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A Composite Dynamic Rule for Machine Scheduling

قاعده ديناميكيه مركبه لجدولة الماكينات

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خلاصة

هذا البحث يقدم قاعده جديده لجدولة الماكينات الفرديه في وجود غرامات تأخير موزونه. وقد تم تصميم هذه القاعده بناءا علي تجريب قاعدتي إرسال ذو كفاءة وهما قاعدتي ATC و COVERT، في مدي واسع لما يعرف ببارامتر التقدم الحر. القاعده المقترحه تعتبر مركبه وديناميكيه في الزمن والتي إستبدل فيها بارامتر التقدم ببارامتر إتزان يمكن تثبيته عند مقدار 0.5. وقد استعرضت التجارب الحسابيه كفائة القاعدة المقترحه وقابليتها للتطوير والإمتداد، والأكثر من ذلك يمكن دمجها كجزء مكمل لأساليب الجدوله الأخري الخاصة بالورش البسيطه أو المركبه.

Abstract

This paper introduces a new rule for scheduling a single machine with weighted tardiness penalties. The design of this rule is based on experimenting two efficient dispatching rules, the ATC and COVERT rules, over a wide range of their free look-ahead parameter. The proposed rule is composite and dynamic in time. Further, it replaces the look-ahead parameter with a balancing parameter which can be fixed at a value of 0.5. The computational experiments exhibit the efficiency of the proposed rule and its availability for extension. Furthermore, it can be integrated as a complementary part of other scheduling approaches of simple or complex shops.

Keywords: Scheduling; Dispatching rules; Weighted tardiness

1. Literature Review

The single machine total weighted tardiness problem (SMTWTP), $1//\sum_j w_j T_j$, can be stated as follows. A set $J = \{J_1, J_2, \ldots, J_n\}$ of n jobs, can be processed, in any schedule, without preemption, on a single machine that can process only one job at a time. Each job j has a processing time p_j, a due date d_j, and a tardiness penalty w_j per unit time. All jobs become available at time zero, i.e. ready at any time, and their processing times are independent of the order. A job is picked as soon as the machine becomes free. The goal is to minimize the total weighted tardiness of jobs. Abstractly, find a schedule S for J, that

$$\text{Minimize} \sum_{j=1}^{n} w_j T_j : st \{(J_i \to J_k) \vee (J_i \leftarrow J_k)\} \forall (J_i, J_k) \in S. \tag{1}$$

Where, $T_j = \max(0, C_j - d_j)$ and $C_j = p_j + \sum_{i \to j} p_i$ are respectively the tardiness and completion time of job j. The SMTWTP is strong NP-hard (Lawler, 1977; Lenstra et al., 1977) and instances of large problems can often not be solved optimally by variety of methods (den Besten et al., 2001; Avci et al., 2003). Early, Emmons (1969) derived dominance rules that used to optimize the unweighted version. He devised a branch and bound (B&B) approach based on his rules (Shwimer, 1972). Rinnooy Kan et al. (1975) and Rachamadugu (1987) applied Emmons' rules to the weighted version. Most of structured approaches for weighted/unweighted version, especially the exact approaches, use Emmons' rules to search the optimal solution (Potts and Van Wassenhove, 1985, 1987; Tansel and Sabuncuoglu, 1997).

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Exact approaches such as B&B of Shwimer (1972), Potts and Van Wassenhove (1985), and Babu et al. (2004) yield optimal solutions for SMTWTP, but need to high computations. Dynamic Programming (DP) approaches were also presented for SMTWTP (Schrage and Baker, 1978). The DP approaches are found inferior than those B&B. Abdul-Razaq et al. (1990), in their survey, examined the exact approaches of SMTWTP using Emmons' rules to form a precedence graph for finding upper and lower bounds. Szwarc (1993) reported a special ordering for earliness-tardiness problem, where penalties of adjacent jobs depend on their start times. Szwarc and Liu (1993) presented a two-stage decomposition mechanism to a special case of the problem, when tardiness weights are proportional to processing times.

The nature of this problem generates a large number of local minima compared with neighborhoods of pairwise interchange. Moreover, the lower bound of solution is based on reducing to other simpler versions of the problem, which may be a virtual bound (Akturk and Yildirim, 1998). Therefore, using exact methods may be unattainable for large problems. Since there is no dominant method for all problem conditions, the researchers have paid more attention to heuristic approaches, instead. Two types of heuristics are widely used—constructive and interchange types—for such problem.

Dispatching rules (static and time-dependent) are the simplest constructive heuristics. More sophisticated constructive heuristics almost employ dispatching rules for making stepwise decisions to assign jobs in fixed positions. For additional information about using dispatching rules, refer to Kondakci et al. (1994), Akturk and Yildirim (1998, 1999), Akturk and Ozdemir (2000), Avci et al. (2003), Della Croce et al. (2004), and Taşgetiren et al. (2005). Interchange heuristics continue with improving an initial solution. Potts and Van Wassenhove (1991) discussed several heuristics, in a comparative study, including dispatching rules reporting that the *pairwise interchange heuristics* perform satisfactorily. After this report, many research followed these heuristics like Akturk and Yildirim (1998) who developed lower bounding scheme based on an adjacent pairwise interchange heuristic. Morton and Pentico (1993) and Pinedo (1995) are two books detail the heuristic scheduling including the SMTWTP.

Metaheuristic approaches have been also proposed to solve the SMTWTP, such as Simulated Annealing (Matsuo et al., 1989; Potts and Van Wassenhove, 1991; Crauwels et al., 1998), Tabu Search (Crauwels et al., 1998), Iterated Local Search (den Besten et al., 2001; Congram et al., 2002; Grosso et al., 2004; Ergun and Orlin, 2005), Genetic Algorithms (Crauwels et al., 1998; Madureira et al., 1999; Avci et al., 2003), Ant Colony Optimization (den Besten et al., 2000; Merkle and Middendorf, 2000), Particle Swarm Optimization and Differential Evolution (Taşgetiren et al., 2004, 2005). Such metaheuristics are mainly based on the local search and some of them use dispatching rules to generate an initial solution such as Congram et al. (2002) who used the ATC rule. For more details on the SMTWTP and metaheuristics, see Taşgetiren et al. (2005).

2. A Proposed Scheduling Rule

Table 1 summarizes the characteristics of the ATC and COVERT dispatching rules. The ATC rule (Rachamadugu and Morton, 1982; Vepsalainen and Morton, 1987) is one of the efficient rules of SMTWTP and superior to other heuristics. The ATC is an iterative rule assigns each job by indexing it in view of completion time t of the preceding jobs, average processing time $\overline{p}$ of all unassigned jobs, and a look-ahead parameter κ. Selection of the free parameter κ is crucial to the ATC rule (see Morton and Pentico, 1993; Morton et al., 1995). Caskey and Storch (1996) tested the efficiency of the ATC and other dispatching rules in different shops. The COVERT rule (see Kanet and Zhou, 1993) is similar to the ATC rule but less efficient. Notice that both rules reduce to the WSPT rule (see Akturk and

Table 1. The ATC and COVERT rules.

Rule	Ellipsis	Priority by the applied criterion
Apparent Tardiness Cost	ATC	$\max\left(\frac{w_j}{p_j}\exp\left(-\frac{\max(0, d_j - t - p_j)}{\kappa\overline{p}}\right)\right)$
Cost Over Time	COVERT	$\max\left(\frac{w_j}{p_j}\max\left(0, 1 - \frac{\max(0, d_j - t - p_j)}{\kappa p_j}\right)\right)$

Yildirim, 1999) after some values of κ. This limiting condition depends on the processing times.

Several studies have reported the superiority of the ATC and COVERT dispatching rules to other heuristics of the unweighted/weighted problem (see for example, Rachamadugu and Morton, 1982; Vepsalainen and Morton, 1987; Akturk and Yildirim, 1998, 1999; Avci et al., 2003). Both rules are composite with good structures and dynamic in time. The ATC rule is reported superior to the COVERT rule. Here, the ATC and COVERT rules are subjected to very large computational analyses over a wide range of κ. As a result, another rule is developed based on the criterion $\psi(tj)$, such that

$$\psi(tj) = \left(\frac{w_j}{p_j}\exp\left(-\frac{\max(0, d_j - t - p_j)}{\alpha\overline{p} + \beta p_j}\right)\right), \tag{2}$$

where, α and β play the role of look-ahead parameters, and t and $\overline{p}$ are as defined former. The highest priority is given to the job having $\max\{\psi(tj)\}$. The effect of a job on its priority increases each time a job is scheduled. Values of α and β can be taken as free constants greater than zero as done for κ. Only one of the two parameters can be zero but not both in the same time. Notice that at $\beta = 0$, the proposed rule reduces to the ATC rule. Here, the principal is to set α and β as functions of the number of all unscheduled jobs at time t. Next are two complement versions of the rule.

$$\psi_1(tj) = \left(\frac{w_j}{p_j}\exp\left(-\frac{\max(0, d_j - t - p_j)}{\lambda\ln(\upsilon)\overline{p} + \exp(-\upsilon^{\lambda})p_j}\right)\right), \tag{3}$$

$$\psi_2(tj) = \left(\frac{w_j}{p_j}\exp\left(-\frac{\max(0, d_j - t - p_j)}{\lambda\ln(\upsilon)p_j + \exp(-\upsilon^{\lambda})\overline{p}}\right)\right), \tag{4}$$

where υ is the number of all unscheduled jobs at time t, $\lambda \geq 0$ is a balancing parameter. Thus, $\lambda\ln(\upsilon)$ and $\exp(-\upsilon^{\lambda})$ represent harmonized look-ahead parameters. The two versions oppose giving the weight to a considered job and all unscheduled jobs.

The computational experiment is continued on the factors α and β. As a result, another two complement versions of the proposed rule are inspired such that

$$\psi_3(tj) = \left(\frac{w_j}{p_j}\exp\left(-\frac{\max(0, d_j - t - p_j)}{\lambda\ln(\upsilon)\overline{p} + \exp(-\lambda\ln(\upsilon))p_j}\right)\right), \tag{5}$$

$$\psi_3(ij) = \left(\frac{w_j}{p_j} \exp\left(-\frac{\max(0, d_j - t - p_j)}{\lambda \ln(\upsilon) p_j + \exp(-\lambda \ln(\upsilon)) \overline{p}} \right) \right). \tag{6}$$

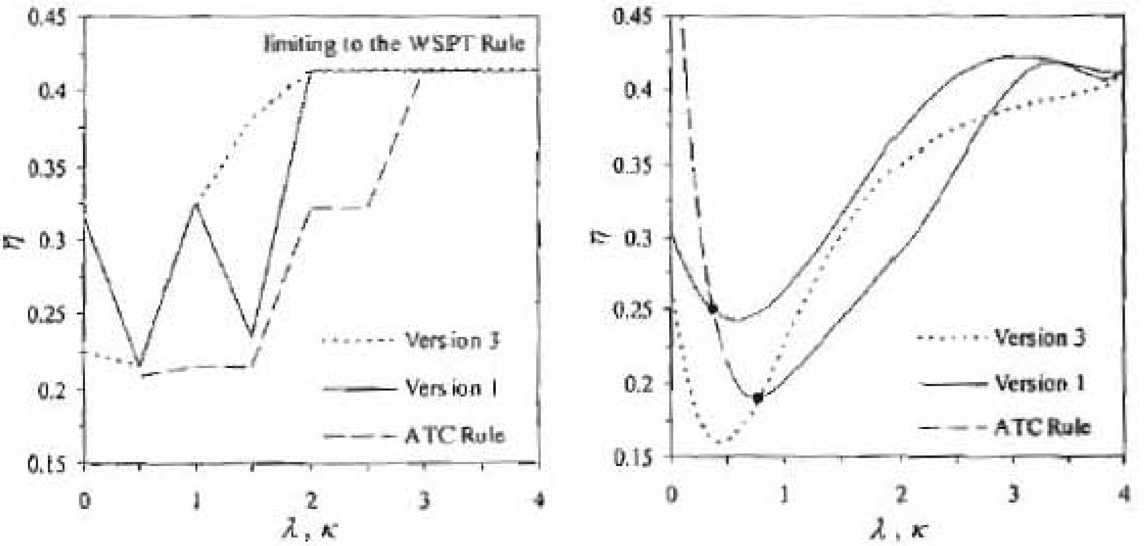
An example problem (Table 2), appeared in Akturk and Yildirim (1998), is processed to display the developed rule (versions 1 and 3) versus the ATC rule over a range of λ and κ, as seen in Fig. (1-a). For the purpose of comparison over different **problems and range** of rules' parameters, the index

$$\eta = \frac{\sum_{j=1}^{n} w_j T_j}{n \sum_{j=1}^{n} p_j}, \tag{7}$$

is proposed to measure the solution deficiency. It is obvious that version 3 yields a monotonic increase of the deficiency for $\lambda \geq 0.5$ while both versions reach the minimum deficiency at $\lambda = 0.5$.

Table 2. A SMTWTP example of ten jobs.

Job	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}
p_j	7	10	10	1	6	3	5	7	9	2
d_j	11	26	26	27	28	31	32	32	32	42
w_j	5	8	1	9	7	9	9	1	7	10



(a) An example SMTWTP. (b) Simulated SMTWTPs (trends).

Fig. 1. Performance of the proposed rule and the ATC rule.

For this example problem, the developed rule alternates the superiority with the ATC rule over the selected range of λ and κ. The ATC rule reduces to the WSPT slower than the proposed rule. This limiting condition isn't necessary to be the same for general. The hindrance of comparison becomes the choice of λ and κ. Such parameters have a significant effect on the solution. Thus, the value of κ becomes an obstacle because it needs to extensive experiments to catch its best value every time a problem is solved. Therefore, the proposed rule is tested for a set of randomly generated problems for a range of λ. This test showed that the best value of λ lies around 0.5 as seen in Fig. (1-b). After some value, the increase of λ doesn't affect the solution because of limiting to the WSPT rule. For the current example, the maximum value of λ is found 2. For large problems, the experiment proved that the proposed rule limits temporarily to the WSPT rule only at the start of solution at $\lambda = 0.5$.

3. Conclusion

Although, the researches inspired structured heuristics for solving the SMTWTP, the dispatching rules stay preferable by the practitioners because of simplicity. Furthermore, even the complex heuristics don't guarantee the quality of solutions. Here, a scheduling rule is developed, based on numerical experiments, for the SMTWTP. Also, an index is proposed to measure the solution deficiency. This rule overcomes the criticality of choosing a look-ahead parameter. For large problems, a slight change of the balancing parameter, around its best value, hasn't that effect as the look-ahead parameter of the ATC and COVERT rules. Nevertheless, the converse may occur for small problems. However, the value of λ is recommended at 0.5. The proposed rule can dominate both rules depending on the choice of the look-ahead parameters. The author sees that it is convenience to call this rule *Harmonized ATC rule* (HATC) because it harmonizes the values of α and β at each scheduling iteration. The proposed rule may become more dynamic and efficient if the balancing parameter is used as a function of the processing time and due date of a considered job besides the number of all unscheduled jobs. Finally, Versions 1 and 2 are recommended for the problems having up to twenty jobs whilst versions 3 and 4 are recommended for all problems. Hence, the latter versions can be applied exclusively or as complementary parts of other constructive scheduling approaches of different shops.

4. Future Insights

The proposed rule will be subjected to further experiments to enhance the relationship between α and β. Furthermore, the performance can be enhanced by linking versions 3 and 4 in a hierarchy with all jobs as seen in Fig. 2. The two versions are weighted to each other and each unscheduled job is weighted by each version in an interactive framework as that developed by Soltan and El-Kassas (2006).

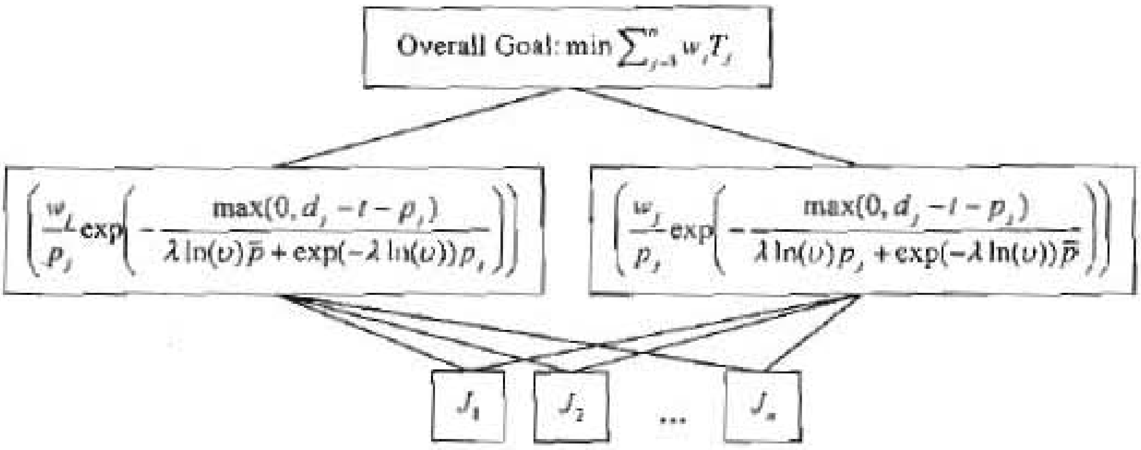


Fig. 2. A weighting hierarchy for enhancing the proposed rule.

Another composite form, Eq. (8), is proposed for experimentation following the same methodology of this paper. This form is only based on the COVERT rule to minimize the total weighted tardiness of a set of jobs. Its composition increases the priority of a tardy job and decreases the priority of an early job at time t.

$$\psi(tj)=\left(\frac{w_j}{p_j}\max\left(0,1+\frac{\max(0,t+p_j-d_j)-\max(0,d_j-t-p_j)}{\alpha\bar{p}+\beta p_j}\right)\right) \tag{8}$$

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