$, $V_1=0$ and $V_2=20$ volt and E_{00}^x . Data for electrooptic materials are indicated in Appendix C. Coupling coefficient (C) increased with absolute value of the difference, $DN=N_1-N_2$ [12] so, C decreased with any parameter which make DN decreased.

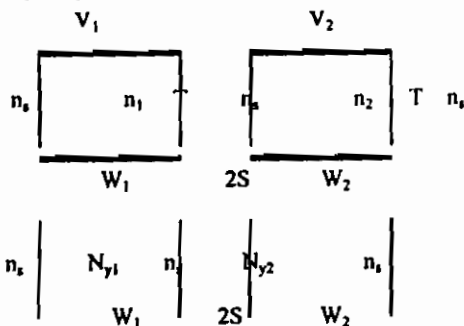


Fig.3 Two rectangular directional coupler and their equivalent two slabs directional coupler.

As suggested [12], C decreased with both waveguide thickness, T , (Table 1), waveguides width, W_1 and W_2 , (Table 2), space between waveguides, S , (Table 3) and refractive index, n_s , (Table 4) but C increased with the operating mode numbers, p and q , (Table 5). Coupling coefficient between the same electrooptic materials ($C_{\text{sym}} : M_1=M_2$) depends on the amplitude of the difference, $V_2 - V_1$, (Fig.4) and C_{sym} (KNbO₃) > C_{sym} (LiTaO₃) > C_{sym} (LiNbO₃) > C_{sym} (BaTiO₅) > C_{sym} (BaTiO₃). C_{asym} is decreased or increased with both V_1 and V_2 as shown in Fig.4 (depend upon the values of DN , Appendix C and D). But the value of C_{sym} becomes very small if it is compared with C_{asym} (coupling coefficient between two different electrooptic materials

$M_1 \neq M_2$) as shown in Figs.4 and 5. Maximum coupling coefficient occurred between LiTaO₃ and BaTiO₅ (Fig.5). Two similar electrooptic materials act as two different electrooptic materials if the orientation case of material 1 dissimilar than that of material 2, also coupling coefficient between two different materials decreased if the orientation cases of the two materials becomes dissimilar (Table 6). By assume that all other parameters are independent upon wavelength, the coupling coefficient depend upon the operating wavelength (λ) [16] (Fig.6) where, N_1 and N_2 are depend on the operating wavelength. Power transferred (P_2) depend on the applied controlled voltages (Fig.4) where, both C and $\Delta\beta$ depend on the applied voltages. Also P_2 depends on both C and $\Delta\beta$ (Fig.7) where, parameters which change $\Delta\beta$ will variable the value of C (exceptional, with especially very difficult conditions). Dependence of P_2 on the coupler length (L) is shown in Fig. 8, which indicated that P_2 increased until L equal L_{max} , (length of coupler at which the maximum power transferred is occurred) dependence on the values of C and $\Delta\beta$ (i.e. V_2). Polarity of the applied voltage effect on the value of coupling coefficient where, C ($V_1=0, V_2=100$) = 7.131577 nm^{-1} , but C ($V_1=0, V_2=-100$) = 7.131100 nm^{-1} . Also coupling coefficient affected by the polarization of the optical field where, C ($V_1=0, V_2=100, E_{00}^x$) = 662.84 nm^{-1} , but C ($V_1=0, V_2=100, E_{00}^y$) = 663.023 nm^{-1} .

Table 1: Effect of waveguide thickness (T) on C with case 1, $M_1=M_2=1, V_1=0, V_2=20$ volt, $W_1=W_2=2 \mu\text{m}, S=1 \mu\text{m}, n_s=1.502, \lambda=0.633 \mu\text{m}$ and E_{pq}^x

T (μm)	2	4	8
C (nm^{-1})	2.860069	1.426220	0.7123947

Table 2: Effect of waveguide widths (W_1 and W_2) on C with case 1, $M_1=M_2=1, V_1=0, V_2=20$ volt, $T=4 \mu\text{m}, S=1 \mu\text{m}, n_s=1.502, \lambda=0.633 \mu\text{m}$ and E_{pq}^x

W_1 (μm)	2	4	8
W_2 (μm)	2	4	8
C (nm^{-1})	1.426220	1.422405	1.421452

Table 3: Effect of space (S) on C with case 1, $M_1=M_2=1, V_1=0, V_2=20$ volt, $W_1=W_2=2 \mu\text{m}, T=4 \mu\text{m}, n_s=1.502, \lambda=0.633 \mu\text{m}$ and E_{pq}^x

S (μm)	0.1	0.25	1.0
C (nm^{-1})	2.347446	1.428673	1.426220

Table 4: Effect of refractive index (n_s) on C with case 1, $M_1=M_2=1$, $V_1=0$, $V_2=20$ volt, $W_1=W_2=2 \mu\text{m}$, $T=4 \mu\text{m}$, $S=1 \mu\text{m}$, $\lambda=0.633 \mu\text{m}$ and E_{∞}^x

n_s	1.50	1.75	2.00
C (nm) ⁻¹	1.426220	1.423359	1.416206

Table 5: Effect of mode numbers (p and q) on the coupling coefficient (C nm⁻¹) with case 1, $M_1=M_2=1$, $V_1=0$, $V_2=20$ volt, $W_1=W_2=2 \mu\text{m}$, $n_s=1.502$, $T=4 \mu\text{m}$, $S=1 \mu\text{m}$, $n_s=1.502$, $\lambda=0.633 \mu\text{m}$ and E_{∞}^x

P	0	0	1	1
Q	0	1	0	1
C	1.426220	1.430035	1.443386	1.446724

Table 6: Effect of the orientation cases on the coupling coefficient (C nm⁻¹) with $V_1=0$ volt, $W_1=W_2=2 \mu\text{m}$, $T=4 \mu\text{m}$, $S=1 \mu\text{m}$, $n_s=1.502$, $\lambda=0.633 \mu\text{m}$ and E_{∞}^x

C for KNbO ₃ * KNbO ₃				
V ₂ (volt)	Cases 1*1	Cases 2*2	Cases 1*2	Cases 2*1
20	2.296	0.642	77.414	80.352
60	6.892	1.922	76.134	84.948
100	11.487	3.203	74.852	89.542
C for KNbO ₃ * BaTiO ₃				
V ₂ (volt)	Cases 1*1	Cases 2*1		
20	724.328	646.274		
60	721.623	643.569		
100	718.919	640.865		

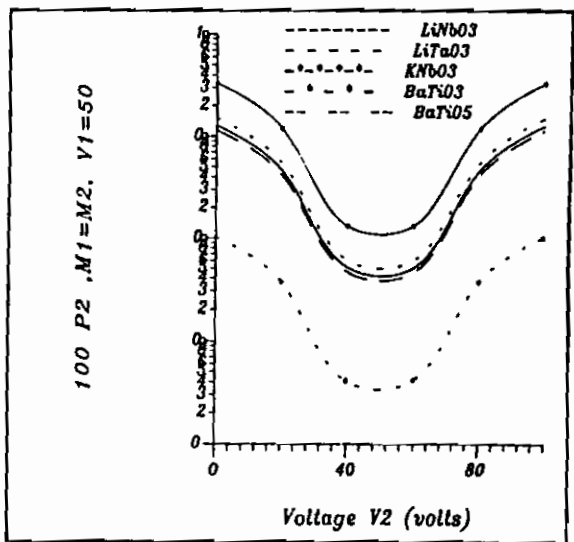
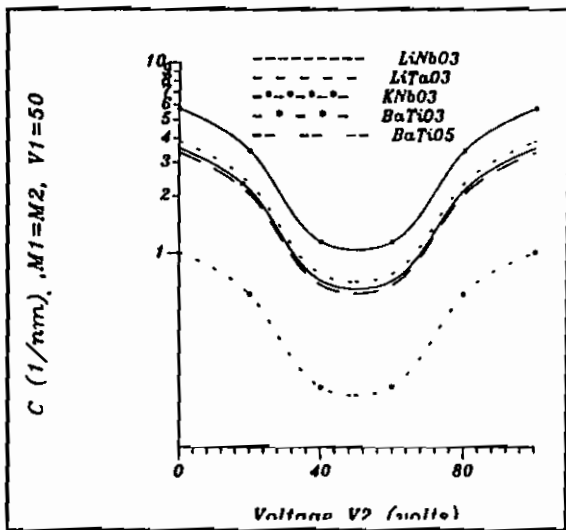
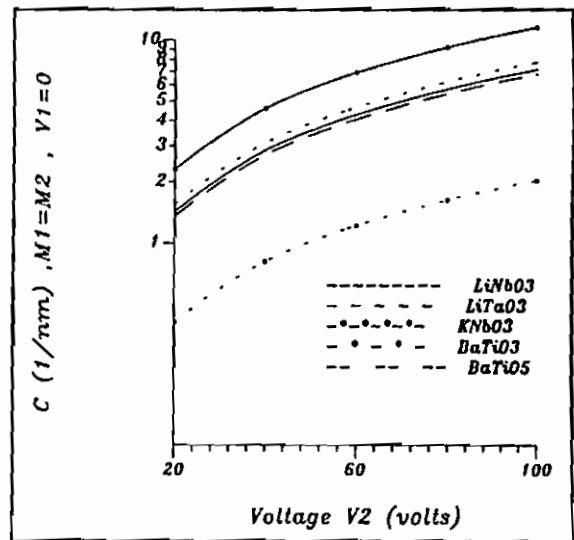
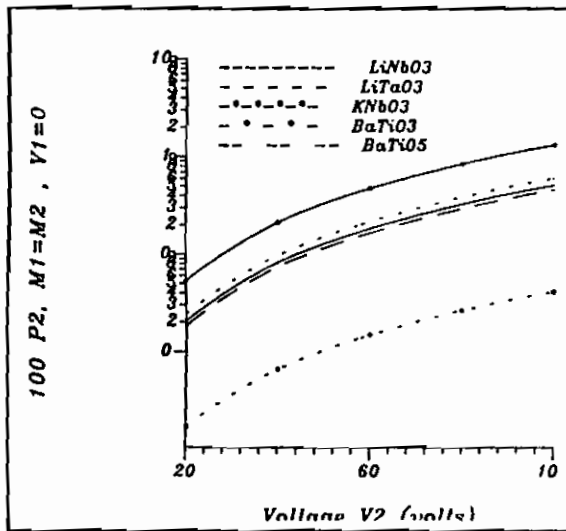


Fig. 4 Coupling coefficient (C) and power transferred (P_2) between two similar waveguides with $W_1=W_2=2 \mu\text{m}$, $T=4 \mu\text{m}$, $S=1 \mu\text{m}$, $n_s=1.502$, $\lambda=0.633 \mu\text{m}$, E_{∞}^x . Upper Figs. With $V_1=0$, lower Figs. $V_1=50$ volts

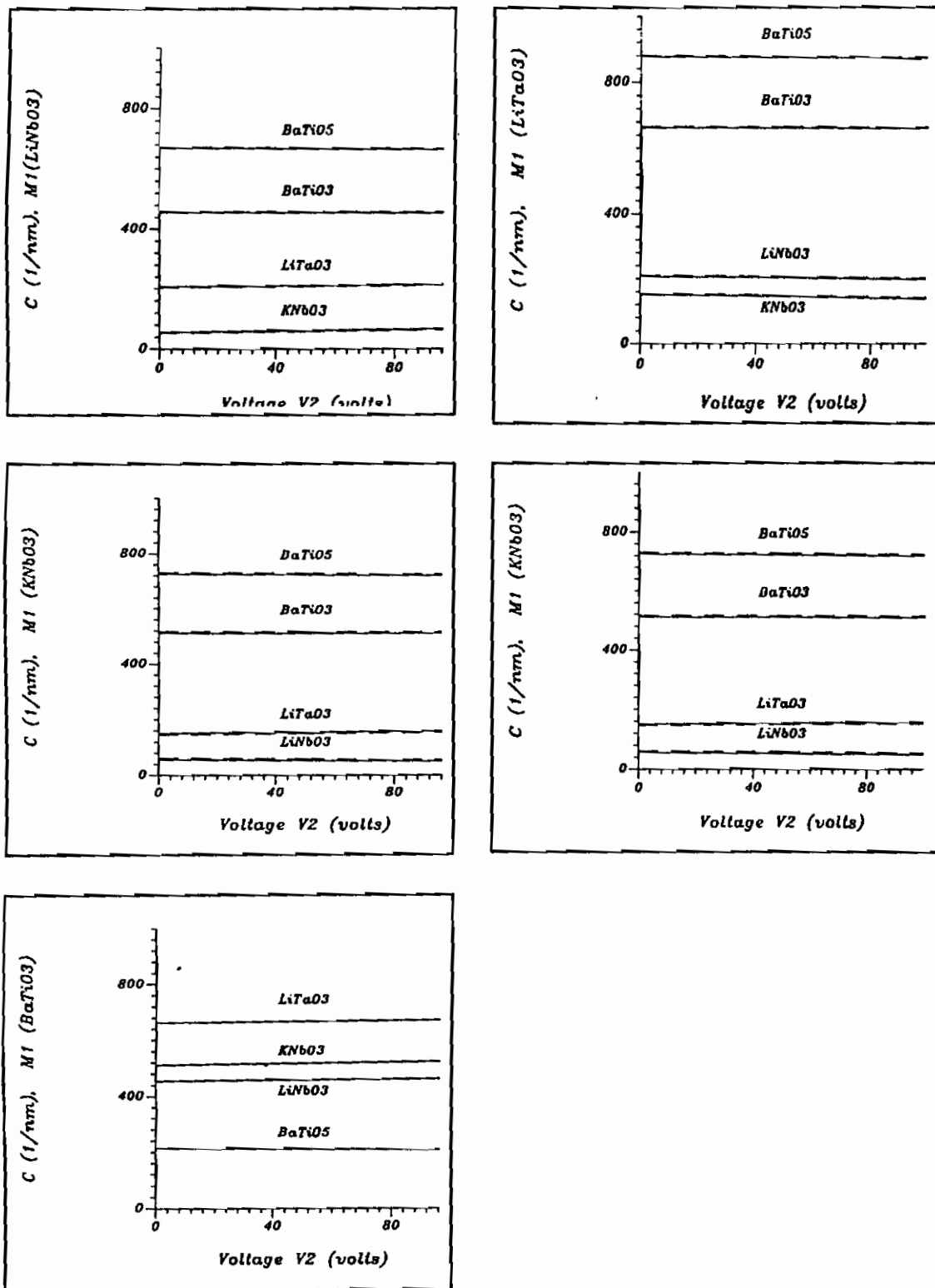
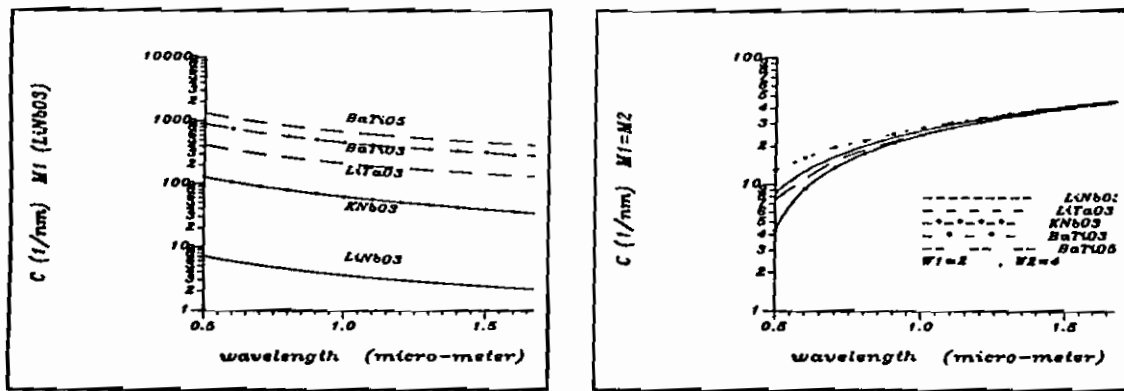


Fig. 5 Effect of the dissimilar electrooptic materials on the coupling coefficient (C) with $W_1=W_2=2\mu m$, $T=4\mu m$, $S=1\mu m$, $n_s=1.502$, $\lambda=0.633\mu m$, E_{∞}^*
 Solid lines with $V_1=0$, dashed lines with $V_1=50$ volts



a) $W_1 = W_2 = 2 \mu\text{m}$ b) $W_1 = 2 \mu\text{m}, W_2 = 4 \mu\text{m}$
 Fig. 6: Effect of the operating wavelength (λ) on the coupling coefficient (C) between two similar wave guide, with $V_1=0, V_2=20$ volts, $T=4 \mu\text{m}, S=1 \mu\text{m}, n_s=1.502, E_{\infty}^x$ (all the other parameters are assumed independent upon wavelength)

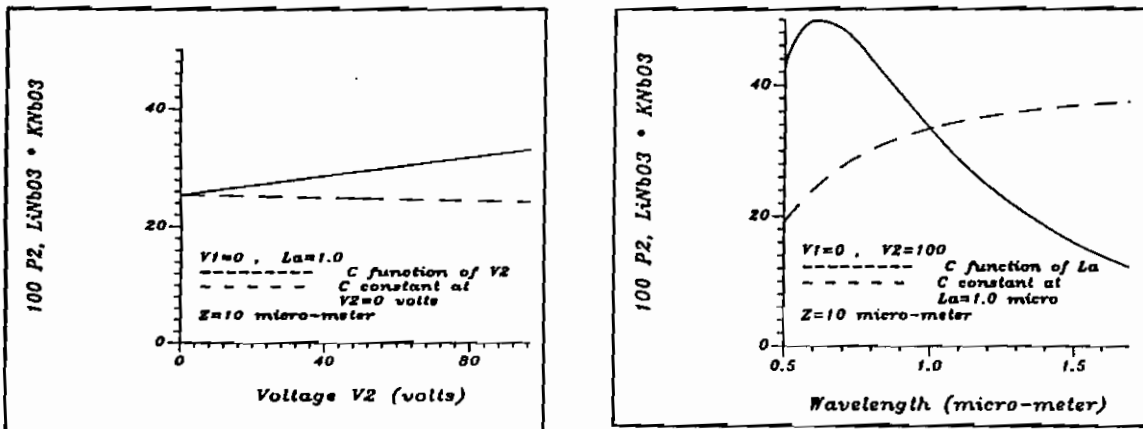


Fig. 7: Comparison between power transferred (P_2) which calculated with: Solid line, both C and $\Delta\beta$ are variables (function of V_2 or λ), dashed line, assuming constant coupling coefficient (C) and variable $\Delta\beta$ (function of V_2 or λ), $W_1=W_2=2 \mu\text{m}, T=4 \mu\text{m}, S=1 \mu\text{m}, n_s=1.502, E_{\infty}^x$, (all the above parameters are assumed independent upon wavelength)

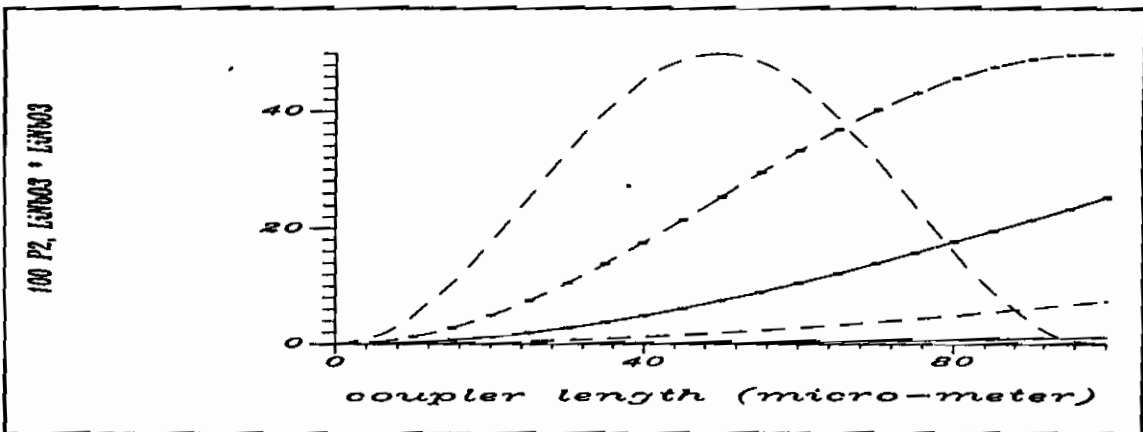


Fig. 8: P_2 as a function of coupler length L at different values of V_2 (i.e. C) with, $V_1=0, W_1=W_2=2 \mu\text{m}, T=4 \mu\text{m}, S=1 \mu\text{m}, n_s=1.502, E_{\infty}^x$.
 Solid ($V_2=10$, i.e. $C=1.124382 \text{ nm}^{-1}$), dashed ($V_2=25$, i.e. $C=2.810478 \text{ nm}^{-1}$),
 Solid with samples ($V_2=50$, i.e. $C=5.619049 \text{ nm}^{-1}$), dashed with samples ($V_2=100$, i.e. $C=11.239050 \text{ nm}^{-1}$),
 Large dashed ($V_2=200$, i.e. $C=22.479060 \text{ nm}^{-1}$)

SUMMARY

In this algorithm, an electrooptic directional coupler is designed using two

similar or dissimilar materials with two controller applied voltages. Anisotropic materials are treated as an isotropic material

with modified both thickness and refractive index of materials. Coupling coefficient (C) is calculated by mixing both effective index method and coupled mode theory. Parameters which change the difference between two propagation constants of the individual waveguides ($\Delta\beta$), also, effect on the value of coupling coefficient and so, power transferred between two waveguides can not be calculated as a function of $\Delta\beta$ only.

The coupling coefficient depends on the value of the modified refractive index of each material before and after the controller voltages (V_1 and V_2). In case of two similar materials, coupling increased with the difference controller voltages (ΔV) and KNbO₃ is the best material. Coupling becomes more if the orientation axes of both similar materials are different. And so, coupling becomes very large with two dissimilar materials. The best pair of materials are LiTaO₃ with BaTiO₅. Coupling between pair of dissimilar materials increased or decreased with the controller voltages.

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Appendix A: Change of refractive index [18-21]

When an electric field applied upon electrooptic (EO) material, a new index ellipsoidal (IE) shape occurred, where there are changed in both scale and orientation from original one. The new IE with linear EO effect (Pockels effect) is [19]

$$(n_a^{-2} + r_{11}E_a + r_{12}E_b + r_{13}E_c) a^2 + (n_b^{-2} + r_{21}E_a + r_{22}E_b + r_{23}E_c) b^2 + (n_c^{-2} + r_{31}E_a + r_{32}E_b + r_{33}E_c) c^2 + 2(r_{41}E_a + r_{42}E_b + r_{43}E_c) bc + 2(r_{51}E_a + r_{52}E_b + r_{53}E_c) ac + 2(r_{61}E_a + r_{62}E_b + r_{63}E_c) ab = 1 \quad (A.1)$$

Where; E_a , E_b and E_c are the electric field components in the directions of crystallographic axes a , b and c , respectively. So, there are six positions (cases) for the orientation of coordinate system axes (x , y and z) with respect to crystallographic axes. Z-cut (Case1: $x//a$, $y//c$, $z//b$, and case3: $x//c$, $y//a$, $z//b$), Y-cut (case2: $x//b$, $y//c$, $z//a$, case4: $x//c$, $y//b$, $z//a$) and X-cut (case5: $x//a$, $y//b$, $z//c$, case 6: $x//b$, $y//a$, $z//c$). For some cases, to eliminate the cross product terms of IE, the normal plane (normal to the propagation direction) of crystallographic axes must be rotate by an angle θ . Change of refractive index as a function of E_y with $E_x=E_z=0$, and numerical values are indicated in Tables A.1 and A.2.

Table A.1: Change of refractive index with constant E_y ($E_x=E_z=0$) and propagation in Z-direction, $A_n=(1/n_x^2 - 1/n^2)$. For any hidden cases the values of θ , Δn_x and Δn_y are zeros

Case	θ	$\Delta n_x / (0.5n^2)$	$\Delta n_y / (0.5n^2)$
LiNbO ₃ , LiTaO ₃			
1	0	$-r_{13}E_y$	$-r_{33}E_y$
2	0	$-r_{23}E_y$	$-r_{33}E_y$
3	$0.5 \tan^{-1}(2r_{31}E_y/A_n)$	$A_n \sin^2(\theta) - r_{31}E_y \sin 2\theta$	$-A_n \sin^2(\theta) + r_{31}E_y \sin 2\theta$
4	$0.5 \tan^{-1}[2r_{42}E_y / (A_n - r_{22}E_y)]$	$A_n \sin^2(\theta) - r_{42}E_y \sin 2\theta - r_{22}E_y \sin^2(\theta)$	$-A_n \sin^2(\theta) + r_{42}E_y \sin 2\theta + r_{22}E_y \sin^2(\theta)$
5	0	$-r_{12}E_y$	$-r_{22}E_y$
6	$0.5 \tan^{-1}(2r_{61}E_y/A_n)$	$A_n \sin^2(\theta) - r_{61}E_y \sin 2\theta$	$-A_n \sin^2(\theta) + r_{61}E_y \sin 2\theta$
KNbO ₃ , BaTiO ₃ , BaTiO ₅			
1	0	$-r_{13}E_y$	$-r_{33}E_y$
2	0	$-r_{23}E_y$	$-r_{33}E_y$
3	$0.5 \tan^{-1}(2r_{31}E_y/A_n)$	$A_n \sin^2(\theta) - r_{31}E_y \sin 2\theta$	$-A_n \sin^2(\theta) + r_{31}E_y \sin 2\theta$
4	$0.5 \tan^{-1}(2r_{42}E_y/A_n)$	$A_n \sin^2(\theta) - r_{42}E_y \sin 2\theta$	$-A_n \sin^2(\theta) + r_{42}E_y \sin 2\theta$

Table A.2 Values of Δn_x and Δn_y with $E_y = 1 \text{ V}/\mu\text{m}$ ($E_x=E_z=0$) and case 1 ($\theta=0$)

	1000 Δn_x	1000 Δn_y
LiNbO ₃	-0.05734	-0.16451
LiTaO ₃	-0.04327	-0.15799
KNbO ₃	-0.16593	-0.32653
BaTiO ₃	-0.13298	-0.18402
BaTiO ₅	-0.11058	-0.74960

Appendix B: Coupling Coefficient C [13-15]

Coupling coefficient between two slabs directional coupler, slabs A and B, (Fig. B1) is defined as [14];

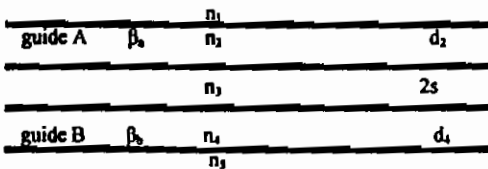


Fig. B.1 Two slabs waveguides directional coupler

$$C^2 = (\beta_e - \beta_o)^2 / 4 - (\beta_a - \beta_b)^2 / 4 \quad (B.1)$$

Where, β_a and β_b are even and odd coupled modes, β_e and β_o are the propagation constant for individual two slabs. The difference $\beta_e - \beta_o$ is defined by coupled mode theory as [14];

$$\beta_e - \beta_o = [k_{oa}^2 k_{ob}^2 k_{ba}^2 + 0.25(\beta_a - \beta_b)^2 (\beta_a + \beta_b)^2 (1 - R^2)^2]^{0.5} / [0.5(\beta_a + \beta_b)(1 - R^2)] \quad (B.2)$$

Then,

$$C = [k_{oa}^2 k_{ob}^2 k_{ba}^2]^{0.5} / [(\beta_a + \beta_b)(1 - R^2)] \quad (B.3)$$

Where, R is overlap integral,

$$R = 0.5 \int_{-\infty}^{\infty} (\psi_a \psi_b + \psi_b \psi_a) dx = (I_1 + I_2 + I_3 + I_4 + I_5) \quad (B.4)$$

k_{ab} is the coupling coefficient between guides A and B,

$$k_{ab} = (n^2 - n_1^2) \int_{s+d/2}^{\infty} \psi_{x2} \psi_{x4} dx - R_{s+d/2} \int_{s+d/2}^{\infty} \psi_{x4}^2 dx + (n^2 - n_2^2) \int_{-\infty}^{s-d/2} \psi_{x2} \psi_{x4} dx - R_{s-d/2} \int_{-\infty}^{s-d/2} \psi_{x4}^2 dx = (n^2 - n_1^2)(I_3 - R I_9) + (n^2 - n_2^2)(I_4 - R I_8) \quad (B.5)$$

k_{ba} is the coupling coefficient between guides B and A,

$$k_{ba} = (n^2 - n_3^2) \int_{-\infty}^{s-d/4} \psi_{x4} \psi_{x2} dx - R_{s-d/4} \int_{-\infty}^{s-d/4} \psi_{x2}^2 dx + (n^2 - n_3^2) \int_{s+d/4}^{\infty} \psi_{x4} \psi_{x2} dx - R_{s+d/4} \int_{s+d/4}^{\infty} \psi_{x2}^2 dx = (n^2 - n_3^2)(I_1 - R I_6) + (n^2 - n_3^2)(I_2 - R I_7) \quad (B.6)$$

By solving eigen value equation for each individual slab, the values of k_j , β_j , N_j , $\gamma_{j-1,j}$ and $\gamma_{j+1,j}$ ($j=1,2$) can be determined and the values of $I_1 \dots I_9$ are,

$$I_1 = w_1 e^{-\gamma_{32}(2s+d/4)} [R_{32} R_{54} (\gamma_{54} + \gamma_{32})] \\ I_2 = w_2 e^{-\gamma_{32} 2s} [e^{-\gamma_{32} d/4} (m^2 \gamma_{54} - m^2 \gamma_{32}) / R_{54} + (m^2 \gamma_{54} - m^2 \gamma_{34}) / R_{34}] / [m^2 R_{32} (k^2 \gamma^2 + \gamma^2)] \\ I_3 = w_1 e^{-\gamma_{32}(2s+d/4)} 2s / (R_{32} R_{34}) \\ I_4 = w_2 e^{-\gamma_{32} 2s} [e^{-\gamma_{32} d/2} (m^2 \gamma_{12} - m^2 \gamma_{34}) / R_{12} + (m^2 \gamma_{12} - m^2 \gamma_{32}) / R_{32}] / [m^2 R_{34} (k^2 \gamma^2 + \gamma^2)] \\ I_5 = w_1 e^{-\gamma_{32}(2s+d/2)} / [R_{34} R_{12} (\gamma_{12} + \gamma_{34})] \\ I_6 = w_1 e^{-\gamma_{32}(4s+2d/4)} / (2\gamma_{32} R^2) \\ I_7 = w_1 e^{-\gamma_{32}(4s+d/4)} \sinh(\gamma_{32} d_4) / (\gamma_{32} R^2) \\ I_8 = w_2 e^{-\gamma_{32}(4s+2d/2)} / (2\gamma_{34} R^2) \\ I_9 = w_2 e^{-\gamma_{32}(4s+d/2)} \sinh(\gamma_{34} d_2) / (\gamma_{34} R^2) \quad (B.7)$$

Where;

$$w = 2m_2 m_4 N_2 N_4 k_2 k_4 / (d_{e12} d_{e14})^{0.5} \\ w_1 = 2m_2^2 N_2^2 k_2^2 / d_{e12}, \quad w_2 = 2m_4^2 N_4^2 k_4^2 / d_{e14} \\ R_{12} = (m_1^4 k_2^2 + m_4^4 \gamma_{12}^2)^{0.5}, \quad R_{32} = (m_3^4 k_2^2 + m_2^4 \gamma_{32}^2)^{0.5} \\ R_{34} = (m_3^4 k_4^2 + m_4^4 \gamma_{34}^2)^{0.5} \\ R_{54} = (m_5^4 k_4^2 + m_4^4 \gamma_{54}^2)^{0.5} \quad \text{and} \\ d_{ij} = d_j + 1 / (q_{j-1,j} \gamma_{j-1,j}) + 1 / (q_{j+1,j} \gamma_{j+1,j}) \quad (B.7)$$

for TE mode $m_j=1$, $q_{i-1,j} = q_{j+1,i} = 1$, For TM mode

$$m_j = n_j, \quad q_{i-1,j} = (N_j/n_j)^2 + (N_j/n_{j-1})^2 - 1, \\ q_{j+1,i} = (N_j/n_j)^2 + (N_j/n_{j+1})^2 - 1 \quad ; j=1,2.$$

Appendix C: Data of used electrooptic materials [18-26]

For the calculation, we need the whole set of EO material parameters, such as elements of ϵ_i , r_{ij} and n_i , we could not find a complete set of these parameters, especially the effect of λ on the last parameters, therefore, we use the following parameters, which we believe, are not too far from the real ones; [27]. Data of EO materials at $\lambda=0.633 \mu\text{m}$ [18-26] are:

LiNbO₃ ($n_a=n_b=2.286$, $n_c=2.200$, $\epsilon_a=\epsilon_b=43$, $\epsilon_c=28$, $r_{12}=-6.8$, $r_{13}=9.6$, $r_{22}=6.8$, $r_{23}=8.6$, $r_{33}=30.9$, $r_{42}=32.6$, $r_{51}=32.6$, $r_{61}=6.8$),
LiTaO₃ ($n_a=n_b=2.176$, $n_c=2.180$, $\epsilon_a=\epsilon_b=42$, $\epsilon_c=41$, $r_{12}=-0.2$, $r_{13}=8.4$, $r_{22}=0.2$, $r_{23}=8.4$, $r_{33}=30.5$, $r_{42}=20$, $r_{51}=20$, $r_{61}=0.2$),
BaTiO₃ ($n_a=n_b=2.410$, $n_c=2.360$, $\epsilon_a=\epsilon_b=2300$, $\epsilon_c=60$, $r_{13}=19$, $r_{23}=19$, $r_{33}=28$, $r_{42}=820$, $r_{53}=820$),
BaTiO₅ ($n_a=n_b=2.480$, $n_c=2.426$, $\epsilon_a=\epsilon_b=4300$, $\epsilon_c=168$, $r_{13}=14.5$, $r_{23}=14.5$, $r_{33}=105$, $r_{42}=1700$, $r_{53}=1700$),
KNbO₃ ($n_a=2.280$, $n_b=2.329$, $n_c=2.169$, $\epsilon_a=160$, $\epsilon_b=1000$, $\epsilon_c=55$, $r_{13}=28$, $r_{23}=1.3$, $r_{33}=64$, $r_{42}=380$, $r_{51}=105$).

LiNbO₃ at $\lambda=1.15 \mu\text{m}$ ($n_a=n_b=2.229$, $n_c=2.150$, $\epsilon_a=\epsilon_b=43$, $\epsilon_c=28$, $r_{12}=-5.4$, $r_{13}=9.6$, $r_{22}=5.4$, $r_{23}=9.6$, $r_{33}=30.9$, $r_{42}=32.6$, $r_{51}=32.6$, $r_{61}=5.4$).

Note, in this paper we assume quasi-static operations.

To study the effect of wavelength, all the above parameters are assumed independent upon wavelength.

Appendix D:

Dependence Of the Values of $DN=N_2-N_1$ On The Applied Voltages V_1 and V_2 For several electroopic materials

with Case 1, $W_1=W_2=2 \mu m$, $T=4 \mu m$, $S=1 \mu m$, $n_r=1.5$, $\lambda=0.633 \mu m$ and E^m .

Table D.1: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=M_2$.

$M_1=M_2$	$V_1=0$		$V_1=50$	
	$V_2=0$	$V_2=100$	$V_2=0$ volt	$V_2=100$
LiNbO ₃	0	0.0022612	-0.0011306	0.0011306
LiTaO ₃	0	0.0024593	-0.0012295	0.0012298
KNbO ₃	0	0.0036421	-0.0018210	0.0018210
BaTiO ₃	0	0.0006413	-0.0003207	0.0003207
BaTiO ₃	0	0.0021434	-0.0010717	0.0010717

Table D.2: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=LiNbO_3$.

M_2	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$	$V_2=100$	$V_2=0$	$V_2=100$
LiTaO ₃	0.06565	0.06811	0.0645	0.06698
KNbO ₃	0.01778	0.02142	0.0167	0.02029
BaTiO ₃	-0.14436	-0.14372	-0.14550	-0.14485
BaTiO ₃	-0.21231	-0.21017	-0.21344	0.21130

Table D.3: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=LiTaO_3$.

M_1	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$ volt	$V_2=100$	$V_2=0$ volt	$V_2=100$
LiNbO ₃	-0.06565	-0.06339	-0.06689	-0.06462
KNbO ₃	-0.04787	-0.04423	-0.04910	-0.04546
BaTiO ₃	-0.21002	-0.20937	-0.21125	-0.21060
BaTiO ₃	-0.27796	-0.27582	-0.27919	-0.27705

Table D.4: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=KNbO_3$.

M_2	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$ volt	$V_2=100$	$V_2=0$ volt	$V_2=100$
LiNbO ₃	-0.01778	-0.01552	-0.01960	-0.01734
LiTaO ₃	0.04787	0.05033	0.04605	0.04851
BaTiO ₃	-0.16215	-0.16150	-0.16397	-0.16332
BaTiO ₃	-0.23009	-0.22795	-0.23191	-0.22977

Table D.5: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=BaTiO_3$.

M_2	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$ volt	$V_2=100$	$V_2=0$ volt	$V_2=100$
LiNbO ₃	0.14436	0.14663	0.14404	0.14631
LiTaO ₃	0.21002	0.2124751	0.2096951	0.21215
KNbO ₃	0.16214	0.1657867	0.1618240	0.16547
BaTiO ₃	-0.06794	-0.06580	-0.06826	-0.066121

Table D.6: Dependence of values of $DN=N_2-N_1$ on the applied voltages V_1 and V_2 with case 1, $M_1=BaTiO_3$.

M_2	V_1 (volt) = 0		V_1 (volt) = 50	
	$V_2=0$ volt	$V_2=100$	$V_2=0$ volt	$V_2=100$
LiNbO ₃	0.21231	0.21457	0.21124	0.02135
LiTaO ₃	0.2779596	0.2804189	0.2768879	0.27935
KNbO ₃	0.2300885	0.2337306	0.2290168	0.23266
BaTiO ₃	0.0679438	0.0685852	0.0668721	0.06751