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Fracture Properties and Process Zone Length of High Strength Concrete Using Size Effect Method.

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FRACTURE PROPERTIES AND PROCESS ZONE LENGTH OF HIGH STRENGTH CONCRETE USING SIZE EFFECT METHOD

خصائص الكسر وطول منطقة الكسر للخرسانة عالية المقاومة باستخدام طريقة تأثير الحجم

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ملخص :- يعرض هذا البحث دراسة خصائص الكسر للخرسانة عالية المقاومة ذات مقاومة ضغط حتى 80 ميجا باسكال. تم تصميم 6 خلطات خرسانية صممت على أساس قيم هبوط اكبر من 100 مم ومقاومة حتى 80 ميجا باسكال. . وكانت المتغيرات في هذه الخلطات هي نسبة السيلكا فوم (0 - 10% - 15%) من وزن الاسمنت ونسبة الركام الكبير (48% ، 65%) بالحجم من المحتوى الكلى للركام. تم تعيين كل من مقاومة الضغط ومقاومة الشد والانحناء وطاقة الكسر (G_f) وطول منطقة الكسر (c_f) باستخدام طريقة تأثير الحجم (size effect). تم صب أكثر من 60 كمرة لاستنتاج معاملات وقيم الكسر . أثبتت النتائج أن طاقة الكسر (G_f) تزداد بزيادة قيم مقاومة الضغط وان طول منطقة الكسر يقل بزيادة قيم مقاومة الخرسانة. أيضا أمكن استنتاج معادلات لحساب مقاومة الانحناء الاسمية لكلا من الخرسانة العادية والعالية المقاومة. كما لوحظ من منحنى تأثير الحجم (size effect). أن سلوك الكسر للخرسانة عالية المقاومة أكثر صلابة من الخرسانة عادية المقاومة. وأيضا أن خصائص الكسر للخرسانة العالية المقاومة تميل إلى الاتجاه الخطى المرن عن الاتجاه الغير خطى.

ABSTRACT

This paper gives a report on a fracture mechanics study of high strength concrete with compressive strength up to 80 MPa. 6 mixes were selected, designed to achieve a slump above 100 mm and strength up to 80 MPa.. The investigation used three levels of silica fume (0, 10%, 20% by weight of cement) and two coarse aggregate proportions of total aggregate (0.48, 0.65 by volume) In each case, compressive strength, splitting tensile strength, flexural strength fracture energy G_f and fracture process zone c_f were determined. A test method for determining fracture energy G_f and process zone length c_f, defined by the size effect law and its generalized theory, has been proposed. It allows the use of specimens of the different size and shape but with same notch lengths. This paper proposes test and data analysis procedures. The results obtained by the Size effect law (SEL) apparently follow the trend of fracture energy increasing as the compressive strength of the concrete increases. On the other hand, process zone length c_f decrease considerably with increase in strength. The Bazant's size effect law gives a very good account of the size dependency of flexural strength for both normal and high strength concrete. As observed in the size effect curve, the fracture behavior of the HSC seems to be more brittle than that of the normal strength concrete. Linear elastic fracture mechanics (LEFM) may be applicable to HSC in the normal size range than nonlinear fracture mechanics.

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INTRODUCTION

It is well known that strengths of concrete structures exhibit some size dependency; strengths of geometrically similar structures decrease with increasing size (1). Intensive studies on the fracture of concrete in the last two decades have clarified that the size effect on strength is related primarily to a relatively large fracture propagation process (FPZ) in concrete (2). Nonlinear fracture models, i.e., the Fictitious Crack Model, the Crack Band Model by Bazant, and the Two-Parameter Model by Jenq and Shah, have been applied to the analysis and prediction of size effects in concrete structures. The size effect law proposed by Bazant (1), although simple, can describe satisfactorily the size dependency of flexural strength. With the advance in concrete technology, high strength concrete (HSC) has been increasingly used in buildings and civil engineering structures and has received enormous research attention. Although HSC tends to be more brittle with increasing strength, the research on the fracture behavior of HSC has been limited. It is felt that the study of the size effect in HSC may help to understand the behavior of HSC and predict load-bearing capacities of different types and sizes of HSC structures. Therefore, an experimental study was carried out to examine the size effect on flexural strengths of HSC.

The size effect law (SEL) and its generalized theory (1) present a nonlinear fracture model of concrete by summarizing the observed size effect on the nominal strength of concrete structures. Therefore, it is natural to exploit the size effect for measuring material fracture parameters G_f and c_f defined in SEL and its general theory. The size effect method, which has been recommended by RILEM (1), proposes to determine G_f and c_f from the maximum loads of geometrically similar beams of different sizes. Since this method only demands measurement of the maximum load, it has been used in both laboratories and construction sites. However, the size effect method needs specimens of different sizes. This specimen requirement brings about difficulties in specimen preparation and testing. Previous study has indicated that, for accurate results, the ratio of the dimension of the largest specimens to that of the smallest specimens should not be less than 2.5. On the other hand, the smallest dimension of the specimens should be at least three to five times as large as the maximum aggregate size. Accordingly, when normal or large coarse aggregates are used, the required largest specimens can be very heavy and very large, so large as to exceed the capability of the testing machine available.

RESEARCH SIGNIFICANCE

The SEL and its generalized theory present a fracture model of concrete in which material fracture parameters G_f and c_f are objective. These parameters are not only necessary, but also sufficient for determining the nominal strength of any concrete structure. Therefore, experimental determination of G_f and c_f is a key issue in application of fracture mechanics of concrete. The size effect method recommended by RILEM requires specimens of different sizes. The objective of this study is to characterize the brittle behavior of the HSC in order to establish adequate safety criteria for the material utilization. The parameters obtained in the experiments can provide a database for calibrating of numerical models and the development of design criteria for HSC structures.

THEORETICAL BACKGROUND

In order to determine the fracture energy, one can apply the recommendation of the Technical Committee RILEM 50-FMC [1] to perform three-point bend tests in notched beams. The fracture energy is defined as the amount of energy necessary to create a crack of unit surface area projected in a plane parallel to the crack direction. As the beam is split in two halves, the fracture energy can be determined dividing the total dissipated energy by the total surface area of the crack. According to Bazant and Pfeiffer [2], the method proposed by RILEM, also known as work-of-fracture method or Hillerborg's method [3], delivers materials fracture characteristics which are ambiguous and, especially, size-dependent. As a consequence, different values for the fracture energy are obtained for specimens of different sizes. In an alternative method proposed by Bazant and Pfeiffer, the fracture energy is determined from the size effect law. In this case, by definition, the value of the fracture energy is independent of the size of the specimens. If geometrically similar beams are used and the load at rupture extrapolated to a beam of infinite dimensions, the fracture energy must have one single value, regardless the type, size or shape of the specimen. This procedure is known as the size effect method (SEL).
Through this asymptotic approach, the problem is now reduced to find and apply the correct law for the size effect. Bazant and Pfeiffer suggested the following relationship:

$$\sigma_n = \frac{B \times f_t}{\sqrt{1+\beta}} \qquad (1)$$

where σ_n is the nominal stress at failure, f_t is the material tensile stress, B is a coefficient obtained through the linear regression plot of the test results, β is the brittleness number

The brittleness number indicates whether the behavior of any structure is related to limit state analysis or to linear elastic fracture mechanic (LEFM) analysis.

$$\beta = \frac{d}{d_o} \qquad (2)$$

where d is the characteristic dimension of the structure (the specimen height in this study) and d_0 is a coefficient determined experimentally. The value of β =1 corresponds to the transition point between the strength approach and the LEFM approach. For values of $\beta \leq 0.1$, the plastic limit analysis should be used for structural design, and for values of $\beta \geq 10$, the LEFM should be used. For $0.1 < \beta < 10$, the nonlinear fracture mechanics should be used for structural design.

To facilitate the evaluation of the constants in the size effect law, Eq. 1 can be transformed into Eq. 3:

$$\left(\frac{f_t}{\sigma_n}\right)^2 = \frac{1}{d_o \times B^2} d + \frac{1}{B^2} \qquad (3)$$

If the experimental data are arranged in a plot of X = d and Y = $(f_t/\sigma_n)^2$, a linear regression equation may be found as Y = AX + C. B and d_0 can be evaluated from A and C as (Eq. 4):

$$B = \frac{1}{\sqrt{C}},$$

$$d_o = \frac{C}{A} \qquad (4)$$

The size range should be sufficiently large (at least 1:2.5) and at least three sizes should be used.

FRACTURE PARAMETER

The SEL (4) was generalized (5, 6) by introducing two material fracture parameters: fracture energy G_f, and process zone length c_f for the infinitely large specimen. When the specimen is infinitely large, the process zone length can always fully develop with no constraint from the specimen boundary; therefore, c_f is a material constant. In addition, c_f is negligible when the initial crack length a_0 is sufficiently long, so linear elastic fracture mechanics (LEFM) applies, and G_f must be a material constant, too. By extending it to the asymptotic case, the case of the infinitely large specimen, SEL can be synthesized as

$$\sigma_n = c_n \left\{ \frac{\dfrac{EG_f}{g'(\alpha_0)}}{c_f + \dfrac{g(\alpha_0)}{g'(\alpha_0)} d} \right\}^{1/2} \quad (5)$$

where E is the elastic modulus, $\alpha_0 = a_0/d$, and σ_n is the nominal strength of the specimen (or structure), defined as

$$\sigma_n = c_n \times \frac{P_u}{b \times d} \quad (6)$$

in which P_u is the maximum load, b is the specimen thickness, d is the specimen dimension (i.e., the depth for the beam), and c_n is an arbitrarily defined constant; and in which α is the ratio of the crack length a to the specimen dimension d, and $g(\alpha)$ is the geometry factor for the stress intensity factor K_I. For a specimen (or structure) of any geometry and any size, function $g(\alpha)$ can be obtained by LEFM analysis. Formulas for specimens or structures of many popular geometries have been obtained and can be found in manuals.(7, 8) Thus, with EG_f and c_f known, the nominal strength of a structure can be determined through the general SEL [Eq. (5)]. It is noted that, although $g(\alpha_0)$ and $g'(a_0)$ are calculated based on LEFM, σ_n of a finite specimen (or structure) determined by Eq. (5) is not from LEFM, but a nonlinear model, that is, the SEL. Only in the infinitely large specimen is LEFM valid. With LEFM, the critical stress intensity factor K_{If} for the infinitely large specimen, and EG_f has the following relation in plane stress

$$(K_{If})^2 = EG_f \quad (7)$$

Since the size effect for specimens of a given geometry is characterized by EG_f and c_f, EG_f and c_f can certainly be calculated by a regression from σ_n values of several geometrically similar specimens of different sizes. This is the theoretical basis of the size effect method. However, RILEM recommended a linear regression for convenience (4).

For specimens of a given geometry, Eq. (5) can be converted to a linear regression

$$Y^* = A^* X^* + C^* \quad (8)$$

Where

$$Y^* = \left\{ \frac{c_n^2}{g'(\alpha_0)\sigma_n^2} \right\} \quad , X^* = \left\{ \frac{g(\alpha_0)}{g'(\alpha_0)} \right\} d ,$$

$$EG_f = \frac{1}{A^*} \quad , c_f = \frac{C^*}{A^*} \quad (9)$$

Based on the SEL, G_f, c_f can be determined from the failure loads of three-point bend specimens. At least three sizes of geometrically similar beams (same thickness and with proportional spans, depth and notch length) have to be tested

EXPERIMENTAL PROGRAME

Materials

Detailed information about the materials used and their characteristics are given in this section.

Cement Portland cement has been found to be adequate for production of high strength concrete. Experience Physical and mechanical properties of cement were measured according to Egyptian standard specification (ESS 373-1963)

Mineral admixtures. Silicafume (sf) was used in this programme and particular care was taken over curing since the pozzolanic reaction takes place over a longer time.

Aggregate Limestone was used as coarse aggregate (C). The coarse aggregate had a maximum size of 16 mm. Limestone batches employed were clean, free from impurity matter. The main properties of limestone were measured according to Egyptian standard specifications (ESS 1109-1971). Table 1 shows the physical properties of Limestone. The fine aggregate (F) used in concrete was desert sand. It was clean and almost free from impurities, silt, lay and saltiness. Main properties of sand were measured according to the Egyptian standard specification (ESS 1109-1971). Table 2 shows the physical properties of sand aggregates.

Chemical Admixtures (Superplasticisers). High range water reducer complies with ASTM C494 type F was used. The admixture is a brown liquid ready to use directly during concrete mixing.

Test specimens and instrumentation

Compressive strength:- Compression tests were carried out on 150 mm cubes according to British Standard BS 1881: Part 116:1983.

Splitting tensile strength Tests were carried out on 150 diameter and 300 mm length cylinders according to British Standard BS 1881: Part 117:1983.

Flexural strength:- Tests were carried out on 100×100×500 mm beams according to British Standard BS 1881: Part 118:1983.

Fracture test specimens:- Beams specimens of three different sizes, three in each sizes, were cast. The specimens (Fig. 1) were 75 mm thick and 400, 200, 100 mm deep. The spans was 2.5 times the depths. Before testing, a notch 2mm wide was cut at midspan of each beam using diamond band saw. The length of the notch was one-six the depth of the beam. The three-point bending test was used due to relatively simple test setup and impossibility of crack bifurcation. The peak loads of the largest specimens were measured by loading them under stroke control in load frame with control system. The other specimens were tested in a smaller load frame with load cell operating. The test setup is shown in Fig. 2 and 3. Test type and specimen dimensions used in the study are shown in Table 3.

Mixing, casting and curing

Mixing was carried out in a 150-litre revolving paddle pan mixer. The concrete was cast into steel moulds using a minimal amount of mould oil and compacted using a vibrating table. The fresh concrete had a slump greater than 100mm as measured with a standard 305 mm cone. The specimens were demoulded and of water curing commenced after 24 hours. Six mixes were selected for the main programme. These mixes involved a range of coarse aggregate ratio (A) and silicafume content. Table 4 gives the composition of selected mixes.

Table 1 Physical properties of Limestone aggregates

Property	Values
Specific gravity	2.61
Unit weight (Kg/m^3)	1660
Absorption	1.58
Crush value %	6.8

Table 2 Physical properties of Sand

Property	Values	E.S.S
Specific gravity	2.67	2.5- 2.75
Unit weight (Kg/m^3)	1678	1600-1800
Fineness modulus	2.64	1.5-3.75
Air voids (%)	35.98	20 – 40
Clay, Silt (by weight %)	0.88	Less than 3%

Table 3 Test methods and specimen size

Test type	Dimension (mm)	Properties
Cube compression	150 × 150 × 150	Compressive strength
Cylinder splitting	Φ 150 × 300	Tensile strength
Three-point bending	100 × 100 × 500	Flexural strength
	75 × 100 × 250 × 350*	Fracture energy and flexural strength
	75 × 200 × 500 × 600*	
	75 ×400 × 1000 × 1100*	

* Hight, Width, Span, and length of beams

Table 4. Composition of mixes used in main programme

Mix No.	c	sf/(c+sf)	w/(c+sf)	SP/(c+sf)	C/(F+C)
	kg/m^3	%		%	
D1	350	0	0.5		0.65
D11	350	0	0.5		0.48
D2	500	10	0.3	2	0.65
D3	500	10	0.3	2	0.48
D4	500	15	0.27	2	0.65
D5	500	15	0.27	2	0.48

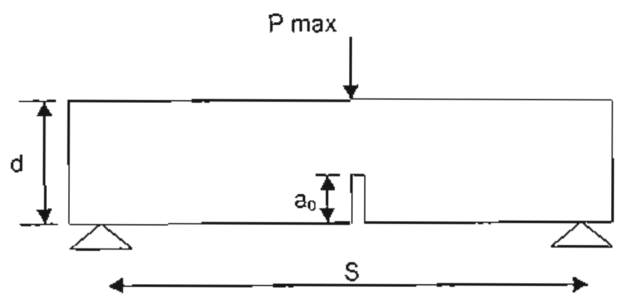

Fig. 1 Sketch of specimens of three point ben

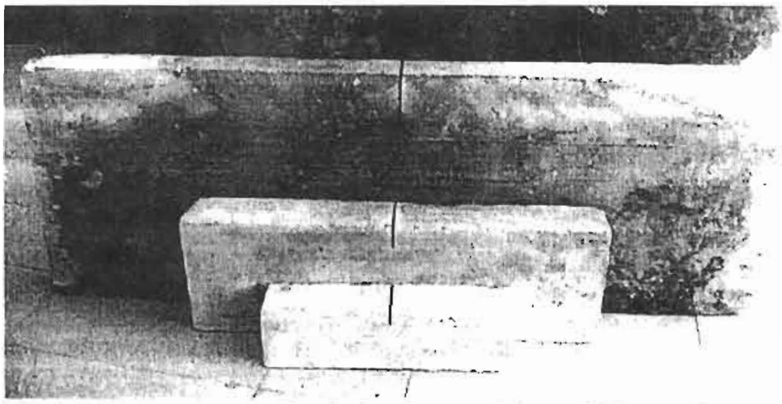

Fig. 2 Specimens of three different size

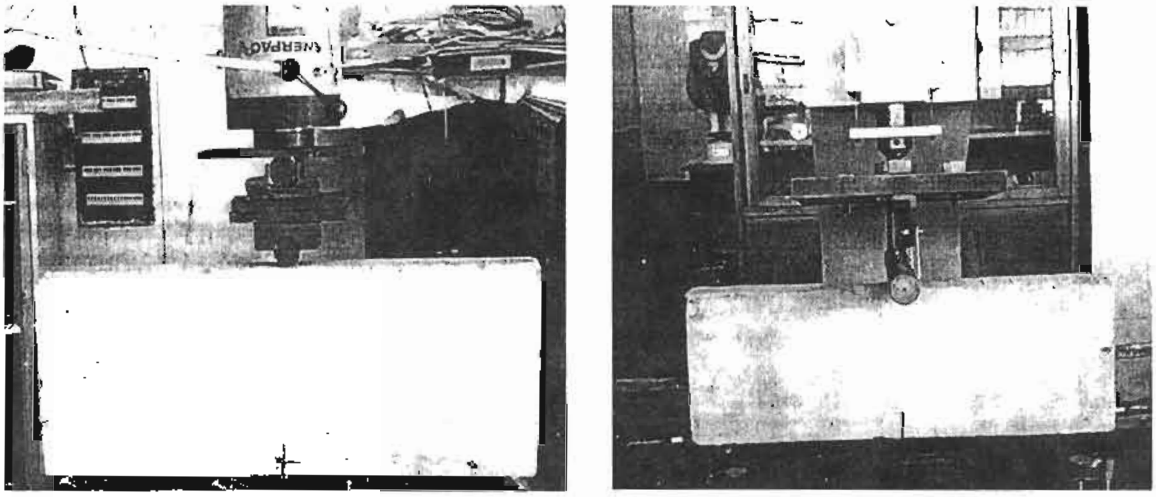

Fig. 3 Test setup

Table 5 Compressive Strength, Splitting tensile strength, Flexural strength and Fracture energy of Concrete

No.	f_c (MPa)	F_{st} (MPa)	f_{fl} (MPa)	E (GPa)	G_f N/m	c_f mm	K_{IC} (MPa mm$^{-1/2}$)	ω_A	ω_C	m
D1	30.50	3.05	4.02	26.12	29.12	6.34	27.58	0.023	0.175	0.057
D11	26.48	2.78	3.54	24.34	26.88	7.39	25.56	0.023	0.148	0.055
D2	68.65	5.00	7.5	34.41	52.03	2.45	42.31	0.007	0.134	0.018
D3	61.78	4.86	6.84	33.00	43.96	1.57	38.09	0.005	0.167	0.015
D4	78.45	5.97	7.8	36.31	58.92	2.27	46.25	0.008	0.169	0.021
D5	64.72	5.14	7.5	33.61	50.39	1.54	41.16	0.005	0.166	0.014

EXPERIMENTAL RESULTS AND DISSCUCTION

The fracture energy obtained by the SEL was computed as specified in RILEM [9]. Values of $g(a_0)$ were computed equal to 6.07 for relative notch length equal to 0.167. The Young's modulus of elasticity was computed from the formula of Carrasquillo et al. (10) $E = 3320 f_c^{1/2} + 6900$ (MPa), at mixes with silicafume. The fracture parameters obtained from the regression analysis are shown in Table 5. As can be seem, all mixes comply the standard requirements concerning the limiting values of the coefficient of variation of the slope of the regression line (ω_A), the coefficient of variation of the intercept of the regression line (ω_C), and the relative width of scatter-band (m). The value of ω_A should not exceed 0.10 and the values of ω_C and m about 0.20.

From the results obtained in Table 5 by SEL method, it is possible to verify that the values obtained for the fracture energy G_f are greatly affected by the component of mixture. Some researches verified the relationship between fracture energy and compressive strength. Xie et al. (11) tested beam specimens by the work-of-fracture method and obtained average values for G_f that increases 13% and 11% for an increase of 53% and 29% in the compressive strength, respectively. Gettu et al. (12) compared results obtained for high-strength concrete and conventional concrete and verified that an increase of 160% in the compressive strength resulted in an increase of only 12% in the fracture energy. These numbers show the necessity of a relative great increase in the compressive strength in order to evidence a small increase in the fracture energy. Some authors, as Rao and Prasad (13) and Gettu et al. (12), concluded that the fracture energy increases as the compressive strength of the concrete increases. Apparently, the results obtained in the present study follow this trend.

Table 5 illustrates the effects of silica fume and aggregate type on the fracture energy. It is shown that increasing coarse aggregate ratio produced higher fracture energy for all mixes. The fracture energy of silica fume concrete increases as percentage of silica fume increases. This may be due to the improvement in the bond which results in fracture developing through aggregates than around the aggregates.

The critical stress intensity factor is used to characterize the fracture toughness of materials. Table 5 shows that K_{Ic} increases linearly with increasing compressive strength. These results are consistent with the observations by Saha (14). Although increasing K_{Ic} values means that the resistance to cracking is increasing fracture behaviour actually becomes more brittle. Therefore K_{Ic} may not be the most appropriate parameter for characterizing the fracture of concrete. The increase of silicafume from 10% to 15% tends to give slightly higher K_{Ic} values at the same other content.

The values of c_f, especially the latter, decrease considerably with increase in strength. This implies that the size of the process zone must be smaller in high strength concrete than in regular concrete, and the crack-tip shielding by the fracture process zone must be weaker. Consequently, the same structure made of high strength concrete is more brittle than that made of regular concrete.

FLEXURAL STRENGTH

The nominal flexural strength of a notched beam subjected to three-point bending was calculated from the following expression (Eq. 10):

$$\sigma_n = \frac{3PS}{2bd^2} \qquad (10)$$

where P is the ultimate load, and S, b, and d are the span, thickness, and height of the beam, respectively. The notch ratio (a/d) was 1/6 and the span/height ratio (S/d) was

2.5. The mean nominal flexural strengths of the normal strength concrete (NSC) and high strength concrete (HSC) measured from different sizes of beams. The flexural strengths of both types of concrete decrease as the specimen size increases, which is similar to the trend observed in normal strength concrete. But a stronger size effect on flexural strength is found in the HSC than in the NSC. When the beam size increases from 100 to 200 mm, and from 200 to 400 mm, the decrease in flexural strength is 21% and 28%, respectively, for the NSC but is 26% and 30% in the case for HSC.

In order to evaluate the size effect law, the experimental data were rearranged in a plot of $X = d$ and $Y = (f_t/\sigma_n)^2$ in Figure 4. The cylinder splitting tensile strengths in Table 5 were used as the material tensile strengths (f_t). The linear regression equations of $X = d$ and $Y = (f_t/\sigma_n)^2$ derived for both types of concrete are given in Fig. 4. For all mixes, the slope A and the intercept C of the linear regression equation are established. For the NSC, the mean of two constants in the size effect law can be calculated as B = $1/\sqrt{C}$ = 3.16 and d_0 = C/A = 39.41 (mm). The size effect law for the NSC can be expressed as (Eq. 11):

$$\sigma_n = \frac{3.16 \times f_t}{\sqrt{1+0.0255d}} \qquad (11)$$

For the HSC the mean of two constants in the size effect law can be calculated as B = $1/\sqrt{C}$ = 5.29 and d_0 = C/A = 11.38 (mm). The size effect law for the HSC can be expressed as (Eq. 12):

$$\sigma_n = \frac{5.29 \times f_t}{\sqrt{1+0.092\,d}} \qquad (12)$$

The experimental data of the normal and high strength concrete are compared in Figure 5 on a double logarithmic scale of σ_n/Bf_t and d/d_0 and the Bazant's size effect law fits the experimental data very well. Clearly, the data points of the HSC shift more towards the prediction by LEFM than those of the NSC, which suggests a more brittle behavior in the HSC than in the NSC. However, LEFM may be applicable to HSC than nonlinear fracture mechanics as β is < 10 in the normal size range.

Table 6 Flexural strength measured from notched beams of three size

	Flexural strength (MPa)		
Mixes	d= 100 mm	d= 200 mm	d=400mm
D1	5.04	4.02	2.88
D11	4.64	3.63	2.66
D2	8.66	6.35	4.56
D3	7.96	5.80	4.13
D4	9.47	6.98	4.98
D5	8.60	6.27	4.46

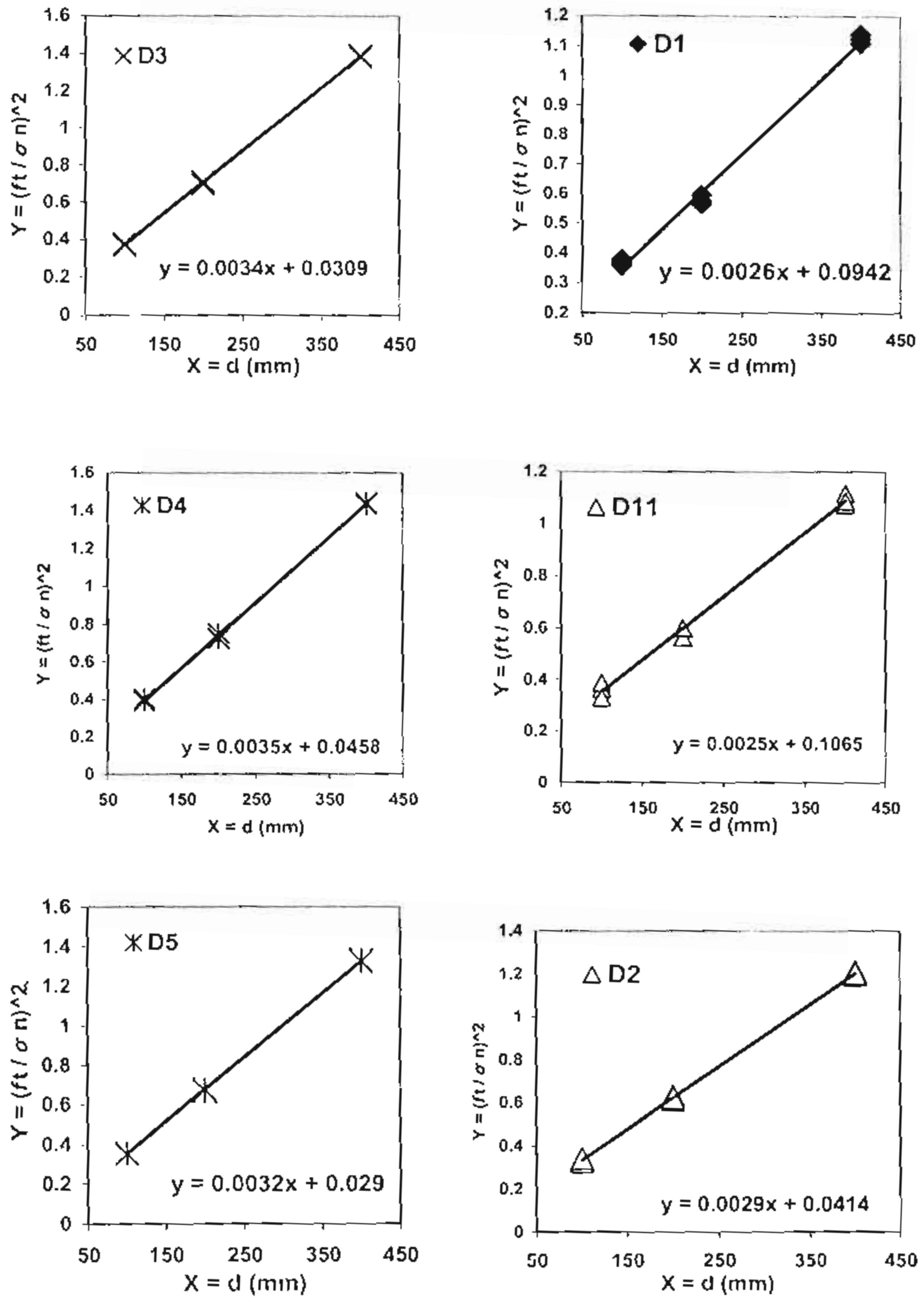

Fig 4 . Linear regression for determining constants in size effect law.

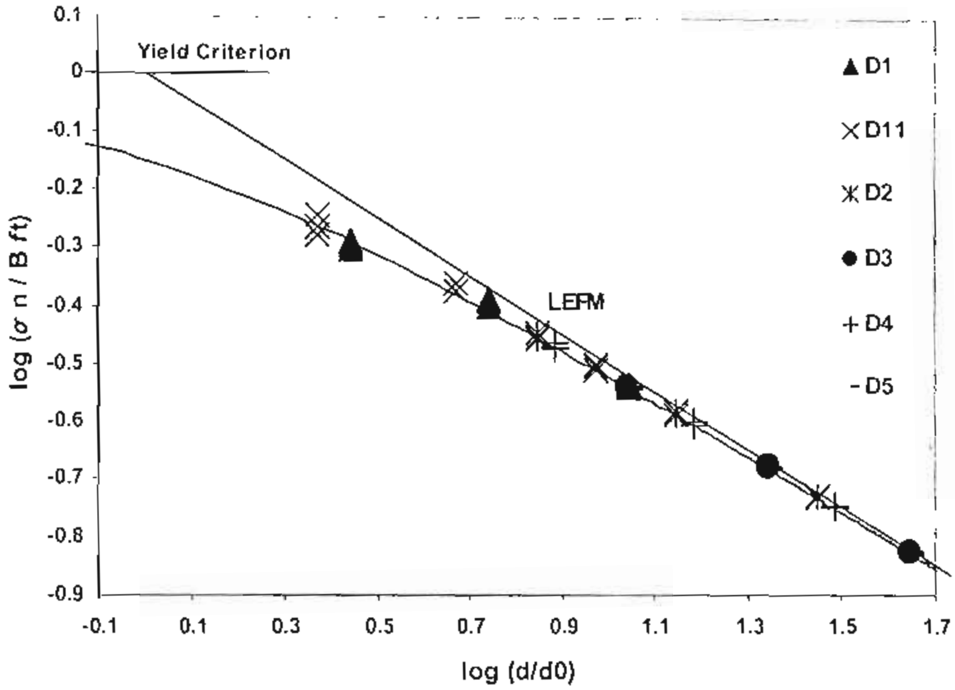

Fig. 5 Size effect law of flexural strength

CONCLUSION

An experimental study has been conducted to investigate the size effect on flexural, of HSC and NSC with normal aggregate (crushed limestone). The following conclusions may be drawn.

1) The Size effect method provides reliable fracture properties of high strength concrete with a very simple experimental setup. These properties, namely, the fracture energy and the effective length of the fracture process zone are size independent and, therefore, true material properties.
2) The results obtained by the SEL apparently follow the trend of fracture energy increasing as the compressive strength of the concrete increases. On the other hand, process zone length cf decrease considerably with increase in strength
3) The Bazant's size effect law gives a very good account of the size dependency of flexural strength for both normal and high strength concrete. As observed in the size effect curve, the fracture behavior of the HSC seems to be more brittle than that of the normal strength concrete. LEFM may be applicable to HSC in the normal size range than nonlinear fracture mechanics.

REFERENCES

1 RILEM 50-FMC, RILEM Draft Recomm. 18 (106) (1985) 285.
2 Z.P. Bazant, P.A. Pfeiffer, ACI Mater. J. (1987 (November–December))463.
3 A. Hillerborg, Mat. Struct. 18 (106) (1985) 291.
4 Bazant, Z. P., "Size Effect in Blunt Fracture: Concrete, Rock, Metal,"

Journal of Engineering Mechanics, V. 110, No. 4, Apr. 1984, pp. 518-535.

5 Bazant, Z. P., and Pfeiffer, P. A., "Determination of Fracture Energy from Size Effect and Brittleness Number," ACI Materials Journal, V. 84, No. 6, Nov.-Dec. 1987, pp. 463-480.

6 Bazant, Z. P., and Kazemi, M. T., "Determination of Fracture Energy, Process Zone Length and Brittleness Number from Size Effect: with Application to Rock and Concrete," International Journal of Fracture, V. 44, No. 2, July 1990, pp. 111-131.

7 Tada, H.; Paris, P. C.; and Irwin, G. R., The Stress Analysis of Cracks Handbooks, 2nd edition, Paris Productions, St. Louis, 1985.

8 Murakami, Y., Stress Intensity Factors Handbook, Pergamon, Oxford, UK, 1987.

9 RILEM Committee on Fracture Mechanics of Concrete—Test Methods, "Size-Effect Method for Determining Fracture Energy and Process Zone Size of Concrete," Materials and Structures, V. 23, No. 138, Nov. 1990, pp. 461-465.

10 R.L. Carrasquillo, A.H. Nilson, F.O. Slate, ACI J. Proc. 78 (3) (1981) 171.

11 J. Xie, A.E. Elwi, J.G. MacGregor, ACI Mater. J. 92 (2) (1995) 135.

12 R. Gettu, Z.P. Bazant, M.E. Karr, ACI Mater. J. 87 (6) (1990) 608.

13 G.A. Rao, B.K.R. Prasad, Cem. Concr. Res. 32 (2) (2002) 247.

14 Saha S. P., Swartz S. E., Ouyang C. , "Fracture Mechanics of concrete", 1995, Hohn Willy & sons Inc., New York, NY.